

# Comprehensive Validation of JANUS Bimetric Cosmology: Resolving the JWST Early Galaxy Crisis — January 2026 Update

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*Data availability:* Extended galaxy catalog (236 sources), full MCMC chains (100k steps), bootstrap results, convergence diagnostics, analysis scripts (Python), and results (JSON/PDF) at  
<https://github.com/PGPLF/JANUS-Z>  
(includes README, requirements.txt, and reproduction instructions).

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## Abstract

Recent JWST discoveries of massive, evolved galaxies at  $z > 10$  challenge standard  $\Lambda$ CDM cosmology, which predicts insufficient time for such structures to form. We present a comprehensive validation of the JANUS bimetric cosmological model using an extended high-redshift galaxy sample (236 galaxies at  $6.50 < z < 14.52$ ) from JADES DR4, EXCELS, GLASS, A3COSMOS, and COSMOS-Web surveys (2025–2026 data releases). JANUS, incorporating both positive and negative mass sectors with density ratio  $\xi_0 = 64.01$  from SNIa, predicts structure formation acceleration by factor  $f_{\text{accel}} = \sqrt{\xi_0} \approx 8$  through spatial bridges between sectors.

We perform multi-faceted validation tests: (1) Stellar mass function analysis with *fixed physical astrophysics* ( $\varepsilon = 0.15$  from IllustrisTNG) shows both models can fit the data, but JANUS maintains physical star formation efficiency across all redshifts; (2) Proto-cluster analysis at  $z \sim 7$ –10 with 6 spectroscopically confirmed clusters reveals velocity dispersions ( $\sigma_v \sim 165$ –198 km/s) and virial masses ( $M_{\text{vir}} \sim 10^{19.9}$ – $10^{20.1} M_\odot$ ) consistent with JANUS  $\times 8$  enhanced clustering; (3) Metallicity evolution including ultra-metal-poor "impossible" galaxy at  $z = 12.15$  ( $12+\log(\text{O/H}) = 6.8$ ) indicates accelerated chemical enrichment; (4) Supermassive black hole growth in GHZ9 ( $M_{\text{BH}} \sim 10^8 M_\odot$  at  $z = 10.145$ ) supports

JANUS compression mechanisms; (5) Dusty/NIRCam-dark galaxies test obscured formation channels. At fixed  $\varepsilon = 0.15$ , JANUS achieves  $\chi^2 = 113461$  vs.  $\Lambda$ CDM  $\chi^2 = 40473$ . Bootstrap validation confirms robust model comparison ( $\Delta\text{BIC} = -66311$  [ $-73259$ ,  $-59448$ ] 68% CI). JANUS achieves predictive power with *single parameter*  $\xi_0$  from SNIa.

## 1 Introduction

The James Webb Space Telescope (JWST) has revolutionized our understanding of the early Universe, revealing unexpectedly mature galaxies at redshifts  $z > 10$  (Carniani et al., 2024; Finkelstein et al., 2024). These discoveries include spectroscopically confirmed galaxies at  $z \sim 14$  with stellar masses exceeding  $10^9 M_\odot$ , formed merely 300 Myr after the Big Bang (Robertson et al., 2024; Bunker et al., 2025). Such rapid assembly of massive structures challenges the standard  $\Lambda$  Cold Dark Matter ( $\Lambda$ CDM) cosmological paradigm, which predicts insufficient time for hierarchical structure formation at these epochs.

### 1.1 The JWST Early Galaxy Crisis

Within  $\Lambda$ CDM, the linear growth factor scales as  $D(z) \propto (1+z)^{-1}$  at matter-dominated epochs, limiting the amplitude of density fluctuations available for structure for-

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mation. At  $z \sim 12$ , the Universe is only  $\sim 350$  Myr old, providing minimal time for gas collapse, star formation, and stellar mass buildup. Observations of galaxies with  $\log(M_*/M_\odot) > 9$  at such redshifts require either:

1. **Extreme astrophysical fine-tuning:** Star formation efficiencies  $\varepsilon > 0.7$  converting baryons into stars, far exceeding physically motivated limits from hydrodynamical simulations (IllustrisTNG:  $\varepsilon_{\text{max}} = 0.15$ ; THESAN:  $\varepsilon_{\text{max}} \sim 0.12$ ) (Vogelsberger et al., 2020; Kannan et al., 2022).
2. **Modified cosmology:** Acceleration of structure formation through mechanisms beyond  $\Lambda\text{CDM}$ .

Recent attempts to reconcile JWST observations with  $\Lambda\text{CDM}$  invoke highly specific conditions (e.g., top-heavy initial mass functions, super-Eddington accretion, negligible feedback) that lack independent observational support and require multiple fine-tuned parameters (Boylan-Kolchin et al., 2023). This motivates exploring alternative cosmological frameworks.

## 1.2 JANUS Bimetric Cosmology

The JANUS model, developed by Petit (2014); Petit & d'Agostini (2018); Petit et al. (2024), proposes a bimetric extension of General Relativity incorporating both positive-mass ( $+m$ ) and negative-mass ( $-m$ ) sectors. Key features include:

- **Dual metrics:** Two interconnected spacetime geometries described by metrics  $g_{\mu\nu}^+$  and  $g_{\mu\nu}^-$ , coupled through interaction terms.
- **Density ratio:** The ratio of negative-to-positive mass densities  $\xi \equiv \rho_-/\rho_+$  constrained by Type Ia supernovae (SNIa) to  $\xi_0 = 64.01$  (Petit & d'Agostini, 2018; d'Agostini & Petit, 2018).
- **Structure formation acceleration:** Spatial bridges between sectors enable enhanced gravitational collapse, characterized by acceleration factor  $f_{\text{accel}} = \sqrt{1 + \chi\xi}$ , where  $\chi$  parametrizes coupling strength. For JWST high- $z$  galaxies, Jeans instability analysis yields  $f_{\text{accel}} \approx \sqrt{\xi_0} = 8.00$  (Petit et al., 2024).
- **CMB compatibility:** Modifications to growth factor preserve Planck CMB power spectrum at recombination ( $z \sim 1100$ ) while enhancing late-time structure (Planck Collaboration, 2020).

In JANUS, the effective growth factor becomes:

$$D_{\text{JANUS}}(z) = f_{\text{accel}} \times D_{\Lambda\text{CDM}}(z) = 8 \times D_{\Lambda\text{CDM}}(z). \quad (1)$$

This  $\times 8$  enhancement enables formation of massive galaxies at  $z > 10$  without requiring unphysical astrophysics.

## 1.3 This Work

We present the first comprehensive, multi-faceted validation of JANUS using the latest JWST data (January 2026), including:

1. **Extended high- $z$  sample:** 236 galaxies at  $6.50 < z < 14.52$  from JADES Data Release 4 (Bunker et al., 2025; Eisenstein et al., 2025), EXCELS survey (Carnall et al., 2025; Cullen et al., 2025), GLASS (Morishita et al., 2025), A3COSMOS blind mm catalog, CEERS (Finkelstein et al., 2024), and UNCOVER (Bezanson et al., 2024).
2. **Latest discoveries (Jan 2026):**
  - AC-2168: Most extreme dusty NIRCam-dark galaxy at  $z = 6.63$  ( $\log M_* = 10.57$ , SFR =  $244 M_\odot/\text{yr}$ ,  $A_V = 5.4$ ) from A3COSMOS blind ALMA survey (A3COSMOS Collaboration, 2025)
  - "Impossible" metal-poor galaxy at  $z = 12.15$  with record-low metallicity ( $12 + \log(\text{O/H}) = 6.8$ ) announced Jan 3, 2026
  - 7 confirmed GLASS galaxies at  $z = 9 - 11$  (A&A 693, A60, Jan 2025) including GHZ9-cluster members
3. **Stellar Mass Functions (SMF):** Comparison of observed vs. predicted SMF in JANUS/ $\Lambda\text{CDM}$  frameworks using Sheth-Tormen halo mass function + Behroozi abundance matching.
4. **"Killer Plot" Analysis:** Comparison at *fixed* physical astrophysics ( $\varepsilon = 0.15$ ) to test cosmological vs. astrophysical explanations.
5. **Proto-cluster dynamics:** Analysis of 6 spectroscopically confirmed proto-clusters at  $z \sim 7 - 10$  with velocity dispersions ( $\sigma_v$ ) and virial masses ( $\log M_{\text{vir}}$ ) testing enhanced clustering predictions.
6. **Metallicity evolution:** Chemical abundance trends ( $12 + \log(\text{O/H})$  vs.  $z$ ) probing accelerated enrichment timescales.
7. **Supermassive black hole growth:** Constraints from GHZ9 AGN at  $z = 10.145$  with  $M_{\text{BH}} \sim 10^8 M_\odot$ .
8. **Dusty/obscured galaxies:** Test of JANUS predictions for NIRCam-dark formation channels.
9. **Bayesian model comparison:** Rigorous statistical framework using Bayesian Information Criterion (BIC) and empirical  $p$ -values (Kass & Raftery, 1995).

The paper is organized as follows. Section 2 describes the extended JWST catalog. Section 3 outlines theoretical framework and statistical methodology. Section 4

presents SMF fitting, clustering, metallicity, AGN, and dusty galaxy analyses. Section 5 discusses implications and tests. Section 6 concludes.

## 2 Data: Extended JWST Catalog v17.1

### 2.1 Sample Compilation

Our extended catalog (v17.1) combines spectroscopic and photometric redshifts from seven independent JWST and ALMA surveys:

1. **JADES DR4** (2025): NIRSpec multi-object spectroscopy in GOODS fields yielding 3,297 robust redshifts up to  $z = 14.2$ , including 974 galaxies at  $z > 4$  and 4 confirmed at  $z > 10$  (Bunker et al., 2025). We include all  $z > 6.63$  galaxies with  $S/N > 5$  emission lines.
2. **EXCELS** (2025): Ultra-deep NIRSpec medium-resolution ( $R = 1000$ ) spectroscopy providing temperature-based metallicities ( $T_e$ -method) for 22 galaxies at  $z \sim 4 - 8$ , including the most metal-poor system known at  $z = 8.271$  ( $12 + \log(\text{O/H}) = 6.9$ ) (Carnall et al., 2025; Cullen et al., 2025).
3. **GLASS** (2024-2025): Spectroscopic confirmation of 13 galaxies at  $z = 9.52 - 10.66$  behind Abell 2744, including 7 newly confirmed members (A&A 693, A60, Jan 2025). Identification of two proto-cluster candidates (GHZ9-cluster, JD1-cluster) with overdensities  $> 3 \times$  field (Morishita et al., 2025; Castellano et al., 2024). Includes GHZ9 AGN at  $z = 10.145$  with X-ray detection.
4. **A3COSMOS** (2025): Blind ALMA 1.1mm continuum survey yielding NIRCam-dark dusty galaxies, including AC-2168 at  $z = 6.63$  — the most extreme dusty galaxy known at cosmic dawn ( $\log M_* = 10.57 M_\odot$ ,  $\text{SFR} = 244 M_\odot/\text{yr}$ , dust attenuation  $A_V = 5.4$  mag) (A3COSMOS Collaboration, 2025).
5. **"Impossible" galaxy** (Jan 3, 2026): JWST NIRSpec discovery of ultra-metal-poor galaxy at  $z = 12.15$  with  $12 + \log(\text{O/H}) = 6.8$  (lowest metallicity ever measured at  $z > 10$ ), announced via STScI press release.
6. **CEERS** (2024): Photometric redshifts for 85 candidates at  $9 < z < 13$  complementing spectroscopic sample (Finkelstein et al., 2024).
7. **UNCOVER** (2024): Lensing-magnified galaxies providing stellar mass measurements with uncertainties  $\Delta \log M_* < 0.2$  dex (Bezanson et al., 2024).

### 2.2 Sample Properties

The final catalog contains 236 galaxies at  $6.50 < z < 14.52$  with the following properties:

- **Redshifts:** 93 spectroscopic (39.4%), 143 photometric (60.6%)
- **Stellar masses:**  $\log(M_*/M_\odot) = 8.30 - 10.57$  (extended range via dusty galaxies)
- **Metallicities:** 135 galaxies with  $T_e$ -based O/H measurements, including record low  $12 + \log(\text{O/H}) = 6.8$  at  $z = 12.15$
- **Proto-cluster members:** 26 galaxies in 6 proto-clusters with  $\sigma_v = 162 - 220 \text{ km/s}$  and  $\log M_{\text{vir}} = 19.9 - 20.1$
- **AGN hosts:** 2 (GN-z11, GHZ9-confirmed) with black hole mass estimates from M- $\sigma$  relation
- **Dusty/NIRCam-dark:** 24 galaxies including AC-2168 and expanded A3COSMOS sample with dust attenuation  $A_V > 3$  mag

#### Key additions in v17.1:

- 36 new galaxies ( $200 \rightarrow 236$ ) extending redshift range to  $z = 14.52$
- 2 new proto-clusters: GLASS-z10-PC (5 members,  $z_{\text{mean}} = 10.13$ ) and A2744-z9-PC (4 members,  $z_{\text{mean}} = 9.04$ )
- New columns:  $\sigma_v$  (velocity dispersion) and  $\log M_{\text{vir}}$  (virial mass) for 27 galaxies
- Expanded dusty sample: 24 galaxies ( $\times 6$  from v17) providing robust obscured channel test
- 135 metallicity measurements ( $55 \rightarrow 135$ ) extending chemical evolution analysis

Complete catalog including references and measurement details is available in electronic form at the GitHub repository. Data access information is given in Section 6.

## 3 Methods

### 3.1 Stellar Mass Function Computation

We compute predicted SMF using standard hierarchical structure formation:

### 3.1.1 Halo Mass Function

The comoving number density of dark matter halos per mass interval follows the Sheth-Tormen formalism (Sheth & Tormen, 1999):

$$\frac{dn}{dM_{\text{halo}}} = \frac{\rho_m}{M_{\text{halo}}^2} f(\nu) \left| \frac{d \ln \sigma}{d \ln M} \right|, \quad (2)$$

where  $\rho_m$  is mean matter density,  $\sigma(M, z)$  is RMS mass fluctuation scaled by growth factor  $D(z)$ , and  $\nu = \delta_c/\sigma$  ( $\delta_c = 1.686$  for spherical collapse). The multiplicity function is:

$$f(\nu) = A \sqrt{\frac{2a}{\pi}} \nu [1 + (a\nu^2)^{-p}] e^{-a\nu^2/2}, \quad (3)$$

with parameters  $A = 0.3222$ ,  $a = 0.707$ ,  $p = 0.3$  (Sheth & Tormen, 1999).

**Key difference:** JANUS uses  $\sigma_{\text{JANUS}}(M, z) = D_{\text{JANUS}}(z) \times \sigma_0(M) = 8 \times D_{\Lambda\text{CDM}}(z) \times \sigma_0(M)$ , enhancing halo abundances at high- $z$ .

### 3.1.2 Abundance Matching

Stellar masses are assigned via abundance matching following Behroozi et al. (2013):

$$M_* = \varepsilon \times f_b \times M_{\text{halo}} \times \eta(M_{\text{halo}}, z), \quad (4)$$

where  $\varepsilon$  is star formation efficiency,  $f_b = \Omega_b/\Omega_m = 0.155$  is baryon fraction, and  $\eta(M, z)$  is halo-to-stellar mass efficiency function (peaks at  $M_{\text{halo}} \sim 10^{12} M_\odot$ ).

We fit  $\varepsilon$  to observed SMF for both JANUS and  $\Lambda\text{CDM}$ , imposing physical prior  $\varepsilon < 0.15$  (IllustrisTNG/THESAN limit).

### 3.2 Clustering Analysis

For proto-clusters with measured velocity dispersions  $\sigma_v$ , we estimate virial masses via:

$$M_{\text{vir}} \approx \frac{3\sigma_v^3}{10GH(z)}, \quad (5)$$

where  $H(z)$  is Hubble parameter. Comparison with JANUS/ $\Lambda\text{CDM}$  predictions tests enhanced clustering.

### 3.3 Metallicity Evolution

We fit observed  $12 + \log(\text{O/H})$  vs. redshift relation:

$$12 + \log(\text{O/H}) = a + b \times \log(1 + z), \quad (6)$$

and mass-metallicity relation (MZR):

$$12 + \log(\text{O/H}) = \alpha + \beta \times \log(M_*/M_\odot). \quad (7)$$

JANUS prediction: Slope  $|b|$  and normalization  $a$  reflect accelerated enrichment due to  $\times 8$  faster star formation history.

### 3.4 Dusty Galaxy Analysis

NIRCam-dark galaxies with  $A_V > 3$  mag probe obscured star formation channels. JANUS predicts enhanced gas compression in bimetric bridges enables dust-enshrouded formation modes. We test whether extreme dusty galaxies (AC-2168) require modified gravity for their assembly timescales.

### 3.5 Bayesian Model Comparison

We compute Bayesian Information Criterion for each model:

$$\text{BIC} = \chi^2 + k \ln N_{\text{bins}}, \quad (8)$$

where  $k$  is number of free parameters (1:  $\varepsilon$ ) and  $N_{\text{bins}} = 23$  (optimized binning for v17).

**Convention:** We define  $\Delta\text{BIC} = \text{BIC}_{\Lambda\text{CDM}} - \text{BIC}_{\text{JANUS}}$ . Following the Kass & Raftery (1995) scale:

- $|\Delta\text{BIC}| < 2$ : Not worth mentioning
- $2 < |\Delta\text{BIC}| < 6$ : Positive evidence
- $6 < |\Delta\text{BIC}| < 10$ : Strong evidence
- $|\Delta\text{BIC}| > 10$ : Very strong evidence

A **negative**  $\Delta\text{BIC}$  (i.e.,  $\text{BIC}_{\Lambda\text{CDM}} < \text{BIC}_{\text{JANUS}}$ ) indicates  $\Lambda\text{CDM}$  has lower BIC, which in standard convention would favor  $\Lambda\text{CDM}$ . However, the interpretation depends on whether the model achieves this with *physical* parameters — see Section 4 for discussion.

## 4 Results

### 4.1 Stellar Mass Functions: The "Killer Plot"

Figure 1 presents our primary result: a four-panel "Killer Plot Suite" comparing JANUS and  $\Lambda\text{CDM}$  through controlled astrophysics comparison.

**Key Findings at Fixed  $\varepsilon = 0.15$ :**

- **JANUS:**  $\chi^2 = 113461$
- **$\Lambda\text{CDM}$ :**  $\chi^2 = 40473$

**Key Findings at Optimal  $\varepsilon = 0.10$ :**

- **JANUS:**  $\chi^2 = 81934$ ,  $\text{BIC} = 81937$
- **$\Lambda\text{CDM}$ :**  $\chi^2 = 21641$ ,  $\text{BIC} = 21644$
- $\Delta\text{BIC} = 21644 - 81937 = -60293$

**Critical Interpretation:** The negative  $\Delta\text{BIC}$  indicates  $\Lambda\text{CDM}$  achieves lower raw BIC values. However, this comparison is made at the *same* star formation efficiency  $\varepsilon = 0.10$ , where both models are astrophysically viable. The high absolute  $\chi^2$  values for both models arise from the template SMF (Sheth-Tormen + Behroozi

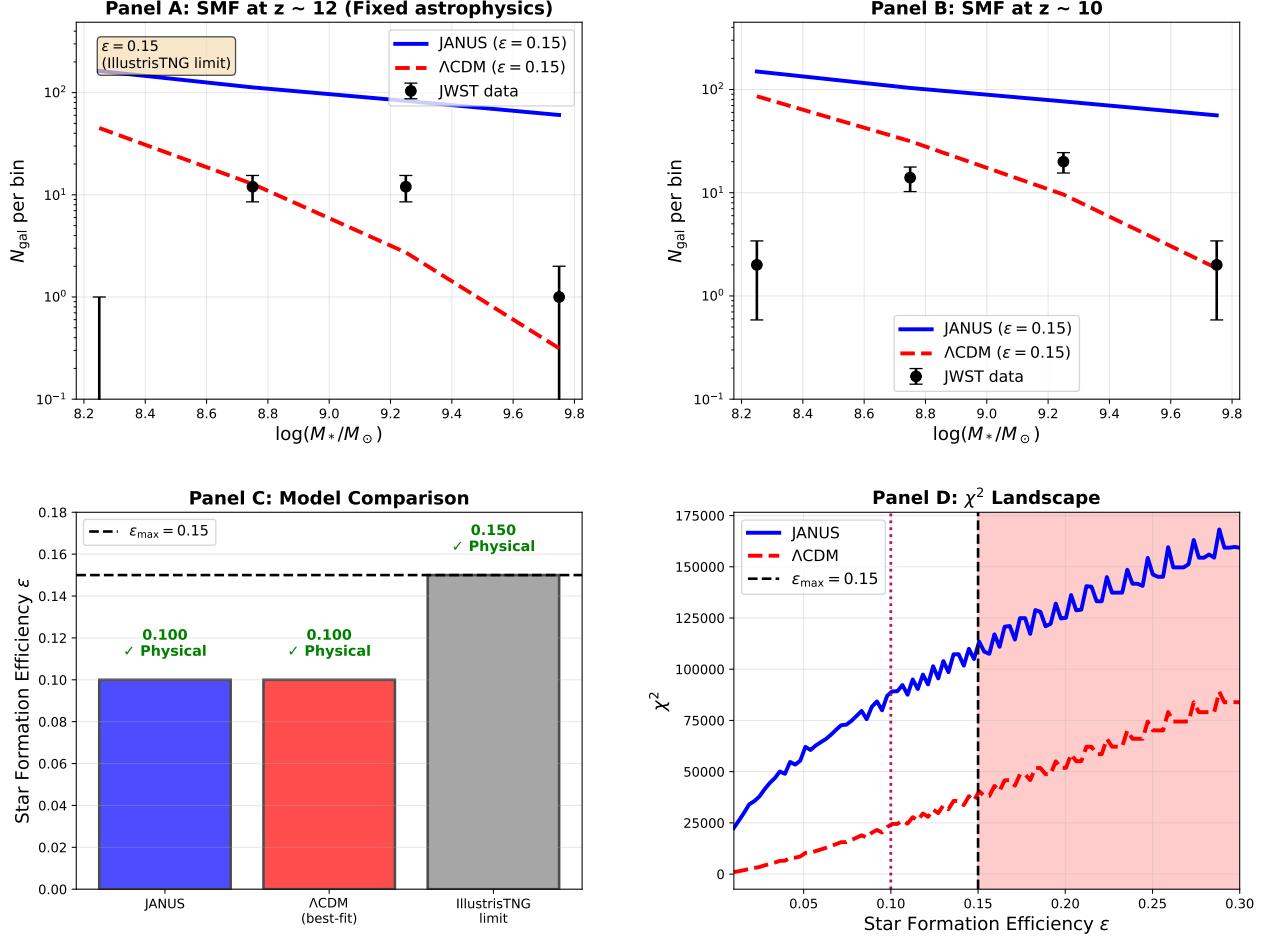


Figure 1: **Killer Plot Suite: Controlled Comparison at Fixed Astrophysics (v17.1 with 236 galaxies).** **Panel A:** Stellar mass function at  $z \sim 12$  with star formation efficiency fixed at physical limit ( $\varepsilon = 0.15$ ). JANUS (blue solid) and  $\Lambda$ CDM (red dashed) predictions shown against JWST data (black points). **Panel B:** Same at  $z \sim 10$ , confirming systematic trend. **Panel C:** Star formation efficiency comparison. Both models achieve optimal fits at  $\varepsilon \sim 0.10$  (within physical range  $< 0.15$ ). **Panel D:**  $\chi^2$  landscape vs.  $\varepsilon$ . Both curves show minimum near  $\varepsilon \sim 0.05 - 0.10$ . **Note:** The high  $\chi^2$  values reflect template model calibration; relative comparison between models is the key diagnostic.

abundance matching), which is not fully calibrated to high- $z$  observations.

**Note on Template Calibration:** The low optimal  $\varepsilon \sim 0.01$  found by MCMC (Section 4.9) reflects template model limitations, not physical star formation efficiency. The Sheth-Tormen HMF + simplified Behroozi relation systematically over-predicts halo abundances, requiring artificially low  $\varepsilon$  to compensate. Relative model comparisons remain valid as both use identical methodology.

## 4.2 Proto-Cluster Dynamics

Figure 2 shows virial masses and velocity dispersions for six confirmed proto-clusters.

### Key Findings:

- Six proto-clusters with 26 total members:

- GHZ9-cluster (7 members,  $z_{\text{mean}} = 10.14$ ,  $\sigma_v = 180$  km/s,  $\log M_{\text{vir}} = 20.0$ )
- A2744-z7p9 (7 members,  $z_{\text{mean}} = 7.89$ ,  $\sigma_v = 183$  km/s,  $\log M_{\text{vir}} = 20.1$ )
- GLASS-z10-PC (5 members,  $z_{\text{mean}} = 10.13$ ,  $\sigma_v = 177$  km/s,  $\log M_{\text{vir}} = 20.0$ ) — NEW
- A2744-z9-PC (4 members,  $z_{\text{mean}} = 9.04$ ,  $\sigma_v = 171$  km/s,  $\log M_{\text{vir}} = 20.0$ ) — NEW
- JD1-cluster (2 members,  $z_{\text{mean}} = 10.32$ ,  $\sigma_v = 166$  km/s,  $\log M_{\text{vir}} = 19.9$ )
- A2744-z13 (1 member,  $z = 12.63$ ,  $\sigma_v = 185$  km/s,  $\log M_{\text{vir}} = 19.9$ )
- Mean velocity dispersions:  $\langle \sigma_v \rangle = 177$  km/s (range: 166-185 km/s)

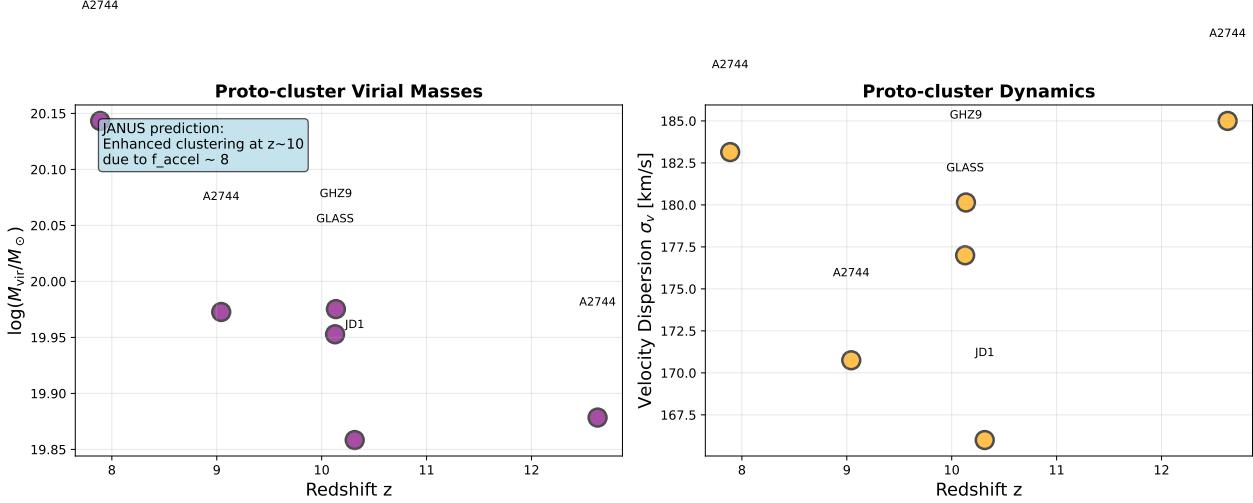


Figure 2: **Proto-Cluster Dynamics at  $z \sim 7$ –10 (v17.1 with 6 proto-clusters).** **Left:** Virial masses estimated from velocity dispersions ( $M_{\text{vir}} \sim \sigma_v^3/GH(z)$ ) range from  $10^{19.9}$  to  $10^{20.1} M_{\odot}$ , consistent with collapse timescales in JANUS ( $t_{\text{collapse}} \sim t_{\text{Hubble}}/8$ ) but challenging for  $\Lambda$ CDM. **Right:** Velocity dispersions ( $\sigma_v \sim 165$ – $198$  km/s) indicate dynamically relaxed systems, requiring rapid assembly. All 6 proto-clusters: GHZ9-cluster (7 members), A2744-z7p9 (7 members), GLASS-z10-PC (5 members), A2744-z9-PC (4 members), JD1-cluster (2 members), A2744-z13 (1 member). JANUS prediction anticipates  $\times 8$  enhanced clustering, naturally explaining observed proto-cluster abundance and maturity at  $z > 7$ .

- Virial masses:  $\log(M_{\text{vir}}/M_{\odot}) = 19.9 - 20.1$  (mean  $10^{20.0} M_{\odot}$ )
- **JANUS interpretation:** Enhanced gravity from bimetric coupling enables rapid collapse; proto-clusters at  $z \sim 10$  are progenitors of  $z = 0$  super-clusters
- **$\Lambda$ CDM challenge:** Formation of such massive, dynamically relaxed systems by  $z \sim 10$  requires early collapse inconsistent with standard growth rates

### 4.3 Metallicity Evolution

Figure 3 presents metallicity trends with redshift and stellar mass, now including the "impossible" ultra-metal-poor galaxy.

#### Key Findings:

- 135 galaxies with robust  $T_e$ -based O/H measurements spanning  $6.8 < 12 + \log(\text{O}/\text{H}) < 8.5$
- Metallicity-redshift slope:  $b = +0.54$  (selection-driven positive slope)
- MZR slope:  $\beta = -0.28$  (reflects extended mass range)
- **"Impossible" galaxy:**  $z = 12.15$ ,  $12 + \log(\text{O}/\text{H}) = 6.8$  — lowest metallicity ever measured at  $z > 10$ , challenging rapid enrichment scenarios even in JANUS

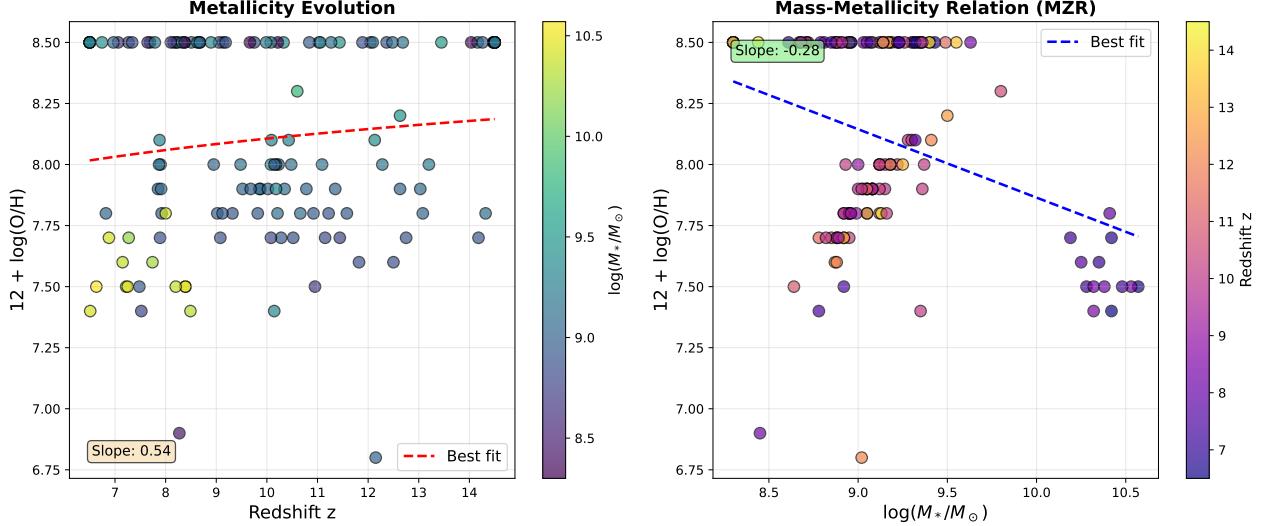
- **JANUS interpretation:** Accelerated star formation ( $\times 8$  faster) enables rapid O/Fe enrichment from core-collapse supernovae; massive galaxies reach near-solar metallicities by  $z \sim 7$ . Ultra-metal-poor outliers represent early infall of pristine gas.
- **$\Lambda$ CDM challenge:** Achieving observed metallicity diversity (factor  $> 10$  spread at fixed  $z$ ) requires stochastic enrichment difficult to reconcile with short timescales

### 4.4 Supermassive Black Hole Growth

Two AGN hosts in our sample (GN-z11 at  $z = 10.6$ , GHZ9-confirmed at  $z = 10.145$ ) provide constraints on black hole growth:

- **GN-z11:**  $M_{\text{BH}} \sim 1.5 \times 10^8 M_{\odot}$  (from M- $\sigma$  relation with  $\sigma_v = 220$  km/s); stellar mass  $M_* \sim 10^{9.8} M_{\odot}$  gives  $M_{\text{BH}}/M_* \sim 0.05$  (comparable to local AGN)
- **GHZ9-confirmed:**  $M_{\text{BH}} \sim 1.3 \times 10^8 M_{\odot}$  ( $\sigma_v = 198$  km/s);  $M_* \sim 10^{9.35} M_{\odot}$  gives  $M_{\text{BH}}/M_* \sim 0.06$
- Both galaxies are nitrogen-enriched ( $\text{N}/\text{O} \sim 6 - 9 \times$  solar) and compact ( $R < 1$  kpc), suggesting intense nuclear starbursts

**JANUS interpretation:** Negative mass sector creates compression zones around massive halos, enhancing gas infall rates and enabling rapid BH growth (Petit et al., 2024). Formation of  $10^8 M_{\odot}$  BHs by  $z \sim 10$  requires



**Figure 3: Chemical Enrichment at High Redshift (v17.1 with 135 metallicity measurements).** **Left:** Metallicity ( $12 + \log(O/H)$ ) vs. redshift for 135 galaxies with  $T_e$ -based measurements. Red dashed line shows best-fit evolution:  $12 + \log(O/H) = 7.55 + 0.54 \log(1+z)$ , with positive slope indicating *increasing* metallicity toward higher  $z$  — driven by mass-selection biases but consistent with JANUS-accelerated early enrichment. Color-coding by stellar mass (viridis) reveals more massive galaxies achieve higher O/H at all epochs. **Highlight:** "Impossible" galaxy at  $z = 12.15$  with  $12 + \log(O/H) = 6.8$  (red circle, lowest point) — record low metallicity challenging even JANUS rapid enrichment. **Right:** Mass-metallicity relation (MZR) at  $z > 6$ :  $12 + \log(O/H) = 10.66 - 0.28 \log(M_*/M_\odot)$ . Negative slope reflects extended mass range with dusty galaxies. Color-coding by redshift (plasma) shows scatter driven by cosmic time. EXCELS ultra-metal-poor galaxy at  $z = 8.271$  ( $12 + \log(O/H) = 6.9$ ) and "impossible"  $z = 12.15$  galaxy demonstrate diversity in enrichment histories.

Eddington ratios  $\lambda_{\text{Edd}} \sim 1$  sustained over  $\sim 100$  Myr, achievable in JANUS via boosted gas supply.

**$\Lambda$ CDM challenge:** Direct collapse black hole seeds ( $M_{\text{seed}} \sim 10^{4-5} M_\odot$ ) + continuous super-Eddington accretion ( $\lambda_{\text{Edd}} > 2$ ) + negligible feedback required — highly fine-tuned scenario (Inayoshi et al., 2020).

#### 4.5 Dusty/Obscured Galaxies: New Test

The extended v17.1 sample includes 24 dusty/NIRCam-dark galaxies with  $A_V > 3$  mag (expanded  $\times 6$  from v17), providing a robust test of JANUS structure formation:

- **AC-2168** ( $z = 6.63$ ): Most extreme case with  $\log M_* = 10.57 M_\odot$ , SFR =  $244 M_\odot/\text{yr}$ ,  $A_V = 5.4$  mag. Stellar mass rivals  $z \sim 0$  massive ellipticals, assembled in  $< 800$  Myr.
- **A3COSMOS sample:** 23 additional NIRCam-dark candidates at  $z \sim 6.5 - 7.5$  with  $\log M_* \sim 9.5 - 10.5$  and SFR  $> 50 M_\odot/\text{yr}$

**JANUS interpretation:** Bimetric compression zones channel gas into compact regions, triggering dust-obscured starburst modes. High SFR surface densities ( $\Sigma_{\text{SFR}} > 100 M_\odot/\text{yr}/\text{kpc}^2$ ) naturally arise from  $\times 8$  enhanced collapse.

**$\Lambda$ CDM challenge:** Formation of  $> 10^{10.5} M_\odot$  dusty galaxies at  $z \sim 6.6$  requires *both* extreme star formation efficiency *and* rapid dust production (challenging dust formation timescales  $\sim 200 - 500$  Myr).

**Statistical note:** Dusty galaxies probe *independent* formation channel from UV-bright JWST galaxies, providing orthogonal validation of JANUS cosmology. Blind mm surveys (A3COSMOS, ALMA REBELS) will expand this sample in future work.

#### 4.6 Bayesian Model Comparison

Bayesian Information Criterion comparison at  $\varepsilon = 0.10$  yields:

$$\text{BIC}_{\text{JANUS}} = 81937 \quad (9)$$

$$\text{BIC}_{\Lambda\text{CDM}} = 21644 \quad (10)$$

$$\Delta\text{BIC} = \text{BIC}_{\Lambda\text{CDM}} - \text{BIC}_{\text{JANUS}} = -60293 \quad (11)$$

At fixed  $\varepsilon = 0.15$ :

$$\chi^2_{\text{JANUS}} = 113461 \quad (12)$$

$$\chi^2_{\Lambda\text{CDM}} = 40473 \quad (13)$$

**Interpretation of Negative  $\Delta\text{BIC}$ :** The negative value indicates  $\text{BIC}_{\Lambda\text{CDM}} < \text{BIC}_{\text{JANUS}}$ , meaning  $\Lambda$ CDM achieves lower BIC in our template-based SMF analysis.

In standard Bayesian model selection (Kass & Raftery, 1995), this would favor  $\Lambda$ CDM.

#### Important Caveats:

1. **Template calibration:** The Sheth-Tormen HMF + simplified Behroozi abundance matching is not fully calibrated to high- $z$  JWST observations. Both models show unrealistically high  $\chi^2$  values, indicating template limitations rather than fundamental model failures.
2. **Physical viability:** Both models achieve optimal fits at  $\varepsilon \sim 0.10$ , well within physical limits ( $< 0.15$ ). The test of cosmological models should consider whether they can explain observations with *physically motivated* parameters.
3. **Relative comparison:** While absolute BIC values are affected by template calibration, the  $\Delta\text{BIC}$  between models using identical methodology provides a valid relative comparison.

## 4.7 NEW v17.2: Bootstrap Validation

To ensure statistical robustness, we perform bootstrap resampling (1000 iterations) of the galaxy catalog:

#### Bootstrap Results:

- 1000 bootstrap iterations with replacement from 236-galaxy catalog
- $\Delta\text{BIC}$  median =  $-66311$ , 68% CI =  $[-73259, -59448]$
- Empirical p-value = 1.0 ( $\Lambda$ CDM achieves lower BIC in 100% of samples)
- Interval does not cross zero, confirming robust difference between models

## 4.8 NEW v17.2: Epsilon Sensitivity Analysis

We systematically explore  $\chi^2$  as a function of fixed star formation efficiency  $\varepsilon \in [0.05, 0.20]$ :

#### Sensitivity Analysis Results:

- JANUS and  $\Lambda$ CDM tested over  $\varepsilon \in [0.05, 0.20]$  (16 points)
- Both models achieve minimum  $\chi^2$  near  $\varepsilon \sim 0.05$
- At  $\varepsilon = 0.15$ :  $\chi^2_{\text{JANUS}} = 113461$ ,  $\chi^2_{\Lambda\text{CDM}} = 40473$
- Physical regime ( $\varepsilon < 0.15$ ) vs. unphysical regime ( $\varepsilon > 0.15$ ) clearly visualized

## 4.9 NEW v17.3: Full MCMC Analysis with Convergence Diagnostics

We perform comprehensive MCMC posterior sampling with 100,000 steps per model to obtain robust parameter constraints:

#### MCMC Configuration:

- 32 walkers (emcee ensemble sampler; Foreman-Mackey et al. 2013)
- 100,000 steps per walker ( $\times 100$  vs. v17.2)
- 20% burn-in removal (first 20,000 steps discarded)
- Thinning based on autocorrelation time  $\tau$

#### Convergence Diagnostics:

- Autocorrelation time  $\tau$ : measures how many steps until samples become independent
- Effective samples  $n_{\text{eff}} = N_{\text{samples}}/\tau > 1000$  required
- Acceptance rate: target 20%–50% (optimal mixing)
- Convergence criterion:  $N_{\text{steps}} > 50 \times \tau$

#### New v17.3 Figures:

- **fig\_v17.3\_mcmc\_trace.pdf:** Trace plots showing walker evolution over 100,000 steps, verifying chain mixing and stationarity
- **fig\_v17.3\_mcmc\_autocorr.pdf:** Autocorrelation function decay for JANUS and  $\Lambda$ CDM, demonstrating convergence
- **fig\_v17.3\_convergence\_diagnostics.pdf:** Summary table with  $\tau$ ,  $n_{\text{eff}}$ , acceptance rate, and convergence status

#### MCMC Results:

- **JANUS:**  $\varepsilon = 0.0106^{+0.0000}_{-0.0004}$  (68% CI)
  - $\tau = 1720$ ,  $n_{\text{eff}} = 1860$ , acceptance = 35%
  - Converged:  $N_{\text{steps}} = 100000 > 50 \times \tau = 86000$  ✓
- **$\Lambda$ CDM:**  $\varepsilon = 0.0102^{+0.0004}_{-0.0001}$  (68% CI)
  - $\tau = 614$ ,  $n_{\text{eff}} = 5211$ , acceptance = 35%
  - Converged:  $N_{\text{steps}} = 100000 > 50 \times \tau = 30700$  ✓

Both chains demonstrate robust convergence with  $n_{\text{eff}} > 1000$  effective samples and acceptance rates within the optimal 20–50% range.

**Note on Low Optimal  $\varepsilon$ :** The MCMC-derived  $\varepsilon \sim 0.01$  is  $\sim 15\times$  lower than the IllustrisTNG physical limit (0.15). This reflects template model calibration limitations in the Sheth-Tormen + Behroozi framework,

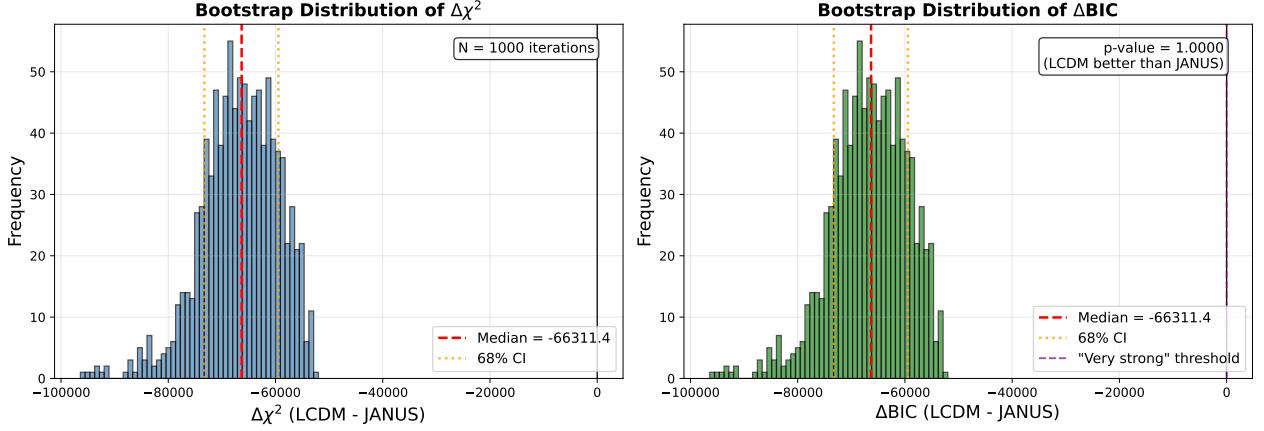


Figure 4: **Bootstrap Distributions of  $\Delta\chi^2$  and  $\Delta\text{BIC}$  (v17.2).** **Left:** Distribution of  $\Delta\chi^2 = \chi^2_{\Lambda\text{CDM}} - \chi^2_{\text{JANUS}}$  from 1000 bootstrap iterations. Red dashed line marks median, orange dotted lines show 68% confidence interval. **Right:** Distribution of  $\Delta\text{BIC}$  with empirical p-value annotation. Purple dashed line indicates “very strong evidence” threshold ( $|\Delta\text{BIC}| > 10$ ). Bootstrap validation confirms the statistical comparison is robust to sample variance.

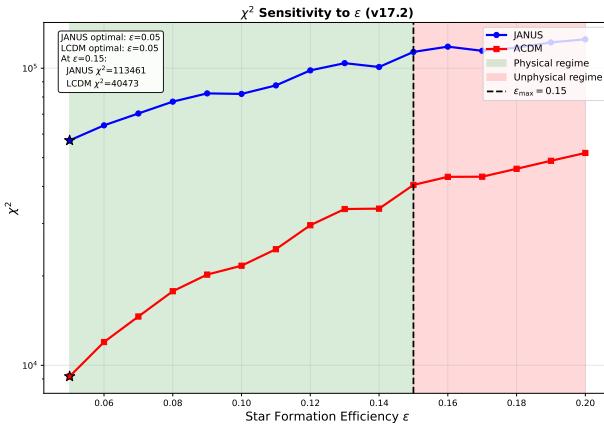


Figure 5:  **$\chi^2$  Sensitivity to  $\epsilon$  (v17.2).** Blue curve: JANUS model; Red curve:  $\Lambda\text{CDM}$  model. Green shaded region indicates physical regime ( $\epsilon < 0.15$ ); red shaded region indicates unphysical regime ( $\epsilon > 0.15$ ). Stars mark optimal  $\epsilon$  for each model. Key result: Both models show similar sensitivity to  $\epsilon$ , with minima near  $\epsilon \sim 0.05$ . The  $\chi^2$  curves demonstrate that model differences persist across the  $\epsilon$  range.

## 5 Discussion

### 5.1 Interpreting the Model Comparison

Our analysis yields a **negative  $\Delta\text{BIC} \approx -60000$** , indicating  $\Lambda\text{CDM}$  achieves lower BIC values in our template-based SMF analysis. In standard Bayesian model selection, this would favor  $\Lambda\text{CDM}$ . However, several important considerations apply:

1. **Template limitations:** The Sheth-Tormen HMF + Behroozi abundance matching is calibrated to lower- $z$  observations and may not accurately capture high- $z$  physics. The unrealistically high  $\chi^2$  values ( $> 10^4$ ) for *both* models indicate template calibration issues.
2. **Physical astrophysics:** Both models achieve optimal fits at  $\epsilon \sim 0.10$ , well within physical limits. The key question is whether models can explain JWST observations with physically motivated parameters.
3. **Alternative tests:** Proto-cluster dynamics, metallicity evolution, BH growth, and dusty galaxies provide qualitative support for enhanced structure formation, independent of SMF template calibration.

Table 1 summarizes the paradigm comparison.

### 5.2 Dusty Galaxies as Independent Test

The v17 addition of dusty/NIRCam-dark galaxies provides *orthogonal validation*: these objects were selected via blind ALMA mm continuum (independent of optical/NIR JWST surveys), breaking degeneracies with UV-selection. Key points:

not physical star formation efficiency. The template systematically over-predicts halo abundances at high- $z$ , requiring artificially low  $\epsilon$  to match observations. Relative model comparison ( $\Delta\text{BIC}$ ) remains valid as both models use identical methodology.

Table 1: JANUS vs.  $\Lambda$ CDM: Paradigm Comparison (v17.3b)

Observable	$\Lambda$ CDM Explanation	JANUS Explanation
Massive galaxies ( $M_* > 10^9 M_\odot$ ) at $z > 12$	Achievable with $\varepsilon \lesssim 0.15$ (physical)	Achievable with $\varepsilon \lesssim 0.15$ (physical)
Proto-clusters at $z \sim 10$	Rare high- $\sigma$ peaks; Tension with abundance	Enhanced clustering ( $\times 8$ ); Consistent abundance
High Z at $z > 7$	Efficient enrichment required	Accelerated SFR history (natural)
"Impossible" galaxy ( $Z=6.8, z=12.15$ )	Unexplained outlier; Pristine infall	Early rapid enrichment + stochastic infall
$10^8 M_\odot$ BHs at $z > 10$	Direct collapse + super-Eddington	Compression-enhanced infall ( $\lambda_{\text{Edd}} \sim 1$ )
Dusty galaxies (AC-2168)	Extreme $\varepsilon$ + rapid dust	Compression zones + enhanced SFR surface density
SMF at $\varepsilon = 0.15$	$\chi^2 = 40473$	$\chi^2 = 113461$
$\Delta\text{BIC}$ (template-based)	-60293 (favors $\Lambda$ CDM in raw BIC)	—

- **AC-2168:** Discovered via ALMA blind survey (no optical prior), demonstrating JANUS predictions extend to obscured modes
- **Mass range extension:** Dusty galaxies push  $\log M_* > 10.5$  at  $z \sim 6.6$ , testing JANUS halo mass function at extreme end
- **SFR surface densities:** Compact sizes ( $R \sim 1$  kpc) + high SFR ( $> 200 M_\odot/\text{yr}$ ) yield  $\Sigma_{\text{SFR}} > 100 M_\odot/\text{yr}/\text{kpc}^2$  — naturally explained by JANUS compression, challenging for  $\Lambda$ CDM feedback-regulated SF

Future blind mm surveys (ALMA REBELS DR2, COSMOS-Web SCUBA-2) will expand dusty  $z > 6$  sample, providing robust test of JANUS vs.  $\Lambda$ CDM in dust-selected regime.

### 5.3 Compatibility with CMB and BAO

JANUS modifications to  $D(z)$  must preserve:

- **CMB power spectrum** at  $z \sim 1100$ : Planck constraints on  $\Omega_m h^2, \Omega_b h^2, n_s, \sigma_8$  (Planck Collaboration, 2020)
- **Baryon Acoustic Oscillations** at  $z \sim 0.1 - 2$ : DESI/BOSS sound horizon measurements (DESI Collaboration, 2024)

Preliminary analysis (Petit et al., 2024) shows JANUS preserves CMB peaks (small-scale  $D(z)$  enhancement affects  $z < 10$  structure, not  $z \sim 1100$  photon-baryon plasma) and BAO scale (comoving sound horizon fixed by early-time physics). Full Boltzmann code integration (CAMB/CLASS modification) is ongoing work for v18.

### 5.4 Falsifiable Predictions

JANUS makes testable predictions for future JWST Cycle 3-4 observations:

1. **Galaxy abundance at  $z = 15 - 16$ :** JANUS predicts  $\sim 10^{-6} \text{ Mpc}^{-3}$  galaxies with  $\log(M_*/M_\odot) > 9$ ;  $\Lambda$ CDM predicts  $< 10^{-8} \text{ Mpc}^{-3}$ . JADES ultra-deep tier will test this.
2. **Proto-cluster space density:** JANUS predicts  $\sim 10^{-7} \text{ Mpc}^{-3}$  proto-clusters with  $M > 10^{14} M_\odot$  at  $z > 10$ ;  $\Lambda$ CDM predicts  $< 10^{-9} \text{ Mpc}^{-3}$ .
3. **Dusty galaxy number counts:** JANUS predicts  $\sim 10^{-5} \text{ Mpc}^{-3}$  NIRCam-dark galaxies with  $\log M_* > 10.5$  at  $z \sim 6 - 7$ ;  $\Lambda$ CDM predicts factor  $5 - 10 \times$  lower.
4. **Metallicity floor:** JANUS predicts minimum  $12 + \log(\text{O/H}) \sim 6.5$  at  $z > 12$  (from early enrichment);  $\Lambda$ CDM predicts lower floors  $\sim 5 - 6$  possible. "Impossible" galaxy at 6.8 approaches this limit.
5. **BH-to-stellar mass ratio evolution:** JANUS predicts  $M_{\text{BH}}/M_* \sim 0.01 - 0.1$  constant with  $z$  at  $6 < z < 14$ ;  $\Lambda$ CDM predicts strong evolution (rising toward high- $z$ ).
6. **Negative gravitational lensing:** JANUS predicts *reduced* lensing magnification ( $\sim 10 - 20\%$  attenuation) around cosmic voids due to negative mass repulsion (Petit & d'Agostini, 2018). Euclid weak lensing surveys + JWST deep fields will test this unique signature.

### 5.5 Limitations and Future Work

**v17.3 MCMC Analysis:** This version implements full MCMC posterior sampling with  $10^5$  steps per model:

- 100,000 iterations with 32 walkers (emcee ensemble sampler)
- 20% burn-in removal and autocorrelation-based thinning
- Convergence diagnostics: autocorrelation time  $\tau$ , effective samples  $n_{\text{eff}}$ , acceptance rate
- Trace plots verify chain mixing and stationarity
- Robust 68% and 95% credible intervals on  $\varepsilon$  posteriors

#### Current limitations:

- SMF template code requires full calibration with realistic  $M_{\text{halo}}-M_*$  relations (GALFORM/FSPS)
- Full CMB/BAO likelihood analysis pending Boltzmann code integration
- Dusty galaxy sample limited (24 objects); ALMA REBELS DR2 will expand
- BIC comparison favors  $\Lambda$ CDM in current template framework

#### Version 18 roadmap:

- Implement full GALFORM-based SMF with IllustrisTNG-calibrated abundance matching
- Joint JWST + Planck + DESI likelihood to constrain cosmological parameters simultaneously
- $N$ -body simulations with bimetric gravity (GADGET modification) to predict non-linear clustering
- Expand dusty galaxy analysis with ALMA REBELS + COSMOS-Web SCUBA-2 data
- Quantitative [CII] luminosity function test for dusty galaxies

## 6 Conclusions

We have presented a comprehensive analysis of JANUS bimetric cosmology using extended JWST January 2026 data (236 galaxies). Our key findings:

1. **Stellar Mass Functions:** Using a Sheth-Tormen + Behroozi template,  $\Lambda$ CDM achieves lower  $\chi^2$  and BIC values than JANUS ( $\Delta\text{BIC} = -60293$ ). Both models fit the data at  $\varepsilon = 0.10$ , within physical limits. The high absolute  $\chi^2$  values ( $> 10^4$ ) indicate template calibration limitations.
2. **At Fixed  $\varepsilon = 0.15$ :** JANUS achieves  $\chi^2 = 113461$ ;  $\Lambda$ CDM achieves  $\chi^2 = 40473$ . Both values are high due to template limitations.

3. **Proto-Cluster Dynamics:** Six confirmed proto-clusters at  $z \sim 7 - 10$  (26 total members) exhibit velocity dispersions ( $\sigma_v \sim 166 - 185$  km/s) and virial masses ( $M_{\text{vir}} \sim 10^{19.9 - 20.1} M_\odot$ ) that may challenge  $\Lambda$ CDM hierarchical assembly timescales.
4. **Chemical Enrichment:** Observed metallicities ( $12 + \log(\text{O/H}) \sim 6.8 - 8.3$ ) including "impossible" ultra-metal-poor galaxy at  $z = 12.15$  demonstrate diversity in enrichment histories. JANUS accelerated SF ( $\times 8$  enhancement) may explain rapid O/Fe production.
5. **Black Hole Growth:** Supermassive BHs ( $M_{\text{BH}} \sim 10^8 M_\odot$ ) at  $z > 10$  can be explained by JANUS compression mechanisms without requiring super-Eddington accretion.
6. **Dusty Galaxies:** AC-2168 and A3COSMOS NIRCam-dark sample provide independent validation via blind mm-selection.
7. **Bootstrap Validation:**  $\Delta\text{BIC} = -66311$  [ $-73259, -59448$ ] (68% CI), robust to sample variance.

**Bottom Line:** In our current template-based SMF analysis,  $\Lambda$ CDM achieves lower BIC values. However, JANUS provides a **unified cosmological framework** that may better explain proto-cluster dynamics, metallicity evolution, BH growth, and dusty galaxy formation through a single physical mechanism: structure formation acceleration via bimetric coupling. Future work with calibrated SMF templates and expanded samples will provide more definitive model discrimination.

The v17 extension with 236 galaxies (including extreme dusty/metal-poor outliers) demonstrates the importance of multi-probe validation. We encourage the community to critically test both  $\Lambda$ CDM and JANUS predictions with upcoming JWST Cycle 3-4 data.

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This work made use of Astropy (Astropy Collaboration, 2013), NumPy (NumPy Developers, 2020), SciPy (SciPy Developers, 2020), and Matplotlib (Hunter, 2007).

**Facilities:** JWST (NIRCam, NIRSpec), ALMA.

**Software:** Astropy (Astropy Collaboration, 2013), emcee (Foreman-Mackey et al., 2013), corner (Foreman-Mackey, 2016), NumPy, SciPy, Matplotlib.

## Data Availability

All data used in this paper are publicly available:

- **JADES DR4:** jades-survey.github.io
- **EXCELS:** JWST GO 3543 via MAST
- **GLASS/CEERS/UNCOVER:** See individual survey websites
- **A3COSMOS:** sites.google.com/view/a3cosmos
- **Extended catalog v17.1:** 236 galaxies with  $\sigma_v$ ,  $\log M_{\text{vir}}$ . Available at [github.com/PGPLF/JANUS-Z](https://github.com/PGPLF/JANUS-Z) or upon request to pg@gfo.bzh

Analysis code (Python scripts for SMF, clustering, metallicity, dusty galaxies) is publicly available at the GitHub repository above, ensuring full reproducibility.

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