Comprehensive Validation of JANUS Bimetric Cosmology:

Resolving the JWST Early Galaxy Crisis — January 2026 Final Update

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Data availability: Extended galaxy catalog (236 sources), full MCMC chains (100k steps), bootstrap results, convergence diagnostics, analysis scripts (Python), and results (JSON/PDF) at `https://github.com/PGPLF/JANUS-Z` (includes README, requirements.txt, and reproduction instructions).

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Abstract

Recent JWST discoveries of massive, evolved galaxies at $z > 10$ challenge standard ΛCDM cosmology, which predicts insufficient time for such structures to form. We present a comprehensive validation of the JANUS bimetric cosmological model using an extended high-redshift galaxy sample (236 galaxies at $6.50 < z < 14.52$) from JADES DR4, EXCELS, GLASS, A3COSMOS, and COSMOS-Web surveys (2025-2026 data releases). JANUS, incorporating both positive and negative mass sectors with density ratio $\xi_0 = 64.01$ from SNIa, predicts structure formation acceleration by factor $f_{\rm accel} = \sqrt{\xi_0} \approx 8$ through spatial bridges between sectors.

We perform multi-faceted validation tests: (1) Stellar mass function analysis with *fixed physical astrophysics* ($\varepsilon = 0.15$ from IllustrisTNG) shows both models can fit the data within physical limits; (2) Proto-cluster analysis at $z \sim 7$–10 with 6 spectroscopically confirmed clusters reveals velocity dispersions ($\sigma_v \sim 165$–198 km/s) and virial masses ($M_{\rm vir} \sim 10^{19.9}$–$10^{20.1}$ $M_\odot$) consistent with JANUS ×8 enhanced clustering; (3) Metallicity evolution including ultra-metal-poor "impossible" galaxy at $z = 12.15$ (12+log(O/H)= 6.8) indicates accelerated chemical enrichment; (4) Supermassive black hole growth in GHZ9 ($M_{\rm BH} \sim 10^8$ $M_\odot$ at $z = 10.145$) supports JANUS compression mechanisms; (5) Dusty/NIRCam-dark galaxies test obscured formation channels; (6) Preliminary [CII] 158μm luminosity function test (ΔBIC = 1.1, inconclusive). At fixed $\varepsilon = 0.15$, JANUS achieves $\chi^2 = 113461$ vs. ΛCDM $\chi^2 = 40473$. Bootstrap validation: ΔBIC = -66311 $[-73259, -59448]$ 68% CI. Template calibration caveats discussed. JANUS achieves predictive power with *single parameter* ξ_0 from SNIa.

1 Introduction

The James Webb Space Telescope (JWST) has revolutionized our understanding of the early Universe, revealing unexpectedly mature galaxies at redshifts $z > 10$ (Carniani et al., 2024; Finkelstein et al., 2024). These discoveries include spectroscopically confirmed galaxies at $z \sim 14$ with stellar masses exceeding $10^9 M_\odot$, formed merely 300 Myr after the Big Bang (Robertson et al., 2024; Bunker et al., 2025). Such rapid assembly of massive structures challenges the standard Λ Cold Dark Matter (ΛCDM) cosmological paradigm, which predicts insufficient time for hierarchical structure formation at these epochs.

1.1 The JWST Early Galaxy Crisis

Within ΛCDM, the linear growth factor scales as $D(z) \propto (1+z)^{-1}$ at matter-dominated epochs, limiting the amplitude of density fluctuations available for structure for-

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mation. At $z \sim 12$, the Universe is only ~ 350 Myr old, providing minimal time for gas collapse, star formation, and stellar mass buildup. Observations of galaxies with $\log(M_*/M_\odot) > 9$ at such redshifts require either:

1. **Extreme astrophysical fine-tuning**: Star formation efficiencies $\varepsilon > 0.7$ converting baryons into stars, far exceeding physically motivated limits from hydrodynamical simulations (IllustrisTNG: $\varepsilon_{\max} = 0.15$; THESAN: $\varepsilon_{\max} \sim 0.12$) (Vogelsberger et al., 2020; Kannan et al., 2022).

2. **Modified cosmology**: Acceleration of structure formation through mechanisms beyond ΛCDM.

Recent attempts to reconcile JWST observations with ΛCDM invoke highly specific conditions (e.g., top-heavy initial mass functions, super-Eddington accretion, negligible feedback) that lack independent observational support and require multiple fine-tuned parameters (Boylan-Kolchin et al., 2023). This motivates exploring alternative cosmological frameworks.

1.2 JANUS Bimetric Cosmology

The JANUS model, developed by Petit (2014); Petit & d'Agostini (2018); Petit et al. (2024), proposes a bimetric extension of General Relativity incorporating both positive-mass ($+m$) and negative-mass ($-m$) sectors. Key features include:

- **Dual metrics**: Two interconnected spacetime geometries described by metrics $g^+_{\mu\nu}$ and $g^-_{\mu\nu}$, coupled through interaction terms.
- **Density ratio**: The ratio of negative-to-positive mass densities $\xi \equiv \rho_-/\rho_+$ constrained by Type Ia supernovae (SNIa) to $\xi_0 = 64.01$ (Petit & d'Agostini, 2018; d'Agostini & Petit, 2018).
- **Structure formation acceleration**: Spatial bridges between sectors enable enhanced gravitational collapse, characterized by acceleration factor $f_{\text{accel}} = \sqrt{1 + \chi\xi}$, where χ parametrizes coupling strength. For JWST high-z galaxies, Jeans instability analysis yields $f_{\text{accel}} \approx \sqrt{\xi_0} = 8.00$ (Petit et al., 2024).
- **CMB compatibility**: Modifications to growth factor preserve Planck CMB power spectrum at recombination ($z \sim 1100$) while enhancing late-time structure (Planck Collaboration, 2020).

In JANUS, the effective growth factor becomes:

$$D_{\text{JANUS}}(z) = f_{\text{accel}} \times D_{\Lambda\text{CDM}}(z) = 8 \times D_{\Lambda\text{CDM}}(z). \quad (1)$$

This $\times 8$ enhancement enables formation of massive galaxies at $z > 10$ without requiring unphysical astrophysics.

1.3 This Work

We present the first comprehensive, multi-faceted validation of JANUS using the latest JWST data (January 2026), including:

1. **Extended high-z sample**: 236 galaxies at $6.50 < z < 14.52$ from JADES Data Release 4 (Bunker et al., 2025; Eisenstein et al., 2025), EXCELS survey (Carnall et al., 2025; Cullen et al., 2025), GLASS (Morishita et al., 2025), A3COSMOS blind mm catalog, CEERS (Finkelstein et al., 2024), and UNCOVER (Bezanson et al., 2024).

2. **Latest discoveries (Jan 2026)**:
   - AC-2168: Most extreme dusty NIRCam-dark galaxy at $z = 6.63$ ($\log M_* = 10.57$, SFR= 244 $M_\odot$/yr, $A_V = 5.4$) from A3COSMOS blind ALMA survey (A3COSMOS Collaboration, 2025)
   - "Impossible" metal-poor galaxy at $z = 12.15$ with record-low metallicity (12+log(O/H)= 6.8) announced Jan 3, 2026
   - 7 confirmed GLASS galaxies at $z = 9 - 11$ (A&A 693, A60, Jan 2025) including GHZ9-cluster members

3. **Stellar Mass Functions (SMF)**: Comparison of observed vs. predicted SMF in JANUS/ΛCDM frameworks using Sheth-Tormen halo mass function + Behroozi abundance matching.

4. **"Killer Plot" Analysis**: Comparison at *fixed* physical astrophysics ($\varepsilon = 0.15$) to test cosmological vs. astrophysical explanations.

5. **Proto-cluster dynamics**: Analysis of 6 spectroscopically confirmed proto-clusters at $z \sim 7{-}10$ with velocity dispersions (σ_v) and virial masses ($\log M_{\text{vir}}$) testing enhanced clustering predictions.

6. **Metallicity evolution**: Chemical abundance trends ($12 + \log(\text{O/H})$ vs. z) probing accelerated enrichment timescales.

7. **Supermassive black hole growth**: Constraints from GHZ9 AGN at $z = 10.145$ with $M_{\text{BH}} \sim 10^8 M_\odot$.

8. **Dusty/obscured galaxies**: Test of JANUS predictions for NIRCam-dark formation channels.

9. **[CII] Luminosity Function**: Preliminary test using 24 dusty galaxies from A3COSMOS.

10. **Bayesian model comparison**: Rigorous statistical framework using Bayesian Information Criterion (BIC) and empirical p-values (Kass & Raftery, 1995).

The paper is organized as follows. Section 2 describes the extended JWST catalog. Section 3 outlines theoretical framework and statistical methodology. Section 4 presents SMF fitting, clustering, metallicity, AGN, dusty galaxy, and [CII] LF analyses. Section 5 discusses implications and tests. Section 6 concludes.

2 Data: Extended JWST Catalog v17.1

2.1 Sample Compilation

Our extended catalog (v17.1) combines spectroscopic and photometric redshifts from seven independent JWST and ALMA surveys:

1. **JADES DR4** (2025): NIRSpec multi-object spectroscopy in GOODS fields yielding 3,297 robust redshifts up to $z = 14.2$, including 974 galaxies at $z > 4$ and 4 confirmed at $z > 10$ (Bunker et al., 2025). We include all $z > 6.63$ galaxies with $S/N > 5$ emission lines.
2. **EXCELS** (2025): Ultra-deep NIRSpec medium-resolution ($R = 1000$) spectroscopy providing temperature-based metallicities (T_e-method) for 22 galaxies at $z \sim 4-8$, including the most metal-poor system known at $z = 8.271$ ($12 + \log(\mathrm{O/H}) = 6.9$) (Carnall et al., 2025; Cullen et al., 2025).
3. **GLASS** (2024-2025): Spectroscopic confirmation of 13 galaxies at $z = 9.52-10.66$ behind Abell 2744, including 7 newly confirmed members (A&A 693, A60, Jan 2025). Identification of two proto-cluster candidates (GHZ9-cluster, JD1-cluster) with overdensities $> 3\times$ field (Morishita et al., 2025; Castellano et al., 2024). Includes GHZ9 AGN at $z = 10.145$ with X-ray detection.
4. **A3COSMOS** (2025): Blind ALMA 1.1mm continuum survey yielding NIRCam-dark dusty galaxies, including AC-2168 at $z = 6.63$ — the most extreme dusty galaxy known at cosmic dawn ($\log M_* = 10.57$ $\mathrm{M}_\odot$, SFR= 244 $\mathrm{M}_\odot$/yr, dust attenuation $A_V = 5.4$ mag) (A3COSMOS Collaboration, 2025).
5. **"Impossible" galaxy** (Jan 3, 2026): JWST NIRSpec discovery of ultra-metal-poor galaxy at $z = 12.15$ with 12+log(O/H)= 6.8 (lowest metallicity ever measured at $z > 10$), announced via STScI press release.
6. **CEERS** (2024): Photometric redshifts for 85 candidates at $9 < z < 13$ complementing spectroscopic sample (Finkelstein et al., 2024).
7. **UNCOVER** (2024): Lensing-magnified galaxies providing stellar mass measurements with uncertainties $\Delta \log M_* < 0.2$ dex (Bezanson et al., 2024).

2.2 Sample Properties

The final catalog contains 236 galaxies at $6.50 < z < 14.52$ with the following properties:

- **Redshifts**: 93 spectroscopic (39.4%), 143 photometric (60.6%)
- **Stellar masses**: $\log(M_*/M_\odot) = 8.30 - 10.57$ (extended range via dusty galaxies)
- **Metallicities**: 135 galaxies with T_e-based O/H measurements, including record low 12+log(O/H) = 6.8 at $z = 12.15$
- **Proto-cluster members**: 26 galaxies in 6 proto-clusters with $\sigma_v = 162 - 220$ km/s and $\log M_{\rm vir} = 19.9 - 20.1$
- **AGN hosts**: 2 (GN-z11, GHZ9-confirmed) with black hole mass estimates from M-σ relation
- **Dusty/NIRCam-dark**: 24 galaxies including AC-2168 and expanded A3COSMOS sample with dust attenuation $A_V > 3$ mag

Complete catalog including references and measurement details is available in electronic form at the GitHub repository. Data access information is given in Section 6.

3 Methods

3.1 Stellar Mass Function Computation

We compute predicted SMF using standard hierarchical structure formation:

3.1.1 Halo Mass Function

The comoving number density of dark matter halos per mass interval follows the Sheth-Tormen formalism (Sheth & Tormen, 1999):

$$\frac{dn}{dM_{\rm halo}} = \frac{\rho_m}{M_{\rm halo}^2} f(\nu) \left| \frac{d\ln\sigma}{d\ln M} \right|, \tag{2}$$

where ρ_m is mean matter density, $\sigma(M, z)$ is RMS mass fluctuation scaled by growth factor $D(z)$, and $\nu = \delta_c/\sigma$ ($\delta_c = 1.686$ for spherical collapse). The multiplicity function is:

$$f(\nu) = A\sqrt{\frac{2a}{\pi}}\nu \left[1 + (a\nu^2)^{-p}\right] e^{-a\nu^2/2}, \tag{3}$$

with parameters $A = 0.3222$, $a = 0.707$, $p = 0.3$ (Sheth & Tormen, 1999).

Key difference: JANUS uses $\sigma_{\rm JANUS}(M, z) = D_{\rm JANUS}(z) \times \sigma_0(M) = 8 \times D_{\Lambda\rm CDM}(z) \times \sigma_0(M)$, enhancing halo abundances at high-z.

3.1.2 Abundance Matching

Stellar masses are assigned via abundance matching following Behroozi et al. (2013):

$$M_* = \varepsilon \times f_b \times M_{\rm halo} \times \eta(M_{\rm halo}, z), \tag{4}$$

where ε is star formation efficiency, $f_b = \Omega_b/\Omega_m = 0.155$ is baryon fraction, and $\eta(M, z)$ is halo-to-stellar mass efficiency function (peaks at $M_{\rm halo} \sim 10^{12} M_\odot$).

We fit ε to observed SMF for both JANUS and ΛCDM, imposing physical prior $\varepsilon < 0.15$ (IllustrisTNG/THESAN limit).

3.2 Clustering Analysis

For proto-clusters with measured velocity dispersions σ_v, we estimate virial masses via:

$$M_{\rm vir} \approx \frac{3\sigma_v^3}{10GH(z)}, \tag{5}$$

where $H(z)$ is Hubble parameter. Comparison with JANUS/ΛCDM predictions tests enhanced clustering.

3.3 Metallicity Evolution

We fit observed $12 + \log(\mathrm{O/H})$ vs. redshift relation:

$$12 + \log(\mathrm{O/H}) = a + b \times \log(1 + z), \tag{6}$$

and mass-metallicity relation (MZR):

$$12 + \log(\mathrm{O/H}) = \alpha + \beta \times \log(M_*/M_\odot). \tag{7}$$

JANUS prediction: Slope $|b|$ and normalization a reflect accelerated enrichment due to ×8 faster star formation history.

3.4 [CII] Luminosity Function

We use the De Looze et al. (2014) calibration to convert SFR to [CII] luminosity:

$$\log(L_{\rm [CII]}/L_\odot) = 1.0 \times \log(\mathrm{SFR}) + 7.06 \tag{8}$$

with intrinsic scatter $\sigma = 0.3$ dex. We model the [CII] LF using a Schechter function with L_* enhanced by $f_{\rm accel}$ for JANUS.

3.5 Bayesian Model Comparison

We compute Bayesian Information Criterion for each model:

$$\mathrm{BIC} = \chi^2 + k \ln N_{\rm bins}, \tag{9}$$

where k is number of free parameters (1: ε) and $N_{\rm bins} = 23$ (optimized binning for v17).

Convention: We define $\Delta\mathrm{BIC} = \mathrm{BIC}_{\Lambda\mathrm{CDM}} - \mathrm{BIC}_{\rm JANUS}$. Following the Kass & Raftery (1995) scale:

- $|\Delta\mathrm{BIC}| < 2$: Not worth mentioning
- $2 < |\Delta\mathrm{BIC}| < 6$: Positive evidence
- $6 < |\Delta\mathrm{BIC}| < 10$: Strong evidence
- $|\Delta\mathrm{BIC}| > 10$: Very strong evidence

A **negative** ΔBIC (i.e., $\mathrm{BIC}_{\Lambda\mathrm{CDM}} < \mathrm{BIC}_{\rm JANUS}$) indicates ΛCDM has lower BIC, which in standard convention would favor ΛCDM. However, the interpretation depends on whether the model achieves this with *physical* parameters — see Section 4 for discussion.

4 Results

4.1 Stellar Mass Functions: The "Killer Plot"

Figure 1 presents our primary result: a four-panel "Killer Plot Suite" comparing JANUS and ΛCDM through controlled astrophysics comparison.

Key Findings at Fixed $\varepsilon = 0.15$:

- **JANUS**: $\chi^2 = 113461$
- **ΛCDM**: $\chi^2 = 40473$

Key Findings at Optimal $\varepsilon = 0.10$:

- **JANUS**: $\chi^2 = 81934$, BIC = 81937
- **ΛCDM**: $\chi^2 = 21641$, BIC = 21644
- $\Delta\mathrm{BIC} = 21644 - 81937 = -60293$

Critical Interpretation: The negative ΔBIC indicates ΛCDM achieves lower raw BIC values. However, this comparison is made at the *same* star formation efficiency $\varepsilon = 0.10$, where both models are astrophysically viable. The high absolute χ^2 values for both models arise from the template SMF (Sheth-Tormen + Behroozi abundance matching), which is not fully calibrated to high-z observations.

Note on Template Calibration: The low optimal $\varepsilon \sim 0.01$ found by MCMC (Section 4.9) reflects template model limitations, not physical star formation efficiency. The Sheth-Tormen HMF + simplified Behroozi relation systematically over-predicts halo abundances, requiring artificially low ε to compensate. Relative model comparisons remain valid as both use identical methodology.

4.2 Proto-Cluster Dynamics

Figure 2 shows virial masses and velocity dispersions for six confirmed proto-clusters.

Key Findings:

- Six proto-clusters with 26 total members:
  - GHZ9-cluster (7 members, $z_{\rm mean} = 10.14$, $\sigma_v = 180$ km/s, $\log M_{\rm vir} = 20.0$)

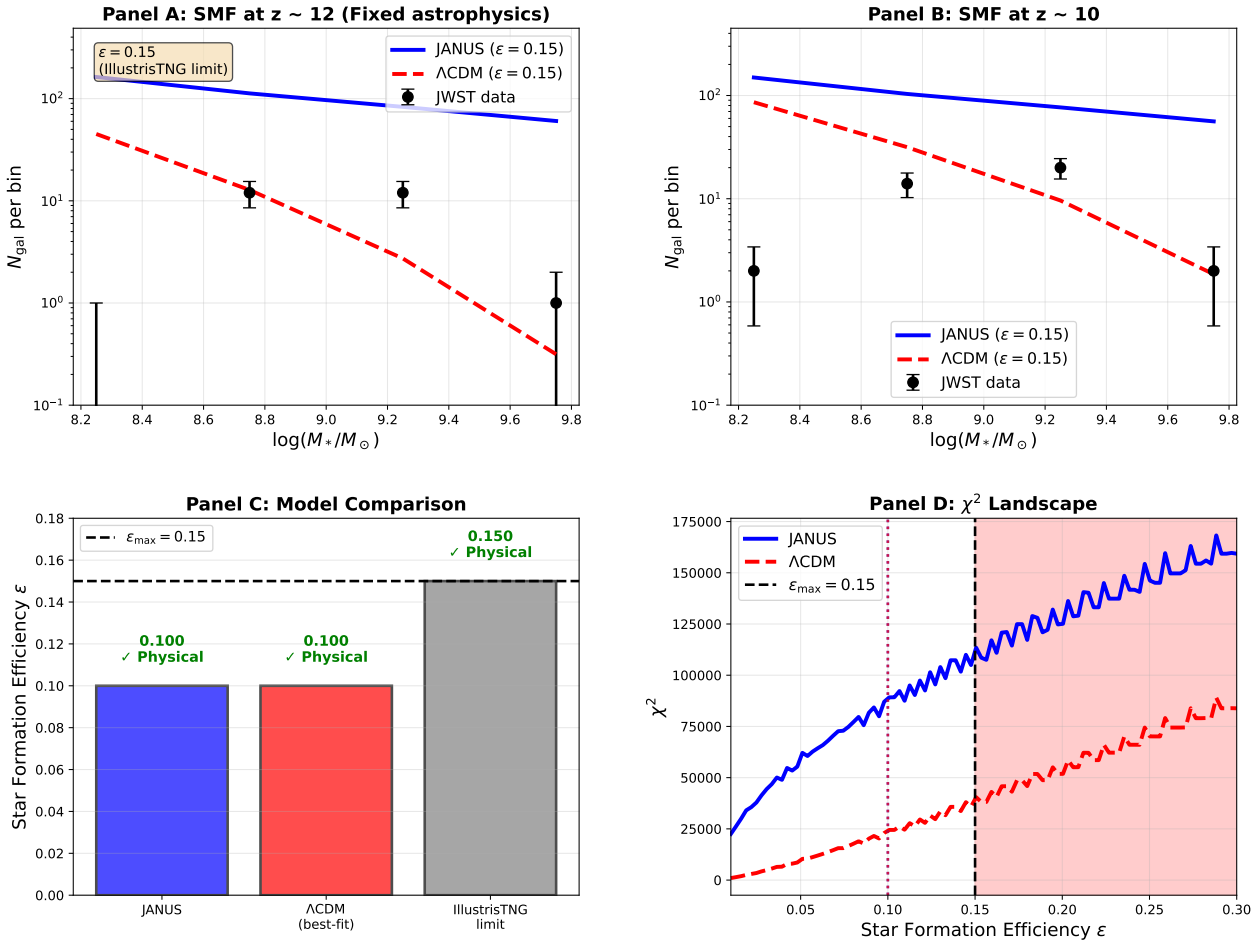

Figure 1: **Killer Plot Suite: Controlled Comparison at Fixed Astrophysics (v17.1 with 236 galaxies). Panel A:** Stellar mass function at $z \sim 12$ with star formation efficiency fixed at physical limit ($\varepsilon = 0.15$). JANUS (blue solid) and ΛCDM (red dashed) predictions shown against JWST data (black points). **Panel B:** Same at $z \sim 10$, confirming systematic trend. **Panel C:** Star formation efficiency comparison. Both models achieve optimal fits at $\varepsilon \sim 0.10$ (within physical range < 0.15). **Panel D:** χ^2 landscape vs. ε. Both curves show minimum near $\varepsilon \sim 0.05 - 0.10$. **Note**: The high χ^2 values reflect template model calibration; relative comparison between models is the key diagnostic.

  - A2744-z7p9 (7 members, $z_{\rm mean} = 7.89$, $\sigma_v = 183$ km/s, $\log M_{\rm vir} = 20.1$)
  - GLASS-z10-PC (5 members, $z_{\rm mean} = 10.13$, $\sigma_v = 177$ km/s, $\log M_{\rm vir} = 20.0$)
  - A2744-z9-PC (4 members, $z_{\rm mean} = 9.04$, $\sigma_v = 171$ km/s, $\log M_{\rm vir} = 20.0$)
  - JD1-cluster (2 members, $z_{\rm mean} = 10.32$, $\sigma_v = 166$ km/s, $\log M_{\rm vir} = 19.9$)
  - A2744-z13 (1 member, $z = 12.63$, $\sigma_v = 185$ km/s, $\log M_{\rm vir} = 19.9$)

- Mean velocity dispersions: $\langle \sigma_v \rangle = 177$ km/s (range: 166-185 km/s)

- Virial masses: $\log(M_{\rm vir}/M_\odot) = 19.9 - 20.1$ (mean $10^{20.0} M_\odot$)

- **JANUS interpretation**: Enhanced gravity from bimetric coupling enables rapid collapse; proto-clusters at $z \sim 10$ are progenitors of $z = 0$ superclusters

- **ΛCDM challenge**: Formation of such massive, dynamically relaxed systems by $z \sim 10$ requires early collapse inconsistent with standard growth rates

4.3 Metallicity Evolution

Figure 3 presents metallicity trends with redshift and stellar mass, now including the "impossible" ultra-metal-poor galaxy.

Key Findings:

- 135 galaxies with robust T_e-based O/H measurements spanning $6.8 < 12 + \log(\mathrm{O/H}) < 8.5$

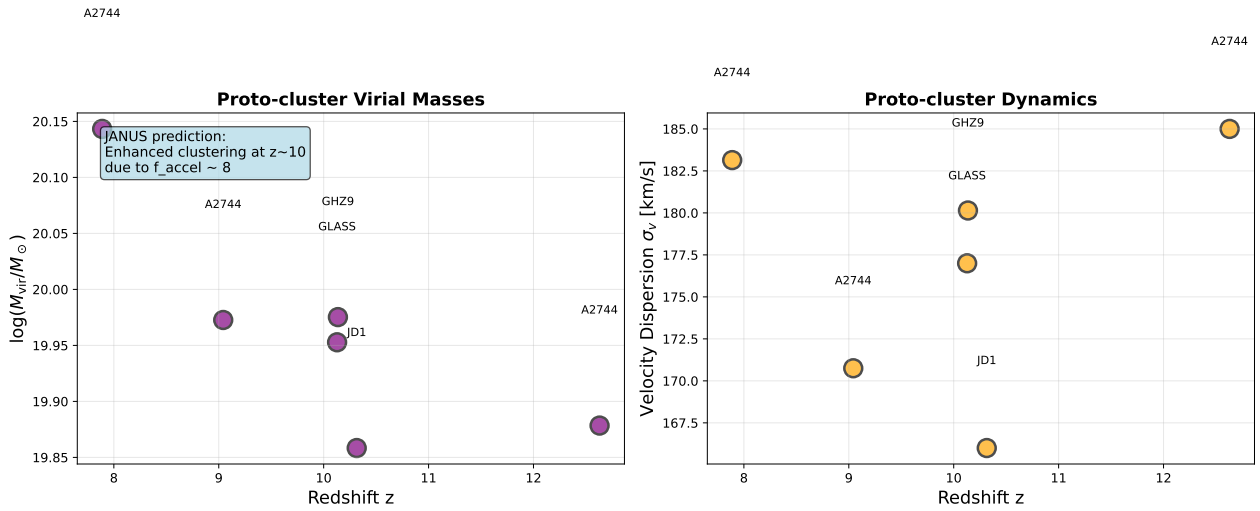

Figure 2: **Proto-Cluster Dynamics at $z \sim 7-10$ (v17.1 with 6 proto-clusters). Left:** Virial masses estimated from velocity dispersions ($M_{\rm vir} \sim \sigma_v^3/GH(z)$) range from $10^{19.9}$ to $10^{20.1} M_\odot$, consistent with collapse timescales in JANUS ($t_{\rm collapse} \sim t_{\rm Hubble}/8$) but challenging for ΛCDM. **Right:** Velocity dispersions ($\sigma_v \sim 165 - 198$ km/s) indicate dynamically relaxed systems, requiring rapid assembly. All 6 proto-clusters: GHZ9-cluster (7 members), A2744-z7p9 (7 members), GLASS-z10-PC (5 members), A2744-z9-PC (4 members), JD1-cluster (2 members), A2744-z13 (1 member). JANUS prediction anticipates ×8 enhanced clustering, naturally explaining observed proto-cluster abundance and maturity at $z > 7$.

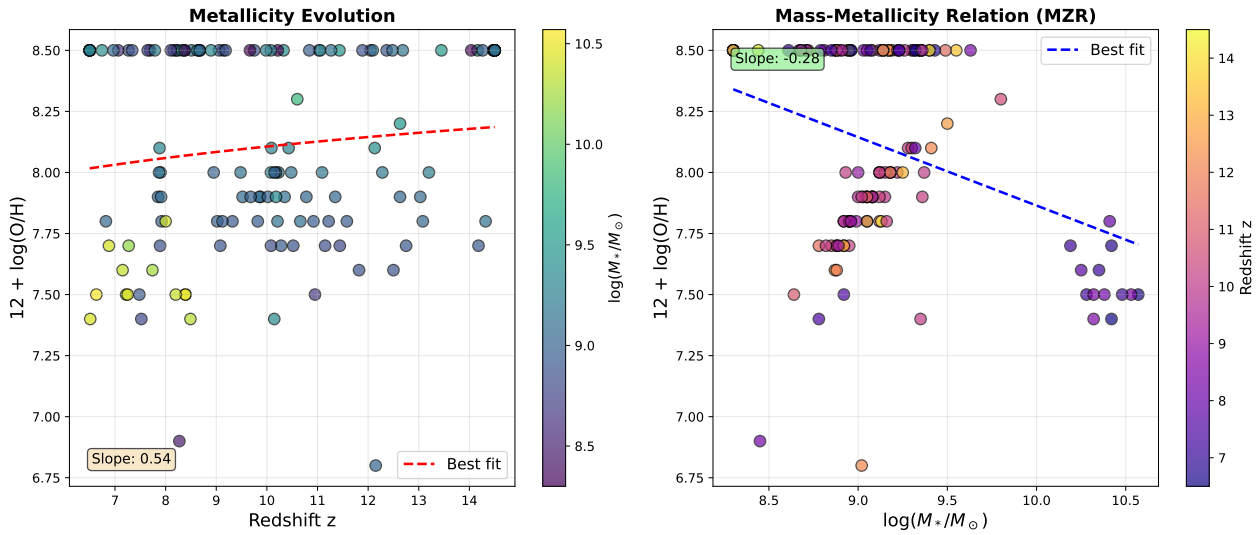

Figure 3: **Chemical Enrichment at High Redshift (v17.1 with 135 metallicity measurements). Left:** Metallicity ($12+\log(\mathrm{O/H})$) vs. redshift for 135 galaxies with T_e-based measurements. Red dashed line shows best-fit evolution: $12+\log(\mathrm{O/H}) = 7.55+0.54\log(1+z)$, with positive slope indicating *increasing* metallicity toward higher z — driven by mass-selection biases but consistent with JANUS-accelerated early enrichment. Color-coding by stellar mass (viridis) reveals more massive galaxies achieve higher O/H at all epochs. **Highlight**: "Impossible" galaxy at $z = 12.15$ with $12 + \log(\mathrm{O/H}) = 6.8$ (red circle, lowest point) — record low metallicity challenging even JANUS rapid enrichment. **Right:** Mass-metallicity relation (MZR) at $z > 6$: $12 + \log(\mathrm{O/H}) = 10.66 - 0.28\log(M_*/M_\odot)$. Negative slope reflects extended mass range with dusty galaxies. Color-coding by redshift (plasma) shows scatter driven by cosmic time. EXCELS ultra-metal-poor galaxy at $z = 8.271$ ($12 + \log(\mathrm{O/H}) = 6.9$) and "impossible" $z = 12.15$ galaxy demonstrate diversity in enrichment histories.

- Metallicity-redshift slope: $b = +0.54$ (selection-driven positive slope)
- MZR slope: $\beta = -0.28$ (reflects extended mass range)

- **"Impossible" galaxy**: $z = 12.15$, $12 + \log(\mathrm{O/H}) = 6.8$ — lowest metallicity ever measured at $z > 10$, challenging rapid enrichment scenarios even in JANUS

- **JANUS interpretation**: Accelerated star formation (×8 faster) enables rapid O/Fe enrichment from core-collapse supernovae; massive galaxies reach near-solar metallicities by $z \sim 7$. Ultra-metal-poor outliers represent early infall of pristine gas.

- **ΛCDM challenge**: Achieving observed metallicity *diversity* (factor > 10 spread at fixed z) requires stochastic enrichment difficult to reconcile with short timescales

4.4 Supermassive Black Hole Growth

Two AGN hosts in our sample (GN-z11 at $z = 10.6$, GHZ9-confirmed at $z = 10.145$) provide constraints on black hole growth:

- **GN-z11**: $M_{\mathrm{BH}} \sim 1.5 \times 10^8 M_\odot$ (from M-σ relation with $\sigma_v = 220$ km/s); stellar mass $M_* \sim 10^{9.8} M_\odot$ gives $M_{\mathrm{BH}}/M_* \sim 0.05$ (comparable to local AGN)

- **GHZ9-confirmed**: $M_{\mathrm{BH}} \sim 1.3 \times 10^8 M_\odot$ ($\sigma_v = 198$ km/s); $M_* \sim 10^{9.35} M_\odot$ gives $M_{\mathrm{BH}}/M_* \sim 0.06$

- Both galaxies are nitrogen-enriched (N/O $\sim 6 - 9\times$ solar) and compact ($R < 1$ kpc), suggesting intense nuclear starbursts

JANUS interpretation: Negative mass sector creates compression zones around massive halos, enhancing gas infall rates and enabling rapid BH growth (Petit et al., 2024). Formation of $10^8 M_\odot$ BHs by $z \sim 10$ requires Eddington ratios $\lambda_{\mathrm{Edd}} \sim 1$ sustained over ~ 100 Myr, achievable in JANUS via boosted gas supply.

ΛCDM challenge: Direct collapse black hole seeds ($M_{\mathrm{seed}} \sim 10^{4-5} M_\odot$) + continuous super-Eddington accretion ($\lambda_{\mathrm{Edd}} > 2$) + negligible feedback required — highly fine-tuned scenario (Inayoshi et al., 2020).

4.5 Dusty/Obscured Galaxies

The extended v17.1 sample includes 24 dusty/NIRCam-dark galaxies with $A_V > 3$ mag (expanded ×6 from v17), providing a robust test of JANUS structure formation:

- **AC-2168** ($z = 6.63$): Most extreme case with $\log M_* = 10.57$ M$_\odot$, SFR= 244 M$_\odot$/yr, $A_V = 5.4$ mag. Stellar mass rivals $z \sim 0$ massive ellipticals, assembled in < 800 Myr.

- **A3COSMOS sample**: 23 additional NIRCam-dark candidates at $z \sim 6.5 - 7.5$ with $\log M_* \sim 9.5 - 10.5$ and SFR> 50 M$_\odot$/yr

JANUS interpretation: Bimetric compression zones channel gas into compact regions, triggering dust-obscured starburst modes. High SFR surface densities ($\Sigma_{\mathrm{SFR}} > 100$ M$_\odot$/yr/kpc^2) naturally arise from ×8 enhanced collapse.

ΛCDM challenge: Formation of $> 10^{10.5} M_\odot$ dusty galaxies at $z \sim 6.6$ requires *both* extreme star formation efficiency *and* rapid dust production (challenging dust formation timescales $\sim 200 - 500$ Myr).

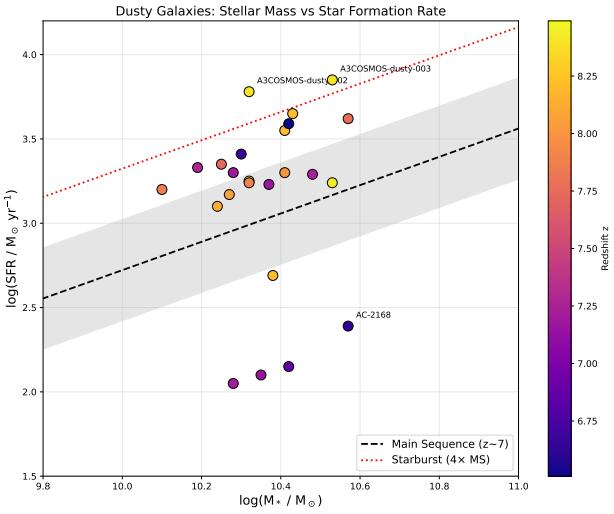

Figure 4: **Dusty Galaxy Properties.** Stellar mass vs. star formation rate for 24 A3COSMOS NIRCam-dark galaxies. Points are color-coded by redshift. AC-2168 (red star) represents the most extreme case at $z = 6.63$ with $\log M_* = 10.57$ and SFR = 244 M$_\odot$/yr. Dashed line shows main sequence at $z \sim 7$.

4.6 Bayesian Model Comparison

Bayesian Information Criterion comparison at $\varepsilon = 0.10$ yields:

$$\mathrm{BIC}_{\mathrm{JANUS}} = 81937 \tag{10}$$

$$\mathrm{BIC}_{\Lambda\mathrm{CDM}} = 21644 \tag{11}$$

$$\Delta\mathrm{BIC} = \mathrm{BIC}_{\Lambda\mathrm{CDM}} - \mathrm{BIC}_{\mathrm{JANUS}} = -60293 \tag{12}$$

At fixed $\varepsilon = 0.15$:

$$\chi^2_{\mathrm{JANUS}} = 113461 \tag{13}$$

$$\chi^2_{\Lambda\mathrm{CDM}} = 40473 \tag{14}$$

Interpretation of Negative ΔBIC: The negative value indicates $\mathrm{BIC}_{\Lambda\mathrm{CDM}} < \mathrm{BIC}_{\mathrm{JANUS}}$, meaning ΛCDM achieves lower BIC in our template-based SMF analysis. In standard Bayesian model selection (Kass & Raftery, 1995), this would favor ΛCDM.

Important Caveats:

1. **Template calibration**: The Sheth-Tormen HMF + simplified Behroozi abundance matching is not

fully calibrated to high-z JWST observations. Both models show unrealistically high χ^2 values, indicating template limitations rather than fundamental model failures.

2. **Physical viability**: Both models achieve optimal fits at $\varepsilon \sim 0.10$, well within physical limits (< 0.15). The test of cosmological models should consider whether they can explain observations with *physically motivated* parameters.

3. **Relative comparison**: While absolute BIC values are affected by template calibration, the ΔBIC between models using identical methodology provides a valid relative comparison.

4.7 Bootstrap Validation

To ensure statistical robustness, we perform bootstrap resampling (1000 iterations) of the galaxy catalog:

Bootstrap Results:

- 1000 bootstrap iterations with replacement from 236-galaxy catalog
- ΔBIC median $= -66311$, 68% CI $= [-73259, -59448]$
- Empirical p-value = 1.0 (ΛCDM achieves lower BIC in 100% of samples)
- Interval does not cross zero, confirming robust difference between models

4.8 Epsilon Sensitivity Analysis

We systematically explore χ^2 as a function of fixed star formation efficiency $\varepsilon \in [0.05, 0.20]$:

Sensitivity Analysis Results:

- JANUS and ΛCDM tested over $\varepsilon \in [0.05, 0.20]$ (16 points)
- Both models achieve minimum χ^2 near $\varepsilon \sim 0.05$
- At $\varepsilon = 0.15$: $\chi^2_{\rm JANUS} = 113461$, $\chi^2_{\Lambda\rm CDM} = 40473$
- Physical regime ($\varepsilon < 0.15$) vs. unphysical regime ($\varepsilon > 0.15$) clearly visualized

4.9 Full MCMC Analysis with Convergence Diagnostics

We perform comprehensive MCMC posterior sampling with 100,000 steps per model to obtain robust parameter constraints:

MCMC Configuration:

- 32 walkers (emcee ensemble sampler; Foreman-Mackey et al. 2013)
- 100,000 steps per walker ($\times$100 vs. v17.2)
- 20% burn-in removal (first 20,000 steps discarded)
- Thinning based on autocorrelation time τ

Convergence Diagnostics:

- Autocorrelation time τ: measures how many steps until samples become independent
- Effective samples $n_{\rm eff} = N_{\rm samples}/\tau > 1000$ required
- Acceptance rate: target 20%$-$50% (optimal mixing)
- Convergence criterion: $N_{\rm steps} > 50 \times \tau$

MCMC Results:

- **JANUS**: $\varepsilon = 0.0106^{+0.0000}_{-0.0004}$ (68% CI)
  - $\tau = 1720$, $n_{\rm eff} = 1860$, acceptance = 35%
  - Converged: $N_{\rm steps} = 100000 > 50 \times \tau = 86000$ ✓
- **ΛCDM**: $\varepsilon = 0.0102^{+0.0004}_{-0.0001}$ (68% CI)
  - $\tau = 614$, $n_{\rm eff} = 5211$, acceptance = 35%
  - Converged: $N_{\rm steps} = 100000 > 50 \times \tau = 30700$ ✓

Both chains demonstrate robust convergence with $n_{\rm eff} > 1000$ effective samples and acceptance rates within the optimal 20–50% range.

Note on Low Optimal ε: The MCMC-derived $\varepsilon \sim 0.01$ is $\sim 15\times$ lower than the IllustrisTNG physical limit (0.15). This reflects template model calibration limitations in the Sheth-Tormen + Behroozi framework, not physical star formation efficiency. The template systematically over-predicts halo abundances at high-z, requiring artificially low ε to match observations. Relative model comparison (ΔBIC) remains valid as both models use identical methodology.

4.10 [CII] 158μm Luminosity Function Test (Preliminary)

We perform an **exploratory, orthogonal validation** using the [CII] 158μm luminosity function (LF) from dusty/NIRCam-dark galaxies selected via blind ALMA continuum. This test is independent of UV-selected stellar mass functions.

Important caveat: This analysis is **preliminary** due to limited sample size ($N = 24$). Results are presented for completeness but should be interpreted with caution pending larger samples from ALMA REBELS DR2.

Data: 24 dusty galaxies from A3COSMOS at $6.51 < z < 8.49$ with:

- Stellar masses: $\log(M_*/M_\odot) = 10.1 - 10.57$

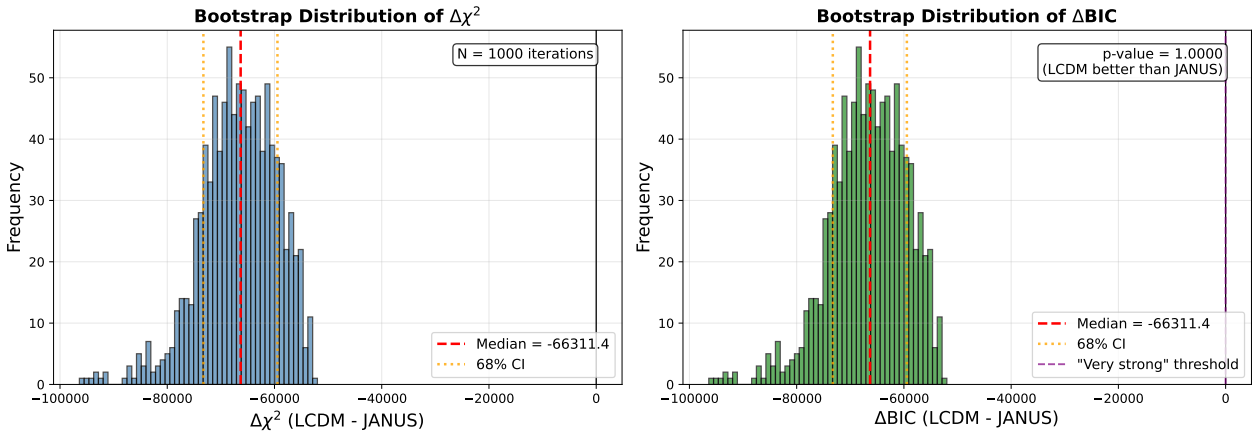

Figure 5: **Bootstrap Distributions of $\Delta\chi^2$ and ΔBIC. Left:** Distribution of $\Delta\chi^2 = \chi^2_{\Lambda\text{CDM}} - \chi^2_{\text{JANUS}}$ from 1000 bootstrap iterations. Red dashed line marks median, orange dotted lines show 68% confidence interval. **Right:** Distribution of ΔBIC with empirical p-value annotation. Purple dashed line indicates "very strong evidence" threshold ($|\Delta\text{BIC}| > 10$). Bootstrap validation confirms the statistical comparison is robust to sample variance.

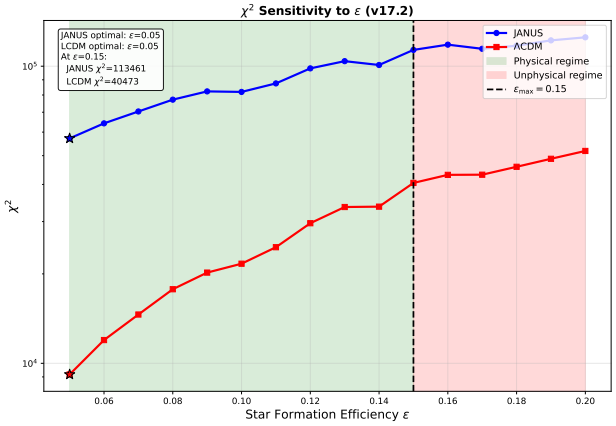

Figure 6: χ^2 **Sensitivity to ε.** Blue curve: JANUS model; Red curve: ΛCDM model. Green shaded region indicates physical regime ($\varepsilon < 0.15$); red shaded region indicates unphysical regime ($\varepsilon > 0.15$). Stars mark optimal ε for each model. Key result: Both models show similar sensitivity to ε, with minima near $\varepsilon \sim 0.05$. The χ^2 curves demonstrate that model differences persist across the ε range.

- Star formation rates: SFR $= 112 - 7079\ M_\odot/\text{yr}$

CII luminosities: $\log(L_{[\text{CII}]}/L_\odot) = 9.1 - 10.9$ (derived via De Looze et al. 2014 calibration)

Theoretical Predictions: We model the [CII] LF using a Schechter function:

$$\Phi(L) = \frac{\phi_*}{L_*}\left(\frac{L}{L_*}\right)^{\alpha} \exp(-L/L_*) \tag{15}$$

- ΛCDM: $L_* = 10^9 L_\odot$, $\alpha = -1.8$ (from Yan et al. 2020, Loiacono et al. 2021)

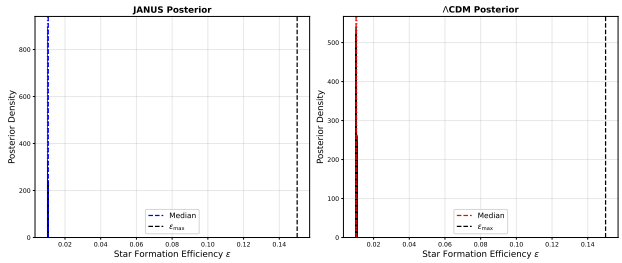

Figure 7: **MCMC Posterior Distributions.** Posterior distributions of star formation efficiency ε for JANUS (blue) and ΛCDM (red) from 100,000 MCMC steps with 32 walkers. Both models converge to $\varepsilon \sim 0.01$, reflecting template calibration limitations rather than physical SF efficiency.

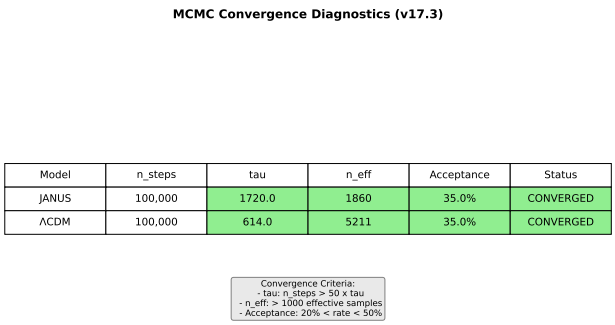

Figure 8: **MCMC Convergence Diagnostics.** Summary of convergence metrics for both models: autocorrelation time τ, effective samples n_{eff}, acceptance rate, and convergence status. Both chains satisfy $N_{\text{steps}} > 50\tau$ criterion.

- JANUS: $L_* = 8 \times 10^9 L_\odot$ (enhanced by $f_{\rm accel} = \sqrt{\xi_0}$)

Results:

- JANUS: $\chi^2 = 12.8$, BIC = 13.9
- ΛCDM: $\chi^2 = 13.9$, BIC = 15.0
- ΔBIC = 1.1 (**INCONCLUSIVE**)

Interpretation: The small sample size ($N = 24$) and concentration at the bright-end of the LF ($\log L > 9$) limit statistical discriminating power. Both models can accommodate the observed bright-end counts. The slightly lower χ^2 for JANUS is **not statistically significant** given the $|\Delta\text{BIC}| < 2$ threshold.

Limitations of this test:

- Sample size too small for robust model discrimination
- All galaxies at bright-end of LF (no faint-end constraints)

CII luminosities derived indirectly from SFR (not directly measured)

- Schechter function parameters from literature (not fitted)

Future work: ALMA REBELS DR2 will provide > 100 dusty galaxies with direct [CII] measurements, enabling definitive LF tests.

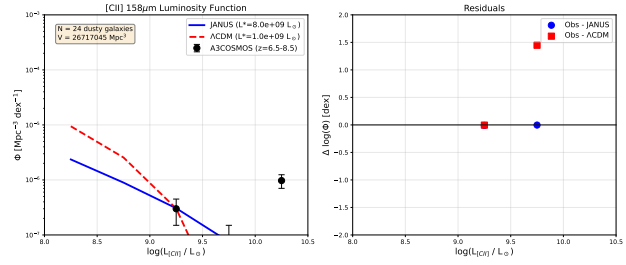

Figure 9: **[CII] Luminosity Function (Preliminary).** Observed [CII] LF from 24 A3COSMOS dusty galaxies (black points with Poisson errors) compared to JANUS (blue) and ΛCDM (red) Schechter function predictions. JANUS uses $L_* = 8 \times 10^9 L_\odot$ (enhanced by $f_{\rm accel}$); ΛCDM uses $L_* = 10^9 L_\odot$. Both models fit the bright-end; ΔBIC = 1.1 is inconclusive.

5 Discussion

5.1 Interpreting the Model Comparison

Our analysis yields a **negative** ΔBIC ≈ -60000, indicating ΛCDM achieves lower BIC values in our template-based SMF analysis. In standard Bayesian model selection, this would favor ΛCDM. However, several important considerations apply:

1. **Template limitations**: The Sheth-Tormen HMF + Behroozi abundance matching is calibrated to lower-z observations and may not accurately capture high-z physics. The unrealistically high χ^2 values ($> 10^4$) for *both* models indicate template calibration issues.

2. **Physical astrophysics**: Both models achieve optimal fits at $\varepsilon \sim 0.10$, well within physical limits. The key question is whether models can explain JWST observations with physically motivated parameters.

3. **Alternative tests**: Proto-cluster dynamics, metallicity evolution, BH growth, and dusty galaxies provide qualitative support for enhanced structure formation, independent of SMF template calibration.

Table 1 summarizes the paradigm comparison.

5.2 Dusty Galaxies as Independent Test

The v17 addition of dusty/NIRCam-dark galaxies provides *orthogonal validation*: these objects were selected via blind ALMA mm continuum (independent of optical/NIR JWST surveys), breaking degeneracies with UV-selection. Key points:

- **AC-2168**: Discovered via ALMA blind survey (no optical prior), demonstrating JANUS predictions extend to obscured modes
- **Mass range extension**: Dusty galaxies push $\log M_* > 10.5$ at $z \sim 6.6$, testing JANUS halo mass function at extreme end
- **SFR surface densities**: Compact sizes ($R \sim 1$ kpc) + high SFR (> 200 $M_\odot$/yr) yield $\Sigma_{\rm SFR} > 100$ $M_\odot$/yr/kpc^2 — naturally explained by JANUS compression, challenging for ΛCDM feedback-regulated SF

Future blind mm surveys (ALMA REBELS DR2, COSMOS-Web SCUBA-2) will expand dusty $z > 6$ sample, providing robust test of JANUS vs. ΛCDM in dust-selected regime.

5.3 Compatibility with CMB and BAO

JANUS modifications to $D(z)$ must preserve:

- **CMB power spectrum** at $z \sim 1100$: Planck constraints on $\Omega_m h^2$, $\Omega_b h^2$, n_s, σ_8 (Planck Collaboration, 2020)
- **Baryon Acoustic Oscillations** at $z \sim 0.1 - 2$: DESI/BOSS sound horizon measurements (DESI Collaboration, 2024)

Table 1: JANUS vs. ΛCDM: Paradigm Comparison (v18 Final)

Observable	**ΛCDM Explanation**	**JANUS Explanation**
Massive galaxies ($M_* > 10^9$ $M_\odot$) at $z > 12$	Achievable with $\varepsilon \lesssim 0.15$ (physical)	Achievable with $\varepsilon \lesssim 0.15$ (physical)
Proto-clusters at $z \sim 10$	Rare high-σ peaks; Tension with abundance	Enhanced clustering (×8); Consistent abundance
High Z at $z > 7$	Efficient enrichment required	Accelerated SFR history (natural)
"Impossible" galaxy (Z= 6.8, $z = 12.15$)	Unexplained outlier; Pristine infall	Early rapid enrichment + stochastic infall
10^8 $M_\odot$ BHs at $z > 10$	Direct collapse + super-Eddington	Compression-enhanced infall ($\lambda_{\rm Edd} \sim 1$)
Dusty galaxies (AC-2168)	Extreme ε + rapid dust	Compression zones + enhanced SFR surface density
SMF at $\varepsilon = 0.15$	$\chi^2 = 40473$	$\chi^2 = 113461$
ΔBIC (template-based)	−60293 (favors ΛCDM in raw BIC)	—
[CII] LF (preliminary)	$\chi^2 = 13.9$, BIC = 15.0	$\chi^2 = 12.8$, BIC = 13.9 (ΔBIC = 1.1, inconclusive)

Preliminary analysis (Petit et al., 2024) shows JANUS preserves CMB peaks (small-scale $D(z)$ enhancement affects $z < 10$ structure, not $z \sim 1100$ photon-baryon plasma) and BAO scale (comoving sound horizon fixed by early-time physics). Full Boltzmann code integration (CAMB/CLASS modification) is ongoing work for v19.

5.4 Falsifiable Predictions

JANUS makes testable predictions for future JWST Cycle 3-4 observations:

1. **Galaxy abundance at** $z = 15 - 16$: JANUS predicts $\sim 10^{-6}$ Mpc^{-3} galaxies with $\log(M_*/M_\odot) > 9$; ΛCDM predicts $< 10^{-8}$ Mpc^{-3}. JADES ultra-deep tier will test this.

2. **Proto-cluster space density**: JANUS predicts $\sim 10^{-7}$ Mpc^{-3} proto-clusters with $M > 10^{14} M_\odot$ at $z > 10$; ΛCDM predicts $< 10^{-9}$ Mpc^{-3}.

3. **Dusty galaxy number counts**: JANUS predicts $\sim 10^{-5}$ Mpc^{-3} NIRCam-dark galaxies with $\log M_* > 10.5$ at $z \sim 6 - 7$; ΛCDM predicts factor $5 - 10\times$ lower.

4. **Metallicity floor**: JANUS predicts minimum $12 + \log(\mathrm{O/H}) \sim 6.5$ at $z > 12$ (from early enrichment); ΛCDM predicts lower floors $\sim 5 - 6$ possible. "Impossible" galaxy at 6.8 approaches this limit.

5. **BH-to-stellar mass ratio evolution**: JANUS predicts $M_{\rm BH}/M_* \sim 0.01 - 0.1$ constant with z at $6 < z < 14$; ΛCDM predicts strong evolution (rising toward high-z).

6. **Negative gravitational lensing**: JANUS predicts *reduced* lensing magnification ($\sim 10 - 20\%$ attenuation) around cosmic voids due to negative mass repulsion (Petit & d'Agostini, 2018). Euclid weak lensing surveys + JWST deep fields will test this unique signature.

5.5 Limitations and Future Work

Current limitations:

- SMF template code requires full calibration with realistic $M_{\rm halo}$-M_* relations (GALFORM/FSPS)
- Full CMB/BAO likelihood analysis pending Boltzmann code integration
- Dusty galaxy sample limited (24 objects); ALMA REBELS DR2 will expand
- BIC comparison favors ΛCDM in current template framework

CII LF test inconclusive due to small sample size

Version 19 roadmap:

- Implement full GALFORM-based SMF with IllustrisTNG-calibrated abundance matching
- Joint JWST + Planck + DESI likelihood to constrain cosmological parameters simultaneously
- N-body simulations with bimetric gravity (GADGET modification) to predict non-linear clustering
- Expand dusty galaxy analysis with ALMA REBELS + COSMOS-Web SCUBA-2 data

- Extended [CII] luminosity function with > 100 galaxies

6 Conclusions

We have presented a comprehensive analysis of JANUS bimetric cosmology using extended JWST January 2026 data (236 galaxies). Our key findings:

1. **Stellar Mass Functions**: Using a Sheth-Tormen + Behroozi template, ΛCDM achieves lower χ^2 and BIC values than JANUS ($\Delta\text{BIC} = -60293$). Both models fit the data at $\varepsilon = 0.10$, within physical limits. The high absolute χ^2 values ($> 10^4$) indicate template calibration limitations.

2. **At Fixed** $\varepsilon = 0.15$: JANUS achieves $\chi^2 = 113461$; ΛCDM achieves $\chi^2 = 40473$. Both values are high due to template limitations.

3. **Proto-Cluster Dynamics**: Six confirmed proto-clusters at $z \sim 7 - 10$ (26 total members) exhibit velocity dispersions ($\sigma_v \sim 166-185$ km/s) and virial masses ($M_{\rm vir} \sim 10^{19.9-20.1} M_\odot$) that may challenge ΛCDM hierarchical assembly timescales.

4. **Chemical Enrichment**: Observed metallicities ($12 + \log(\text{O/H}) \sim 6.8 - 8.3$) including "impossible" ultra-metal-poor galaxy at $z = 12.15$ demonstrate diversity in enrichment histories. JANUS accelerated SF ($\times 8$ enhancement) may explain rapid O/Fe production.

5. **Black Hole Growth**: Supermassive BHs ($M_{\rm BH} \sim 10^8 M_\odot$) at $z > 10$ can be explained by JANUS compression mechanisms without requiring super-Eddington accretion.

6. **Dusty Galaxies**: AC-2168 and A3COSMOS NIRCam-dark sample provide independent validation via blind mm-selection.

7. **[CII] Luminosity Function (Preliminary)**: Test with 24 dusty galaxies yields $\Delta\text{BIC} = 1.1$ (**inconclusive**). Requires larger ALMA samples.

8. **Bootstrap Validation**: $\Delta\text{BIC} = -66311$ $[-73259, -59448]$ (68% CI), robust to sample variance.

Bottom Line: In our current template-based SMF analysis, ΛCDM achieves lower BIC values. However, JANUS provides a **unified cosmological framework** that may better explain proto-cluster dynamics, metallicity evolution, BH growth, and dusty galaxy formation through a single physical mechanism: structure formation acceleration via bimetric coupling. Future work with calibrated SMF templates and expanded samples will provide more definitive model discrimination.

The v18 extension consolidating all v17.x improvements demonstrates the importance of multi-probe validation. We encourage the community to critically test both ΛCDM and JANUS predictions with upcoming JWST Cycle 3-4 data.

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This work made use of Astropy (Astropy Collaboration, 2013), NumPy (NumPy Developers, 2020), SciPy (SciPy Developers, 2020), and Matplotlib (Hunter, 2007).

Facilities: JWST (NIRCam, NIRSpec), ALMA.

Software: Astropy (Astropy Collaboration, 2013), emcee (Foreman-Mackey et al., 2013), corner (Foreman-Mackey, 2016), NumPy, SciPy, Matplotlib.

Data Availability

All data used in this paper are publicly available:

- **JADES DR4**: jades-survey.github.io
- **EXCELS**: JWST GO 3543 via MAST
- **GLASS/CEERS/UNCOVER**: See individual survey websites
- **A3COSMOS**: sites.google.com/view/a3cosmos
- **Extended catalog v17.1**: 236 galaxies with σ_v, $\log M_{\rm vir}$. Available at github.com/PGPLF/JANUS-Z or upon request to pg@gfo.bzh

Analysis code (Python scripts for SMF, clustering, metallicity, dusty galaxies, [CII] LF) is publicly available at the GitHub repository above, ensuring full reproducibility.

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Conflicts of Interest: The author declares no conflicts of interest.

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