# Problem Set 3

*Pieter Quinton*

*2/26/2020*

**Question 1**

*a)*

If the returner knows the server will serve to the forehand, their optimal strategy is to cheat to the forehand because their win probability is 0.5 which is higher than 0.2 for cheating backhand.

If the server knows the returner will cheat to the forehand, their optimal strategy is to serve to the backhand because their win probability is 0.7 which is higher than 0.4 for serving to the forehand.

This does not represent a pure strategy Nash equilibrium because there is no one strategy that is optimal for both players.

*b)*

Payoff for the returner if they cheat forehand: $(x) * (0.5) + (1 - x) * (0.3)$

Payoff for the returner if they cheat backhand: $(x) * (0.2) + (1 - x) * (0.6)$

Indifference point:

$(x) * (0.5) + (1 - x) * (0.3) = (x) * (0.2) + (1 - x) * (0.6)$

$0.2x + 0.3 = 0.6 - 0.4x$

$0.6x = 0.3$

$x = 0.5 = \hat{x}$

*c)*

Payoff for the server if they serve forehand: $(y) * (0.5) + (1 - y) * (0.8)$

Payoff for the server if they serve backhand: $(y) * (0.7) + (1 - y) * (0.4)$

Indifference point:

$(y) * (0.5) + (1 - y) * (0.8) = (y) * (0.7) + (1 - y) * (0.4)$

$1.3 - 0.3y = 0.3y + 1.1$

$0.6y = 0.2$

$y = 0.05 = \hat{y}$

*d)*

**Question 2**

*i)*

Expected goals from 20 shots: $20 * 0.057 = 1.14$ goals

*ii)*

Expected points from taking the one goal: 1 (from the overtime average)

Expected points from taking the shots: This can be calculated by taking the probability of scoring more than one goal times two points for a win plus the probability of scoring exactly one goal times one point for the overtime point average plus the probability of scoring no goals times zero points for a loss. To calculate this, I simulated the twenty shots ten-thousand times with a scoring probability of 5.7% and found the proportion of times the team won, lost, or went to overtime based on the number of goals scored.

Based off the simulation, the probability of a loss is 0.307, the probability of going to overtime is 0.3731 and the probability of a win is 0.3199.

Thus, the expected points per game from taking the shots is:

$0.307 * 0 + 0.3731 * 1 + 0.3199 * 2 = 1.0129$

Because $1.0129 > 1$, the team should take the 20 shots. It is worth noting, however, that if the 10,000 simulations are run repeatedly, there are simulations in which the expected points are below 1. However, on the whole, the value is usually greater than 1.

*iii)*

The methodology is largely the same. However, the expected points for taking the goal rises to 1.5 because the average points for going to overtime is $(2+1)/2$ which is 1.5.

The simulation for determining the expected points for taking the shots remains the same. The only thing that changes is the point values for going to overtime. A win still gets 2 points, going to overtime gets 1.5, and a loss still gets 0.

The new value of points per game from taking the shots is:

$0.307 * 0 + 0.3731 * 1.5 + 0.3199 * 2 = 1.19945$

Because $1.19945 < 1.5$, the team should take the guaranteed goal and go to overtime.

*iv)*

The expected points for taking the goal remains at 1.5 but the value for a regulation win has gone up to 3 so the expected points for taking the shots needs to be re-calculated.

Again using the 10,000 replication of 20 shots simulation data, the only change is the point value for a regulation win (or scoring more than 1 goal in regulation).

$0.307 * 0 + 0.3731 * 1.5 + 0.307, 0.3731, 0.3199 * 3 = 1.51935$

Because $1.51935 > 1.5$, the team should opt for the 20 shots over the guaranteed goal.

*v)*

The expected points for taking the goal returns to 1.

The simulation needs to be re-done for the new goal scoring percentage.

Based off the new simulation with the percentage at 5.5, the probability of a loss is 0.3236, the probability of going to overtime is 0.3826 and the probability of a win is 0.2938.

The new value of points per game from taking the shots is:

$0.307 * 0 + 0.3731 * 1.5 + 0.3199 * 2 = 1.19945$

Because $1.0129 < 1$, the team should take the guaranteed goal and go to overtime.