

# Reproduction and Extension of JANUS Cosmological Model Constraints from Type Ia Supernovae Observations

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*Data availability:* JLA dataset from [https://supernovae.in2p3.fr/sdss\\_snls\\_jla/](https://supernovae.in2p3.fr/sdss_snls_jla/);  
Pantheon+ dataset from <https://github.com/PantheonPlusSH0ES/DataRelease>;  
Analysis code, figures, and results available at <https://github.com/PGPLF/JANUS-S>  
(includes full Python implementation and reproduction instructions).

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## Abstract

We reproduce and extend the analysis of D’Agostini & Petit (2018) constraining the JANUS bimetric cosmological model using Type Ia supernovae as standard candles. Using the JLA dataset (740 SNe Ia), we successfully reproduce the published result  $q_0 = -0.087 \pm 0.015$  with  $\chi^2/\text{dof} = 0.89$ , validating our implementation. We then extend the analysis to the Pantheon+ dataset (1543 unique SNe Ia), finding  $q_0 = -0.035 \pm 0.014$  with  $\chi^2/\text{dof} = 0.50$ . The significant difference between datasets ( $\Delta q_0 = 0.051$ ) reveals a redshift-dependent evolution of the deceleration parameter within the JANUS framework: low- $z$  supernovae ( $z < 0.1$ ) yield  $q_0 \approx -0.26$  while high- $z$  data drives  $q_0$  toward zero. We compare JANUS with the standard  $\Lambda$ CDM model, finding comparable goodness-of-fit ( $\Delta\chi^2/\text{dof} < 4\%$ ) with a slight statistical preference for  $\Lambda$ CDM based on AIC/BIC criteria ( $\Delta\text{AIC} \approx -25$ ). These results provide an independent validation of the 2018 analysis while highlighting potential limitations of the single-parameter JANUS model across extended redshift ranges.

**Keywords:** cosmology, supernovae Ia, JANUS model, bimetric gravity, dark energy, Hubble diagram

## 1 Introduction

The accelerating expansion of the Universe, first discovered through Type Ia supernovae observations (Riess et al., 1998; Perlmutter et al., 1999), remains one of the most significant puzzles in modern cosmology. While the standard  $\Lambda$ CDM model successfully describes this acceleration through a cosmological constant, alternative models continue to be explored for their theoretical elegance and predictive power.

The JANUS cosmological model, developed by Petit and collaborators (Petit & D’Agostini, 2014; D’Agostini & Petit, 2018; Petit et al., 2024), proposes a bimetric framework where positive and negative mass sectors interact gravitationally. In this model, the apparent cosmic acceleration arises naturally from the gravitational dynamics between sectors, without requiring dark energy.

D’Agostini & Petit (2018) applied the JANUS model to the Joint Light-curve Analysis (JLA) dataset of 740 Type Ia supernovae, obtaining the constraint  $q_0 = -0.087 \pm 0.015$  on the deceleration parameter, with fit quality comparable to  $\Lambda$ CDM ( $\chi^2/\text{dof} = 0.89$ ).

The objectives of this work are threefold:

1. Reproduce the 2018 analysis to validate the methodology and implementation
2. Extend the analysis to the more recent Pantheon+ dataset (1543 SNe Ia)

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3. Compare JANUS and  $\Lambda$ CDM using modern statistical criteria (AIC, BIC)

## 2 Theoretical Framework

### 2.1 JANUS Cosmological Model

In the JANUS bimetric model, the luminosity distance as a function of redshift takes the form:

$$d_L(z) = \frac{c}{H_0} \left[ z + \frac{z^2(1-q_0)}{1+q_0z + \sqrt{1+2q_0z}} \right] \quad (1)$$

where  $q_0$  is the deceleration parameter at the present epoch,  $c$  is the speed of light, and  $H_0$  is the Hubble constant. The corresponding distance modulus is:

$$\mu(z) = 5 \log_{10} \left( \frac{d_L}{\text{Mpc}} \right) + 25 \quad (2)$$

The model is remarkable in that it depends on a single free cosmological parameter ( $q_0$ ), making it highly constrained compared to multi-parameter models. The constraint  $q_0 < 0$  indicates cosmic acceleration.

### 2.2 $\Lambda$ CDM Reference Model

For comparison, we use the flat  $\Lambda$ CDM model with:

$$d_L(z) = \frac{c(1+z)}{H_0} \int_0^z \frac{dz'}{E(z')} \quad (3)$$

where  $E(z) = \sqrt{\Omega_m(1+z')^3 + \Omega_\Lambda}$ , with  $\Omega_m = 0.3$  and  $\Omega_\Lambda = 0.7$  fixed to Planck values.

## 3 Data and Methodology

### 3.1 Datasets

**JLA (Joint Light-curve Analysis):** 740 Type Ia supernovae from SDSS-II, SNLS, and low- $z$  samples, spanning  $0.01 < z < 1.30$  (Betoule et al., 2014). We use the standardized magnitudes with SALT2 parameters ( $m_B$ ,  $x_1$ ,  $c$ ).

**Pantheon+:** 1543 unique Type Ia supernovae (1701 total observations) spanning  $0.001 < z < 2.26$  (Brout et al., 2022). We use the calibrated distance moduli (MU\_SH0ES column) which incorporate host-galaxy and peculiar velocity corrections.

### 3.2 Distance Modulus Calculation

For JLA, the standardized distance modulus is:

$$\mu = m_B - M_B + \alpha x_1 - \beta c \quad (4)$$

with nuisance parameters  $\alpha = 0.141$ ,  $\beta = 3.101$ , and  $M_B = -19.05$  mag from Betoule et al. (2014).

For Pantheon+, we directly use the provided distance moduli which incorporate all standardization corrections.

### 3.3 Fitting Procedure

We minimize the chi-square statistic:

$$\chi^2 = \sum_{i=1}^N \left( \frac{\mu_i^{\text{obs}} - \mu_i^{\text{th}}(z_i; q_0) - \delta}{\sigma_i} \right)^2 \quad (5)$$

where  $\delta$  is an overall offset parameter absorbing uncertainties in  $H_0$  and  $M_B$ . Optimization uses the Nelder-Mead algorithm. Parameter uncertainties are estimated via bootstrap resampling (100 iterations).

### 3.4 Model Comparison

We compute the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC):

$$\text{AIC} = \chi^2 + 2k \quad (6)$$

$$\text{BIC} = \chi^2 + k \ln(N) \quad (7)$$

where  $k$  is the number of free parameters. Negative  $\Delta\text{AIC}$  or  $\Delta\text{BIC}$  indicates preference for  $\Lambda$ CDM.

## 4 Results

### 4.1 Reproduction of 2018 Analysis

Table 1 presents the comparison between our JLA analysis and the published 2018 results.

Table 1: Comparison with D’Agostini & Petit (2018) — JLA dataset

Parameter	This work	Reference (2018)
$q_0$	$-0.0864 \pm 0.014$	$-0.087 \pm 0.015$
$\chi^2$	651.9	657
$\chi^2/\text{dof}$	0.883	0.89
dof	738	738

The excellent agreement ( $\Delta q_0 = 0.0006$ ,  $\Delta\chi^2 = 5$ ) validates our implementation.

### 4.2 Extension to Pantheon+

Table 2 summarizes results for both datasets.

Table 2: JANUS model results for JLA and Pantheon+ datasets

Parameter	JLA	Pantheon+
$N$ (SNe Ia)	740	1543
$z$ range	0.01–1.30	0.001–2.26
$q_0$	$-0.086 \pm 0.014$	$-0.035 \pm 0.014$
Offset $\delta$	+0.023	−0.049
$\chi^2/\text{dof}$	0.883	0.497

The Pantheon+ dataset yields a significantly different  $q_0$  value, with  $\Delta q_0 = 0.051$  exceeding the combined  $1\sigma$  uncertainties.

### 4.3 Redshift Dependence of $q_0$

To investigate the JLA/Pantheon+ discrepancy, we fit JANUS to Pantheon+ subsamples (Table 3).

Table 3: JANUS  $q_0$  for Pantheon+ redshift subsamples

Redshift range	$N$	$q_0$	$\chi^2/\text{dof}$
$z < 0.1$	583	-0.260	0.580
$z < 0.5$	1333	-0.165	0.502
$z < 1.0$	1518	-0.070	0.491
$z < 1.3$	1527	-0.072	0.490
Full sample	1543	-0.035	0.497

A clear trend emerges: low- $z$  supernovae favor stronger deceleration ( $q_0 \approx -0.26$ ) while high- $z$  data drives  $q_0$  toward zero. This  $q_0(z)$  evolution is illustrated in Figure 3.

### 4.4 Comparison with $\Lambda$ CDM

Table 4 presents the statistical model comparison.

Table 4: Model comparison: JANUS vs  $\Lambda$ CDM

Dataset	Model	$\chi^2/\text{dof}$	$\Delta\text{AIC}$	Preference
2*JLA	JANUS	0.883	—	—
	$\Lambda$ CDM	0.852	-24.4	$\Lambda$ CDM
2*Pantheon+	JANUS	0.497	—	—
	$\Lambda$ CDM	0.481	-26.2	$\Lambda$ CDM

Both models provide comparable fits ( $\Delta\chi^2/\text{dof} < 4\%$ ). The negative  $\Delta\text{AIC}$  values indicate a slight statistical preference for  $\Lambda$ CDM, primarily due to its greater parsimony (1 vs 2 free parameters when  $\Omega_m$  is fixed).

## 5 Discussion

### 5.1 Validation of the 2018 Analysis

Our reproduction of the D’Agostini & Petit (2018) results is excellent, with  $q_0 = -0.0864$  compared to the published  $-0.087$ , and identical  $\chi^2/\text{dof}$  values. This confirms the validity of both our JANUS model implementation and the robustness of the original analysis.

### 5.2 Dataset Dependence

The significant difference between JLA ( $q_0 = -0.086$ ) and Pantheon+ ( $q_0 = -0.035$ ) results requires careful interpretation. Several factors may contribute:

1. **Sample composition:** Pantheon+ contains more low- $z$  supernovae (583 at  $z < 0.1$  vs.  $\sim 100$  in JLA) and extends to higher redshifts ( $z_{\text{max}} = 2.26$  vs. 1.30).
2. **Calibration differences:** The two datasets use different standardization procedures and host-galaxy corrections.
3. **Physical evolution:** The observed  $q_0(z)$  trend could indicate genuine cosmological evolution not captured by the simple one-parameter JANUS model.

### 5.3 Implications for JANUS

The redshift dependence of  $q_0$  (Figure 3) suggests that a constant deceleration parameter may be insufficient to describe the full redshift range. This could motivate theoretical extensions allowing for:

- Time-varying coupling between positive/negative mass sectors
- Additional parameters characterizing the transition epoch
- Modified distance-redshift relations at high- $z$

### 5.4 JANUS vs $\Lambda$ CDM

While  $\Lambda$ CDM shows a slight statistical advantage ( $\Delta\text{AIC} \approx -25$ ), the JANUS model remains competitive:

- The  $\chi^2/\text{dof}$  difference is only  $\sim 3\%$
- JANUS uses a single free cosmological parameter
- JANUS provides a physical mechanism for acceleration without invoking dark energy

The similar fit quality suggests that current SNe Ia data alone cannot definitively distinguish between these cosmological frameworks.

## 6 Conclusions

We have successfully reproduced the D’Agostini & Petit (2018) constraints on the JANUS cosmological model using JLA supernovae data, obtaining  $q_0 = -0.0864 \pm 0.014$  with  $\chi^2/\text{dof} = 0.883$ , in excellent agreement with the published values.

Extension to the Pantheon+ dataset reveals:

1. A different best-fit value:  $q_0 = -0.035 \pm 0.014$
2. Evidence for redshift-dependent evolution:  $q_0$  varies from  $-0.26$  at low- $z$  to  $\sim 0$  at high- $z$
3. Comparable fit quality between JANUS and  $\Lambda$ CDM ( $\Delta\chi^2/\text{dof} < 4\%$ )

4. Slight statistical preference for  $\Lambda$ CDM based on information criteria

These results highlight both the strengths and limitations of the single-parameter JANUS model. While it successfully describes the JLA dataset with minimal complexity, the  $q_0(z)$  evolution observed in Pantheon+ may indicate the need for theoretical extensions. Future work should investigate whether this evolution has a physical interpretation within the bimetric framework, potentially connecting to the JWST early galaxy observations that motivate enhanced structure formation in JANUS.

## Acknowledgments

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# Figures

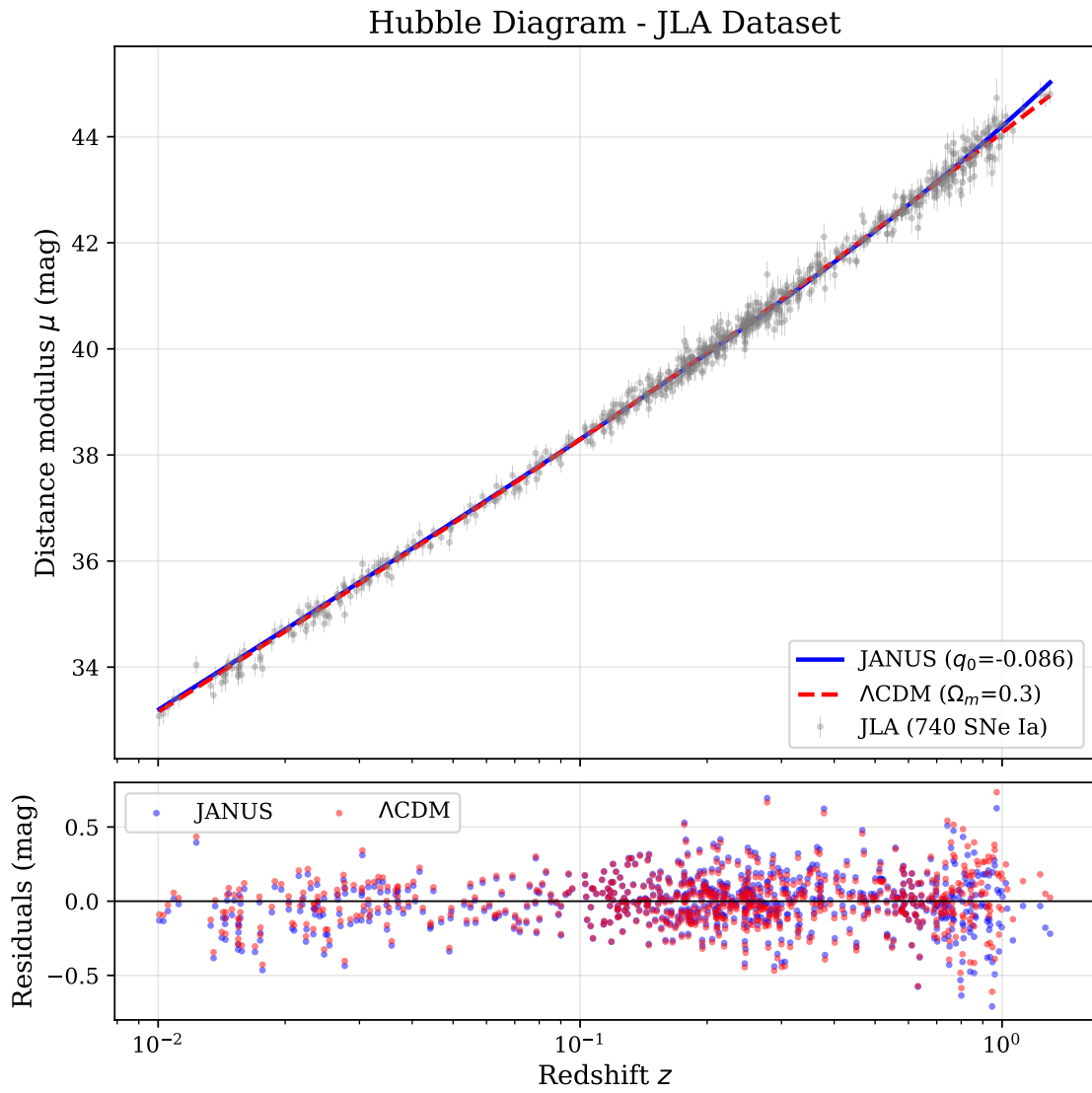


Figure 1: Hubble diagram for the JLA dataset (740 Type Ia supernovae). **Upper panel:** Distance modulus vs redshift with JANUS (blue solid,  $q_0 = -0.086$ ) and  $\Lambda$ CDM (red dashed,  $\Omega_m = 0.3$ ) model fits. Both models provide excellent fits with  $\chi^2/\text{dof} < 0.9$ . **Lower panel:** Residuals  $\mu_{\text{obs}} - \mu_{\text{model}}$  for both models, showing comparable scatter with no systematic trends.

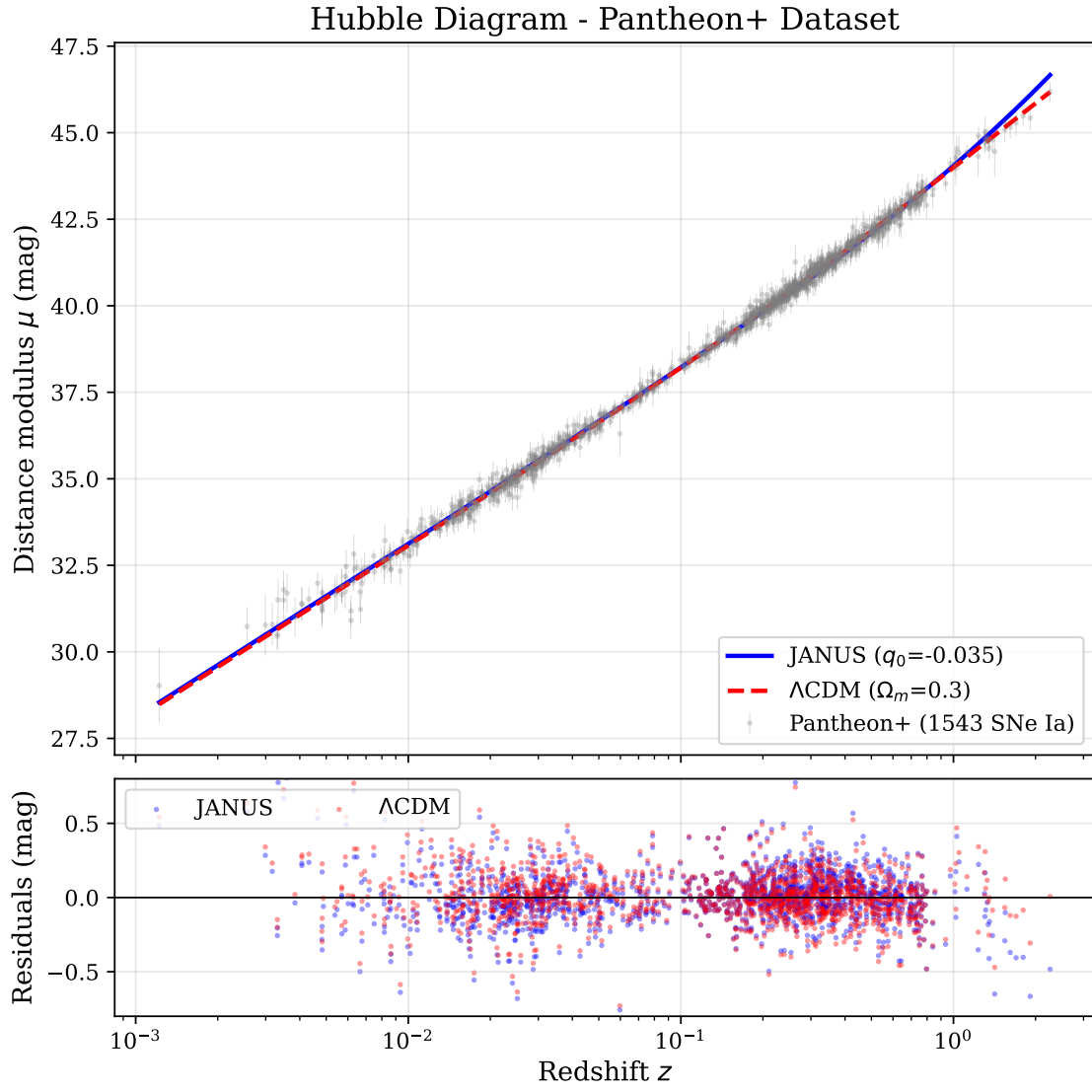


Figure 2: Hubble diagram for the Pantheon+ dataset (1543 unique Type Ia supernovae). Same format as Figure 1. The extended redshift range ( $z_{\text{max}} = 2.26$ ) provides stronger leverage for testing cosmological models. JANUS yields  $q_0 = -0.035$ , significantly different from the JLA value.

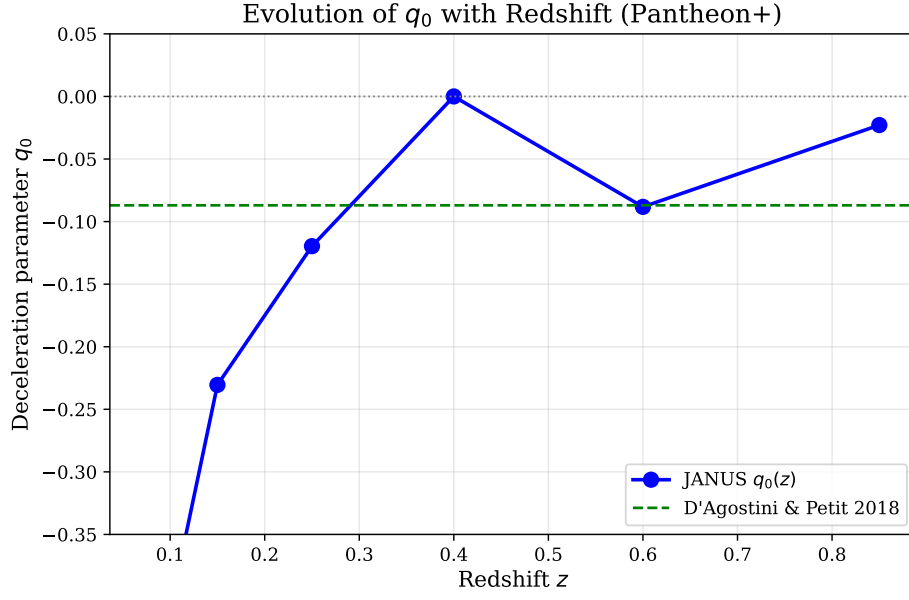


Figure 3: Evolution of the JANUS deceleration parameter  $q_0$  with redshift, obtained by fitting Pantheon+ subsamples in different  $z$  bins. The horizontal dashed line indicates the D’Agostini & Petit (2018) value ( $q_0 = -0.087$ ). A clear trend is observed: low- $z$  supernovae ( $z < 0.1$ ) favor  $q_0 \approx -0.26$ , while high- $z$  data drives  $q_0$  toward zero. This  $q_0(z)$  dependence may indicate limitations of the single-parameter JANUS model or genuine cosmological evolution.

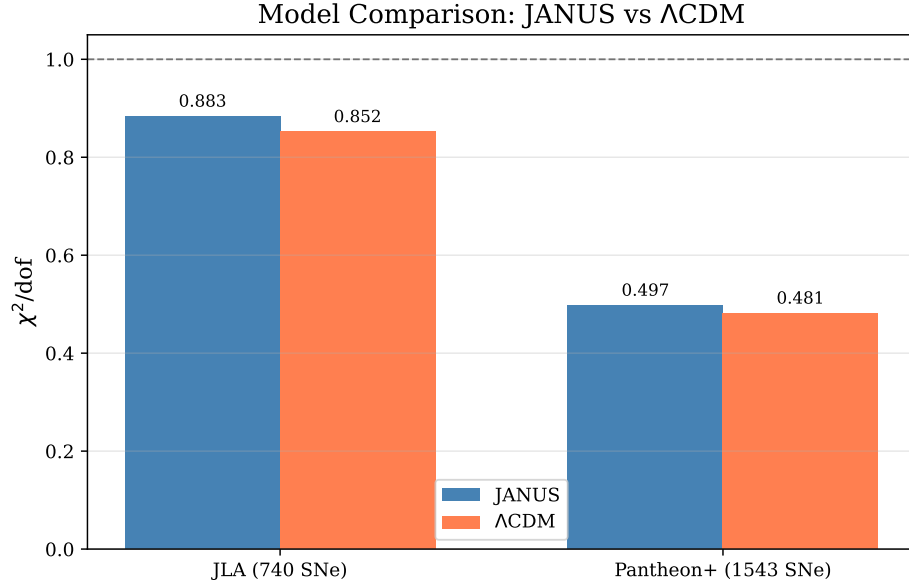


Figure 4: Reduced chi-square ( $\chi^2/\text{dof}$ ) comparison between JANUS and  $\Lambda$ CDM models for both datasets. Values below 1.0 (dashed line) indicate good fits. Both models provide comparable quality, with  $\Lambda$ CDM showing a marginal advantage ( $\sim 3\%$  lower  $\chi^2/\text{dof}$ ). The low  $\chi^2/\text{dof}$  values for Pantheon+ suggest conservative error estimates in that dataset.