Long-term forecasting of heat and water consumption

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Summary

The goal of the thesis is to use a method, developed for electricity forecasts, to forecast heat and water consumption in one-family households. The method uses RLS with either exponential or variable forgetting. To account for weekly variation, Fourier series are added to the model. The building dynamics are modeled using a low-pass filter on the temperature difference between the indoor and outdoor temperatures. Data from Helsingør, Denmark and Catalonia, Spain has been made available to investigate relations between heat consumption and weather, as well as producing the forecasts. It is found that heat consumption depends mostly on the air temperature and partly sun radiation. Water consumption does not follow a clear pattern and can be difficult to predict. Forecasts are based one 1 year of training data and the forecast horizon is 92 days. Heat forecasts are good overall and within a reasonable margin of error. Water forecasts are less accurate on a daily basis but reasonable on a monthly basis. It is investigated whether optimizing the smoothing factor in the low-pass filter can improve the forecasts. The results show either near identical forecasts compared to regular forecasts, or very poor forecasts. For the Helsingør-data, the summer period is removed from the training data to improve heat forecasts in the colder months. The results show very similar forecasts compared to the regular forecasts. Also for the Helsingør-data, weather data from the cold year of 2012 is given as input to produce heat forecasts for colder winters. The resulting forecasts expect higher heat consumption compared to 2017. Finally, it is concluded that the RLS method is useful for heat and water forecasts. For further work, it is proposed to find a method to determine when a household turns on the heat in the beginning of winter.

Preface

This thesis was prepared at DTU Compute in partial fulfillment of the requirements for acquiring a M.Sc. in Engineering. The work was carried out from September 4th 2017 to February 23rd 2018 under supervision of Peder Bacher and Jan Kloppenborg Møller. The thesis represents a workload of 30 ECTS points.

I would like to thank Peder Bacher (DTU) for setting up the project, weekly meetings and helpful feedback. Also Jan Kloppenborg Møller (DTU) was very helpful towards the end of the project. I would also like to thank Claus Brandt Jensen (FH) and Liv Kure Molin (FH) for providing the heat and water data, as well as Anders Spur Hansen (SEAS-NVE) for providing weather data for the project. I would like to thank Jon Anders Reichert Liisberg (DTU/SEAS-NVE) for providing the code for the method and helping me understand it. Finally, I would like to thank Jordi Cipriano (CIMNE-UPC) for providing the BECA data to make use of the time between data transfers.

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List of symbols

Symbol	Description
m	Number of variables
n	Number of observations
I	Identity matrix
y_t	Response variable, observation at time t, scalar
x_t	Explanatory variables, observation at time t, $m \times 1$
$\widehat{\theta}_t$	Model parameters at time t, $m \times 1$
ε	Model error, assumes $\varepsilon_{t+k\|t} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ and i.i.d.
e	Residuals/realizations of model error ε
λ	Forgetting factor, $\lambda \in [0, 1]$

Abbreviations

RLS	Recursive Least Squares
OLS	Ordinary Least Squares
RMSE	Root Mean Square Error
i.i.d.	independent and identically distributed
BIC	Bayesian information criterion
AIC	Akaike information criterion
Exp.	Exponential forgetting
Var.	Variable forgetting
UTC	Coordinated Universal Time
ACF	Autocorrelation Function
PACF	Partial Autocorrelation Function
AR	Autoregressiv

Contents

Chapter 1

Introduction

1.1 Motivation

Smart meters are becoming more common in households in Denmark, and having software collect data on electricity, heat and water consumption opens the door for new opportunities. SEAS-NVE has been working on forecasting electricity consumption and, as a result, implemented a method in the app, "Watts". The next step is collaboration with providers of district heating and water to expand the functionality of the app. Forsyning Helsingør(FH) is the first company that have agreed to deliver heat and water consumption data for developing the forecasting method.

1.1.1 Watts - Energiassistent

Watts is an app for iOS and Android devices developed by SEAS-NVE to provide consumers with an overview of their electricity consumption as well as forecasts[1]. As of today, the app can only do forecasts of electricity consumption.

1.2 Aim of the thesis

The overall aim of the thesis is: using a method developed for electricity forecasts, to forecast heat and water consumption in one-family households. More specifically, the thesis aims to answer the following questions:

- Is heat consumption affected by weather?
- Can the method developed by SEAS-NVE be used to forecast heat and water consumption?
- Can the forecasts be improved by optimizing the smoothing factor in the low-pass filter?
- Can the forecasts be improved by keeping two separate models for summer and winter periods?
- Given "cold weather"-input, can the models predict heat consumption in colder winters?

1.3 Thesis outline

The thesis contains analyses of two data sets. To help the reader, the chapters are organized such that the data sets are kept separate.

Methods Presents the model structure and RLS for estimating the parameters. Low-pass filtering of temperature and finding a suitable Fourier series. Finally, the model selection process is described.

Data: Forsyning Helsingør(FH) Description and initial analysis of heat and water data from FH. Training data is used to investigate relations between heat consumption and weather.

Data: BECA Description and initial analysis of data from Catalonia, Spain. Training data is used to investigate relations between heat consumption and weather.

Results: Forsyning Helsingør(FH) Forecasts of heat and water consumption using data from FH, including optimization of smoothing factor, separate models for winter period and "cold weather" forecasts.

Results: BECA Forecasts of heat and water consumption using data from Catalonia, Spain, including optimization of smoothing factor.

Discussion and conclusion The data and forecasts are discussed, as well as the performance of the methods. Finally, the conclusions of the thesis are presented.

Chapter 2

Methods

This chapter describes which methods are used to model the data.

2.1 Model structure

The chosen model structure is the k-step linear regression model of the form,

$$y_{t+k|t} = x_{t+k|t}^T \widehat{\theta} + e_{t+k|t}, \tag{2.1}$$

where $y_{t+k|t}$ is the response variable, $x_{t+k|t}$ is the explanatory variables, $\widehat{\theta}$ is the parameter vector and $e_{t+k|t}$ is the realizations of the i.i.d. error, $\varepsilon_{t+k|t} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$.

The explanatory variables, $x_{t+k|t}$, can be exogenous, e.g. weather data, or Fourier series.

2.2 Parameter estimation

Parameter estimation is usually done using ordinary least squares (OLS). However, by using a recursive method the model parameters can be updated when new data is available without having to fit all the data again. This is useful for updating the forecasts in Watts.

2.2.1 Recursive Least Squares (RLS)

RLS is an adaptive filter algorithm which computes the parameters, $\widehat{\theta}_t$, that minimizes the cost function, $S_t(\theta)$. This can be written,

$$\widehat{\theta}_t = \arg\min_{\theta} S_t(\theta), \tag{2.2}$$

where the cost function, $S_t(\theta)$, is defined as,

$$S_t(\theta) = \sum_{s=1}^{t} \left(y_s - x_s^T \theta \right)^2 . \tag{2.3}$$

Thus, $\widehat{\theta}_t$ minimizes the sum of squared errors.

The recursive update of the parameters is carried out for each horizon k, as defined in [2] as,

$$R_t = R_{t-1} + x_{t|t-k} x_{t|t-k}^T, \tag{2.4a}$$

$$\widehat{\theta}_t = \widehat{\theta}_{t-1} + R_t^{-1} x_{t|t-k} \left(y_t - x_{t|t-k}^T \widehat{\theta}_{t-1} \right) \tag{2.4b}$$

Initial values for $\widehat{\theta}_0$ are usually zeros and R_0 is chosen to be relatively large.
The k-step prediction is defined as

$$\widehat{y}_{t+k|t} = x_{t+k|t}^T \widehat{\theta}_t \tag{2.5}$$

2.2.2 RLS with exponential forgetting

Until now, the parameters, $\widehat{\theta}_t$, have been assumed constant in time. However, it may prove useful to weight the observations differently when estimating the parameters. The cost function for the weighted least squares estimate in Equation (2.2) is defined as,

$$S_t(\theta_t) = \sum_{s=1}^{t} \lambda^{t-s} \left(y_s - x_s^T \theta_t \right)^2, \tag{2.6}$$

where λ is the forgetting factor and, for now, assumed constant.
The recursive update is defined similarly as in Equation (2.4),

$$R_t = \lambda R_{t-1} + x_{t|t-k} x_{t|t-k}^T, \tag{2.7a}$$

$$\widehat{\theta}_t = \widehat{\theta}_{t-1} + R_t^{-1} x_{t|t-k} \left(y_t - x_{t|t-k}^T \widehat{\theta}_{t-1} \right), \tag{2.7b}$$

with the k-step prediction being,

$$\widehat{y}_{t+k|t} = x_{t+k|t}^T \widehat{\theta}_t \tag{2.8}$$

2.2.3 RLS with variable forgetting

Introducing variable forgetting with covariance resetting[3]. The RLS algorithm in Equation (2.4) has to be restructured to

$$\widehat{\theta}_t = \widehat{\theta}_{t-1} + K_t \left(y_t - x_{t-k}^T \widehat{\theta}_{t-1} \right), \tag{2.9a}$$

$$K_t = \frac{P_{t-1} x_t}{\lambda + x^T P_{t-1} x_t}, \tag{2.9b}$$

$$P_t = \frac{1}{\lambda} \left(P_{t-1} - K_t x_t^T P_{t-1} \right), \tag{2.9c}$$

where K_t is the gain vector and $P_t = R_t^{-1}$ is the inverse covariance. However, the implementation is done by replacing Equation (2.9b) and Equation (2.9c) with

$$K_t = \frac{P_{t-1}x_t}{1 + x_t^T P_{t-1} x_t}, \tag{2.10a}$$

$$\lambda_t = 1 + \frac{\left(y_t - x_{t-k}^T \widehat{\theta}_{t-1}\right)^2}{\sigma\left(1 + x_t^T P_{t-1} x_t\right)}, \tag{2.10b}$$

$$W_t = P_{t-1} - K_t x_t^T P_{t-1}, \tag{2.10c}$$

if $\quad \textbf{trace}\left(\frac{W_t}{\lambda_t}\right) \leq C \quad$ then

$$\bar{P}_t = \frac{W_t}{\lambda_t}, \tag{2.10d}$$

else

$$\bar{P}_t = W_t. \tag{2.10e}$$

C is chosen as an upper bound on the covariance matrix to avoid covariance windup. σ is defined such that $\frac{\sigma}{\sigma_w} = 1000$ where σ_w is chosen to be the standard deviation of the residuals,

$$\sigma_w(t) = \sqrt{\frac{\sum_{i=k}^{t}(y_i - \widehat{y}_i)^2}{(n-k)(1 + x_t^T P_{t-1} X_t)}}. \tag{2.11}$$

To prevent the covariance matrix from decaying to zero, a lower bound is introduced by adding a scalar to the diagonal as described below.

If $\textbf{trace}(\bar{P}_t) < c_{min}$ then

$$P_t = \bar{P}_t + Q, \tag{2.12}$$

where $Q = cI$. c is a suitable scalar and I is the identity matrix.

If $\textbf{trace}(\bar{P}_t) \geq c_{min}$ then

$$P_t = \bar{P}_t. \tag{2.13}$$

The lower bound depends on the choice of c_{min} and c.

2.3 Filtering of temperature

Most households prefer a comfortable room temperature that is kept constant - usually only shifting a few degrees. Thus, the assumption that the indoor temperature is at a constant 22 °C, is introduced.

The relation between indoor and outdoor temperature has been investigated in [4] and [5], where building dynamics have been described by using an RC low-pass filter of the form,

$$y_t = a_{T_a} y_{t-1} + (1 - a_{T_a}) x_t, \tag{2.14}$$

where y_t is the filtered temperature difference at time t, x_t is the measured temperature difference at time t and a_{T_a} is a smoothing factor. This is also known as exponential smoothing. The smoothing factor is for hourly observations.
As the model will use daily values of the temperature difference, the smoothing factor needs to be calculated for a sample period of $\Delta_{24} = 24$ hours.

The time constant, τ, is constant for all sampling periods and is defined as

$$\tau = \Delta_1 \frac{a_{T_a}}{1 - a_{T_a}}, \tag{2.15}$$

where Δ_1 denotes a sampling period of 1 hour. Equation (2.15) can be rearranged to

$$a_{T_a} = \frac{\tau}{\tau + \Delta_1}. \tag{2.16}$$

The smoothing factor for daily observations can be defined in a similar way,

$$a_{T_{a,24}} = \frac{\tau}{\tau + \Delta_{24}}. \tag{2.17}$$

Inserting Equation (2.15) into Equation (2.17) and multiplying by 1 yields,

$$a_{T_{a,24}} = \frac{\Delta_1 \frac{a_{T_a}}{1 - a_{T_a}}}{\Delta_1 \frac{a_{T_a}}{1 - a_{T_a}} + \Delta_{24}} \frac{\frac{1 - a_{T_a}}{a_{T_a}}}{\frac{1 - a_{T_a}}{a_{T_a}}}, \tag{2.18}$$

which reduces to

$$a_{T_a,24} = \frac{\Delta_1}{\frac{\Delta_{24}}{a_{T_a}} - \Delta_{24} + \Delta_1}. \tag{2.19}$$

Optimizing the smoothing factor

One can settle with using a fixed value of $a_{T_a} = 0.95$ for all houses/apartments. However, it is worth investigating if another value can improve the performance of the model. A good choice of smoothing factor is the one that minimizes the root mean squared error (RMSE), defined as,

$$RMSE = \sqrt{\frac{\sum_{t=k}^{n} (y_t - \widehat{y}_t)^2}{n-k}}, \tag{2.20}$$

where n is the number of observations and k is the horizon. It has been decided to minimize the 1-step prediction error, i.e. $k = 1$ in the RMSE. The optimization problem will then be expressed as,

$$a^*_{T_a} = \arg\min_{a_{T_a}} RMSE(a_{T_a}), \text{ for } a_{T_a} \in [0.5, 1]. \tag{2.21}$$

The RMSE will serve as the objective function and the search interval for a_{T_a} is $[0.5, 0.999]$ with initial value $a_{T_a} = 0.95$. It should be noted that it is the ϕ for hourly time steps that is being optimized. This value is than converted to daily time steps.

2.4 Harmonics

Whether it is water, heat or electricity, it is very likely that the daily consumption changes during the week - especially if comparing workdays and weekends. To account for this weekly variation, Fourier series are introduced. This will add more variables to the model such that the parameter vector, $\widehat{\theta}_t$, consists of two parts, $\widehat{\theta}^p_t = \left(\widehat{\alpha}_t, \widehat{\beta}^p_t\right)$, where $\widehat{\alpha}_t$ represents parameters associated with the exogenous variables and $\widehat{\beta}_t^p$ are the p pairs of Fourier coefficients. The Fourier series are

$$\sum_{i=1}^{p} \beta_{i_{sin}} \sin\left(\frac{2\pi t}{r}\right) + \beta_{i_{cos}} \cos\left(\frac{2\pi t}{r}\right), \tag{2.22}$$

where t is time, r is the period and p is the number of sine and cosine pairs. For $p = 0$, the Fourier series is not included in the model and $\widehat{\theta}_t^0 = \widehat{\alpha}_t$.

The model will at most use 3 sine/cosine pairs, i.e. 6 parameters to describe the weekly variation. Using more than 3 pairs, e.g. 4 pairs, would result in more than 7 parameters that describe the variation in periods of 7 days and that would be overfitting.

2.5 Model selection

This section describes the model selection process.

2.5.1 Overview of algorithms

To get an overview of the method(s), the process has been expressed as an algorithm. Algorithm 1 uses RLS with exponential forgetting and Algorithm 2 uses RLS with variables forgetting.

Algorithm 1 Recursive least squares estimation with exponential forgetting

Require: Data set (y_t, x_t)
1: **Initialization** Forgetting factor, $\lambda = 0.998$. Compute temperature difference, $T_{\text{diff}} = 22 - T_{\text{air}}$
2: Define forecast horizon, N_h.
3: **for** k:=1 **to** N_h **do**
4: Prepare data: Compute low-pass filtering of T_{diff}, denoted T_{lpf}. Compare BIC values of linear models using either T_{diff} or T_{lpf}.
5: Determine significant climate variables using forward selection
6: Compute harmonics
7: Determine optimal number of harmonics using RMSE
8: Compute model parameters, $\widehat{\theta}_{k,t}$, using fixed λ
9: **end for**

Algorithm 2 Recursive least squares estimation with variable forgetting

Require: Data set (y_t, x_t)
1: **Initialization** Forgetting factor, $\lambda = 0.998$. Compute temperature difference, $T_{\text{diff}} = 22 - T_{\text{air}}$
2: Define forecast horizon, N_h.
3: **for** k:=1 **to** N_h **do**
4: Prepare data: Compute low-pass filtering of T_{diff}, denoted T_{lpf}. Compare BIC values of linear models using either T_{diff} or T_{lpf}.
5: Determine significant climate variables using forward selection
6: Compute harmonics
7: Determine optimal number of harmonics using RMSE
8: Compute model parameters, $\widehat{\theta}_{k,t}$, using variable λ
9: **end for**

2.5.2 Model selection process

Given training data in the form (y_t, x_t), where y_t is the response, i.e. water or heat consumption, and x_t is the explanatory variables. The two methods only have minor differences, as shown in Algorithm 1 and Algorithm 2, thus the model selection process is quite similar for both algorithms.

Initially, the forgetting factor is set, $\lambda = 0.998$, and the temperature difference between the measured air temperature and the assumed in-door temperature is defined as, $T_{\text{diff}} = 22 - T_{\text{air}}$. As mentioned in section 2.3, low-pass filtering of the temperature is used to account for building dynamics. The low-pass filtered temperature difference is denoted T_{lpf}. Two simple linear models are fitted with T_{diff} and T_{lpf}, separately, and the BIC values of both models are compared. By fitting the temperature variables separately colinearity is avoided.

When the appropriate temperature variable has been chosen, the remaining explanatory variables, e.g. sun radiation, are fitted to a linear regression and this is then reduced using forward selection to determine which weather variables should be included in the final model.

The harmonics are computed as per section 2.4. A model is fitted for every p harmonics, such that the third model includes 3 sine/cosine pairs. The model fitting routine used in this is RLS with the appropriate type of forgetting. The RMSE is evaluated for each model to determine the optimal p sine/cosine pairs to include in the final model.

The final model consists of the appropriate weather variables and number of sine/cosine pairs.

$$y_t = \mu + \alpha x_t + \sum_{i=1}^{p} \left(\beta_{i_{sin}} \sin\left(\frac{2\pi t}{7}\right) + \beta_{i_{cos}} \cos\left(\frac{2\pi t}{7}\right)\right) + \varepsilon_t \tag{2.23}$$

The parameter vector, $\widehat{\theta}$, will contain the following parameters,

$$\widehat{\theta} = [\mu, \alpha_1, ..., \alpha_n, \beta_{1_{sin}}, \beta_{1_{cos}}, ..., \beta_{p_{sin}}, \beta_{p_{cos}}]. \tag{2.24}$$

As the model can at most include $p = 3$ sine and cosine pairs, the maximum number of parameters in the model is $m + p + 1 = m + 4$.

The final step is to estimate the parameters in the final model using RLS with either exponential or variable forgetting.

CHAPTER 3

Data: Forsyning Helsingør(FH)

Figure 3.1: Area in Helsingør similar to the area of which the data was obtained.

3.1 Description

The data from Forsyning Helsingør A/S includes observations of heat and water readings from 33(heat)/47(water) households from a neighborhood in Helsingør. The households have been labeled with a number such that household number 10 will be referred to as ID: 10.

The variable names are translated to English and the time stamps of the observations are rounded to nearest hour and converted to UTC. The data sets will then have the format as shown in Table 3.1. The data covers a period from 2016-10-01 to 2017-12-31 where the first year is used for training and the remaining 3 months are reserved for testing.

Variable	**Description**
Date	Date and time of observation (YYYY-MM-DD HH:MM:SS UTC).
Description	Data type (Not used)
OBIS	Internal code (Not used)
Value	Reading from meter
Unit	Wh or m^3 (depends on data type)
ID	Installation ID of monitor

Table 3.1: Description of variables in data from Forsyning Helsingør A/S.

From the hourly readings, the daily consumption is computed and stored as a new variable: *Consumption*.

3.2 Weather data

The data provided by SEAS-NVE is weather forecasts from a weather station in Kvistgaard - not far from Helsingør. The forecasts are provided by ECMWF (European Centre for Medium-Range Weather Forecasts) and covers a period from 2011-12-31 to 2018-01-21. The variables are listed in Table 3.2. The forecasts are hourly but for the initial analysis and final results, the daily average will be used instead.

Variable	Description
Location ID	Internal ID of weather station
Date	Date and time of observation (YYYY-MM-DD HH:MM:SS UTC)
SurfaceSolarRadiationDownwards	Solar radiation received on Earth [W/m^2]
TotalSkyDirectSolarRadiationAtSurface	Direct solar radiation received on a horizontal surface[W/m^2]
AirTemperature	Air temperature [°C]
WindSpeed	Wind speed measured at 10m above sea level [m/s]
WindDirectionDegree	Wind direction in degrees, $[0, 360]$, measured at 10 m
WindDirection (Not used)	Wind direction (factor) (e.g. N for North)
HighAltitudeWindSpeed	Wind speed measured at 100m above sea level [m/s]
HighAltitudeWindDirection	Wind direction in degrees measured at 100 m above sea level
AtmophericPressure	Atmospheric pressure at 0 m [Pa]

Table 3.2: Description of variables in weather data provided by SEAS-NVE.

The weather forecasts are updated frequently such that forecasts from past dates are reasonably accurate.

As mentioned in Section 2.3, the temperature difference, *temp.diff*, is introduced as, $temp.diff = 22 - AirTemperature$.

Figure 3.2 shows a pairs plot of every variable to see correlations.

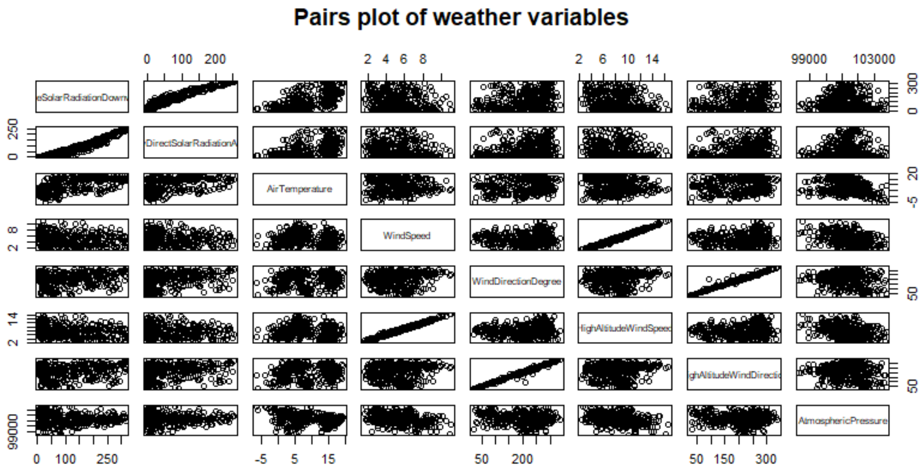


Figure 3.2: Weather data: Pairs plot of variables.

The two sun radiation variables, *SurfaceSolarRadiationDownwards* and *TotalSkyDirectSolarRadiationAtSurface*, appear to be very correlated. Intuitively, there should be some correlation between the two wind speed variables, *WindSpeed* and *HighAltitudeWindSpeed*, which is also present in the plot. In the case of wind direction at both altitudes, correlation can be observed as well. The remaining variables do not appear to be correlated with one another.

While not necessarily associated with weather, the length of night, using Læsø as reference, has also been provided by Jon Liisberg. Simply put: the time between sunset and sunrise in [hours].

3.3 Heat consumption

Figure 3.3 shows daily consumption from four households in training and test periods.

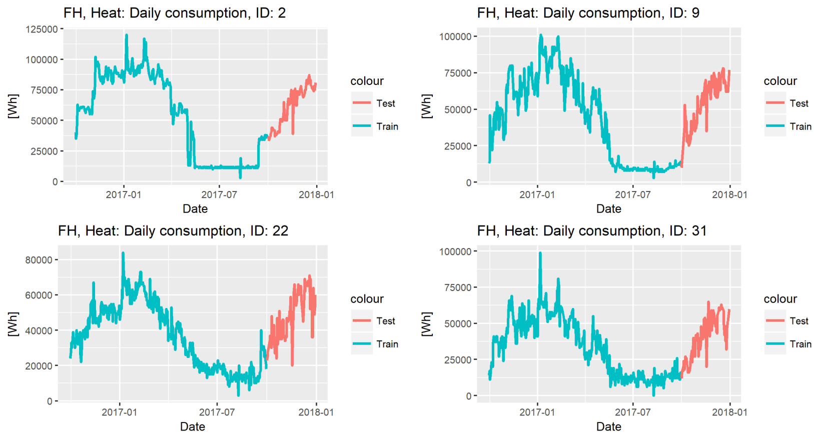


Figure 3.3: FH, Heat: Daily consumption of select IDs in training and test periods.

The consumption follows a clear pattern where heat consumption peaks during the cold months and flattens out during the summer.

3.4 Water consumption

Figure 3.4 shows daily consumption from four households in training and test periods.

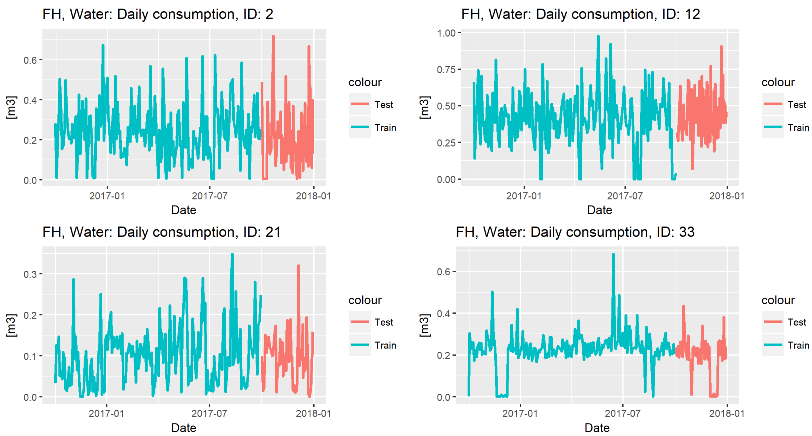


Figure 3.4: FH, Water: Daily consumption of select IDs in training and test periods.

Water consumption appears to be quite random and patterns are not noticeable. Observations with zero consumption suggests that the household was unoccupied at that time.

3.5 Initial analysis

In this analysis, the relation between heat consumption and the weather data is investigated using the training period. OLS is used to fit the models to the data and forward selection is used to reduce the models. The penalty used in the forward selection is, $k = \log(n)$, which corresponds to BIC. BIC is used instead of AIC because there is a larger penalty on the number of parameters($k = \log(365) \approx 5.9$). This ensures that the model is as simple as possible and only includes variables that provide the most information. The variables that are included in the models will be summarized in a bar plot. A single ID will be chosen for a more detailed analysis. The procedure of the detailed analysis is:

1. Fit a model using all variables.
2. Select a suitable model using forward selection.
3. Check model assumptions: Distribution of residuals, autocorrelation.
4. Check correlation between residuals and temperature difference.

5. F-test of the selected model and a simpler model.

Water consumption should be mostly constant on a weekly basis and not depend on the weather. Therefore, the initial analysis only involves heat data.

3.5.1 Heat consumption

A linear regression is fitted using all variables for each ID - 33 IDs in total. The models are then reduced by using forward selection with BIC as selection criteria.

The variables included in the reduced models are summarized in Figure 3.5.

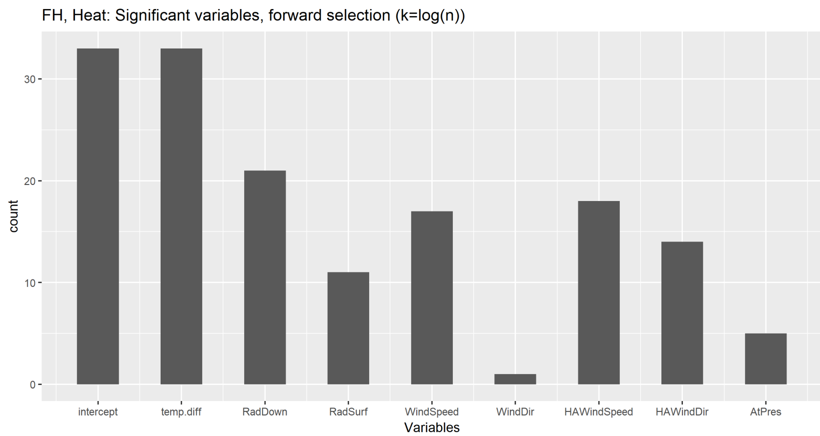


Figure 3.5: FH, Heat: Bar plot of significant variables based on BIC.

All models include an intercept and the temperature difference. More than half of the models include solar radiation, either "downwards" or "at surface" or both. Wind speed in either 10 m or 100 m altitude is included in several models. Some models also include high altitude wind direction. Finally, a few models also include atmospheric pressure. Table A.1 shows which variables are included in each model. Some IDs have some problems when fitting the data due to missing observations. The four worst examples are shown in Figure 3.6.

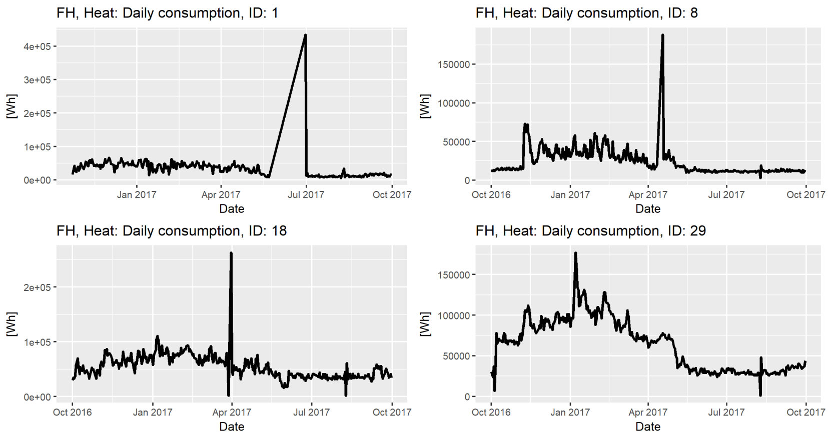


Figure 3.6: FH, Heat: Select IDs which have missing data resulting in unrealistic consumption. Note different scales on y-axis.

The method used to compute daily consumption does not work when observations are missing - fortunately, there are only a few cases with missing observations. As a result, the consumption data that is "lost" is accumulated and allocated to the first available date, leading to high spikes in consumption. Developing a method that accounts for missing observations is beyond the scope of this project.

Detailed analysis of ID: 10

This ID has been chosen for a more detailed analysis. This ID is a good representation of the data overall because there are few, if not zero, missing observations, as is the case for at least 50% of households, and the heat consumption is not unusual in the winter and summer periods.

The heat consumption for the training period is shown in Figure 3.7.

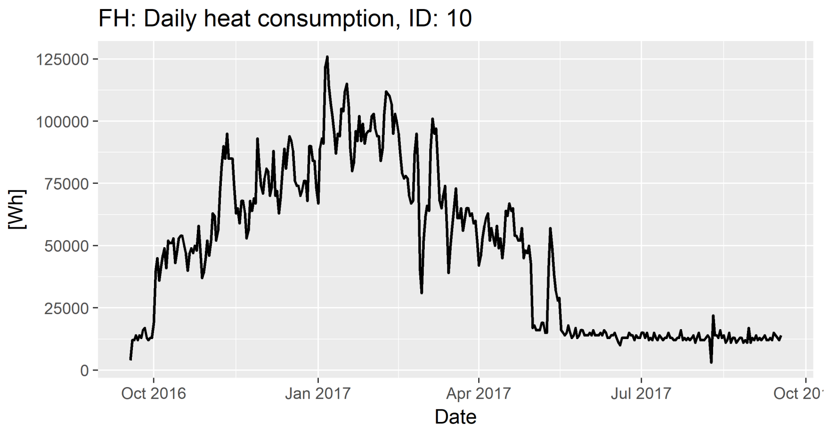


Figure 3.7: FH, Heat: 1 year of daily consumption for ID: 10.

The heat consumption increases significantly during the winter period compared to the summer months. The heat consumption during the summer is most likely water heating etc. Thus, it may require 2 different models: one for the winter period and one for the summer period - the model for the summer period could very well be a constant.

The models that have been tested for this ID are shown in Table 3.3. The model chosen by forward selection has been highlighted with bold font. Both AIC and BIC have been computed for each model. Model formula notation: This report uses R notation to define models, i.e. "Response" $\sim$ "Explanatory variables".

Model	AIC	BIC
Consumption ~ (intercept) + temp.diff + SurfaceSolarRadiationDownwards + TotalSkyDirectSolarRadiationAtSurface + WindSpeed + WindDirectionDegree + WindDirection + HighAltitudeWindSpeed + HighAltitudeWindDirection + AtmosphericPressure	7735.4	7774.4
Consumption ~ (intercept) + temp.diff + SurfaceSolarRadiationDownwards + WindSpeed + WindDirection + HighAltitudeWindSpeed + HighAltitudeWindDirection + AtmosphericPressure	**7731.7**	**7763.0**
Consumption ~ (intercept only)	8619.0	8626.8
Consumption ~ (intercept) + temp.diff	7829.9	7841.6
Consumption ~ (intercept) + temp.diff + SurfaceSolarRadiationDownwards	7771.4	7787.0

Table 3.3: Heat: Models tested for ID: 10.

The model chosen by forward selection includes temperature difference, solar radiation, wind speed/direction in both altitudes and atmospheric pressure - a somewhat complex model. It has the lowest AIC and BIC of all the models which is interesting as BIC should favor simpler models.

A normal QQ-plot and a histogram of the residuals of the reduced model are shown in Figure 3.8.

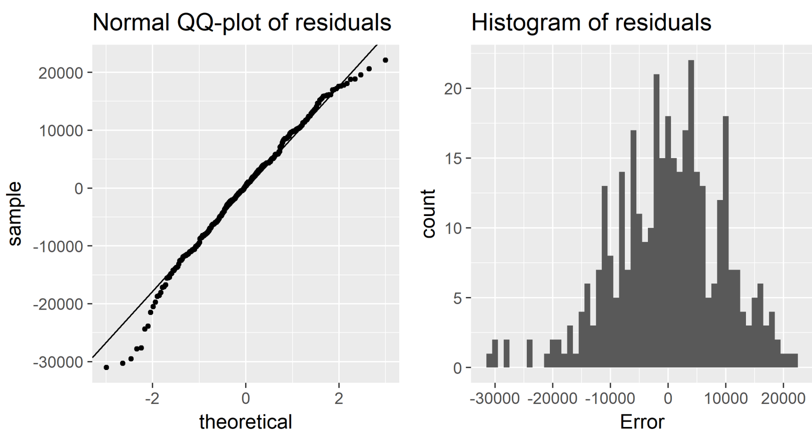


Figure 3.8: FH, Heat: Normal QQ-plot and histogram of residuals. ID: 10. Note: Unit of Error is [Wh].

The figure shows a slightly left skewed distribution of the residuals suggesting that they may not be normally distributed. The residuals in the histogram are mostly centered around zero.

In Figure 3.9 the temperature difference is plotted against the residuals to investigate dependence.

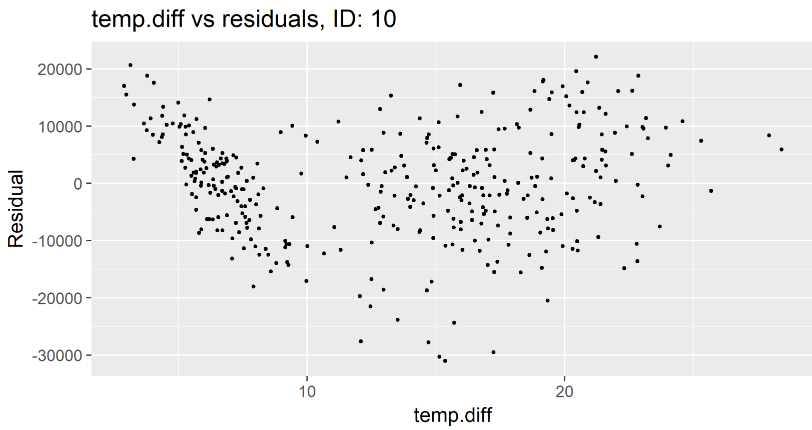


Figure 3.9: FH, Heat: Temperature difference vs residuals. ID: 10.

The residuals at low temperature differences resemble a pattern and is most likely related to the summer period where the temperature should have no influence on the heat consumption. The remaining data points look random.

Figure 3.10 shows the residuals, the sample ACF and PACF of the residuals.

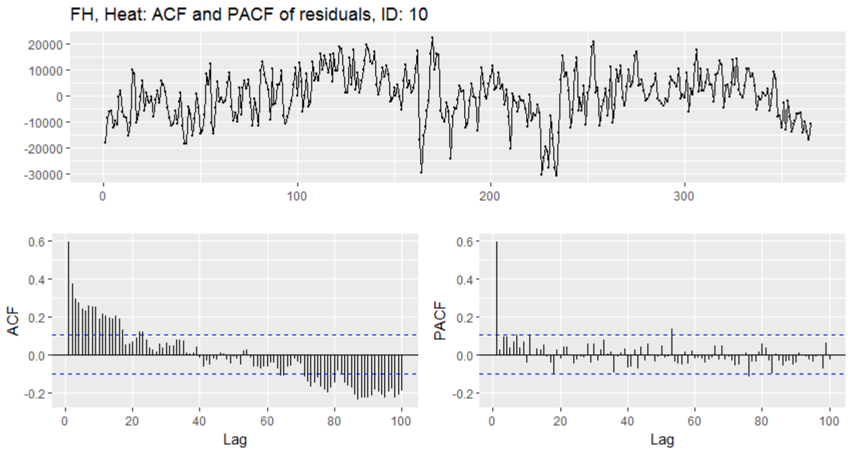


Figure 3.10: FH, Heat: Residuals, ACF and PACF of residuals. ID: 10.

The ACF shows that the residuals are autocorrelated suggesting that an AR part should be included in the model.

Finally, a model using only temperature difference is compared to the reduced model using an F-test. As the F-test assumes that both populations are normally distributed, the result may not be conclusive. However, the test still provides useful information.

Listing 3.1: Result from F-test between the temperature model and the final model.

```
    F test to compare two variances

data:  mtemponly and mreduced
F = 1.3263, num df = 363, denom df = 358, p-value = 0.007508
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
 1.078460 1.630861
sample estimates:
ratio of variances
          1.326305
```

The F-test computes the ratio of the variances and in this case it is ≈ 1.3. The p-value is 0.0075 and the null hypothesis is rejected. The final model should be the one chosen by forward selection.

However, the statistical tests do not tell the whole story and even though the tests say that wind speed/direction have a significant effect on the heat consumption, it simply does not reflect reality. In addition, the heat consumption during summer is more affected by the behavior than the weather. Thus it makes sense to focus the analysis on the winter period where the weather has a larger effect on the heat consumption.

3.5.2 Heat consumption: Winter period

This analysis focuses on heat consumption during winter. The winter period has been defined as 2016-10-15 - 2017-04-01 which is a longer period than the traditional winter period. However, heat consumption in the period just before and after the traditional winter period may also contain information. As before, a model is fitted with all variables for each ID.

Figure 3.11 shows which variables have been included in the reduced models after forward selection.

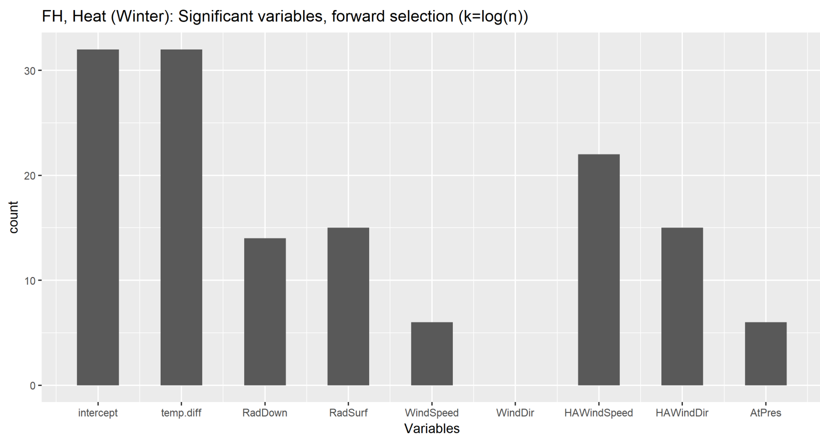


Figure 3.11: FH, Heat (Winter): Bar plot of significant variables based on BIC.

All models include an intercept and the temperature difference. Several models include solar radiation, either "downwards" or "at surface". High altitude wind speed/direction has been included in several models as well. Atmospheric pressure only appear in a few of the models. Table A.2 shows which variables have been included in each model.

Detailed analysis of ID: 10

As before, a single ID has been chosen for a more detailed analysis. The heat consumption in the winter period is shown in 3.12.

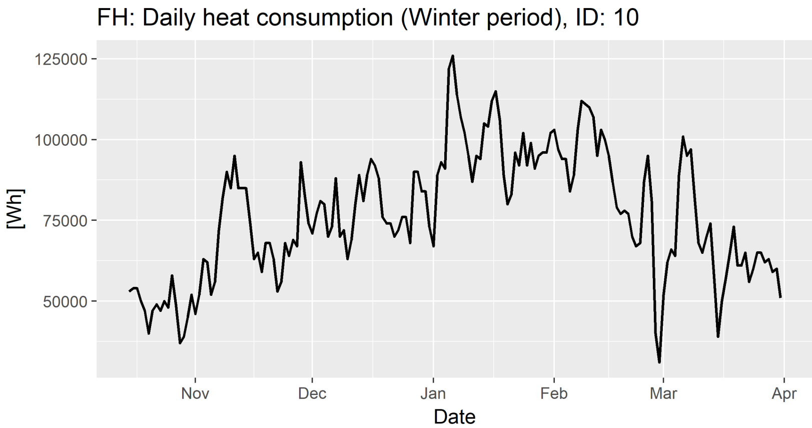


Figure 3.12: FH, Heat (Winter): Daily consumption for ID: 10.

In this period the heat consumption fluctuates more than during the summer months. The models tested for this ID is shown in Table 3.4.

Model	AIC	BIC
Consumption ∼ (intercept) + temp.diff + SurfaceSolarRadiationDownwards + TotalSkyDirectSolarRadiationAtSurface + WindSpeed + WindDirectionDegree + WindDirection + HighAltitudeWindSpeed + HighAltitudeWindDirection + AtmosphericPressure	3543.1	3574.3
Consumption ∼ (intercept) + temp.diff + SurfaceSolarRadiationDownwards + HighAltitudeWindSpeed + HighAltitudeWindDirection + AtmosphericPressure	**3543.2**	**3565.1**
Consumption ∼ (intercept only)	3799.8	3806.0
Consumption ∼ (intercept) + temp.diff	3604.5	3613.8
Consumption ∼ (intercept) + temp.diff + SurfaceSolarRadiationDownwards	3564.5	3577.0

Table 3.4: FH, Heat (Winter): Models tested for ID: 10.

The model chosen by forward selection has been highlighted with bold font. The significant variables are: the temperature difference, solar radiation, high altitude wind speed/direction and atmospheric pressure. This model is somewhat complex while still having the lowest BIC value and the second lowest AIC value.

A QQ-plot and histogram of the residuals of the highlighted model is shown in Figure 3.13.

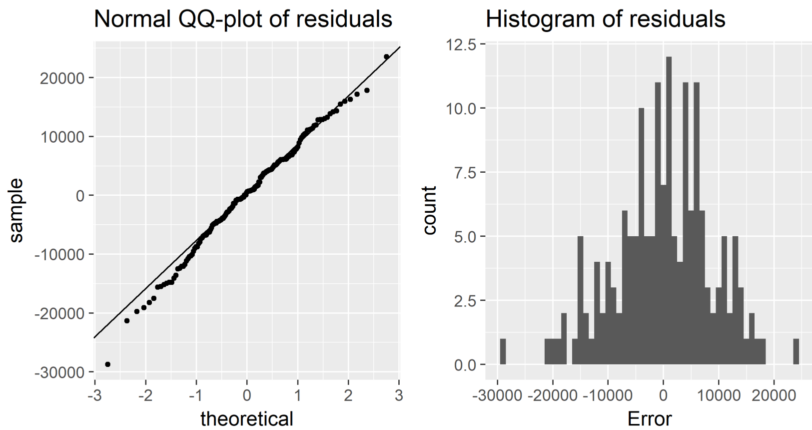


Figure 3.13: FH, Heat (Winter): Normal QQ-plot and histogram of residuals. ID: 10. Note: Unit of Error is [Wh].

There is a little hint of a tail in the QQ-plot but not as noticeable as seen previously suggesting that the residuals are normally distributed. The histogram shows that the residuals are mostly centered around zero. Figure 3.14 shows a plot of the temperature difference against the residuals.

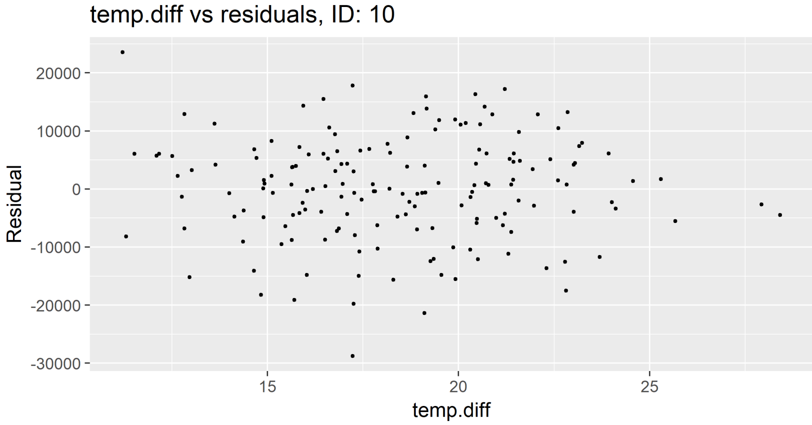


Figure 3.14: FH, Heat (Winter): Temperature difference vs residuals. ID: 10.

The points appear evenly and randomly spread out - there is no obvious pattern.

The residuals, ACF and PACF of the residuals are shown in Figure 3.15.

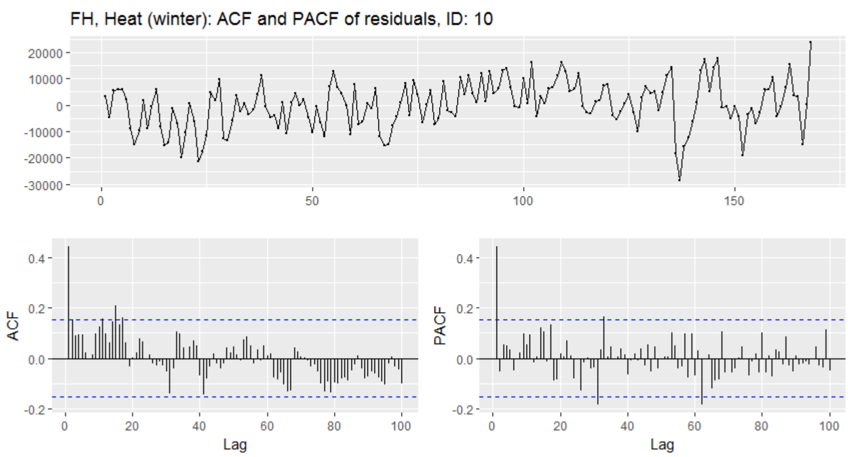


Figure 3.15: FH, Heat (Winter): Residuals, ACF and PACF of residuals. ID: 10.

The ACF shows high autocorrelation at lag 1.

Finally, an F-test is used to compare the forward selection model with a simpler model that only includes the temperature difference. The result of the test is shown in Listing 3.2.

Listing 3.2: Result from F-test between the temperature model and the final model.

```
    F test to compare two variances

data:  mtemponly and mreduced
F = 1.4738, num df = 166, denom df = 162, p-value = 0.01353
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
 1.083721 2.003247
sample estimates:
ratio of variances
          1.473847
```

The ratio of variances is about 1.47 and the p-value is 0.01. The null hypothesis is rejected, suggesting that the more complex model is better at describing the variance in the data.

The forward selection method has chosen to include wind speed/direction in some of the models which will be overruled as this does not reflect reality. Wind direction is particularly problematic as it has a range of $[0, 360]$ which is discontinuous. Forward selection is also prone to include variables that has a significant but small effect on the response.

Even though the statistical tests show that the heat consumption depends on wind speed, the model selection will be restricted to the following variables: *temperature difference*, *TotalSkyDirectSolarRadiationAtSurface* and *SurfaceSolarRadiationDownwards*. Atmospheric pressure has also been omitted as it only appeared in a few models and the effect may be negligible.

3.6 Conclusion

Forward selection with BIC has been used to select which weather variables that should be included in the models for the households in Helsingør. A full year and a winter period have been used to test weather dependence. The residuals have been investigated for a single ID and and a simpler model has been used as a comparison. The analysis of the residuals suggests that the assumptions of the linear regression are not always satisfied. This may also cause problems when using the F-test for model selection. The model selection will be restricted to only include: the *temperature difference*, *TotalSkyDirectSolarRadiationAtSur-*

face and *SurfaceSolarRadiationDownwards*.

Chapter 4

Data: BECA

4.1 Description

A data set was provided by Jordi Cipriano to further investigate the method's performance. BECA consists of heat, electricity and water consumption, as well as weather data, for 44 apartments in Catalonia, Spain[6]. The data covers a period from 2012-07-16 to 2013-12-31. A description of the data, after it has been reshaped, is given in Table 4.1.

Variable	**Description**
Date	Date and time of observation (YYYY-MM-DD HH:MM:SS UTC).
Electricity	Electricity consumption [kWh] (Not used)
SpaceHeating	Heat consumption [kWh]
Water	Water consumption [Liter]
HotWater	Hot water consumption [Liter] (Not used)
ID	Apartment ID
InteriorTemperature	Temperature inside apartment [°C]
Temperature	Air temperature outside apartments [°C]
Radiation	Solar radiation received [W/m^2]
Wind_Velocity_avrg	Average wind speed [m/s]
Wind_Direction_avrg	Average wind direction [°]
Precipitation	Precipitation [mm]

Table 4.1: Description of variables in data from Spain.

A more in-depth description and analysis of the variables are available in [6]. An important note about space heating: The resolution for this variable is 1 kWh, as the meters in the apartments are configured this way.

The temperature difference is defined as, $\mathit{temp.diff} = \mathit{InteriorTemperature} - \mathit{Temperature}$.

4.2 Heat consumption

Figure 4.1 shows daily heat consumption for 4 apartments. The training period is 1 year. 3 months of the data are reserved for testing.

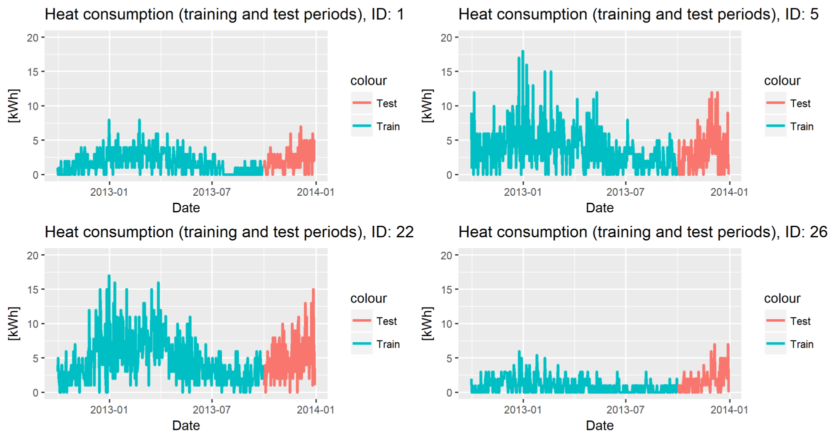


Figure 4.1: Daily heat consumption for 4 apartments.

There is a noticeable difference in consumption between the four apartments. Also, there is some seasonal variation - higher consumption during winter months and lower during the summer. However, there should be no need of space heating during the summer, especially in Spain, so there may be some errors in the data.

4.3 Water consumption

Figure 4.2 shows daily water consumption for the same four apartments as used previously. Training and test periods are the same.

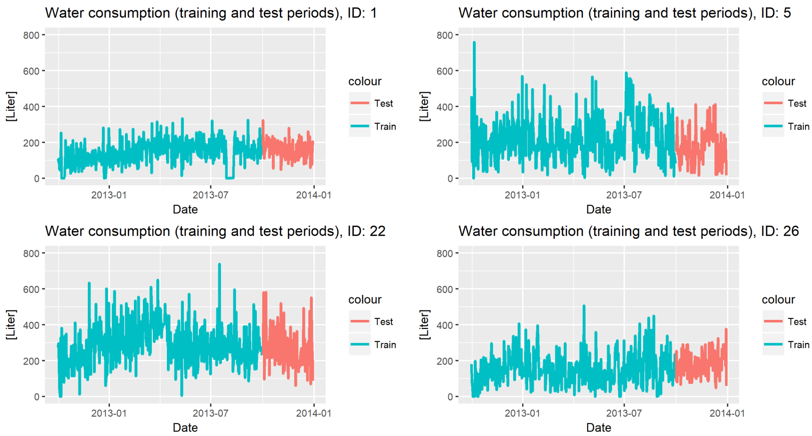


Figure 4.2: Daily heat consumption for 4 apartments.

The four apartments show different needs in water consumption. Also worth noting is the short periods with zero consumption, as these periods can be random and difficult to account for in the model. Finally, seasonal trends are not apparent in the data - as opposed to the heat consumption.

4.4 Initial analysis

As previously mentioned, the space heating data has a resolution of 1 kWh. Combined with an overall low daily heat consumption means that the linear regression model is not ideal. However, the linear regression model is still a good approximation for this purpose.

Water consumption should be mostly constant on a weekly basis and not depend on the weather. Therefore, the initial analysis only involves heat data.

4.4.1 Heat consumption

A linear regression is fitted for each apartment using all variables available. Next, forward selection is used to reduce the models by using BIC as selection criteria. Figure 4.3 shows the significant variables in the final models.

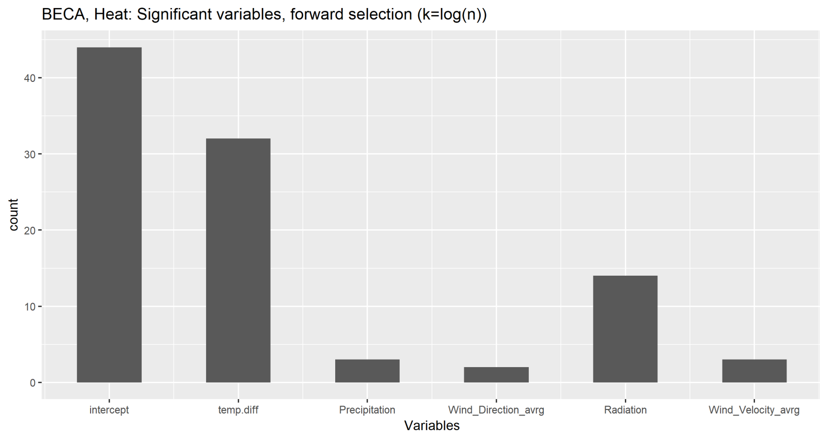


Figure 4.3: BECA, Heat: Bar plot of significant variables based on BIC.

All models include an intercept. Most models include the temperature difference as well. Some models include radiation. Finally, there is a few cases of precipitation and wind direction/speed. Table B.1 in Appendix B.1 shows which variables have been included in all 44 models. A few models stand out from the rest: The models that include precipitation or wind speed/direction.

The heat consumption of four apartments are shown in Figure 4.4.

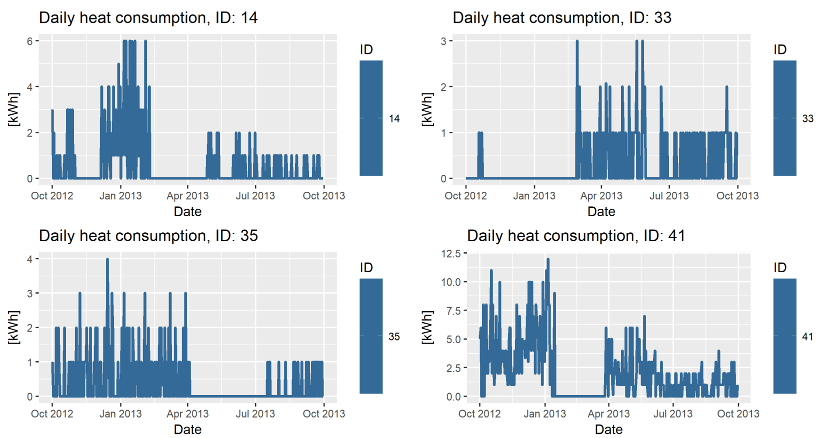


Figure 4.4: BECA: Heat consumption for apartments 14, 33, 35 and 41.

The common trend for these four apartments is the long periods of zero consumption. The lack of data in these apartments makes it difficult to conclude anything.

Apartment 29

The daily heat consumption for apartment 29 is shown in Figure 4.5

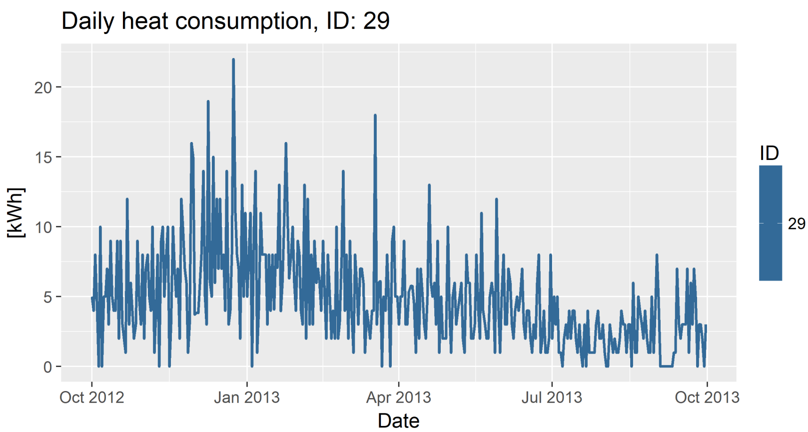


Figure 4.5: BECA: Daily heat consumption for apartment 29.

Overall, the heat consumption of apartment 29 is what you would expect. Higher consumption during the winter and very few zero observations.

Table 4.2 shows which models have been considered for apartment 29.

Model	**AIC**	**BIC**
Consumption $\sim$ (intercept) + temp.diff + Radiation + Wind_Velocity_avrg + Wind_Direction_avrg + Precipitation	1905.8	1933.2
Consumption $\sim$ (intercept) + temp.diff + Precipitation	1904.6	1920.2
Consumption $\sim$ (intercept only)	1984.0	1991.8
Consumption $\sim$ (intercept) + temp.diff	**1907.8**	**1919.5**

Table 4.2: Models tested for apartment 29.

The model with the lowest BIC includes only the temperature difference.

The AIC for each model has been included to illustrate the difference between AIC and BIC, and why BIC has been chosen to be a deciding factor. According to the AIC, the best model uses the temperature difference and precipitation. However, the final model chosen by forward selection only includes the temperature difference.

A QQ-plot and histogram of the residuals are shown in Figure 4.6.

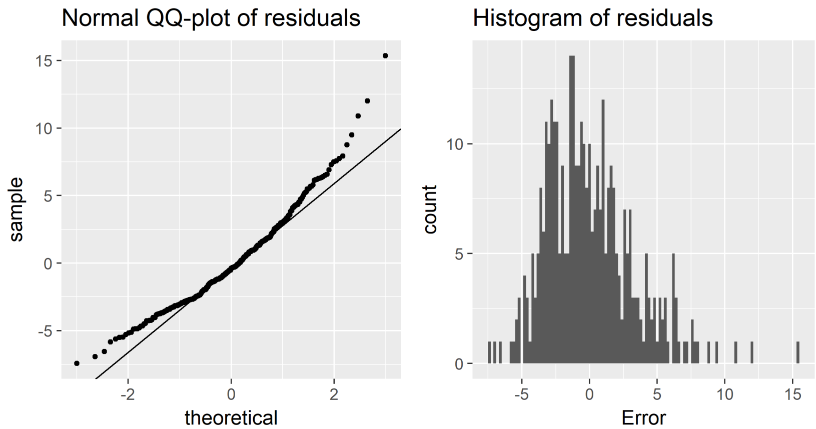


Figure 4.6: BECA: QQ-plot and histogram of residuals. Note: Unit of Error is [kWh].

The QQ-plot shows some deviation from the theoretical line indicating that the normality assumption is not completely satisfied. This is not surprising as the observations are (positive) integers and the fitted values are real numbers. The histogram shows that the distribution is slightly skewed to the right.

To investigate whether the error is correlated with the temperature difference, the temperature difference is plotted against the residuals, as seen in Figure 4.7.

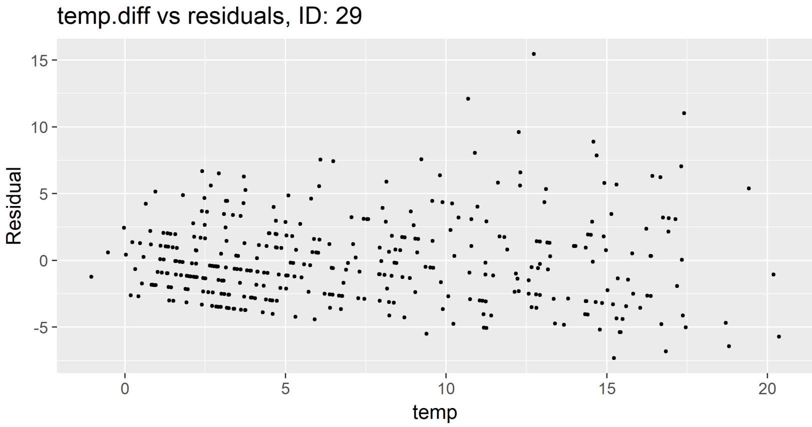


Figure 4.7: BECA, Heat: Temperature difference plotted against residuals. Apartment 29. Note: Unit of temperature difference is [°C].

A subtle pattern appears in the plot which could be due to the resolution of the heat consumption. Otherwise, the points look random and somewhat evenly distributed across the temperature difference axis.

The residuals, ACF and PACF of the residuals are shown in Figure 4.8.

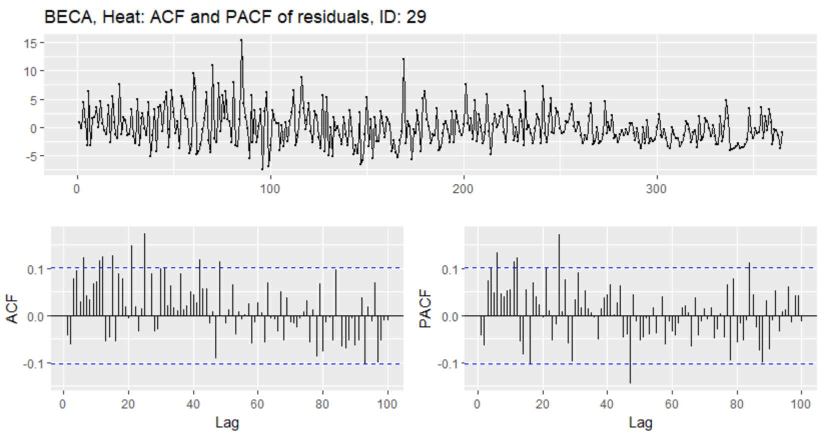


Figure 4.8: BECA, Heat: Residuals, ACF and PACF of residuals. Apartment 29.

While the autocorrelation for some lags are higher than the significance band, the ACF and PACF resemble white noise.

A simple null model is fitted to the data and compared to the reduced model using an F-test.

Listing 4.1: Result from F-test between the null model and the reduced model.

```
    F test to compare two variances

data:  mnull and mtemp
F = 1.2356, num df = 364, denom df = 363, p-value = 0.04404
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
 1.005669 1.518164
sample estimates:
ratio of variances
          1.235643
```

The F-test computes the ratio of the variances from the two models and it is larger than 1. The p-value, 0.044, is below the usual significance level of 0.05 and the null hypothesis is rejected. The null model does not describe the variation in the data better than the model using temperature difference. Thus, the final model using the temperature difference is the best model. However, as the normality assumption is not satisfied this result is not conclusive.

4.4.2 Heat consumption: Winter period

During the summer months, there should be little to no demand of heating, if water heating is excluded. Thus it would be interesting to focus on the colder months of the year where the outside temperature should have a larger impact on the heat consumption. The winter period has been defined as 2012-10-16 to 2013-03-16. As a result, there are 151 observations for all 44 apartments. A linear regression with all weather variables is fitted to the data for each apartment.

Forward selection is used to reduce the models using BIC. The significant variables that remain in each model are shown in Table B.2 in Appendix B.1. Figure 4.9 shows the significant variables.

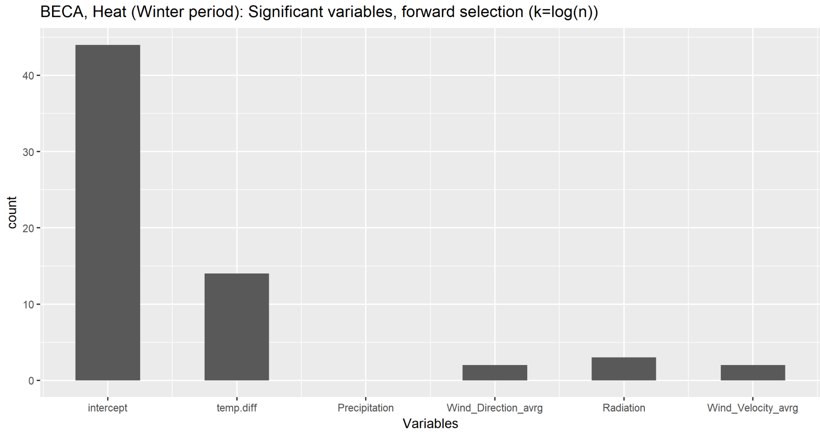


Figure 4.9: BECA, Heat(Winter): Bar plot of significant variables based on BIC.

All models include an intercept but only 14 models include the temperature difference. A few models include wind speed/direction or solar radiation.

Analysis of apartment 29

The heat consumption during the winter period is shown in Figure 4.10.

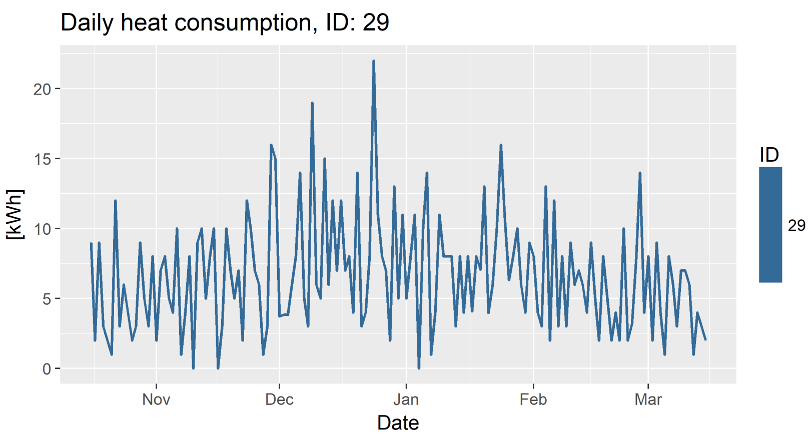


Figure 4.10: BECA, Heat(Winter): Heat consumption in apartment 29 during the winter period.

While the consumption varies a great deal during this period, there is a hint of a trend, i.e. higher consumption in December and January. A day with zero consumption at various times are also observed.

Model	**AIC**	**BIC**
Consumption ∼ (intercept) + temp.diff + Radiation + Wind_Velocity_avrg + Wind_Direction_avrg + Precipitation	850.5	871.6
Consumption ∼ (intercept)	852.3	858.3
Consumption ∼ (intercept) + temp.diff	**848.4**	**857.5**

Table 4.3: Winter: Models tested for apartment 29.

The final model includes the temperature difference and has the lowest BIC value of the bunch - and in this case the lowest AIC as well.

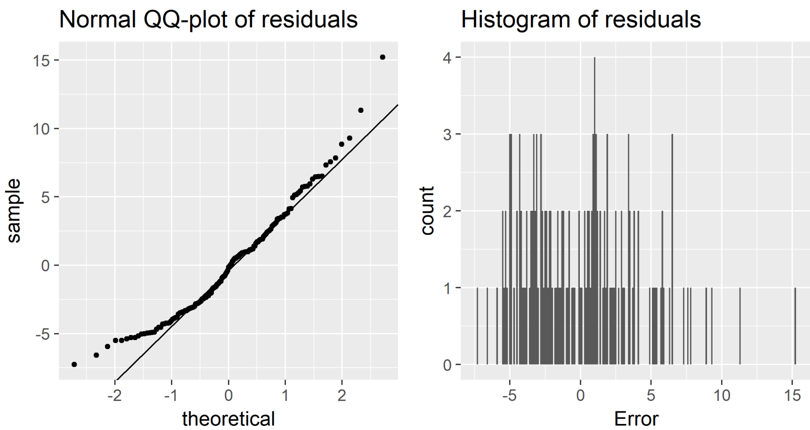


Figure 4.11: BECA, Heat(Winter): Normal QQ-plot and histogram of residuals. Apartment 29. Note: Unit of Error is [kWh].

The residuals in the normal QQ-plot in Figure 4.11 have lightly skewed tails which could be due to the data being discrete. The residuals in the histogram appear to be skewed to the right.

In Figure 4.12 the temperature difference is plotted against the residuals.

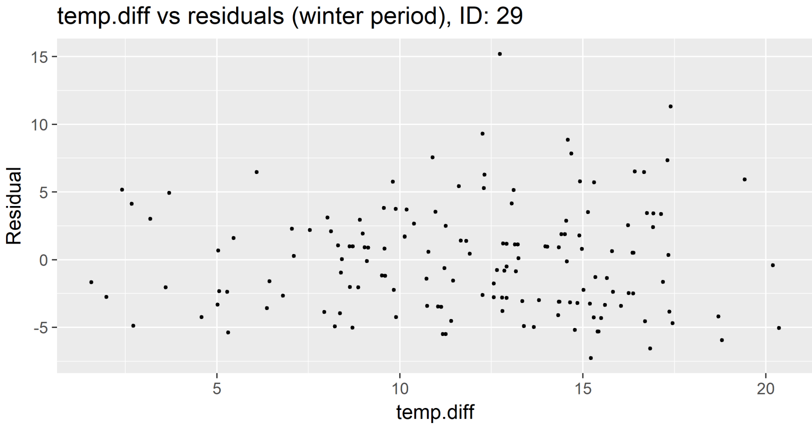


Figure 4.12: BECA, Heat(Winter): Temperature difference plotted against residuals. Apartment 29.

There is no obvious pattern in the plot. The residuals appear to spread out independently from the temperature difference. Of course, most of the residuals are at roughly 10-15 degrees Celsius, as these observations of the temperature difference are more frequent in the data.

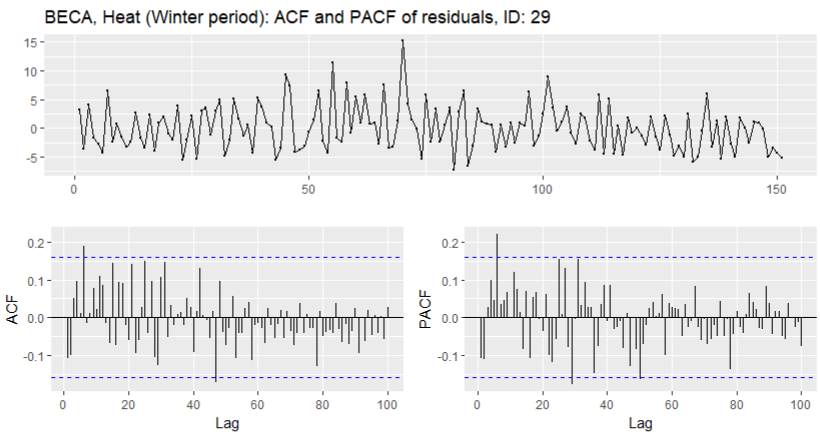


Figure 4.13: BECA, Heat(Winter): Residuals, ACF and PACF of residuals. Apartment 29.

The top plot in Figure 4.13 is the residuals. The plots below are the ACF and PACF of the residuals, respectively. Both the ACF and PACF do not show any obvious patterns or high values suggesting that the residuals are white noise.

As a final check, the F-test is used to compare the final model and the null model. The result from the F-test is shown in Listing 4.2.

Listing 4.2: Result from F-test between the null model and the final model.

```
    F test to compare two variances

data:  mnull and mtemp
F = 1.0326, num df = 150, denom df = 149, p-value = 0.8451
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
 0.7483674 1.4244490
sample estimates:
ratio of variances
          1.032569
```

The ratio of the variances is 1.03 - which is close to 1. The p-value is 0.85 meaning the null hypothesis cannot be rejected. As a result, the final model does not provide a better fit than the null model. Relying only on forward selection to chose the best model may not be the best choice in this case. However, as the residuals are not normally distributed, the result is difficult to interpret.

4.4.3 Heat consumption: Non-zero observations

Several apartments in the data set have many zero observations and occasionally long periods of zero observations, as was seen in Figure 4.4, which make analyses difficult. In this section, the relation between heat consumption and weather is investigated using non-zero observations only.

The analysis is the same as before. A linear regression with all weather variables is fitted to data from each apartment.

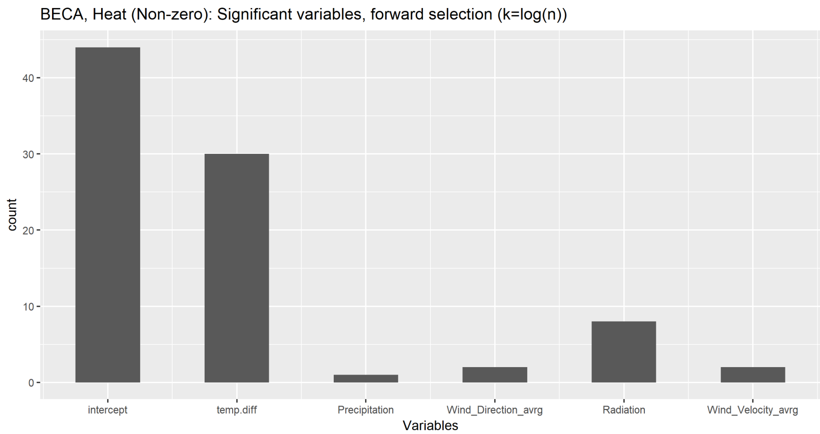


Figure 4.14: BECA, Heat (Non-zero): Bar plot of significant variables based on BIC.

Figure 4.14 show which variables are significant in the reduced models. All models contain an intercept. 30 models contain the temperature difference while only a few contain radiation. There are still a few models that contain precipitation or wind speed/direction.

Apartment 29

Apartment 29 is, again, used as an example to further investigate which model has the best fit. The non-zero observations are plotted in Figure 4.15.

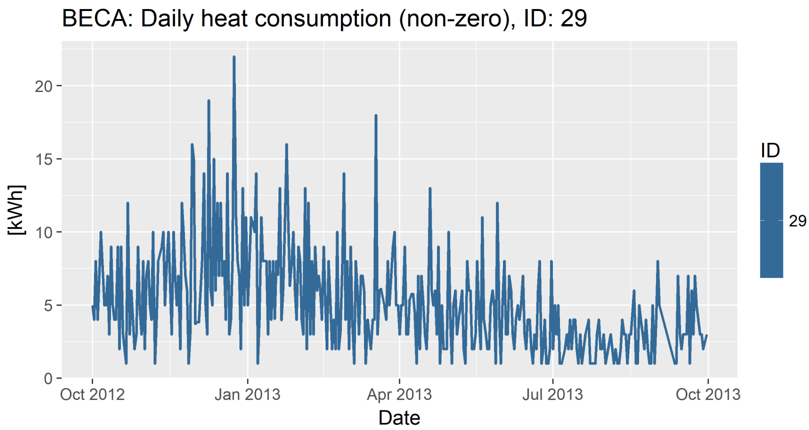


Figure 4.15: BECA, Heat (non-zero): Daily non-zero heat consumption for apartment 29.

Each observation is still connected by a line in the plot making spotting the missing observations difficult. However, it is clear that all observations are larger than zero. Table 4.4 shows which models have been tested.

Model	**AIC**	**BIC**
Consumption ∼ (intercept) + temp.diff + Radiation + Wind_Velocity_avrg + Wind_Direction_avrg + Precipitation	1769.6	1796.4
Consumption ∼ (intercept)	1837.7	1845.4
Consumption ∼ (intercept) + temp.diff + Radiation + Precipitation	**1767.4**	**1786.6**
Consumption ∼ (intercept) + temp.diff	1775.8	1787.3

Table 4.4: Heat (non-zero): Models tested for apartment 29.

The final model is highlighted with bold font and contains the variables: temp.diff, Radiation and Precipitation. The Normal QQ-plot and histogram of the residuals are shown in Figure 4.16

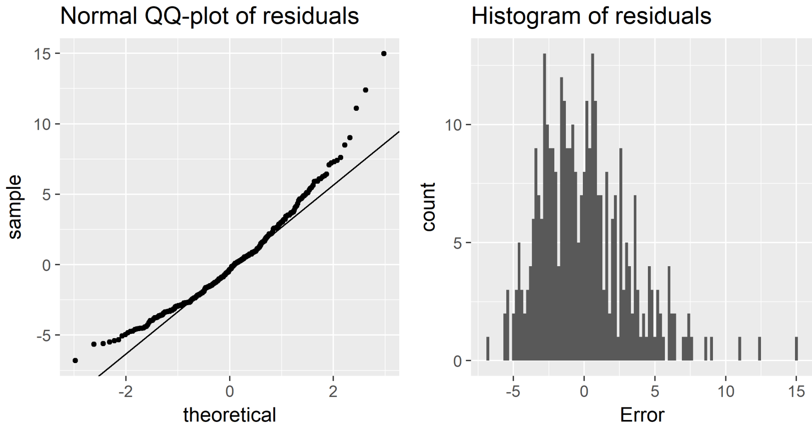


Figure 4.16: BECA, Heat (non-zero): Normal QQ-plot and histogram of residuals. Apartment 29. Note: Unit of Error is [kWh].

The normal QQ-plot is slightly skewed suggesting that the normality assumption is not satisfied. The errors are mostly centered around zero in the histogram. However, the distribution looks a little skewed as well. In Figure 4.17 the temperature difference is plotted against the residuals.

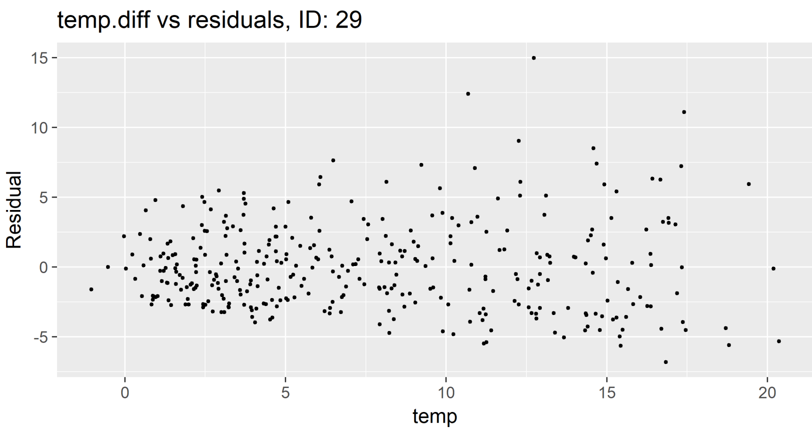


Figure 4.17: BECA, Heat (non-zero): Temperature difference vs residuals. Apartment 29.

The variance of the residuals appear to be increasing with the temperature difference. One should question the chosen model's performance when presented with data like this. However, an important thing to keep in mind is that the heat consumption also depends on the behavior of the occupants.

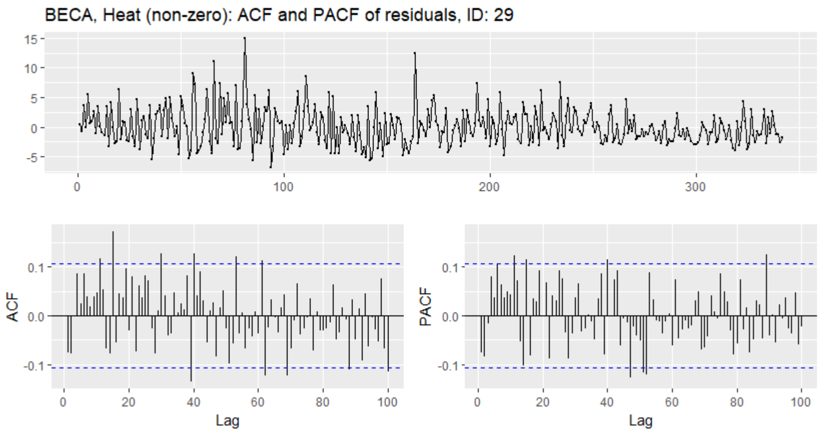


Figure 4.18: BECA, Heat (non-zero): Residuals, ACF and PACF of residuals. Apartment 29.

The variance of the residuals is a little higher during the winter period, which is also seen in Figure 4.17. There are no obvious patterns in the ACF and PACF in Figure 4.18, suggesting that the residuals are white noise. However, as previously stated, the residuals may not be normally distributed.

As a final check, the final model chosen by forward selection is compared to a model that includes only the temperature difference. The F-test is used to compare the two models and the result is shown in Listing 4.3.

Listing 4.3: Result from F-test between the temperature model and the final model.

```
    F test to compare two variances

data:  mtemponly and mreduced
F = 1.0307, num df = 340, denom df = 338, p-value = 0.7809
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
 0.8327301 1.2756752
sample estimates:
ratio of variances
          1.030711
```

The ratio of the variances is 1.03 and p-value is 0.78. The null hypothesis cannot be rejected and this result suggests that the final model does not describe the variance in the data any better than the simpler model. However, the residuals may not be normally distributed.

Regardless of the results of the F-test, the model selection will be restricted to the variables: temperature difference and radiation.

4.5 Conclusion

Forward selection with BIC has been used to select which weather variables that should be included in the models for the apartments in Spain. The residuals have been investigated for a single apartment and a simpler model has been used as a comparison. The model selection will be restricted to only include: the temperature difference and radiation. Forward selection alone may not be the best method for choosing the appropriate variables for the models. The distribution of residuals have been a problem throughout the analyses, as it cannot be assumed normal.

Chapter 5

Results: FH

In this section the results obtained with data from Forsyning Helsingør(FH) are presented. This includes forecasts of heat and water consumption using the two methods: RLS with exponential or variable forgetting. Finally, a single ID is used for a detailed analysis.

The overall results are presented with the following elements:

- Actual consumption compared to forecasts.
- RMSE for each horizon.
- Histogram of accumulated prediction error.

A single ID has been chosen for detailed analysis, presented with the following elements:

- Actual consumption and forecasts for chosen ID.
- 95% prediction interval for both methods.
- Plot of parameter values of 1-step model for each RLS step.

- Plot fit and forecast of 1-step model on full data set.
- Residuals: Distribution, correlation and temperature dependence(if applicable).

An extra result obtained with this data is so-called "cold winter" forecasts. These forecasts use weather data from 2012 as input to simulate heat consumption during a colder winter.

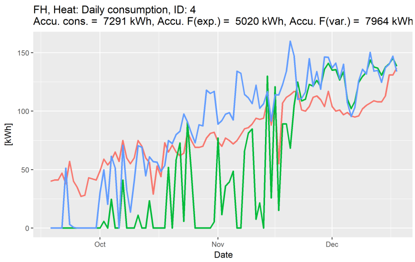
5.1 Forecasts

5.1.1 Forecasting heat consumption

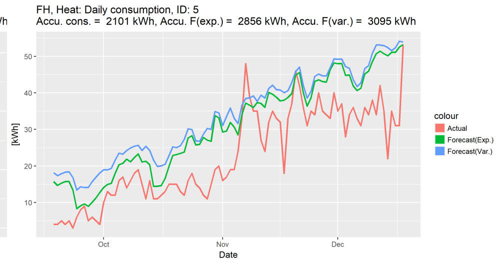
The weather variables that can be included in the model are: *temp.diff*, *TotalSkyDirectSolarRadiationAtSurface* and *SurfaceSolarRadiationDownwards*. Furthermore, the length of night, in [hours], is saved in the variable, *length.night*. The method can also include up to 3 pairs of sine/cosine pairs.

1-step models for all IDs are listed in Table A.3 and Table A.4 for exponential and variable forgetting methods, respectively.

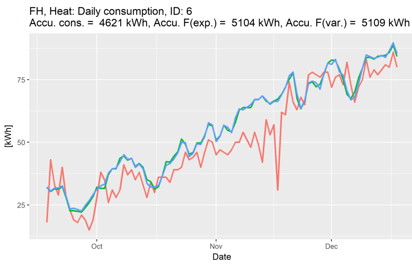
Forecasts and consumption for the test period, described in Section 3.5.1, for select IDs are presented in Figure 5.1.



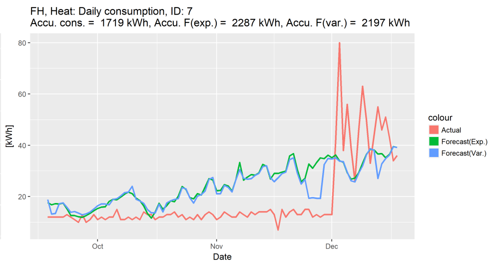
(a) ID 4.



(b) ID 5.



(c) ID 6.



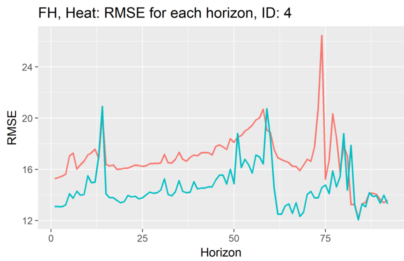
(d) ID 7.

Figure 5.1: FH: Actual heat consumption and forecasts of select IDs. Note: Different scales on y-axis.

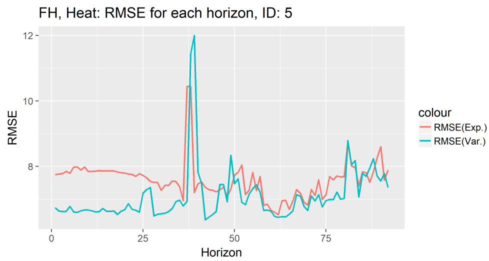
In Figure 5.1a, the 2 forecasts differ significantly the first 60 days. Exponential forgetting adapts slower than variable forgetting and could possibly be more affected by the summer period in the short horizons. In Figure 5.1b, the forecasts appear to be influenced by the summer period in the beginning and then slowly increases. In Figure 5.1c, forecasts follow the trend very well and matches the time the heat is turned on. As the increase in consumption is quite steady, both methods deliver good results. There is a short period in the middle of November where the consumption drops significantly which could indicate that the house was unoccupied. In Figure 5.1d, the household uses very little heat in the first two months and both forecasts did not expect this. However, in December the heat consumption increases and is close to the forecasts.

Boxplots of prediction errors for all IDs are available in Appendix A.2.4.

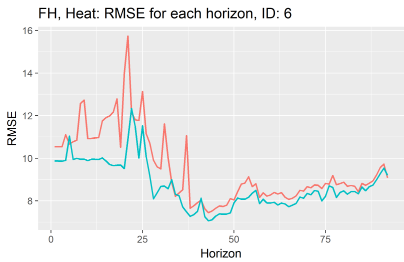
The RMSE for each horizon, i.e. each model, based on the training period is shown in Figure 5.2 for both methods for the same IDs.



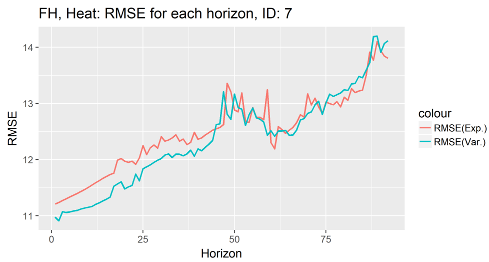
(a) ID 4.



(b) ID 5.



(c) ID 6.



(d) ID 7.

Figure 5.2: FH: RMSE from training period for each horizon for select IDs. Note: Different scales on y-axis.

In general, the RMSE is expected to increase with the horizon as the uncertainty increases when forecasting further ahead. In Figure 5.2a, the RMSE for both methods behave differently than expected. After roughly 60 days, the RMSE drops and increases again. In Figure 5.2b, the RMSE is increasing slowly. A large spike occurs at horizon ≈ 40 that could be due to a period of zero consumption in the training data. In Figure 5.2c, the RMSE decreases for both methods until day 40 where it increases slowly again. In Figure 5.2d, the RMSE increases steadily until day 50 where it drops a little and then increases again.

The accumulated prediction error for both methods is calculated for each ID, as forecast minus actual consumption, and the histograms are shown in Figure 5.3.

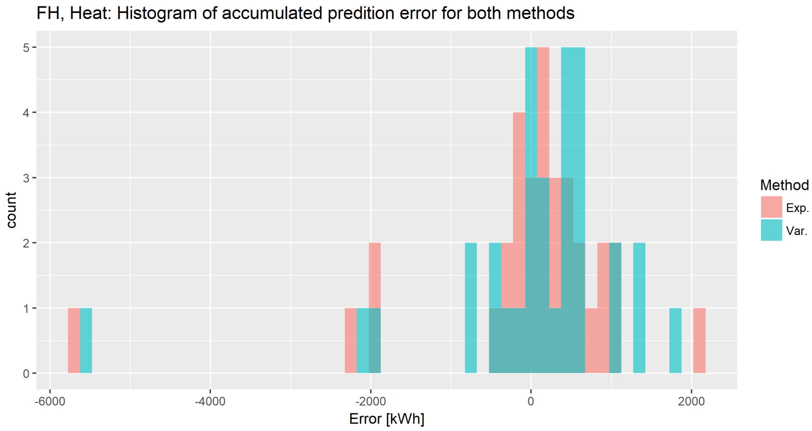


Figure 5.3: FH, Heat: Histograms of accumulated prediction error for both methods.

The histograms are more or less on top of each other with only minor differences. The majority of the error is larger than zero indicating that most of the accumulated forecasts are larger than the actual accumulated consumption.

The outlier with accumulated prediction error of around 6000 kWh is ID 23 which is missing 8 months worth of data. As a result, the date following the missing period contains the accumulated consumption of 5674 kWh.

In Table A.5, the mean RMSE and accumulated forecast for all IDs are available.

Analysis of ID 10

To get a better understanding of the performance of the methods, a single ID has been chosen for a more detailed analysis - ID 10.

In Figure 5.4, the consumption and forecasts are plotted.

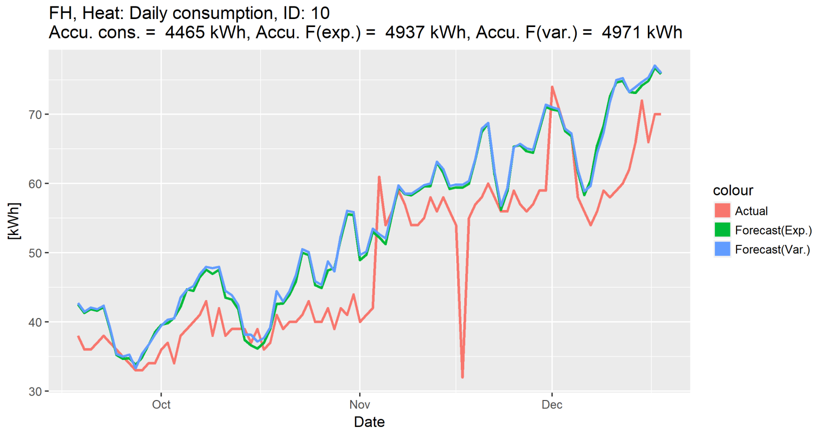
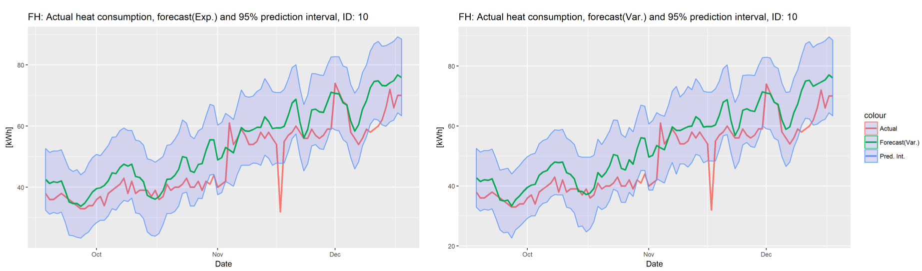


Figure 5.4: FH: Actual heat consumption and forecasts for ID 10.

The forecasts are perhaps slightly higher than the actual consumption but the methods have captured the trend quite well, overall. It could be that this period is warmer than last year. There is a short period of low consumption in the middle of November that was also observed in Figure 5.1c which, again, could mean that the heat was turned off and the house was unoccupied or there are errors in the data.

Figure 5.5 shows the forecasts with 95% prediction intervals, based on the normal distribution.



(a) Forecast with prediction interval. Exponential forgetting.

(b) Forecast with prediction interval. Variable forgetting.

Figure 5.5: FH: Heat forecasts with 95% prediction interval.

For both methods, almost all observations are within the prediction interval.

In the following, the 1-step model for both methods has been selected for analysis. Figure 5.6 shows the fit and forecast of the 1-step model for both methods.

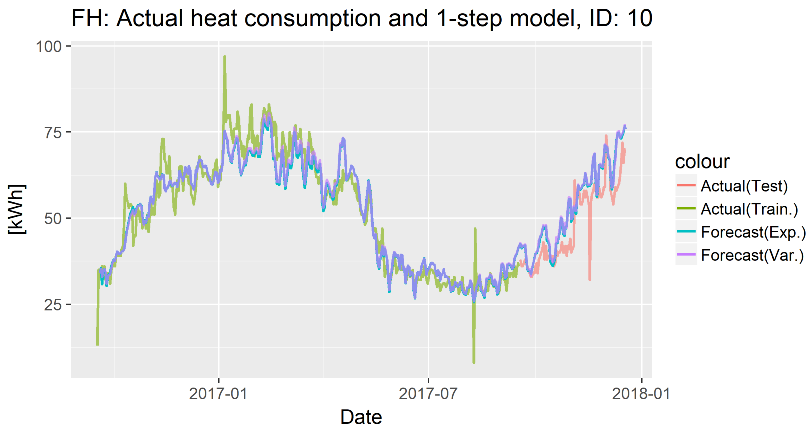
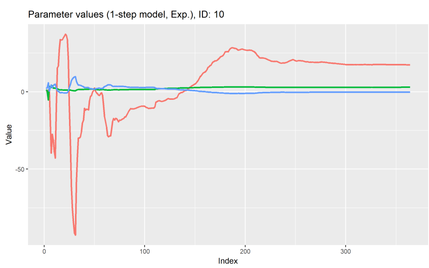


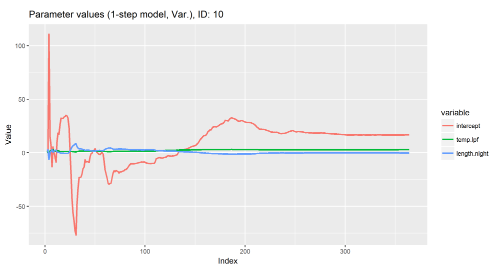
Figure 5.6: FH: Heat consumption and 1-step model for ID 10.

The models follow the trend in the data quite well. The 1-step model appears to perform similar to the k-step model for large horizons.

The estimated parameters are plotted for both methods in Figure 5.7.



(a) Parameter values of 1-step model(exp.) in training period.



(b) Parameter values of 1-step model(var.) in training period.

Figure 5.7: FH, Heat: Parameter values of 1-step models. Note: Different scales on y-axis.

For both methods, the burn-in period for the parameters is roughly 200 observations, then the estimates become more steady. The estimates are almost identical for both models in this case. Variable forgetting is just as stable as exponential forgetting is this case.

Residuals for 1-step model (Exp.)

The residuals for the 1-step model using exponential forgetting is plotted together with the ACF and PACF in Figure 5.8.

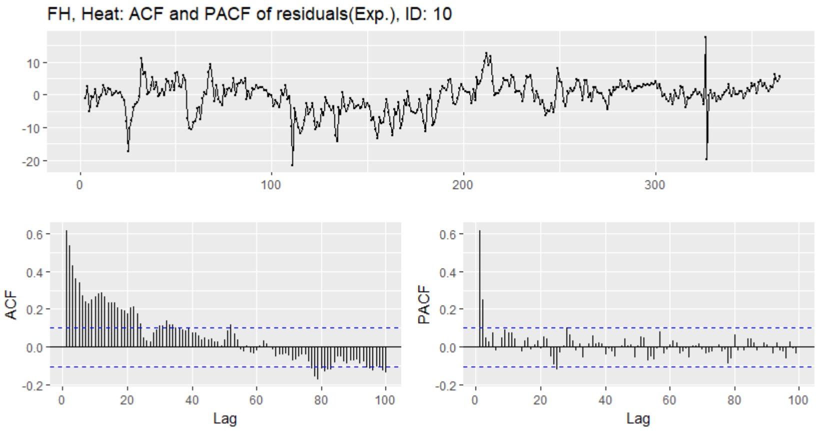


Figure 5.8: FH, Heat: ACF and PACF for 1-step model(Exp.) residuals for ID 10.

The residuals are negative during the winter months and closer to zero in the summer months. Both ACF and PACF show autocorrelation is present.

Figure 5.9 shows a QQ-plot and a histogram of the residuals.

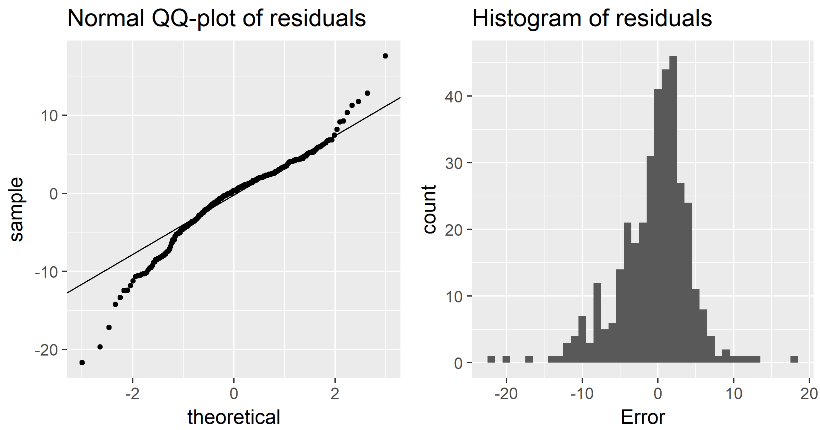


Figure 5.9: FH, Heat: Histogram and QQ-plot of residuals. Exponential forgetting. Note: Unit of Error is [kWh].

The distribution of the residuals is a little skewed to the left indicating that the assumption of normality is not completely satisfied. In Figure 5.10 the temperature difference is plotted against the residuals.

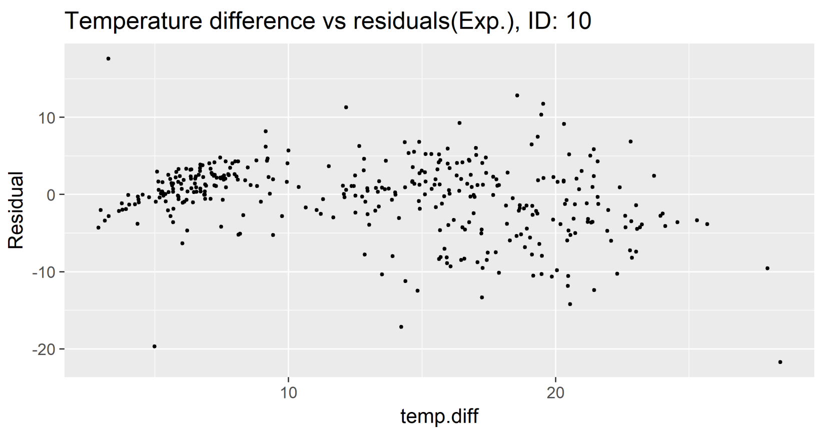


Figure 5.10: FH, Heat: Temperature difference vs 1-step model(Exp.) residuals for ID 10.

The residuals appear to be increasing with the temperature difference, i.e. the model performs worse when the temperature difference is high, as it is in the winter.

Residuals for 1-step model (Var.)

The residuals for the 1-step model using variable forgetting is plotted together with the ACF and PACF in Figure 5.11.

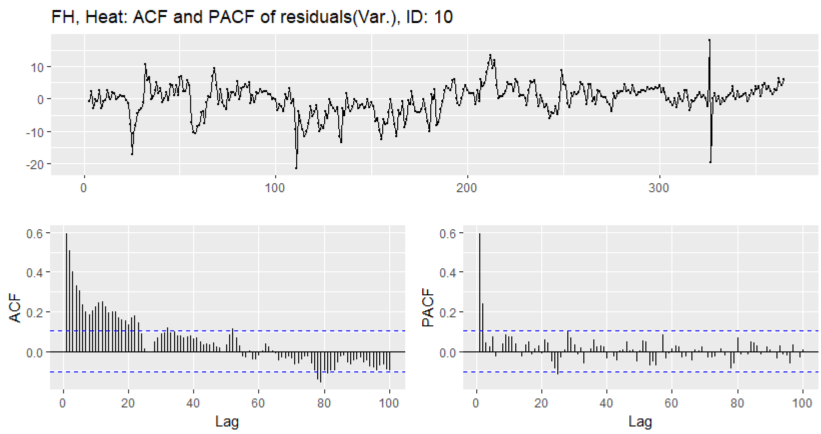


Figure 5.11: FH, Heat: ACF and PACF for 1-step model(Var.) residuals for ID 10.

The residuals are largest during the winter period. The ACF shows that there is some autocorrelation present.

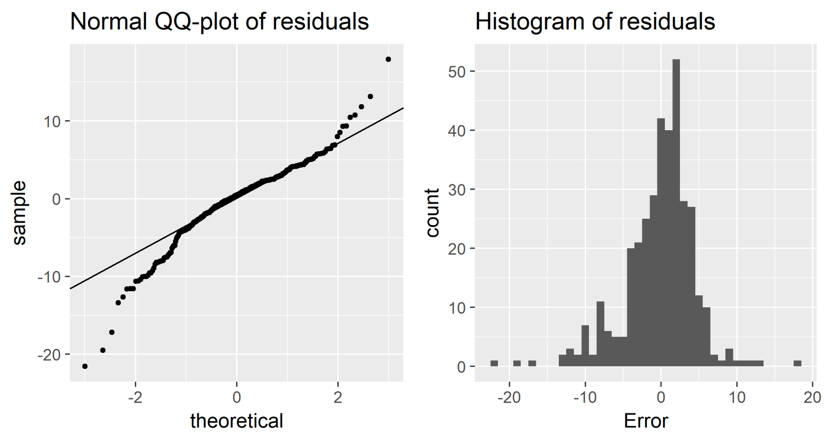


Figure 5.12: FH, Heat: Histogram and QQ-plot of residuals. Variabel forgetting. Note: Unit of Error is [kWh].

Again, the distribution is a little skewed to the left but there also appears to be a heavy tail in the QQ-plot further indicating that the assumption of normality is not completely satisfied.

Finally, the temperature difference is plotted against the residuals, in Figure 5.13.

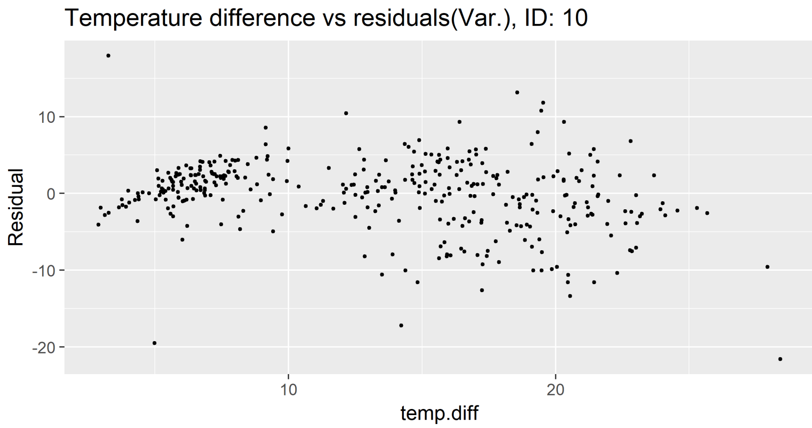


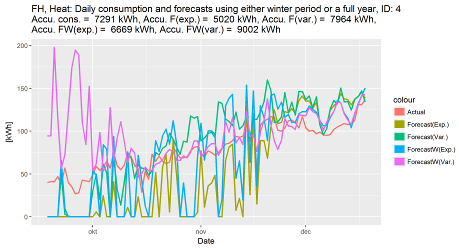
Figure 5.13: FH, Heat: Temperature difference vs 1-step model(Var.) residuals for ID 10.

As seen previously, the residuals increase with the temperature difference. Thus, this model has difficulties during the winter period, as well.

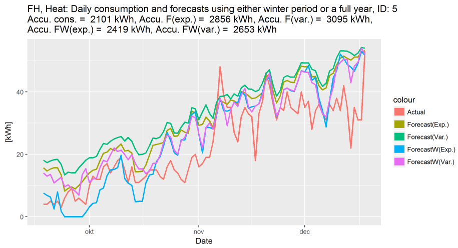
5.1.2 Forecasting heat consumption using winter period

In Section 3.5, weather dependence on heat consumption was investigated using either a full year or a winter period as training data. The same idea is used in this case where forecasting is based on a winter period instead of the full year - it is more about removing the summer period where the heat consumption does not depend on the outside temperature. The winter period is 2016-09-18 - 2017-05-18.

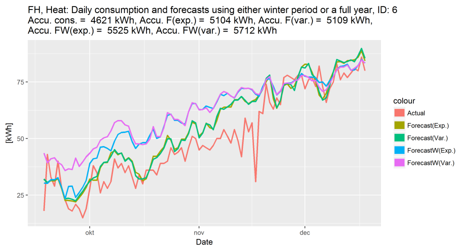
The actual consumption together with the forecasts using either a full year or a winter period, for both methods, for select IDs is shown in Figure 5.14.



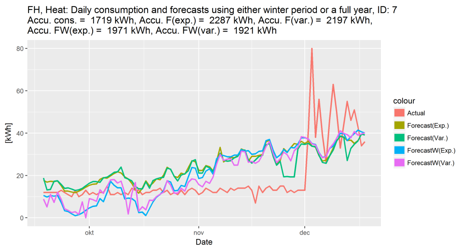
(a) ID 4.



(b) ID 5.



(c) ID 6.



(d) ID 7.

Figure 5.14: FH, Heat: Forecasts of select IDs using either a full year or winter period for training. Note: Different scales on y-axis.

The four plots are a good representation of the results. In many cases the forecasts based on the winter period will be slightly above or below the "full year" forecasts.

The accumulated prediction error for exponential forgetting is calculated for each ID, as forecast minus actual consumption, and the histogram is shown in Figure 5.15. Note: It is the forecasts using either a full year or winter period, that are compared in the figure.

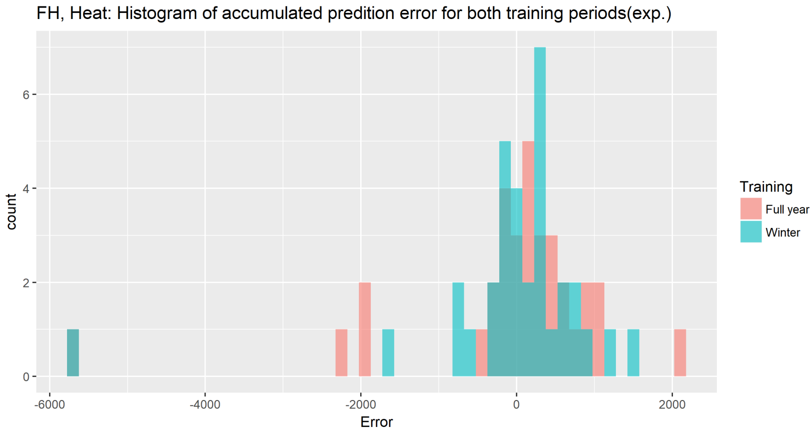


Figure 5.15: FH, Heat: Histogram of accumulated prediction error for both training periods. Exponential forgetting method.

The majority of the errors are larger than zero indicating that many of the forecasts are larger than the actual consumption. Only a few outliers are present. The "full year" accumulated errors appear to be more centered around zero compared to "winter".

This histogram of accumulated prediction error for variable forgetting is shown in Figure 5.16

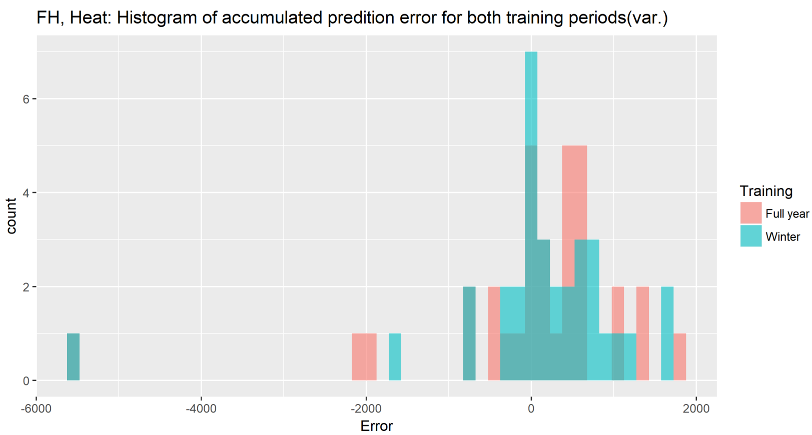


Figure 5.16: FH, Heat: Histogram of accumulated prediction error for both training periods. Variable forgetting method.

The histogram is only slightly different than the previous, which is expected as the forecasts of the two methods are close in most cases. Again, the majority of the errors are larger than zero.

5.1.3 Forecasting water consumption

Due to missing observations, only between 70% and 85% of the observations expected in a year are available for most households.

Actual water consumption and forecasts for select IDs are shown in Figure 5.17.

(a) ID 2.

(b) ID 3.

(c) ID 4.

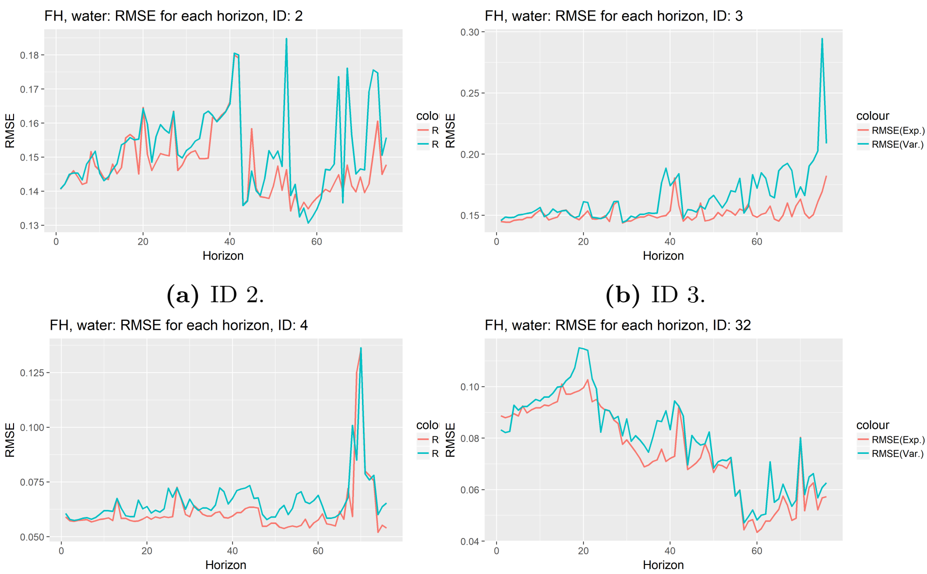
(d) ID 32.

Figure 5.17: FH: Actual water consumption and forecasts of select IDs. Note: Slightly different scales on y-axis.

For all IDs, the consumption varies a lot. Some have long periods of zero consumption most likely due to the residents being away. The exponential forgetting method is mostly constant. In Figure 5.17a and 5.17b, the exponential forgetting method varies a little but stays mostly around 0.2 m^3. The variable forgetting method fluctuates more. In Figure 5.17c, the variable forgetting method is similar to exponential forgetting. In Figure 5.17d, the variable forgetting method varies a little more than exponential forgetting.

Boxplots of prediction errors for all IDs are available in Appendix A.2.9.

Overall, the methods struggle with describing and predicting the dynamics in water consumption - as it is quite random. The RMSE from the training data for all horizons for the four IDs are shown in Figure 5.18.



(a) ID 2.

(b) ID 3.

(c) ID 4.

(d) ID 32.

Figure 5.18: FH: RMSE for training data for each horizon for select IDs. Note: Different scales on y-axis.

Overall, the RMSE graphs look unusual, as RMSE should not be decreasing ahead in time. The RMSE for the exponential forgetting method is lower, suggesting that a mean value might be sufficient for forecasting water consumption.

Histograms of the accumulated prediction error is shown in Figure 5.19.

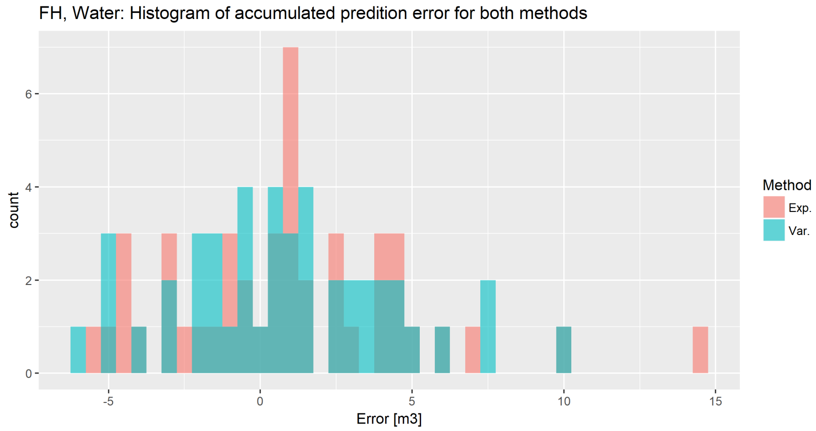


Figure 5.19: FH, Water: Histogram of accumulated prediction error for both methods.

Since the two histograms are mostly on top of each other, it is difficult to distinguish the performance of the two methods.

In Table A.9, the mean RMSE and accumulated forecast for all IDs are available.

Analysis of ID 16

ID 16 has been selected for further analysis because of few zero consumption observations. Water consumption and forecasts are shown in Figure 5.20.

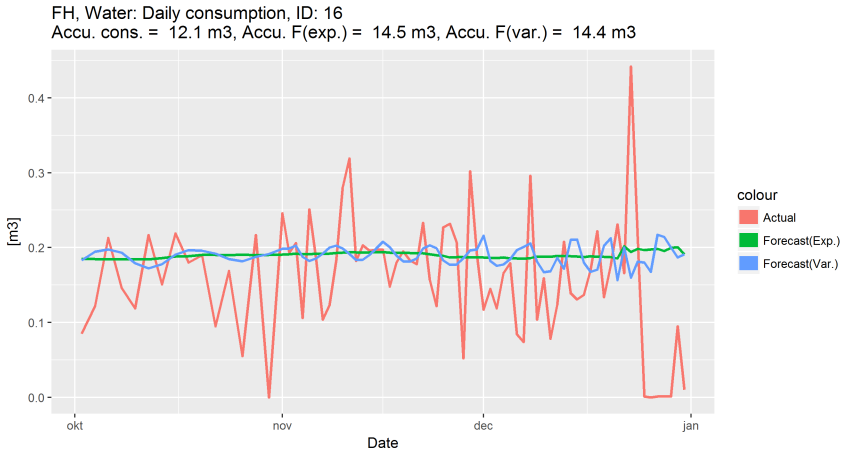
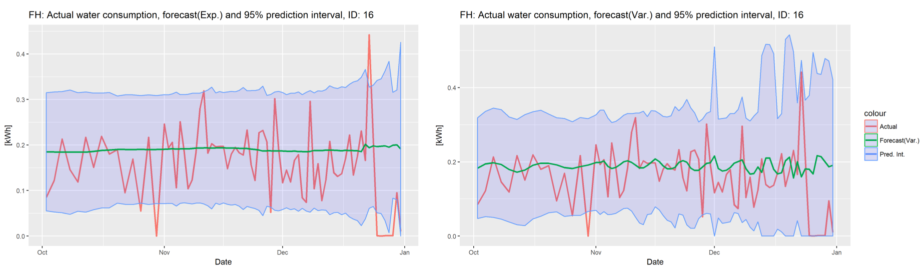


Figure 5.20: FH: Actual water consumption and forecasts for ID 16.

It is not very different from what has been seen previously: The exponential forgetting method is mostly constant where variable forgetting fluctuates more. Figure 5.21 shows forecasts and 95% prediction intervals for both methods. Note that the prediction interval has been restricted to nonnegative values.



(a) Forecast with prediction interval. Exponential forgetting.
(b) Forecast with prediction interval. Variable forgetting.

Figure 5.21: FH: Water forecast with 95% prediction interval.

In Figure 5.21a, the zero consumption periods are not covered by the prediction interval but the remaining observation are. While the interval is wide, it does not increase noticeably with time. In Figure 5.21b, almost all observations are covered by the prediction interval with the exception of a few zero consumption observations. A few large spikes appear in the later half of December.

The 1-step model for both methods is shown in Figure 5.22

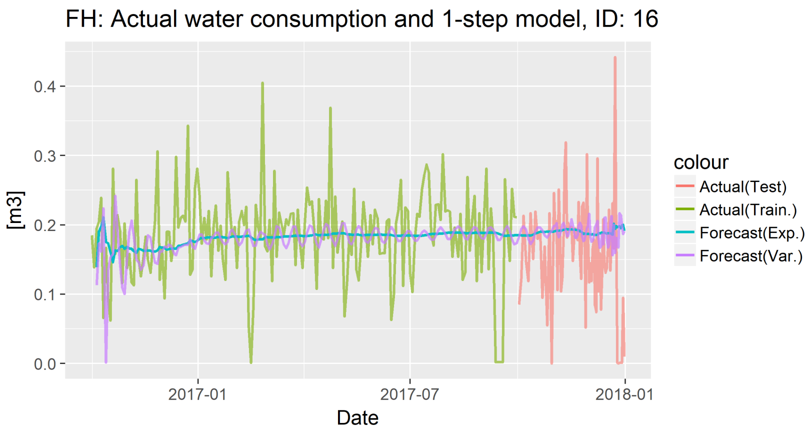
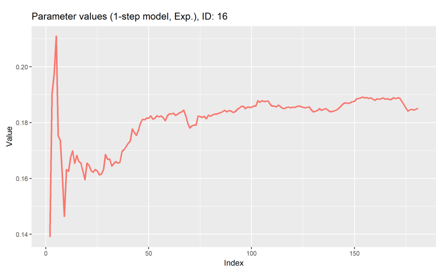


Figure 5.22: FH: Water consumption and 1-step model for ID 16.

The exponential forgetting method is quite steady throughout the training and test periods. The variable forgetting method appears to fluctuate a little around the forecast of exponential forgetting because it includes a sine/cosine pair. Both methods become more stable after December 2016, showing the effect of the burn-in period in the first couple of months.

Parameter values for the 1-step model for both methods are shown in Figure 5.23.



(a) Parameter values of 1-step model in training period.

(b) Parameter values of 1-step model in training period.

Figure 5.23: FH, Water: Parameter values of 1-step models in training period. Note: Different scales on y-axis.

In Figure 5.23a, the only parameter is the intercept and the value is somewhat steady after 50 observations. In Figure 5.23b, one sine/cosine pair has been included besides the intercept. All three parameters are steady after 50 observations.

Residuals for 1-step model (Exp.)

A QQ-plot and histogram of the residuals of the exponential forgetting method is shown in Figure 5.24.

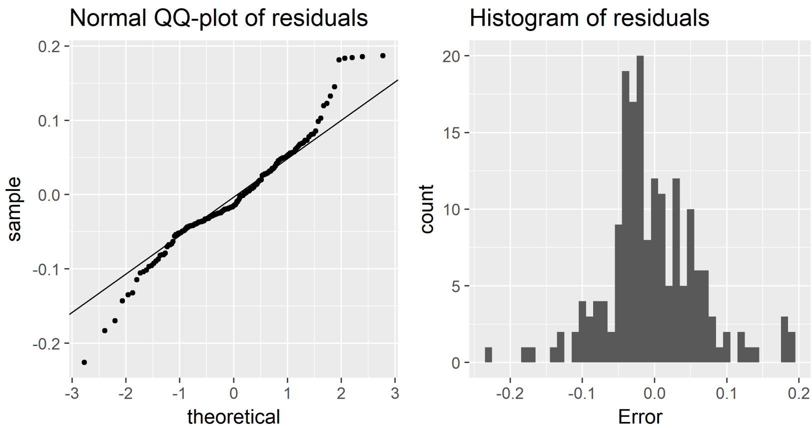


Figure 5.24: FH, Water: Histogram and QQ-plot of residuals. Exponential forgetting. Note: Unit of Error is [m^3].

The distribution of the residuals is a little skewed to the left. The QQ-plot shows this as well, suggesting that the assumption of normality is not satisfied.

The residuals, ACF and PACF are shown in Figure 5.25.

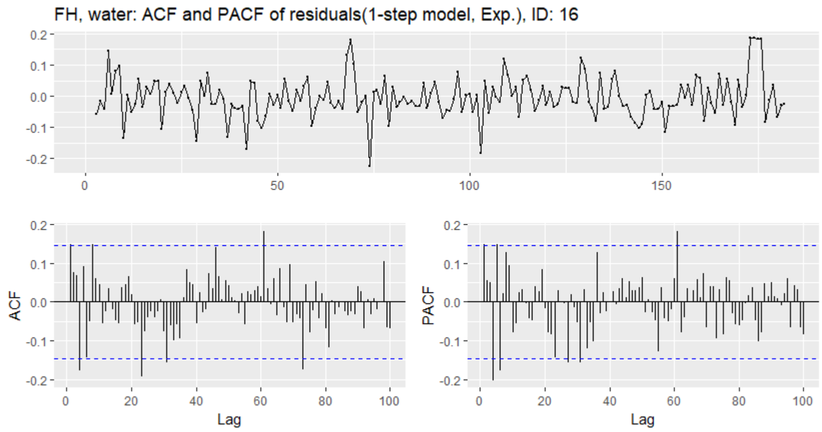


Figure 5.25: FH, Water: ACF and PACF for 1-step model(Exp.) residuals for ID 16.

The residuals are quite large due to the model being mostly constant around 0.2 m^3 and the consumption ranges from 0 m^3 to 0.4 m^3. The ACF and PACF look like white noise.

Residuals for 1-step model (Var.)

A QQ-plot and histogram of the residuals of the variable forgetting method is shown in Figure 5.26.

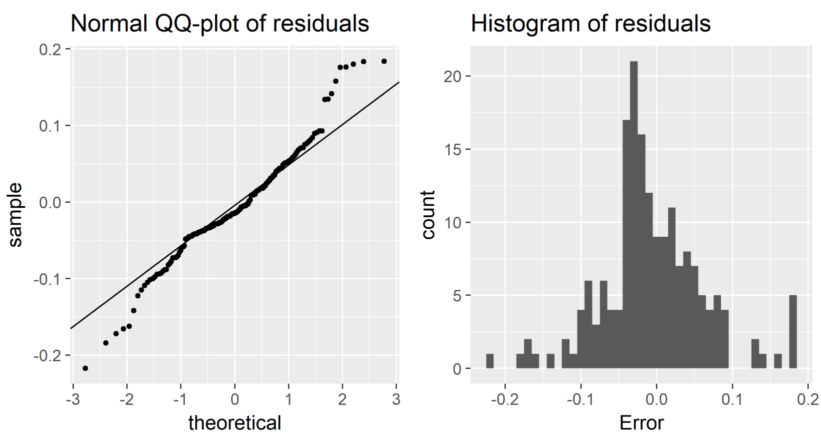


Figure 5.26: FH, Water: Histogram and QQ-plot of residuals. Variable forgetting. Note: Unit of Error is [m^3].

The distribution of the residuals are a little skewed to the left. The QQ-plot shows this as well, suggesting that the assumption of normality is not satisfied.

The residuals, ACF and PACF are shown in Figure 5.27.

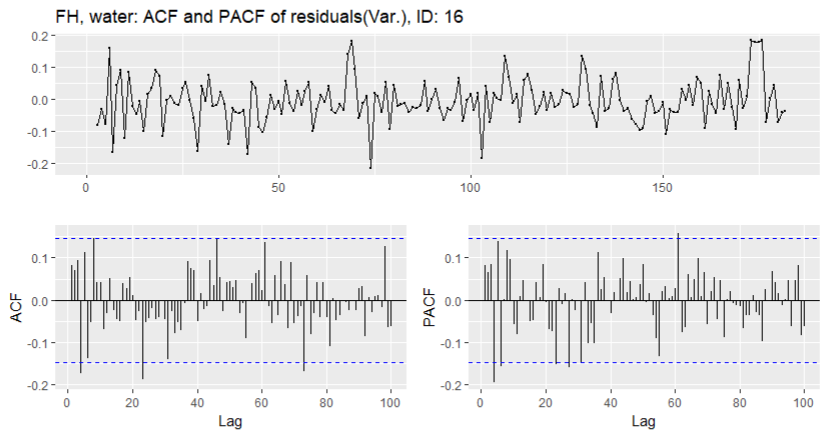
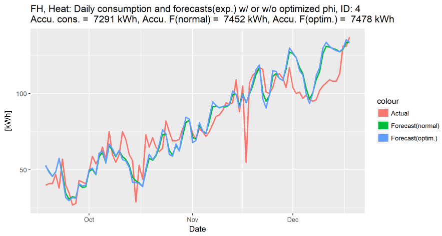


Figure 5.27: FH, Water: ACF and PACF for 1-step model(Var.) residuals for ID 16.

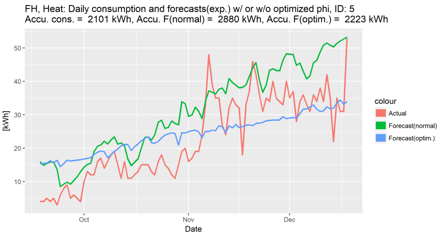
The residuals are quite large, even though the model tries to capture all the variation in the observations. The ACF and PACF look like white noise.

5.1.4 Optimizing ϕ in low-pass filter

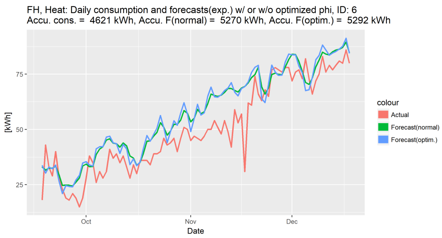
Section 2.3 introduces low-pass filtering of the temperature difference. The filter uses $\phi = 0.95$ as a smoothing factor and this value is currently fixed for all households. The following is the results from optimizing the smoothing factor for each household and then use RLS to compute the forecasts. Figure 5.28 shows forecasts from using the normal fixed value of ϕ and the optimized value.



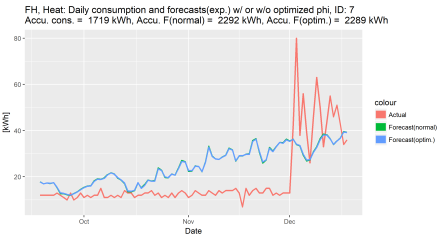
(a) ID 4.



(b) ID 5.



(c) ID 6.



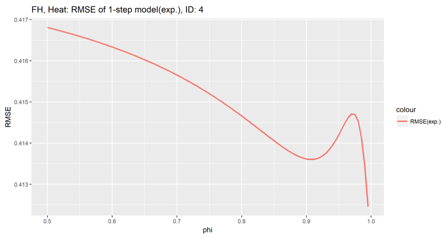
(d) ID 7.

Figure 5.28: FH: Actual heat consumption and forecasts(exp.) of select IDs using normal or optimized ϕ. Note: Different scales on y-axis.

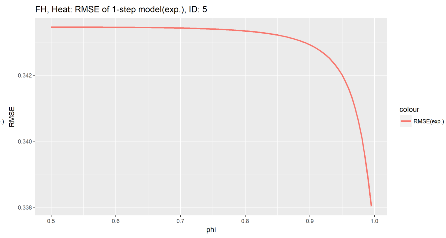
The four plots represent the overall result quite well. With the exception of Figure 5.28b, the "optimized ϕ" forecasts only differ slightly from the "normal ϕ" forecasts. In Figure 5.28b the forecast is completely different and performs poorly. The "optimized ϕ" forecasts using exponential forgetting will either be almost identical to "normal ϕ" forecasts or be worse.

In Table A.6, the mean RMSE and accumulated forecast for all IDs are available.

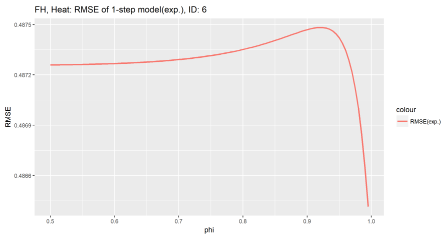
In Figure 5.29 the RMSE as a function of ϕ is plotted for the select IDs to see which value of ϕ minimizes the RMSE. 100 values of $\phi \in [0.5, 0.999]$ has been used.



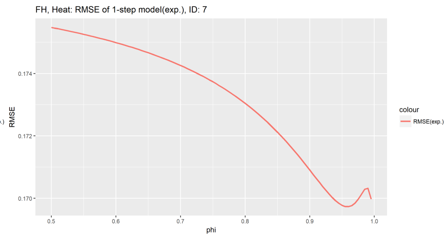
(a) ID 4.



(b) ID 5.



(c) ID 6.

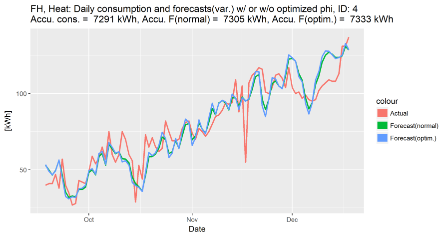


(d) ID 7.

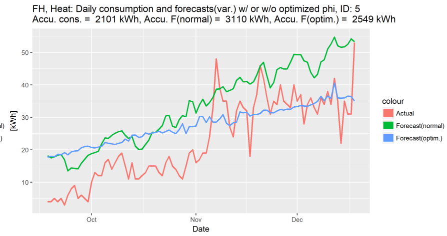
Figure 5.29: FH, Heat: RMSE as a function of ϕ for select IDs. Note: Different scales on y-axis.

For Figures 5.29a, 5.29b and 5.29c: The RMSE is lowest when $\phi \approx 0.99$ which is slightly higher than the standard value of 0.95. In Figure 5.29d, the RMSE is lowest at $\phi \approx 0.96$.

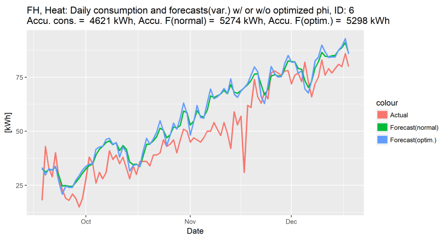
In Figure 5.30, the forecasts using variable forgetting are shown.



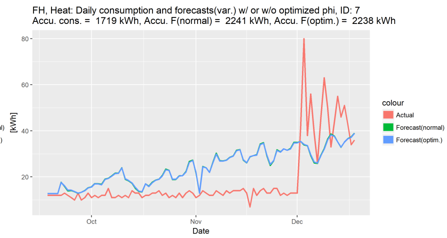
(a) ID 4.



(b) ID 5.



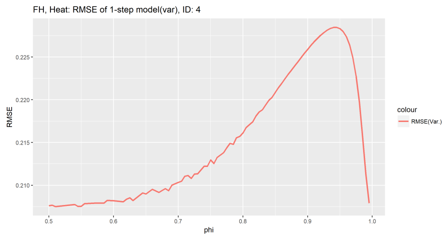
(c) ID 6.



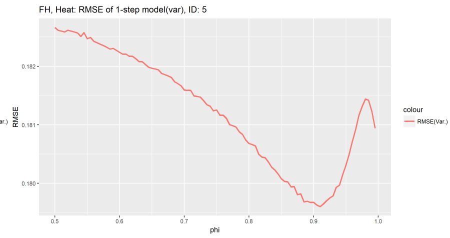
(d) ID 7.

Figure 5.30: FH: Actual heat consumption and forecasts(var.) of select IDs using fixed or optimized ϕ. Note: Different scales on y-axis.

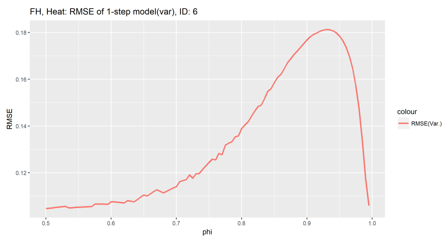
The results are similar to the ones of exponential forgetting. The forecasts are either very similar, if not identical, or worse when using "optimized ϕ". The RMSE are plotted in Figure 5.31.



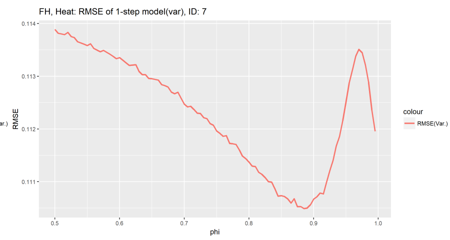
(a) ID 4.



(b) ID 5.



(c) ID 6.



(d) ID 7.

Figure 5.31: FH, Heat: RMSE as a function of ϕ for select IDs. Note: Different scales on y-axis.

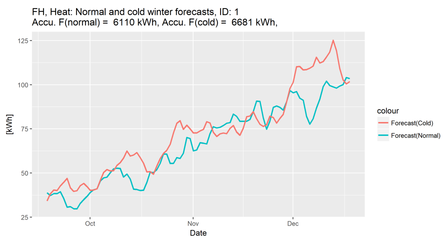
The curves are not as smooth as exponential forgetting. In Figures 5.31a and 5.31c, the RMSE is lowest when $\phi \approx 0.99$. In Figures 5.31b and 5.31d, the RMSE is lowest at $\phi \approx 0.90$.

5.1.5 Cold winter forecasts

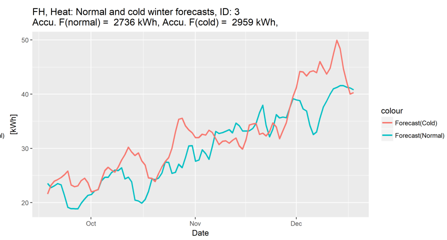
An idea for a feature in Watts: Beside regular forecasts of heat consumption, a "cold winter" forecast could be provided. If a winter period is expected to be colder than the 30 year average, the heat consumption of a household is expected to increase and thus is the expenses of keeping warm. Given a model for a household, forecasting is based on weather data from a "cold winter" model. The forecasts should show higher consumption during the winter and give the user an idea of how much extra heat is needed.

In this example, the cold winter data is represented by the winter of 2012/2013 as this data was available and this winter is the coldest winter in Denmark in the past 5 years[7].

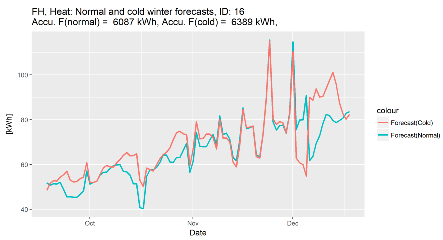
Figure 5.32 shows normal and cold winter forecasts of select IDs using exponential forgetting.



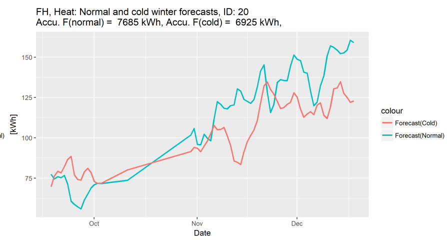
(a) ID 1.



(b) ID 3.



(c) ID 16.



(d) ID 20.

Figure 5.32: FH, Heat: Normal and cold winter forecasts of select IDs using exponential forgetting. Note: Different scales on y-axis.

Most of the cold winter forecasts look as expected. The forecasts are noticeably higher in December where the temperature is usually low. However, in Figure 5.32d, the cold winter forecast is lower than the normal forecast in November and December. Still, the idea shows potential for giving users insight about heat consumption during colder winters.

5.2 Conclusion

Forecasts of heat consumption are accurate overall. Change in occupant behavior is difficult to capture, whether it is leaving home for a day or two, or turning on the heat later than expected. Reducing the training period to winter data for forecasting winter months did not improve the forecasts significantly. Optimizing ϕ in the low-pass filter did not improve forecasts and in some cases the results were worse than before. Cold winter forecasts show potential for providing insight to users about heat consumption during colder winters. Water consumption is very random and difficult to model. The water forecasts give decent results on a monthly level.

Chapter 6

Results: BECA

In this section the results obtained with data from Catalonia, Spain are presented. This includes forecasts of heat and water consumption using the two methods: RLS with exponential or variable forgetting. Finally, a single ID is used for a detailed analysis.

The overall results are presented with the following elements:

- Actual consumption compared to forecasts.
- RMSE for each horizon is presented.
- Histogram of accumulated prediction error.

A single ID has been chosen for detailed analysis, presented with the following elements:

- Actual consumption and forecasts for chosen ID.
- 95% prediction interval for both methods.
- Plot of parameter values of 1-step model for each RLS step.

- Plot fit and forecast of 1-step model on full data set.
- Residuals: Distribution, correlation and temperature dependence(if applicable).

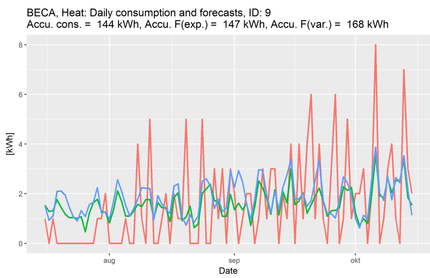
6.1 Forecasts

6.1.1 Forecasting heat consumption

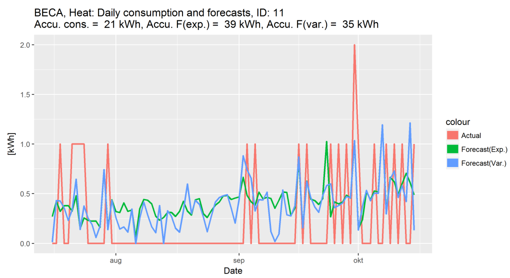
The weather variables that can be included in the model are: *temp.diff* and *Radiation*. The method can also include up to 3 pairs of sine/cosine pairs.

1-step models for all apartments are listed in Table B.4 and Table B.5 for exponential and variable forgetting methods, respectively.

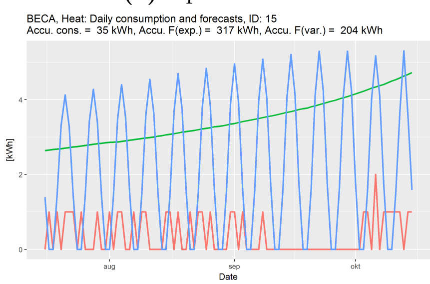
Forecasts and consumption for select apartments are presented in Figure 6.1.



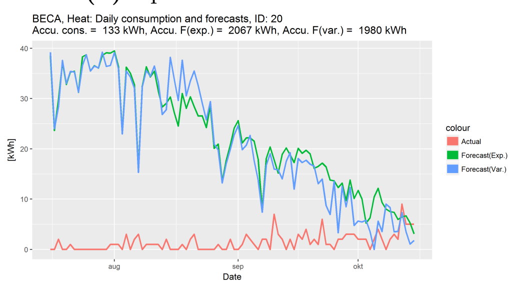
(a) Apartment 9.



(b) Apartment 11.



(c) Apartment 15.



(d) Apartment 20.

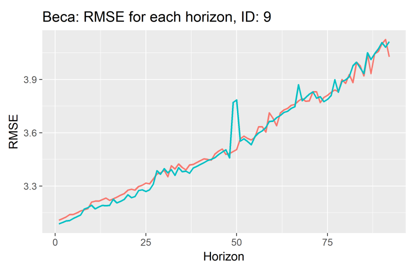
Figure 6.1: BECA: Actual heat consumption and forecasts of select apartments. Note: Different scales on y-axis.

In Figure 6.1a, the forecast follows the trend somewhat. The method has difficulties with long periods of zero consumption. The method with variable forgetting varies more than the method using exponential forgetting. This is an example where the methods perform quite well. In Figure 6.1b, again, the method struggles with zero consumption periods but also an overall low heat consumption. The forecasts are mostly between 0 and 1 where only integers are observed in the data. In Figure 6.1c, both methods completely miss by a significant margin. This can be due to a change in behavior in the apartment and this is very difficult to account for in the methods. The exponential forgetting method has chosen a mean value for the entire period, unfortunately, it is quite larger than the actual consumption. In Figure 6.1d, the forecasts are much higher than the actual consumption. This can, again, be due to a change in behavior.

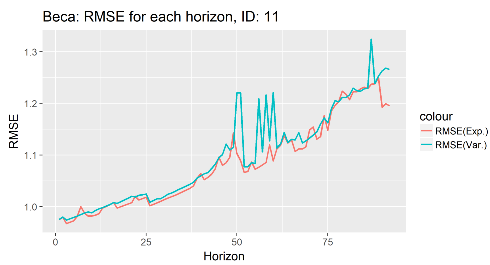
In Table B.6, the mean RMSE and accumulated forecast for all IDs are available.

Boxplots of prediction errors for all IDs are available in Appendix B.2.4.

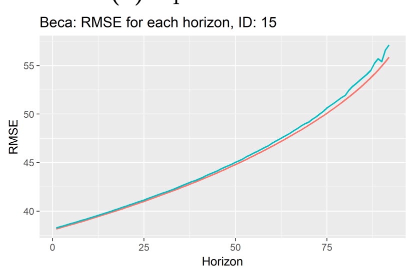
The RMSE from the training data is plotted is Figure 6.2 for each horizon for both methods for the same apartments as previously.



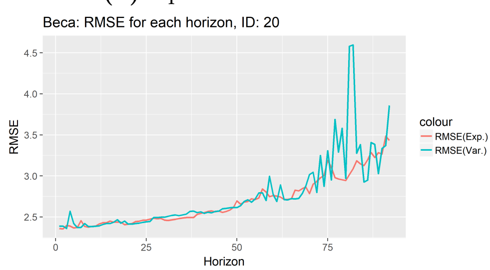
(a) Apartment 9.



(b) Apartment 11.



(c) Apartment 15.



(d) Apartment 20.

Figure 6.2: BECA: RMSE for each horizon for select apartments. Note: Different scales on y-axis.

In general, the RMSE is expected to increase with the horizon as the uncertainty increases when forecasting further ahead. In Figure 6.2a, the RMSE for both methods are very close with the exception of horizon ≈ 50 where the variable forgetting method has a larger RMSE. In Figure 6.2b, the RMSE takes a small dive after horizon 25 but overall it is increasing. Again, the variable forgetting method has some problems after horizon 50. In Figure 6.2c, the RMSE looks very smooth and increasing steadily with the horizon. The RMSE for exponential forgetting is slightly lower than variable forgetting. However, the RMSE is very large. In Figure 6.2d, the RMSE looks okay until horizon ≈ 60 where variable forgetting has some large spikes. This could be due to a change in behavior already happening in the end of the training data.

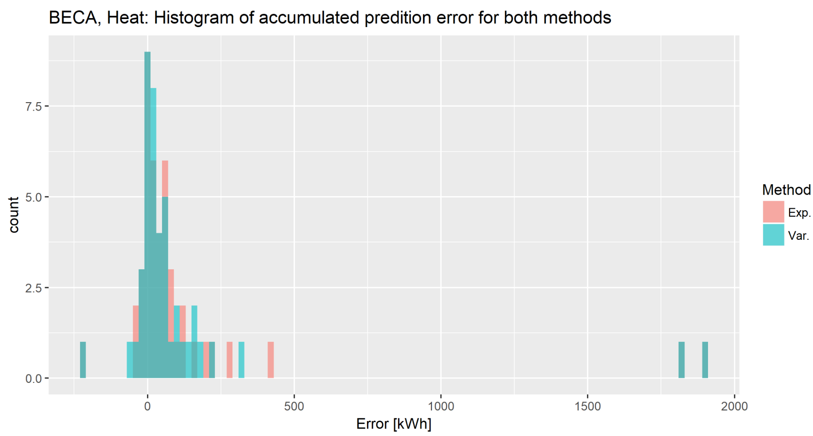


Figure 6.3: BECA, Heat: Histogram of accumulated prediction error for both methods.

The two histograms are very similar and makes it difficult to distinguish the performance of the two methods.

Outliers are IDs 20 and 32 which both have few non-zero observations and sometimes daily consumption of more than 10 kWh. These IDs are examples of where the method does not perform well.

Analysis of apartment 29

To get a better understanding of the performance of the methods, a single apartment has been chosen for a more detailed analysis - apartment 29.

In Figure 6.4, the consumption and forecasts are shown.

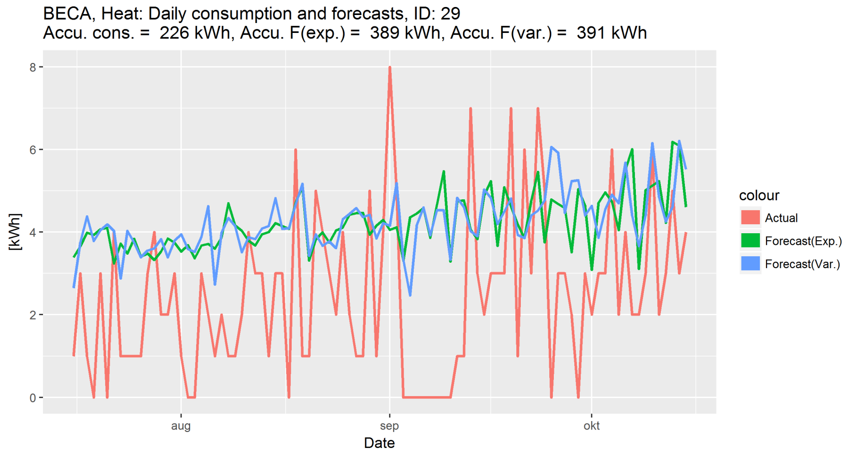
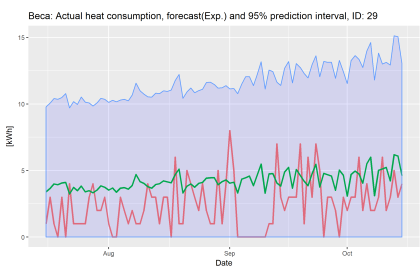


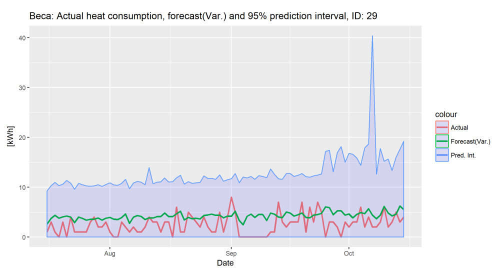
Figure 6.4: BECA: Actual heat consumption and forecasts for apartment 29.

The forecasts are perhaps slightly higher than the actual consumption but the methods have captured the trend quite well, overall. There is a period of zero consumption that was obviously not expected.

Figure 6.5 shows the forecasts with 95% prediction intervals, based on the normal distribution. It should be noted that the interval has been restricted to positives values, as negative consumption can not be observed.



(a) Forecast with prediction interval. Exponential forgetting.



(b) Forecast with prediction interval. Variable forgetting.

Figure 6.5: BECA: Heat forecast with 95% prediction interval. Note: Different scales on y-axis.

For both methods, all observations are within the prediction interval. In Figure 6.5b, there is a large spike in the prediction interval in the middle of October. This could be uncertainty from the training data.

In the following, the 1-step model for both methods has been selected for analysis. Figure 6.6 shows the fit and forecast of the 1-step model for both methods.

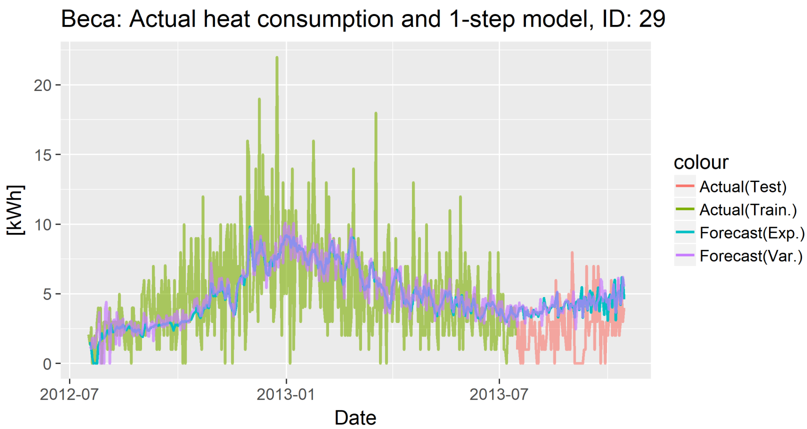
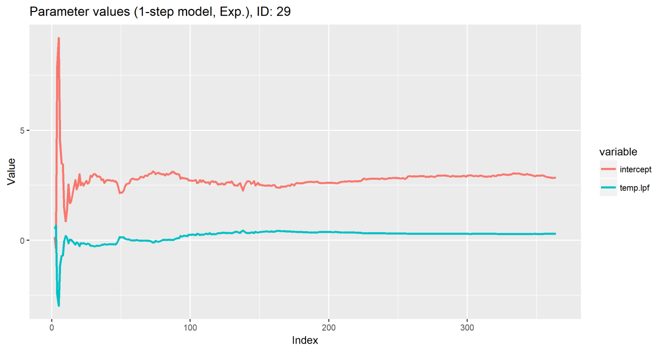


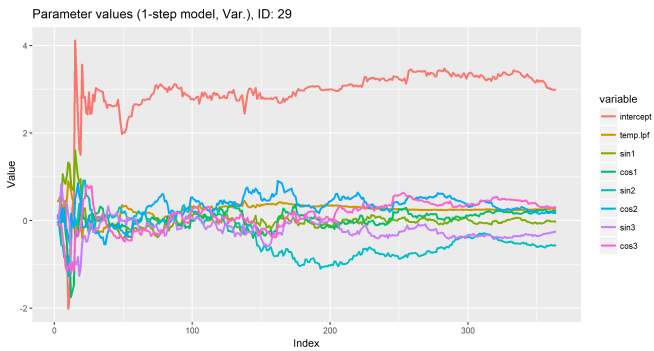
Figure 6.6: BECA: Heat consumption and 1-step model for apartment 29.

The models follow the trend in the data well. The 1-step model appears to perform worse than the k-step model for large horizons. The variable forgetting method fluctuates more than the exponential forgetting method.

The estimated parameters are plotted for both methods in Figure 6.7.



(a) Parameter values of 1-step model(exp.) in training period.



(b) Parameter values of 1-step model(var.) in training period.

Figure 6.7: BECA, Heat: Parameter values of 1-step models. Note: Different scales on y-axis.

For both methods, the burn-in period for the parameters is roughly 50 observations, then the estimates become more steady. Note that the exponential forgetting method only uses and intercept and the temperature difference and variable forgetting uses 3 sine/cosine pairs. The estimates for variable forgetting varies quite a lot during the whole period.

Residuals for 1-step model (Exp.)

The residuals for the 1-step model using exponential forgetting is plotted together with the ACF and PACF in Figure 6.8.

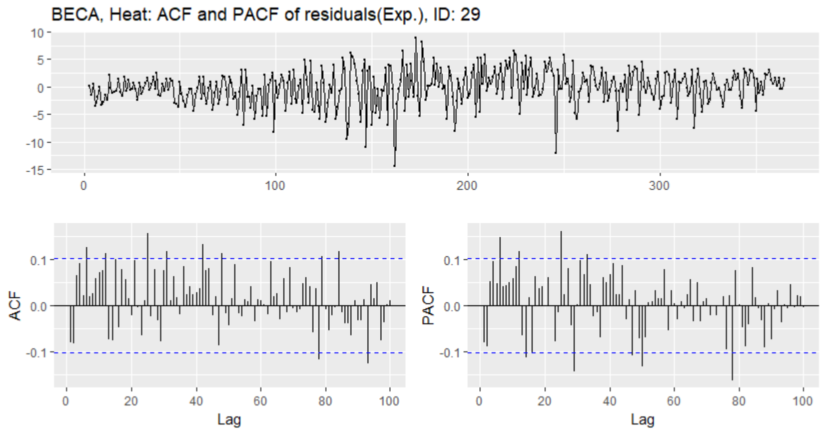


Figure 6.8: BECA, Heat: ACF and PACF for 1-step model(Exp.) residuals for apartment 29.

The residuals are larger in the winter period and smaller in the warmer months. Both ACF and PACF look like white noise. Due to the chosen training period and the climate in Spain, the winter period, i.e. the period where heat consumption may depend on the outside temperature, is short. Outside of this period there should not be any correlation between the observations, as can be seen in the ACF.

Figure 6.9 shows a QQ-plot and a histogram of the residuals

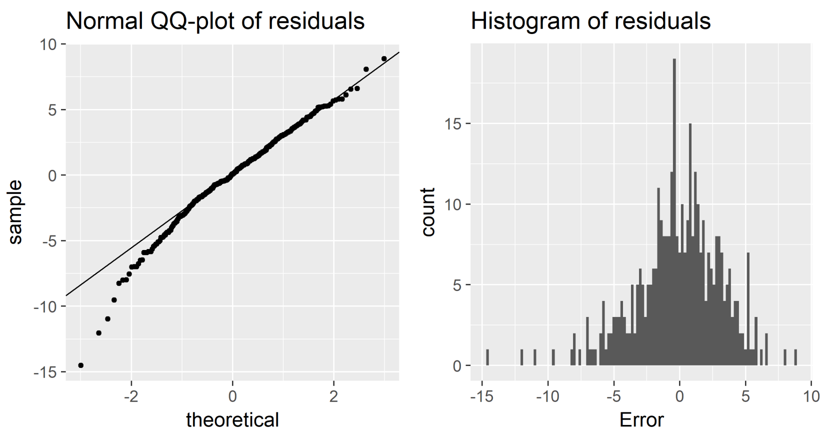


Figure 6.9: BECA, Heat: Histogram and QQ-plot of residuals. Exponential forgetting. Note: Unit of Error is [kWh].

The distribution of the residuals is a little skewed to the left indicating that the assumption of normality is not completely satisfied. In Figure 6.10 the temperature difference is plotted agains the residuals.

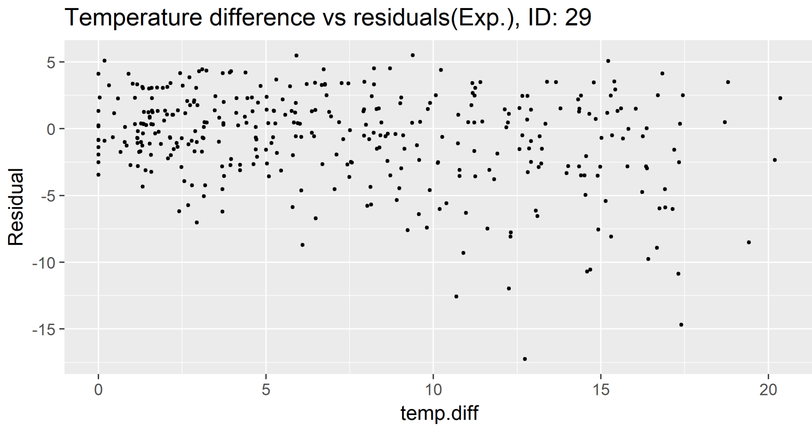


Figure 6.10: BECA, Heat: Temperature difference vs 1-step model(Exp.) residuals for apartment 29.

The residuals appear to be increasing with the temperature difference, i.e. the model performs worse when the temperature difference is high, as it is in the winter.

Residuals for 1-step model (Var.)

The residuals for the 1-step model using variable forgetting is plotted together with the ACF and PACF in Figure 6.11.

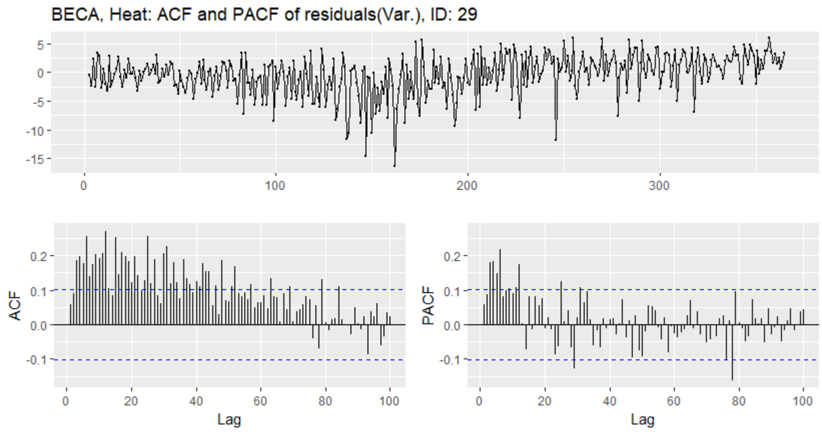


Figure 6.11: BECA, Heat: ACF and PACF for 1-step model(Var.) residuals for apartment 29.

The residuals are largest during the winter period. The ACF shows that there is some autocorrelation present.

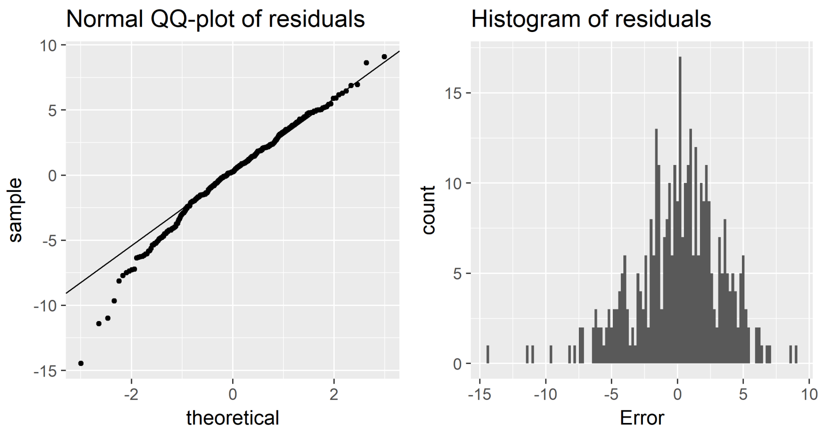


Figure 6.12: BECA, Heat: Histogram and QQ-plot of residuals. Variabel forgetting. Note: Unit of Error is [kWh].

Again, the distribution is a little skewed to the left but there also appears to be a heavy tail in the QQ-plot further indicating that the assumptions of normality is not completely satisfied.

Finally, the temperature difference is plotted against the residuals, in Figure 6.13.

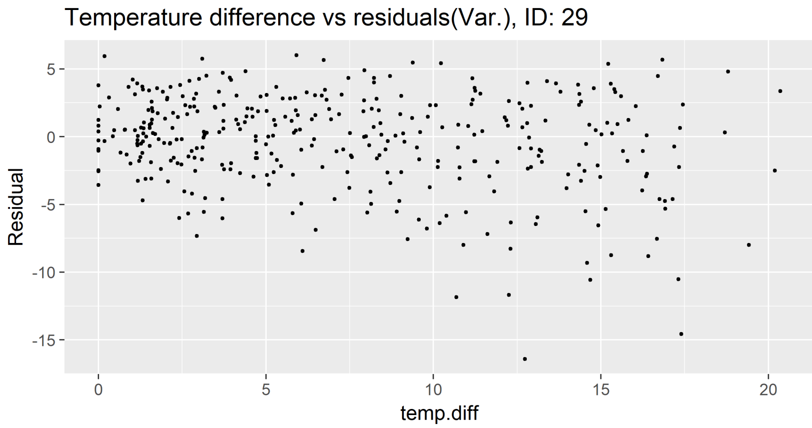
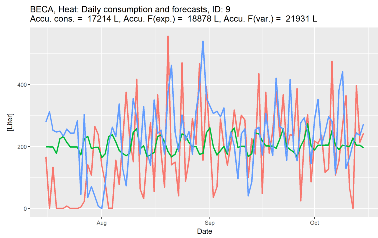


Figure 6.13: BECA, Heat: Temperature difference vs 1-step model(Var.) residuals for apartment 29.

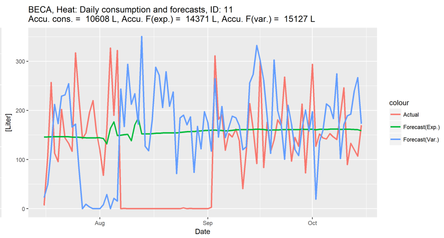
As seen previously, the residuals increase with the temperature difference. Thus, this model has difficulties during the winter period, as well.

6.1.2 Forecasting water consumption

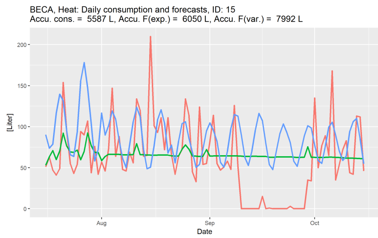
Actual water consumption and forecasts for select apartments are shown in Figure 6.14.



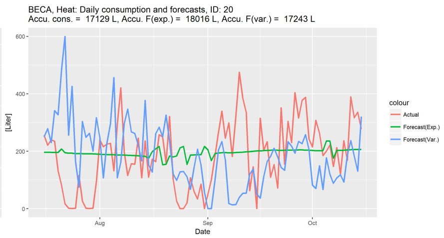
(a) Apartment 9.



(b) Apartment 11.



(c) Apartment 15.



(d) Apartment 20.

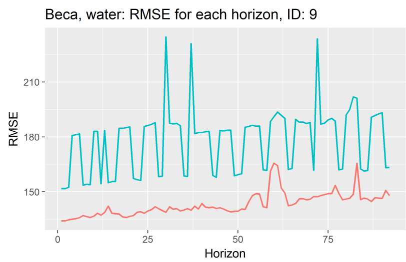
Figure 6.14: BECA: Actual water consumption and forecasts of select apartments. Note: Different scales on y-axis.

For all apartments, the consumption varies a lot. Some have long periods of zero consumption most likely due to the residents being away. The exponential forgetting method is mostly constant. In Figure 6.14a, the exponential forgetting method varies a little but stays mostly around 200 liters. The variable forgetting method fluctuates more. In Figure 6.14b, there is a long period of zero consumption and the variable forgetting method forecasts a zero consumption period just before that. In Figure 6.14c, the variable forgetting method forecasts low water consumption in September.

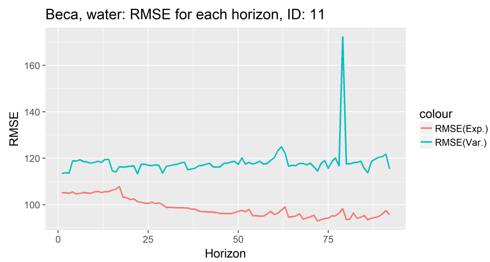
Overall, the methods struggle with describing and predicting the dynamics in water consumption.

Boxplots of prediction errors for all IDs are available in Appendix B.2.9.

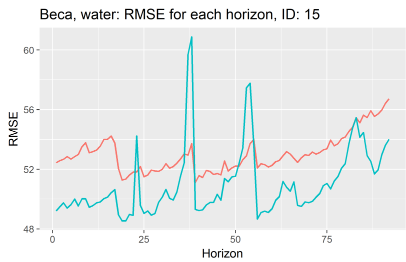
The RMSE for the training data for all horizons for the four apartments are shown in Figure 6.15.



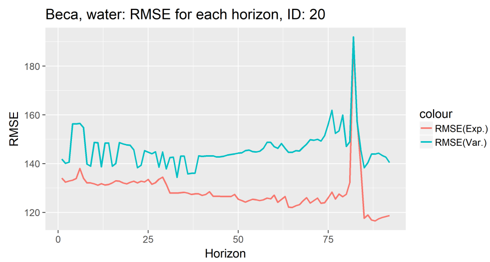
(a) Apartment 9.



(b) Apartment 11.



(c) Apartment 15.



(d) Apartment 20.

Figure 6.15: BECA: RMSE for each horizon for select apartments. Note: Different scales on y-axis.

Overall, the RMSE graphs look unusual. With the exception of apartment 15 in Figure 6.15c, the RMSE for the exponential forgetting method is lower, suggesting that a mean value may be sufficient for forecasting water consumption.

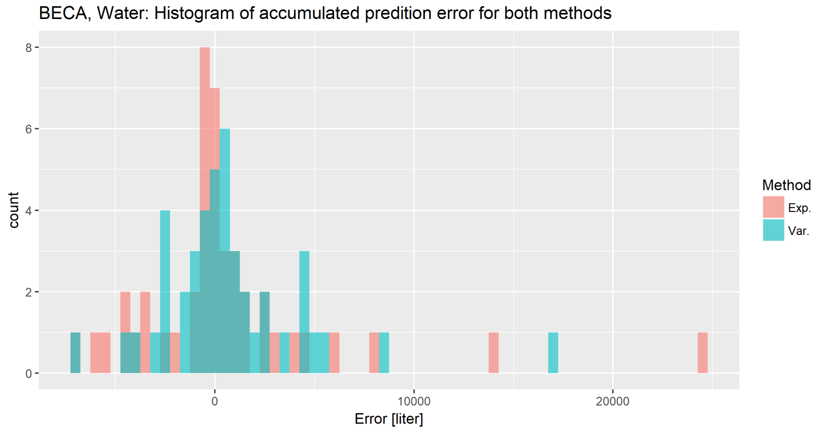


Figure 6.16: BECA, Water: Histogram of accumulated prediction error for both methods.

Apart from a couple of outliers the histogram of exponential forgetting appears to be a bit more narrow, suggesting that a mostly constant mean value is sufficient.

Outlier is ID 23 which has almost zero water consumption in the test period where the forecast is much higher.

In Table B.8, the mean RMSE and accumulated forecast for all IDs are available.

Analysis of apartment 29

Apartment has been selected for further analysis. Water consumption and forecasts are shown in Figure 6.17

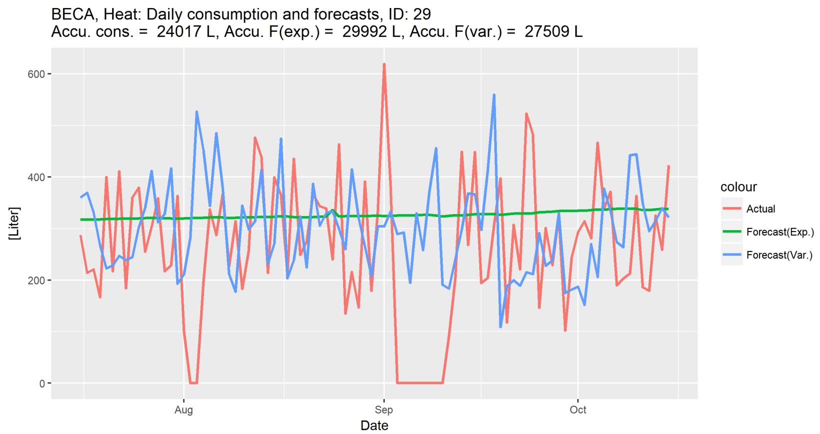
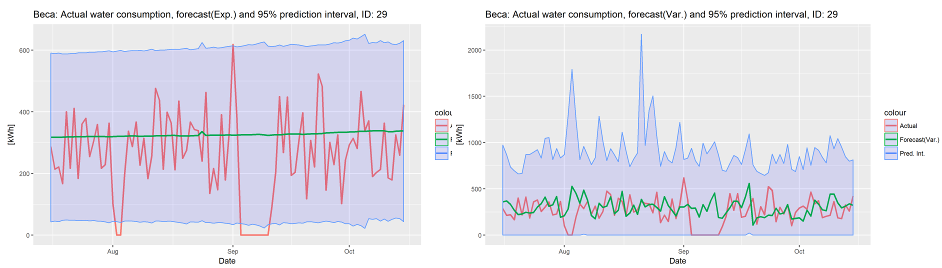


Figure 6.17: BECA: Actual water consumption and forecasts for apartment 29.

It is not very different from what has been seen previously: The exponential forgetting method is mostly constant where variable forgetting fluctuates more. Figure 6.18 shows forecasts and 95% prediction intervals for both methods.



(a) Forecast with prediction interval. Exponential forgetting.

(b) Forecast with prediction interval. Variable forgetting.

Figure 6.18: BECA: Water forecast with 95% prediction interval. Note: Different scales on y-axis.

In Figure 6.18a, the zero consumption periods are not covered by the prediction interval but the remaining observation are. While the interval is wide, it does not increase noticeably with time, as one would expect. In Figure 6.18b, all observations are covered by the prediction interval. Some large spikes appear in the beginning and end of August.

The 1-step model for both methods is shown in Figure 6.19

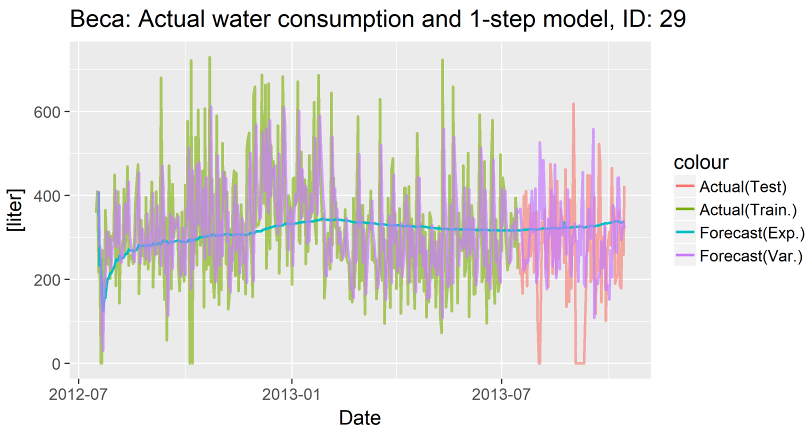
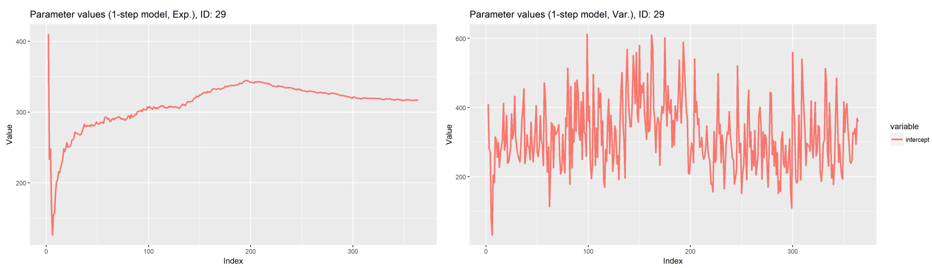


Figure 6.19: BECA: Water consumption and 1-step model for apartment 29.

The exponential forgetting method is steady throughout the training and test periods. The variable forgetting method appears to cover most of the observations on the training period and this could be a sign of overfitting as this does not carry over in the test period. As a result, the variable forgetting method performs poorly.

Parameter values for the 1-step model for both methods are shown in Figure 6.20.



(a) Parameter values of 1-step model in training period.

(b) Parameter values of 1-step model in training period.

Figure 6.20: BECA, Water: Parameter values of 1-step models in training period. Note: Different scales on y-axis.

In Figure 6.20a, the only parameter is the intercept and the value stabilizes after roughly 200 observations. In Figure 6.20b, again, the only parameter is the intercept but the value does not converge. It appears that the variable forgetting period has difficulties getting good parameter estimates when there is no trend in the observations.

Residuals for 1-step model (Exp.)

A QQ-plot and histogram of the residuals of the exponential forgetting method is shown in Figure 6.21.

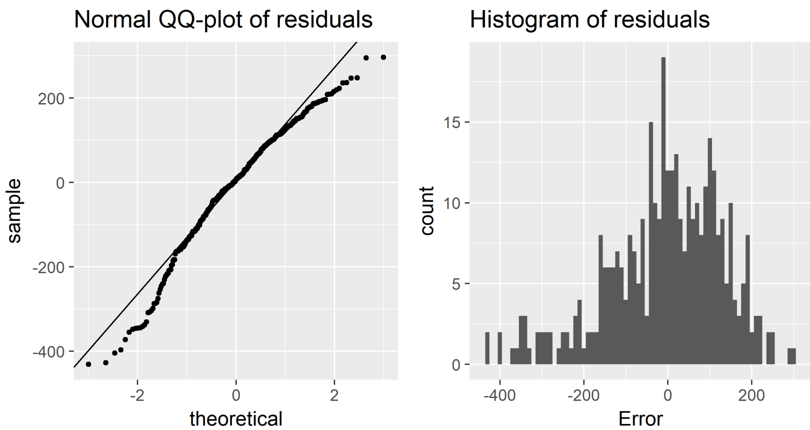


Figure 6.21: BECA, Water: Histogram and QQ-plot of residuals. Exponential forgetting.

The distribution of the residuals are a little skewed to the left. The QQ-plot shows this as well, suggesting that the assumption of normality is not satisfied.

The residuals, ACF and PACF are shown in Figure 6.22.

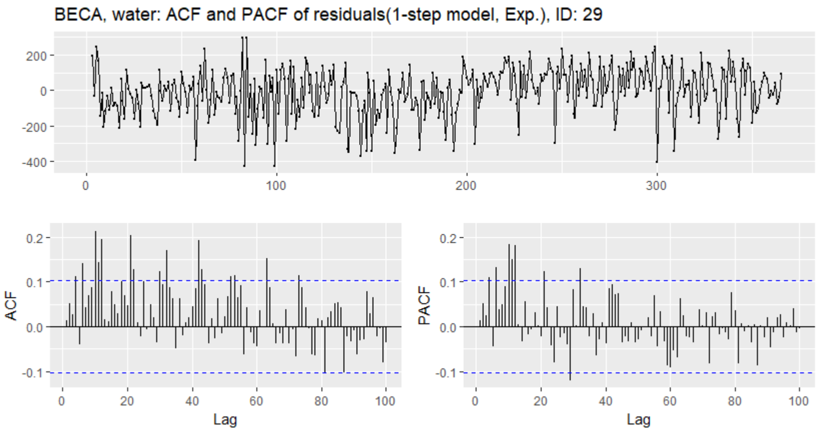


Figure 6.22: BECA, Water: ACF and PACF for 1-step model(Exp.) residuals for apartment 29.

The residuals are quite large, as one would expect, due to the model being mostly constant around 300 liters daily and the daily consumption ranges from 0 liters to 650 liters. The ACF and PACF look like white noise.

Residuals for 1-step model (Var.)

A QQ-plot and histogram of the residuals of the exponential forgetting method is shown in Figure 6.23.

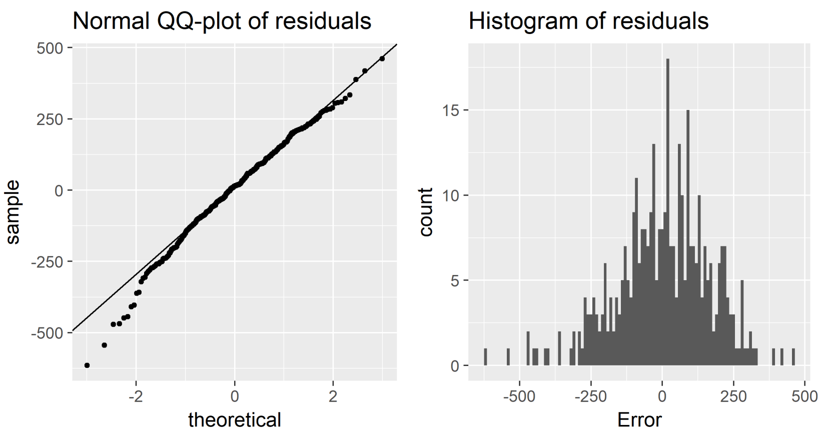


Figure 6.23: BECA, Water: Histogram and QQ-plot of residuals. Variable forgetting.

The distribution of the residuals are a little skewed to the left. The QQ-plot shows this as well, suggesting that the assumption of normality is not satisfied.

The residuals, ACF and PACF are shown in Figure 6.24.

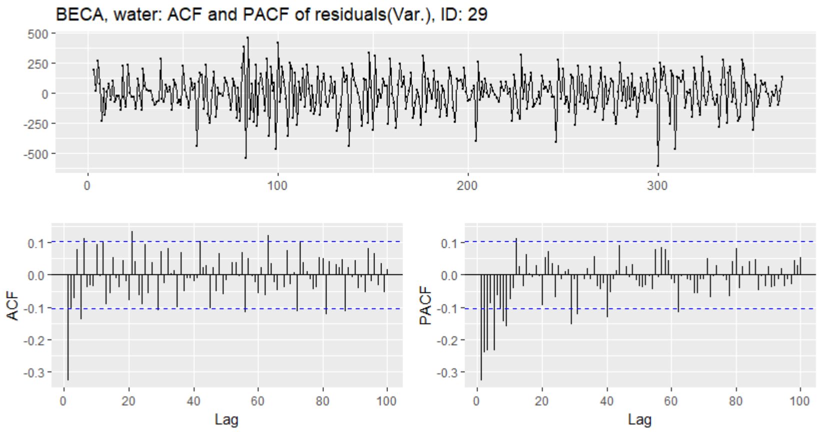


Figure 6.24: BECA, Water: ACF and PACF for 1-step model(Var.) residuals for apartment 29.

The residuals are quite large, even though the model tries to capture all the variation in the observations. The ACF shows dependency on lag 1.

6.1.3 Optimizing ϕ in low-pass filter

The following is the results from optimizing the smoothing factor for each apartment and then use RLS to compute the heat forecasts. Figure 6.25 shows forecasts from using the normal fixed value of ϕ and the optimized value.

(a) Apartment 9.

(b) Apartment 11.

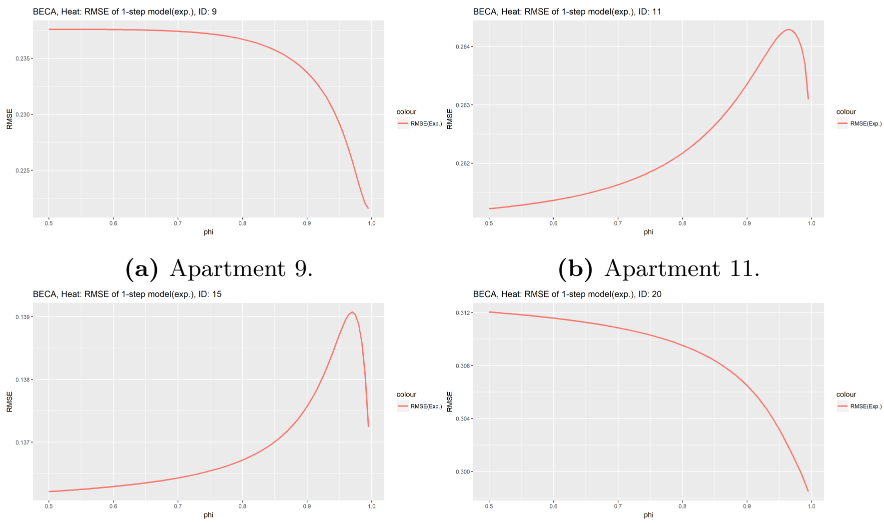
(c) Apartment 15.

(d) Apartment 20.

Figure 6.25: BECA: Actual heat consumption and forecasts(exp.) of select apartments using optimized ϕ. Note: Different scales on y-axis.

The forecasts are either identical of very similar when using optimized ϕ. The RMSE as a function of ϕ is plotted in Figure 6.26

In Table B.7, the mean RMSE and accumulated forecast for all IDs are available.

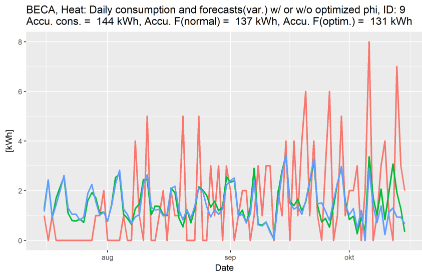


(a) Apartment 9. **(b)** Apartment 11.

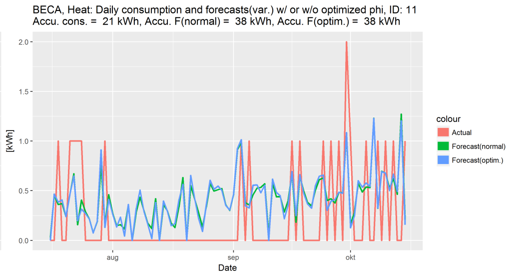
(c) Apartment 15. **(d)** Apartment 20.

Figure 6.26: BECA: RMSE as a function of ϕ. Exponential forgetting. Note: Different scales on y-axis.

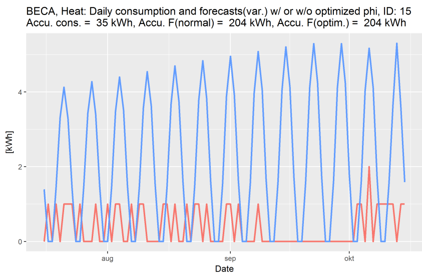
In Figures 6.28a and 6.28d the RMSE is lowest at $\phi = 0.999$. In Figures 6.28b and 6.28c the RMSE is lowest at $\phi = 0.5$. Figure 6.25 shows the forecasts from using variable forgetting.



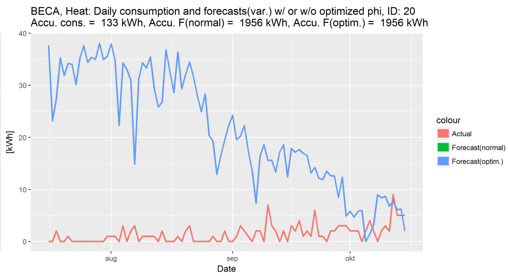
(a) Apartment 9.



(b) Apartment 11.



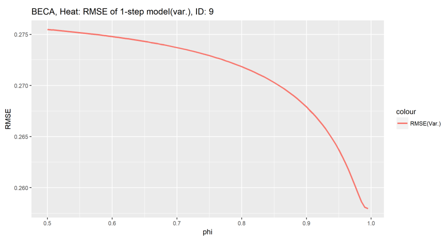
(c) Apartment 15.



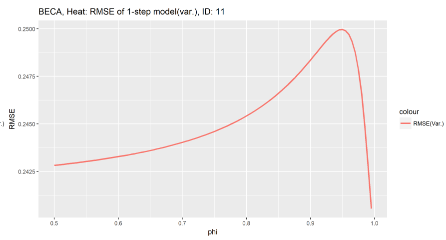
(d) Apartment 20.

Figure 6.27: BECA: Actual heat consumption and forecasts of select apartments. Note: Different scales on y-axis.

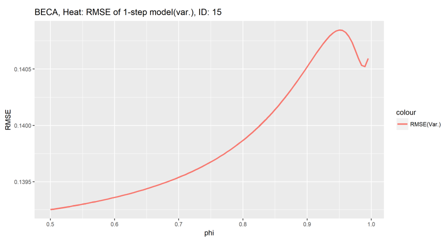
Again, the forecasts are either similar or identical. The RMSE is plotted in Figure 6.28.



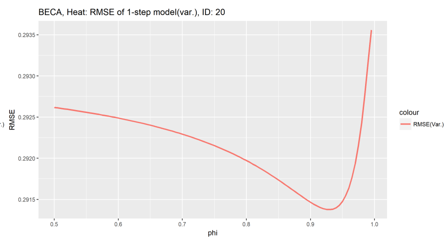
(a) Apartment 9.



(b) Apartment 11.



(c) Apartment 15.



(d) Apartment 20.

Figure 6.28: BECA: RMSE as a function of ϕ. Variable forgetting. Note: Different scales on y-axis.

For Figure 6.28a and 6.28b, the RMSE is lowest at $\phi = 0.999$. In Figure 6.28c the RMSE is lowest at $\phi = 0.5$ and in Figure 6.28d the RMSE is lowest at $\phi \approx 0.93$.

6.2 Conclusion

For most apartments, the heat forecasts are reasonably accurate. Low or zero consumption present some challenges for the method. Water consumption is more random and difficult to forecast but can be done fairly well on a monthly level using a mean value. Optimizing ϕ in the low-pass filter did not provide better heat forecasts.

Chapter 7

Discussion and conclusion

The most relevant topics are discussed and proposed for future work. Finally, the results of the thesis are concluded.

7.1 Discussion and future work

The topics chosen for discussion.

7.1.1 Weather data

The weather data is from a Numerical Weather Prediction (NWP) model. However, the predictions are updated frequently such that the most recent predictions can be very close to an actual measurement. As a result, the past weather predictions that are used in the analysis and heat forecasts are quite accurate, although an actual measurement of the prediction error has not been provided.

In reality, the weather data is predictions of a 3 month period and while the first week might have low error, the uncertainty will increase significantly for the longer horizons, e.g. 60 and 90 days. This will in the end affect the heat forecasts.

7.1.2 FH

The resolution in the heat data is 1000 Wh which was not a huge problem as the overall consumption was quite large. Some periods with missing data did occur in the heat data, but not as bad as the water data.

The missing observations in the water data has made it difficult to forecast longer horizons as the test data for most houses only contained roughly 70 days of data. One way of dealing with this in the future is developing a method that can identify periods with missing data and fill out these periods with e.g. the average consumption.

Heat forecasts are okay, overall. Some households turn on the heat at different times in the test period which is difficult for the method to predict. There were some cases of low consumption for a single day which could be errors in the data, or the house was unoccupied which, in both cases, is difficult to account for in the method.

Water forecasts were mainly mean values giving decent results on a monthly basis but unable to deliver accurate forecasts on a daily basis. Periods with zero consumption usually indicates that the house was unoccupied and this is very difficult, if not impossible, to account for in the method.

Regarding outliers: Some IDs had either missing or unusual data. An example of unusual data could be a long period with zero heat/water consumption. These IDs are not handled well by the method and should by analyzed individually to determine how they should be handled.

7.1.3 BECA

A major challenge with this data has been the resolution of 1 kWh for heat consumption and, overall, low or zero consumption. Even though, it is stated that the heat consumption is only space heating, it might also include water heating, due to a faulty meter, which could explain the relatively high heat consumption during the summer. Rumor has it that some residents would use alternate ways of heating, e.g. turning on the oven and let the oven door stay open. It should be noted that this has not been confirmed by the owner of the data.

Regarding results: Change in resident behavior has made forecasting difficult for some apartments. Overall, the forecasts are okay and within a reasonable margin of error. Heat consumption depend mostly on the outside temperature. For apartments with low consumption, the method struggles.

Water consumption is quite random and difficult to model. A mean value for the whole period gives low accumulated errors. By using a mean value, less computations are required compared to RLS.

7.1.4 Model selection

The model selection uses forward selection with BIC as selection criteria. In most cases, this method will chose the best model, i.e. the simplest model without losing too much information. In the initial analysis, the best model according to forward selection was compared to a simpler model using an F-test. One result showed that the simpler model explained the variation in the data just as well as the model chosen by forward selection which suggests that the F-test might be a good selection criteria. In general, it might be good to be more conservative in the model selection as simpler models tend to give better results for long horizons.

The models used assume normally distributed residuals and in most cases this assumption is not satisfied which makes the use of the F-test questionable.

7.1.5 RLS

As the implementation of the variable forgetting method is tailored to electricity consumption where values are expected to be in a certain interval, e.g. $[0, 100]$ kWh per day, some challenges arose for some households with high heat consumption, e.g. 100000+ Wh per day, because the values were too large. This was solved by adding a prefix to the unit of heat consumption, i.e. changing Wh to kWh.

The method has sometimes struggled with changing seasons, i.e. when changing from summer/fall to winter. When changing to winter after the summer, the latest parameters are heavily influenced by the summer period. To get parameters that were estimated from winter data, the summer period was removed from the training data in Section 5.1.2. As a result, the forecasts were only slightly different than before and did not improve.

Water consumption has been observed from many different households and does not follow a particular pattern, making it difficult to predict. What can be said, however, is that on average the water consumption does not change significantly during a year. While the RLS method has been modified to work for water data, it may not be necessary to use in practice, since a mean value for the whole period could be sufficient. It may be beneficial to use RLS to update estimates if changes occur in occupant behavior, e.g. new residents, but a k-step model might not be much better than a 1-step model.

Finally, one could investigate if optimizing the forgetting factor for each household could improve the forecasts.

Overall, RLS has given good results. The method is somewhat computationally expensive but the C++ implementation makes up for that. Also, the recursive element in the method makes it useful for frequent updates of the models. However, if the models are only updated when new forecasts are needed, i.e. every quarter, perhaps RLS is not necessary and could be replaced by OLS or WLS (Weighted Least Squares).

7.1.6 Optimizing ϕ in low-pass filter

Optimizing the smoothing factor did not seem to improve the forecast - in some cases they got worse. It may be that the effect of the low-pass filter is too small and thus the gain of optimizing the smoothing factor is small, as well.

7.1.7 Switching between summer and winter models

During the summer, the heat consumption is mostly constant, however, when the weather gets colder the heat consumption changes with the temperature difference, as described in Section 3.5. Some cases have been observed where the summer period has influenced the forecasts a bit too much resulting in relatively high prediction errors. This leads to the idea of keeping separate models for summer and winter. Here, the challenge is knowing when to switch between them, as this can be different for each year and it can certainly be different for each household.

In Section 5.1.2, model parameters were estimated using a training period restricted to "winter"-data to forecast heat consumption. As the test period covers the months: October, November and December, it is mix of fall and winter which could be why forecasts were not significantly different from the regular forecasts in Section 5.1.1.

For future work, it would be worth investigating if a household turns on the heat when the air temperature falls below a certain value. A more advanced study could include hidden Markov models to model the switch between the two states: "heat on" and "heat off", to decide if the method should use the summer model or the winter model.

7.1.8 Assumptions

The linear regression model assumes identically and independently normally distributed residuals with mean zero.

Comments regarding independence and distribution of residuals have been summarized as:

- FH, Heat: Autocorrelation present, especially at lag 1. Slightly skewed distribution.
- FH, Water: Independent residuals. Slightly skewed distribution.
- BECA, Heat: Some autocorrelation present. Slightly skewed distribution.
- BECA, Water: Some autocorrelation, depends on type of forgetting. Skewed distribution.

If the aim of modeling was to test hypotheses, e.g. of values of parameters, then the results would not be accepted, since the assumptions are not satisfied. However, in engineering, approximations are necessary and while the residuals may not look perfect, the model delivers acceptable results.

Regarding autocorrelation in heat forecasts: one input in most of the models is the temperature difference which is directly related to the air temperature, which is autocorrelated. Using autocorrelated predictors will consequently introduce autocorrelation in the residuals. To account for this, an AR-part should be included in the model.

7.2 Conclusion

The RLS method has been presented with both exponential and variable forgetting along with the Fourier series to account for weekly variation and the low-pass filter used to account for the heat dynamics of the building. Data from Spain and Helsingør have been presented including analyses determining which weather variables influence heat consumption. In all cases the heat consumption depends significantly on the outside temperature and in about half the cases also on the sun radiation. Heat and water forecasts for both data sets were produced using RLS with exponential and variable forgetting. Overall, the forecasts were within a reasonable margin of error. Some errors were due to specific houses/apartments being more influenced by the summer period in the parameter estimation. Water consumption is difficult to forecasts on a daily basis but can be done on a monthly basis with reasonable accuracy. Heat forecasts using an optimized ID-specific smoothing factor for the low-pass filter has been produced and discussed. These forecasts were determined to be either very similar or poor compared to the regular heat forecasts. For the FH-data, the training period was reduced to winter-data in order to improve the heat forecasts, which ended up not being significantly different from the regular heat forecasts. Again, for the FH-data, the heat consumption models were given input from 2012 in order to estimate heat consumption in a colder winter. In most cases, these forecasts looked promising and gave an idea of how much weather can affect the heat consumption during colder winters. Finally, the primary challenges in heat forecasting were found to be: knowing when a house turns on the heat in the fall and if a house has been unoccupied for a certain period. For that reason, it has been suggested that a method for estimating when a house turns on the heat would be relevant for future work.

Appendix A

FH

A.1 Initial analysis

A.1.1 Heat: Models, forward selection

ID	intercept	temp.diff	RadDown	RadSurf	WindSpeed	WindDir	HAWindSpeed	HAWindDir	AtPres
1	1	1	1	0	0	0	0	0	0
2	1	1	1	1	1	1	1	0	0
3	1	1	1	0	0	0	0	0	1
4	1	1	1	1	1	0	1	1	0
5	1	1	0	1	1	0	0	0	0
6	1	1	1	0	0	0	0	1	0
7	1	1	0	0	1	0	1	1	0
8	1	1	0	0	0	0	0	0	0
9	1	1	1	0	1	0	1	0	0
10	1	1	1	0	1	0	1	1	1
11	1	1	1	1	1	0	1	1	0
12	1	1	1	0	0	0	0	1	0
13	1	1	1	0	0	0	0	0	0
14	1	1	1	0	1	0	1	1	0
15	1	1	0	0	0	0	0	0	0
16	1	1	0	0	1	0	1	0	0
17	1	1	0	0	0	0	0	0	0
18	1	1	0	0	0	0	0	0	0
19	1	1	0	1	0	0	1	1	1
20	1	1	0	1	0	0	0	1	0
21	1	1	0	0	1	0	1	0	0
22	1	1	1	0	1	0	1	1	0
23	1	1	1	0	1	0	1	0	1
24	1	1	0	0	0	0	1	1	0
25	1	1	1	0	1	0	1	1	0
26	1	1	1	1	0	0	0	0	1
27	1	1	1	0	1	0	1	0	0
28	1	1	1	1	1	0	1	0	0
29	1	1	1	1	0	0	0	1	0
30	1	1	1	0	1	0	1	1	0
31	1	1	1	1	0	0	0	0	0
32	1	1	1	1	1	0	1	0	0
33	1	1	0	0	0	0	0	0	0

Table A.1: FH, Heat: Table with weather variables included in each model. 1 indicates inclusion and 0 indicates omission.

A.1.2 Heat (Winter): Models, forward selection

ID	intercept	temp.diff	RadDown	RadSurf	WindSpeed	WindDir	HAWindSpeed	HAWindDir	AtPres
1	1	1	0	0	0	0	1	0	0
2	1	1	0	0	1	0	1	1	0
3	1	1	0	1	0	0	1	0	0
4	1	1	1	1	0	0	1	1	1
5	1	1	0	1	0	0	0	0	0
6	1	1	0	0	0	0	1	1	0
7	1	1	0	0	1	0	1	1	0
8	1	1	1	0	0	0	0	0	0
9	1	1	0	1	0	0	0	0	0
10	1	1	1	0	0	0	1	1	1
11	1	1	1	1	0	0	1	1	0
12	1	1	0	0	0	0	1	1	0
13	1	1	0	0	0	0	0	0	0
14	1	1	1	1	1	0	1	1	0
15	1	1	0	0	0	0	0	0	0
16	1	1	1	1	0	0	1	1	0
17	1	1	1	1	0	0	0	1	0
18	1	1	0	0	0	0	0	0	0
19	1	1	0	0	1	0	0	0	1
20	1	1	1	1	0	0	1	0	0
21	1	1	0	0	1	0	1	0	1
22	1	1	0	1	0	0	1	1	0
23	1	1	0	1	0	0	1	0	1
24	1	1	0	0	0	0	1	1	0
25	1	1	0	0	0	0	1	1	0
26	1	1	1	1	0	0	0	0	0
27	1	1	0	1	0	0	1	0	0
28	1	1	1	0	0	0	1	0	0
29	1	1	1	0	0	0	0	1	0
30	1	1	1	0	0	0	1	1	0
31	1	1	1	1	1	0	1	0	0
32	1	1	1	1	0	0	1	0	1

Table A.2: FH, Heat (Winter): Table with weather variables included in each model. 1 indicates inclusion and 0 indicates omission.

A.2 Results

A.2.1 Heat: Models (Exp.), 1-step

ID	intercept	temp.diff	temp.lpf	SSRD	TSDSRAS	l.night	sin1	cos1	sin2	cos2	sin3	cos3
1	1	0	1	0	0	1	1	1	1	1	0	0
2	1	0	1	1	0	0	0	0	0	0	0	0
3	1	0	1	0	0	1	0	0	0	0	0	0
4	1	0	1	1	1	1	0	0	0	0	0	0
5	1	0	1	0	0	1	0	0	0	0	0	0
6	1	0	1	0	0	1	0	0	0	0	0	0
7	1	0	1	0	0	0	0	0	0	0	0	0
8	1	0	1	1	0	0	0	0	0	0	0	0
9	1	0	1	0	1	1	1	1	0	0	0	0
10	1	0	1	0	0	1	0	0	0	0	0	0
11	1	0	1	0	1	0	0	0	0	0	0	0
12	1	1	0	1	0	0	0	0	0	0	0	0
13	1	0	1	0	0	1	1	1	0	0	0	0
14	1	1	0	0	0	1	0	0	0	0	0	0
15	1	0	1	1	0	0	0	0	0	0	0	0
16	1	0	1	0	0	0	0	0	0	0	0	0
17	1	0	1	0	0	0	0	0	0	0	0	0
18	1	0	1	0	0	0	0	0	0	0	0	0
19	1	0	1	0	0	0	0	0	0	0	0	0
20	1	0	1	0	0	1	0	0	0	0	0	0
21	1	0	1	0	0	1	0	0	0	0	0	0
22	1	0	1	0	0	1	1	1	0	0	0	0
23	1	0	1	0	0	0	0	0	0	0	0	0
24	1	0	1	0	0	1	0	0	0	0	0	0
25	1	0	1	0	0	1	0	0	0	0	0	0
26	1	0	1	0	0	1	1	1	1	1	1	1
27	1	0	1	0	0	1	0	0	0	0	0	0
28	1	0	1	0	0	1	1	1	1	1	1	1
29	1	0	1	0	0	1	0	0	0	0	0	0
30	1	0	1	0	0	1	0	0	0	0	0	0
31	1	0	1	0	0	1	0	0	0	0	0	0
32	1	0	1	1	1	1	1	1	1	1	0	0

Table A.3: FH, Heat: Variables included in 1-step models using exponential forgetting.

A.2.2 Heat: Models (Var.), 1-step

ID	intercept	temp.diff	temp.lpf	SSRD	TSDSRAS	l.night	sin1	cos1	sin2	cos2	sin3	cos3
1	1	0	1	0	0	1	0	0	0	0	0	0
2	1	0	1	1	0	0	0	0	0	0	0	0
3	1	0	1	0	0	1	0	0	0	0	0	0
4	1	0	1	1	1	1	0	0	0	0	0	0
5	1	0	1	0	0	1	1	1	1	1	1	1
6	1	0	1	0	0	1	0	0	0	0	0	0
7	1	0	1	0	0	0	1	1	0	0	0	0
8	1	0	1	1	0	0	0	0	0	0	0	0
9	1	0	1	0	1	1	1	1	0	0	0	0
10	1	0	1	0	0	1	0	0	0	0	0	0
11	1	0	1	0	1	0	1	1	1	1	1	1
12	1	1	0	1	0	0	0	0	0	0	0	0
13	1	0	1	0	0	1	1	1	0	0	0	0
14	1	1	0	0	0	1	0	0	0	0	0	0
15	1	0	1	1	0	0	1	1	1	1	0	0
16	1	0	1	0	0	0	1	1	0	0	0	0
17	1	0	1	0	0	0	1	1	0	0	0	0
18	1	0	1	0	0	0	0	0	0	0	0	0
19	1	0	1	0	0	0	0	0	0	0	0	0
20	1	0	1	0	0	1	0	0	0	0	0	0
21	1	0	1	0	0	1	0	0	0	0	0	0
22	1	0	1	0	0	1	1	1	0	0	0	0
23	1	0	1	0	0	0	0	0	0	0	0	0
24	1	0	1	0	0	1	0	0	0	0	0	0
25	1	0	1	0	0	1	0	0	0	0	0	0
26	1	0	1	0	0	1	1	1	0	0	0	0
27	1	0	1	0	0	1	0	0	0	0	0	0
28	1	0	1	0	0	1	0	0	0	0	0	0
29	1	0	1	0	0	1	0	0	0	0	0	0
30	1	0	1	0	0	1	0	0	0	0	0	0
31	1	0	1	0	0	1	0	0	0	0	0	0
32	1	0	1	1	1	1	0	0	0	0	0	0

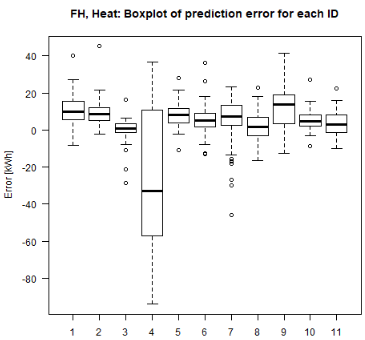
Table A.4: FH, Heat: Variables included in 1-step models using variable forgetting.

A.2.3 Heat: Forecast statistics

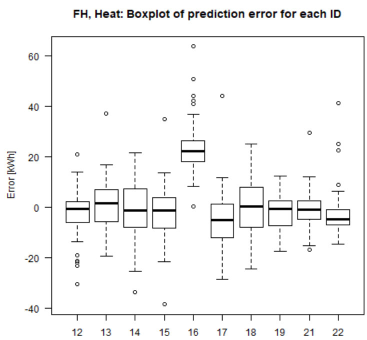
ID	Mean RMSE(Exp.)	Mean RMSE(Var.)	Accu. forecast(Exp.)	Accu. forecast(Var.)	Accu. consumption
1	9.0	8.6	6110.3	6220.4	5148.0
2	21.8	21.8	5421.7	5256.2	4591.0
3	4.5	4.4	2735.8	2753.5	2700.0
4	16.9	14.6	5019.9	7963.9	7291.0
5	7.6	7.1	2855.6	3094.6	2101.0
6	9.5	8.7	5103.5	5109.1	4621.0
7	12.5	12.4	2286.6	2196.9	1719.0
8	7.7	7.8	4360.0	4294.5	4146.0
9	10.2	9.7	5579.3	5812.1	4526.0
10	5.8	5.7	4937.4	4970.6	4465.0
11	9.5	8.9	2937.9	2666.3	2629.0
12	12.0	12.1	3755.4	3779.2	3969.0
13	9.5	9.2	6756.3	6819.4	6646.0
14	16.7	16.9	3009.4	3136.9	3113.0
15	10.8	10.2	6284.1	6118.1	6530.0
16	87.0	87.2	6087.3	5770.5	4001.0
17	16.4	16.5	5152.0	4872.0	5630.0
18	12.1	11.5	4383.3	4022.0	4425.0
19	6.4	6.2	2680.1	2067.8	2873.0
20	11.0	10.6	7685.0	7618.5	9630.0
21	6.2	6.1	3904.5	3992.1	4007.0
22	7.6	7.5	4653.7	4638.3	4958.0
23	4.9	4.6	2060.4	2206.3	7721.0
24	9.4	8.7	6219.8	6446.2	5170.0
25	1.9	1.9	187.6	186.1	0.0
26	11.2	10.9	6265.3	6560.0	6050.0
27	9.0	8.9	6434.7	6503.9	5936.0
28	10.5	10.0	7091.1	7323.9	6732.0
29	7.6	7.3	4222.4	4332.0	3904.0
30	6.6	6.4	3575.8	3610.2	3379.0
31	6.5	6.5	5064.2	5085.4	4510.0
32	28.5	28.9	1526.3	1434.6	3527.0

Table A.5: FH, Heat: Mean RMSE, accumulated forecast for both methods and accumulated consumption.

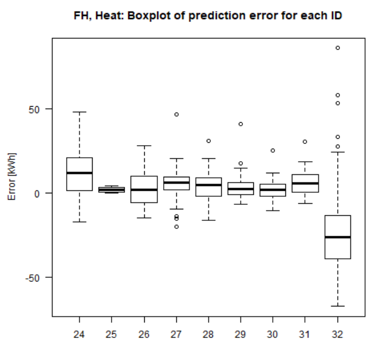
A.2.4 Heat: Boxplots of prediction error



(a) IDs 1 to 11.

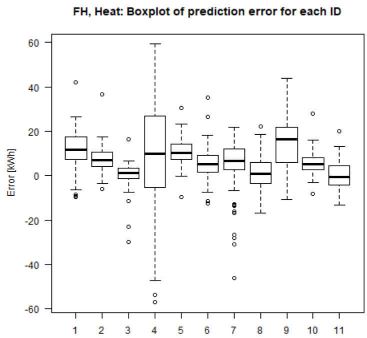


(b) IDs 12 to 22, excluding 20.

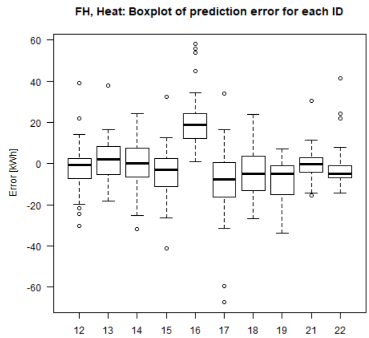


(c) IDs 24 to 32.

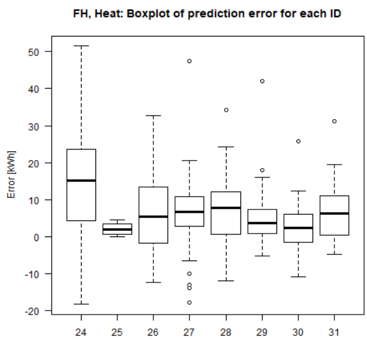
Figure A.1: FH, Heat: Boxplots of prediction error. Exponential forgetting. Note: The following IDs were omitted due to large errors that would have made the plots unreadable: 20,23.



(a) IDs 1 to 11.



(b) IDs 12 to 22, excluding 20.



(c) IDs 24 to 32.

Figure A.2: FH, Heat: Boxplots of prediction error. Variable forgetting. Note: The following IDs were omitted due to large errors that would have made the plots unreadable: 20,23,32.

A.2.5 Heat, Optimize phi: Forecast statistics

ID	Mean RMSE(Exp.)	Mean RMSE(Var.)	Accu. forecast(Exp.)	Accu. forecast(Var.)	Accu. consumption
1	0.0	0.0	6268.3	6375.0	5148.0
2	0.0	0.0	5461.1	5233.9	4591.0
3	0.0	0.0	2793.8	2811.3	2700.0
4	0.0	0.0	7478.3	7332.9	7291.0
5	0.0	0.0	2223.5	2548.8	2101.0
6	0.0	0.0	5292.3	5298.1	4621.0
7	0.0	0.0	2288.8	2237.7	1719.0
8	0.0	0.0	4435.9	4519.4	4146.0
9	0.0	0.0	4616.5	5064.3	4526.0
10	0.0	0.0	4842.1	4876.4	4465.0
11	0.0	0.0	2451.0	2105.6	2629.0
12	0.0	0.0	3581.7	4352.5	3969.0
13	0.0	0.0	6888.3	6949.7	6646.0
14	0.0	0.0	3009.4	3136.9	3113.0
15	0.0	0.0	6755.3	6976.1	6530.0
16	0.0	0.0	6291.3	5653.7	4001.0
17	0.0	0.0	5179.4	4656.6	5630.0
18	0.0	0.0	4425.5	3671.2	4425.0
19	0.0	0.0	2708.8	1962.0	2873.0
20	0.0	0.0	7692.0	7603.4	9630.0
21	0.0	0.0	3923.5	4010.3	4007.0
22	0.0	0.0	4760.9	4774.6	4958.0
23	0.0	0.0	2060.1	2261.6	7721.0
24	0.0	0.0	6414.9	6694.7	5170.0
25	0.0	0.0	193.5	192.1	0.0
26	0.0	0.0	5606.3	6110.3	6050.0
27	0.0	0.0	6558.9	6642.3	5936.0
28	0.0	0.0	6313.5	6769.9	6732.0
29	0.0	0.0	3470.4	3660.8	3904.0
30	0.0	0.0	3659.9	3685.2	3379.0
31	0.0	0.0	5135.3	5155.3	4510.0
32	0.0	0.0	3778.5	2966.2	3527.0

Table A.6: FH, Heat: RMSE, forecasts using optimized ϕ and actual consumption for comparison.

A.2.6 Water: Models (Exp.), 1-step

ID	intercept	sin1	cos1	sin2	cos2	sin3	cos3
1	1	0	0	0	0	0	0
2	1	1	1	0	0	0	0
3	1	0	0	0	0	0	0
4	1	0	0	0	0	0	0
5	1	0	0	0	0	0	0
6	1	0	0	0	0	0	0
7	1	1	1	0	0	0	0
8	1	0	0	0	0	0	0
9	1	0	0	0	0	0	0
10	1	0	0	0	0	0	0
11	1	0	0	0	0	0	0
12	1	0	0	0	0	0	0
13	1	0	0	0	0	0	0
14	1	0	0	0	0	0	0
15	1	0	0	0	0	0	0
16	1	0	0	0	0	0	0
17	1	0	0	0	0	0	0
18	1	1	1	1	1	1	1
19	1	0	0	0	0	0	0
20	1	0	0	0	0	0	0
21	1	0	0	0	0	0	0
22	1	1	1	1	1	0	0
23	1	0	0	0	0	0	0
24	1	0	0	0	0	0	0
25	1	0	0	0	0	0	0
26	1	0	0	0	0	0	0
27	1	1	1	1	1	0	0
28	1	0	0	0	0	0	0
29	1	0	0	0	0	0	0
30	1	0	0	0	0	0	0
31	1	0	0	0	0	0	0
32	1	0	0	0	0	0	0
33	1	0	0	0	0	0	0
34	1	0	0	0	0	0	0
35	1	1	1	1	1	1	1
36	1	0	0	0	0	0	0
37	1	0	0	0	0	0	0
38	1	0	0	0	0	0	0
39	1	1	1	1	1	1	1
40	1	1	1	0	0	0	0
41	1	0	0	0	0	0	0
42	1	0	0	0	0	0	0
43	1	0	0	0	0	0	0
44	1	0	0	0	0	0	0
45	1	0	0	0	0	0	0

Table A.7: FH, Water: Variables included in 1-step models using exponential forgetting.

A.2.7 Water: Models (Var.), 1-step

ID	intercept	sin1	cos1	sin2	cos2	sin3	cos3
1	1	1	1	0	0	0	0
2	1	1	1	0	0	0	0
3	1	1	1	1	1	0	0
4	1	1	1	1	1	1	1
5	1	1	1	0	0	0	0
6	1	1	1	0	0	0	0
7	1	1	1	0	0	0	0
8	1	1	1	0	0	0	0
9	1	0	0	0	0	0	0
10	1	1	1	0	0	0	0
11	1	1	1	0	0	0	0
12	1	1	1	0	0	0	0
13	1	1	1	0	0	0	0
14	1	1	1	0	0	0	0
15	1	1	1	0	0	0	0
16	1	1	1	0	0	0	0
17	1	0	0	0	0	0	0
18	1	1	1	1	1	1	1
19	1	1	1	1	1	1	1
20	1	0	0	0	0	0	0
21	1	0	0	0	0	0	0
22	1	1	1	1	1	0	0
23	1	1	1	1	1	1	1
24	1	1	1	1	1	1	1
25	1	1	1	0	0	0	0
26	1	1	1	0	0	0	0
27	1	1	1	1	1	0	0
28	1	1	1	0	0	0	0
29	1	1	1	1	1	1	1
30	1	1	1	0	0	0	0
31	1	1	1	1	1	1	1
32	1	0	0	0	0	0	0
33	1	0	0	0	0	0	0
34	1	1	1	1	1	1	1
35	1	1	1	1	1	1	1
36	1	0	0	0	0	0	0
37	1	1	1	1	1	1	1
38	1	1	1	0	0	0	0
39	1	1	1	1	1	1	1
40	1	1	1	0	0	0	0
41	1	0	0	0	0	0	0
42	1	1	1	0	0	0	0
43	1	1	1	1	1	0	0
44	1	1	1	0	0	0	0
45	1	1	1	0	0	0	0

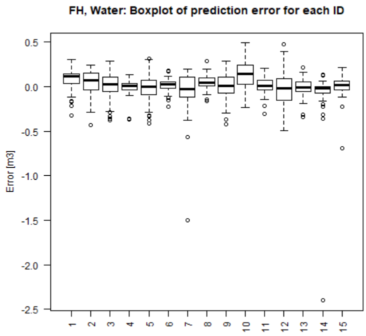
Table A.8: FH, Water: Variables included in 1-step models using variable forgetting.

A.2.8 Water: Forecast statistics

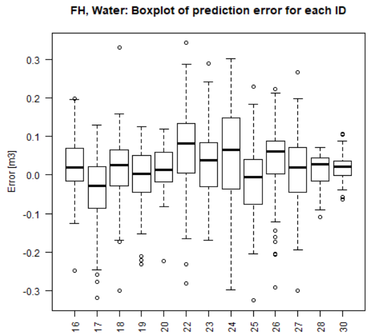
ID	Mean RMSE(Exp.)	Mean RMSE(Var.)	Accu. forecast(Exp.)	Accu. forecast(Var.)	Accu. consumption
1	0.11	0.12	21.92	21.90	15.86
2	0.15	0.15	17.61	18.40	14.81
3	0.15	0.16	19.99	19.80	18.60
4	0.06	0.07	7.52	7.44	7.76
5	0.13	0.14	23.77	23.33	24.63
6	0.10	0.11	13.61	13.34	12.63
7	0.15	0.15	9.70	9.64	12.78
8	0.25	0.25	16.40	15.78	12.46
9	0.19	0.19	24.26	26.82	24.00
10	1.53	1.53	27.75	27.76	17.60
11	0.08	0.08	14.89	14.95	13.70
12	0.20	0.21	31.15	31.25	33.49
13	0.28	0.29	14.35	14.30	14.98
14	0.09	0.10	10.45	10.74	15.63
15	0.10	0.11	14.82	14.24	13.82
16	0.07	0.07	14.46	14.40	12.08
17	0.11	0.12	8.12	8.92	11.17
18	0.10	0.11	25.43	24.94	24.62
19	0.09	0.10	9.55	9.44	9.90
20	0.07	0.07	8.19	8.68	6.95
21	0.19	0.16	30.07	16.57	15.67
22	0.12	0.13	26.57	26.67	21.85
23	0.21	0.21	13.46	13.35	10.88
24	0.13	0.15	20.50	20.55	16.34
25	0.12	0.13	18.79	18.78	20.01
26	0.14	0.15	16.24	16.59	13.64
27	0.09	0.09	18.59	18.44	17.92
28	0.11	0.11	5.05	5.41	4.07
29	0.13	0.14	22.78	22.77	26.58
30	0.04	0.04	7.98	7.94	6.61
31	0.14	0.14	7.69	7.57	3.62
32	0.07	0.08	18.07	18.02	13.64
33	0.13	0.14	25.75	28.13	20.72
34	0.15	0.15	22.54	23.29	23.95
35	0.12	0.12	14.06	14.26	17.25
36	0.05	0.06	11.02	11.24	10.14
37	0.19	0.21	38.86	38.88	34.48
38	0.13	0.13	35.17	34.97	37.14
39	0.25	0.26	16.97	16.23	21.46
40	0.06	0.06	7.96	7.90	9.19
41	0.22	0.21	31.15	34.06	35.48
42	0.19	0.21	48.89	47.82	53.62
43	0.11	0.12	16.74	16.34	16.49
44	0.13	0.14	31.73	31.94	37.16
45	0.13	0.14	22.38	22.48	15.17

Table A.9: FH, Water: Mean RMSE, accumulated forecast for both methods and accumulated consumption.

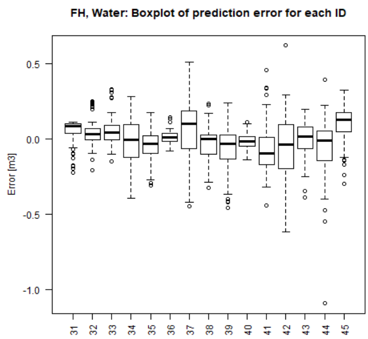
A.2.9 Water: Boxplots of prediction error



(a) IDs 1 to 15.

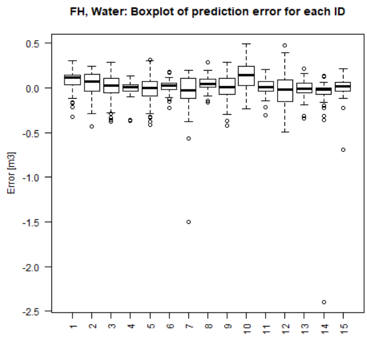


(b) IDs 16 to 30, excluding 29.

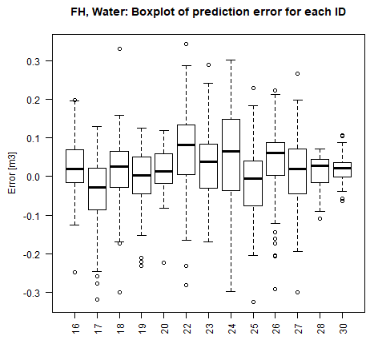


(c) IDs 31 to 45.

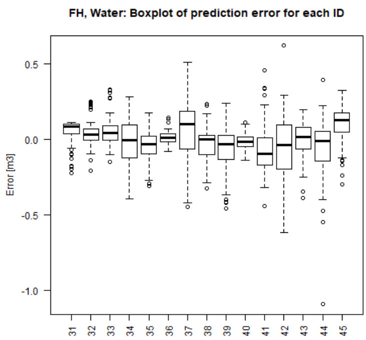
Figure A.3: FH, Water: Boxplots of prediction error. Exponential forgetting. Note: The following IDs were omitted due to large errors that would have made the plots unreadable: 21,29.



(a) IDs 1 to 15.



(b) IDs 16 to 30, excluding 21 and 29.



(c) IDs 31 to 45.

Figure A.4: FH, Water: Boxplots of prediction error. Variable forgetting. Note: The following IDs were omitted due to large errors that would have made the plots unreadable: 21,29.

Appendix B

BECA

B.1 Initial analysis

B.1.1 Heat: Models, forward selection

ID	intercept	temp.diff	Precipitation	Wind_Direction_avrg	Radiation	Wind_Velocity_avrg
1	1	1	0	0	0	0
2	1	0	0	0	1	0
3	1	1	0	0	0	0
4	1	1	0	0	0	0
5	1	1	0	0	0	0
6	1	1	0	0	0	0
7	1	1	0	0	0	0
8	1	1	0	0	1	0
9	1	1	0	0	0	0
10	1	0	0	0	0	0
11	1	1	0	0	0	0
12	1	1	0	0	0	0
13	1	1	0	0	1	0
14	1	0	0	1	1	0
15	1	0	0	0	1	1
16	1	0	0	0	0	0
17	1	1	0	0	0	0
18	1	1	0	0	0	0
19	1	0	0	0	0	0
20	1	1	0	0	1	1
21	1	0	0	0	0	0
22	1	1	0	0	0	0
23	1	1	0	0	0	0
24	1	1	0	0	1	0
25	1	1	0	0	0	0
26	1	1	0	0	0	0
27	1	1	0	0	1	0
28	1	1	0	0	0	0
29	1	1	0	0	0	0
30	1	1	0	0	0	0
31	1	1	0	0	0	0
32	1	1	1	0	1	0
33	1	0	1	0	1	0
34	1	0	0	0	1	0
35	1	0	1	0	1	1
36	1	1	0	0	0	0
37	1	1	0	0	0	0
38	1	0	0	0	1	0
39	1	1	0	0	0	0
40	1	1	0	0	0	0
41	1	0	0	1	1	0
42	1	1	0	0	0	0
43	1	1	0	0	0	0
44	1	1	0	0	0	0

Table B.1: BECA, Heat: Table with weather variables included in each model. 1 indicates inclusion and 0 indicates omission.

B.1.2 Heat (Winter): Models, forward selection

ID	intercept	temp.diff	Precipitation	Wind_Direction_avrg	Radiation	Wind_Velocity_avrg
1	1	0	0	0	0	0
2	1	0	0	0	0	0
3	1	0	0	0	0	0
4	1	0	0	0	0	0
5	1	0	0	0	0	0
6	1	0	0	0	0	0
7	1	0	0	0	0	0
8	1	1	0	0	0	0
9	1	1	0	0	0	0
10	1	0	0	0	0	0
11	1	0	0	0	0	0
12	1	0	0	0	0	0
13	1	1	0	0	0	0
14	1	0	0	1	0	0
15	1	0	0	0	0	1
16	1	0	0	0	0	0
17	1	0	0	0	0	0
18	1	0	0	0	0	0
19	1	0	0	0	0	0
20	1	0	0	0	0	0
21	1	0	0	0	0	0
22	1	1	0	0	0	0
23	1	1	0	0	0	0
24	1	0	0	0	0	0
25	1	1	0	0	0	0
26	1	0	0	0	0	0
27	1	1	0	0	0	0
28	1	1	0	0	0	0
29	1	1	0	0	0	0
30	1	0	0	0	0	0
31	1	1	0	0	0	0
32	1	1	0	0	0	0
33	1	0	0	0	1	1
34	1	0	0	0	0	0
35	1	0	0	0	0	0
36	1	0	0	0	0	0
37	1	1	0	0	0	0
38	1	0	0	0	1	0
39	1	0	0	0	0	0
40	1	1	0	0	0	0
41	1	0	0	1	0	0
42	1	1	0	0	1	0
43	1	0	0	0	0	0
44	1	0	0	0	0	0

Table B.2: BECA, Heat (Winter period): Table with weather variables included in each model. 1 indicates inclusion and 0 indicates omission.

B.1.3 Heat (non-zero): Models, forward selection

	intercept	temp.diff	Precipitation	Wind_Direction_avrg	Radiation	Wind_Velocity_avrg
1	1	1	0	0	0	0
2	1	0	0	0	0	0
3	1	1	0	0	0	0
4	1	1	0	0	0	0
5	1	1	0	0	0	0
6	1	1	0	0	0	0
7	1	1	0	0	1	1
8	1	1	0	0	0	0
9	1	1	0	0	0	0
10	1	0	0	0	0	0
11	1	1	0	0	0	0
12	1	0	0	0	1	0
13	1	1	0	0	0	0
14	1	1	0	0	0	0
15	1	0	0	0	0	1
16	1	0	0	0	0	0
17	1	1	0	0	0	0
18	1	1	0	0	0	0
19	1	0	0	0	0	0
20	1	1	0	0	0	0
21	1	0	0	0	0	0
22	1	1	0	0	0	0
23	1	1	0	0	0	0
24	1	0	0	0	0	0
25	1	1	0	0	0	0
26	1	1	0	0	0	0
27	1	1	0	0	1	0
28	1	1	0	0	0	0
29	1	1	1	0	1	0
30	1	1	0	0	0	0
31	1	1	0	1	0	0
32	1	1	0	0	0	0
33	1	1	0	0	0	0
34	1	1	0	0	1	0
35	1	0	0	0	0	0
36	1	0	0	0	0	0
37	1	1	0	0	0	0
38	1	0	0	0	0	0
39	1	0	0	0	1	0
40	1	1	0	0	0	0
41	1	1	0	1	1	0
42	1	1	0	0	0	0
43	1	0	0	0	1	0
44	1	0	0	0	0	0

Table B.3: BECA, Heat (non-zero): Table with weather variables included in each model. 1 indicates inclusion and 0 indicates omission.

B.2 Results

B.2.1 Heat: Models (Exp.), 1-step

	intercept	temp.diff	temp.lpf	Radiation	sin1	cos1	sin2	cos2	sin3	cos3
1	1	0	1	0	0	0	0	0	0	0
2	1	0	0	1	1	1	0	0	0	0
3	1	0	1	0	1	1	1	1	0	0
4	1	1	0	0	0	0	0	0	0	0
5	1	0	1	0	0	0	0	0	0	0
6	1	0	1	0	1	1	1	1	1	1
7	1	0	1	0	0	0	0	0	0	0
8	1	0	1	1	0	0	0	0	0	0
9	1	1	0	0	0	0	0	0	0	0
10	1	0	0	1	0	0	0	0	0	0
11	1	0	1	0	0	0	0	0	0	0
12	1	0	1	0	1	1	1	1	1	1
13	1	0	1	0	1	1	1	1	1	1
14	1	0	0	1	0	0	0	0	0	0
15	1	0	0	0	0	0	0	0	0	0
16	1	0	0	0	0	0	0	0	0	0
17	1	0	1	0	1	1	1	1	1	1
18	1	0	1	0	1	1	1	1	1	1
19	1	0	1	0	0	0	0	0	0	0
20	1	0	1	1	1	1	1	1	1	1
21	1	0	0	0	0	0	0	0	0	0
22	1	0	1	1	1	1	1	1	1	1
23	1	0	1	0	0	0	0	0	0	0
24	1	0	1	1	0	0	0	0	0	0
25	1	0	1	0	1	1	1	1	0	0
26	1	0	1	0	0	0	0	0	0	0
27	1	0	1	1	0	0	0	0	0	0
28	1	0	1	0	0	0	0	0	0	0
29	1	0	1	0	0	0	0	0	0	0
30	1	0	1	0	1	1	1	1	0	0
31	1	0	1	0	1	1	0	0	0	0
32	1	0	1	1	0	0	0	0	0	0
33	1	0	0	1	0	0	0	0	0	0
34	1	0	1	0	1	1	0	0	0	0
35	1	0	0	1	1	1	1	1	1	1
36	1	0	1	0	0	0	0	0	0	0
37	1	0	1	0	0	0	0	0	0	0
38	1	0	0	1	0	0	0	0	0	0
39	1	0	1	0	0	0	0	0	0	0
40	1	0	1	0	0	0	0	0	0	0
41	1	0	0	1	0	0	0	0	0	0
42	1	0	1	0	0	0	0	0	0	0
43	1	0	1	0	1	1	1	1	1	1
44	1	1	0	0	0	0	0	0	0	0

Table B.4: BECA, Heat: Variables included in 1-step models using exponential forgetting.

B.2.2 Heat: Models (Var.), 1-step

	intercept	temp.diff	temp.lpf	Radiation	sin1	cos1	sin2	cos2	sin3	cos3
1	1	0	1	0	1	1	1	1	1	1
2	1	0	0	1	1	1	0	0	0	0
3	1	0	1	0	1	1	1	1	0	0
4	1	1	0	0	1	1	1	1	1	1
5	1	0	1	0	1	1	1	1	1	1
6	1	0	1	0	1	1	1	1	1	1
7	1	0	1	0	1	1	1	1	1	1
8	1	0	1	1	1	1	1	1	1	1
9	1	1	0	0	1	1	0	0	0	0
10	1	0	0	1	1	1	0	0	0	0
11	1	0	1	0	1	1	1	1	1	1
12	1	0	1	0	1	1	1	1	1	1
13	1	0	1	0	1	1	1	1	1	1
14	1	0	0	1	1	1	0	0	0	0
15	1	0	0	0	1	1	0	0	0	0
16	1	0	0	0	1	1	0	0	0	0
17	1	0	1	0	1	1	1	1	1	1
18	1	0	1	0	1	1	1	1	0	0
19	1	0	1	0	1	1	1	1	1	1
20	1	0	1	1	1	1	1	1	1	1
21	1	0	0	0	1	1	0	0	0	0
22	1	0	1	1	1	1	1	1	0	0
23	1	0	1	0	0	0	0	0	0	0
24	1	0	1	1	0	0	0	0	0	0
25	1	0	1	0	1	1	1	1	0	0
26	1	0	1	0	1	1	1	1	1	1
27	1	0	1	1	1	1	0	0	0	0
28	1	0	1	0	0	0	0	0	0	0
29	1	0	1	0	1	1	1	1	1	1
30	1	0	1	0	1	1	1	1	0	0
31	1	0	1	0	1	1	0	0	0	0
32	1	0	1	1	1	1	1	1	0	0
33	1	0	0	1	1	1	0	0	0	0
34	1	0	1	0	1	1	0	0	0	0
35	1	0	0	1	1	1	1	1	1	1
36	1	0	1	0	1	1	1	1	1	1
37	1	0	1	0	1	1	1	1	1	1
38	1	0	0	1	1	1	0	0	0	0
39	1	0	1	0	1	1	0	0	0	0
40	1	0	1	0	1	1	1	1	1	1
41	1	0	0	1	0	0	0	0	0	0
42	1	0	1	0	1	1	1	1	1	1
43	1	0	1	0	1	1	1	1	1	1
44	1	1	0	0	1	1	1	1	1	1

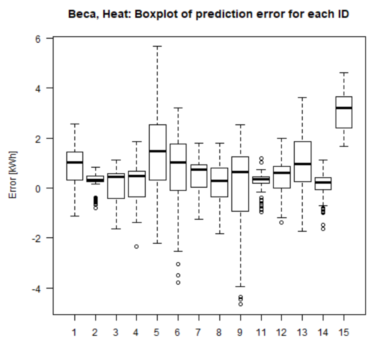
Table B.5: BECA, Heat: Variables included in 1-step models using variable forgetting.

B.2.3 Heat: Forecast statistics

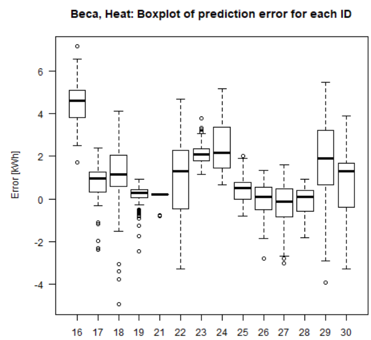
	ID	Mean RMSE(Exp.)	Mean RMSE(Var.)	Accu. forecast(Exp.)	Accu. forecast(Var.)	Accu. consumption
1	1	1.6	1.6	143.6	153.9	63.0
2	2	1.0	1.0	36.8	36.8	12.0
3	3	1.0	1.0	52.9	54.4	41.0
4	4	1.1	1.1	62.9	63.3	43.0
5	5	3.3	3.3	303.0	320.8	174.0
6	6	2.7	2.7	165.1	193.4	98.0
7	7	1.0	1.0	93.0	92.3	43.0
8	8	2.1	3.0	92.7	4.7	72.0
9	9	3.5	3.5	146.9	168.1	144.0
10	10	1.7	1.7	59.7	59.2	276.0
11	11	1.1	1.1	38.7	35.5	21.0
12	12	1.3	1.3	73.6	71.5	34.0
13	13	1.9	2.0	253.3	260.3	162.0
14	14	1.3	1.3	33.0	28.6	27.0
15	15	45.0	45.4	317.3	204.3	35.0
16	16	44.4	45.1	519.5	418.0	101.0
17	17	1.7	1.7	104.8	110.2	36.0
18	18	3.2	3.3	239.7	259.7	144.0
19	19	4.1	4.2	34.4	3.8	26.0
20	20	2.7	2.8	2066.5	1980.2	133.0
21	21	0.5	0.5	18.8	17.1	9.0
22	22	3.4	3.4	327.9	297.8	237.0
23	23	2.1	2.1	199.7	196.6	4.0
24	24	2.5	2.5	221.4	221.2	0.0
25	25	1.1	1.1	70.1	71.6	34.0
26	26	1.1	1.1	54.0	56.0	55.0
27	27	1.3	1.4	105.9	68.6	125.0
28	28	1.1	1.1	32.6	61.8	43.0
29	29	3.7	3.9	388.5	391.0	226.0
30	30	3.0	3.1	149.4	166.4	82.0
31	31	1.6	1.6	113.2	114.8	71.0
32	32	1.3	1.3	1817.5	1864.9	63.0
33	33	0.5	0.5	31.0	22.9	41.0
34	34	0.6	0.6	21.9	17.9	19.0
35	35	0.8	0.8	25.1	25.0	28.0
36	36	0.7	0.7	33.7	31.2	16.0
37	37	1.4	1.4	179.3	182.5	72.0
38	38	1.6	1.6	19.4	17.6	0.0
39	39	2.9	3.0	112.7	106.3	55.0
40	40	0.9	0.9	36.1	38.4	43.0
41	41	2.8	2.8	166.0	159.7	92.0
42	42	0.7	0.7	18.3	17.5	17.0
43	43	1.2	1.2	86.9	89.0	35.0
44	44	0.7	0.7	14.9	16.0	7.0

Table B.6: BECA, Heat: Mean RMSE, accumulated forecast for both methods and accumulated consumption.

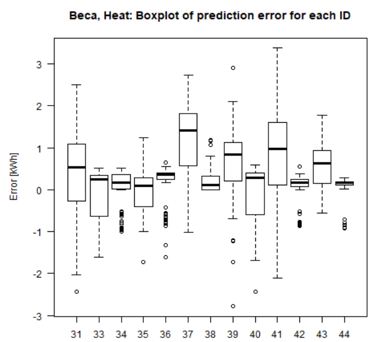
B.2.4 Heat: Boxplots of prediction error



(a) IDs 1 to 11.

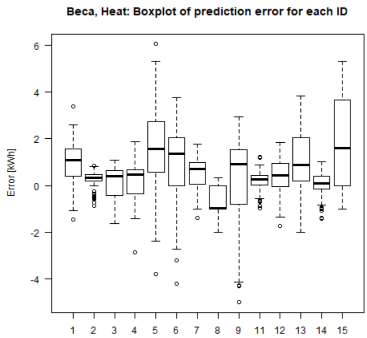


(b) IDs 12 to 22, excluding 20.

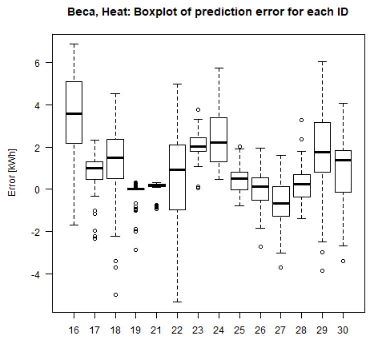


(c) IDs 24 to 32.

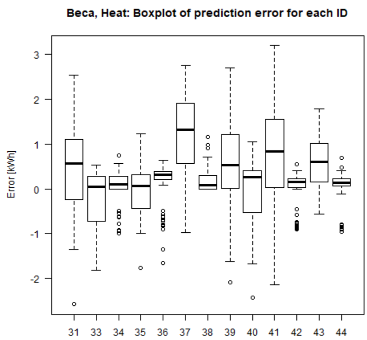
Figure B.1: BECA, Heat: Boxplots of prediction error. Exponential forgetting. Note: The following IDs were omitted due to large errors that would have made the plots unreadable: 10,20,32.



(a) IDs 1 to 11.



(b) IDs 12 to 22, excluding 20.



(c) IDs 24 to 32.

Figure B.2: BECA, Heat: Boxplots of prediction error. Variable forgetting. Note: The following IDs were omitted due to large errors that would have made the plots unreadable: 10,20,32.

ID	Mean RMSE(Exp.)	Mean RMSE(Var.)	Accu. forecast(Exp.)	Accu. forecast(Var.)	Accu. consumption
1	1.6	1.6	143.8	153.4	63.0
2	1.0	1.0	36.8	36.8	12.0
3	1.0	1.0	52.9	54.5	41.0
4	1.1	1.1	62.9	63.3	43.0
5	3.3	3.3	303.0	320.8	174.0
6	2.7	2.7	164.2	192.5	98.0
7	1.0	1.0	93.1	92.4	43.0
8	2.1	3.0	92.9	5.0	72.0
9	3.5	3.5	146.9	168.1	144.0
10	1.7	1.7	59.7	59.2	276.0
11	1.1	1.1	38.7	35.7	21.0
12	1.3	1.3	70.3	68.4	34.0
13	1.9	2.0	234.5	238.6	162.0
14	1.3	1.3	33.0	28.6	27.0
15	45.0	45.4	317.3	204.3	35.0
16	44.4	45.1	519.5	418.0	101.0
17	1.7	1.7	103.6	109.3	36.0
18	3.2	3.3	229.2	253.4	144.0
19	4.1	4.2	33.5	2.6	26.0
20	2.7	2.8	2066.9	1980.7	133.0
21	0.5	0.5	18.8	17.1	9.0
22	3.4	3.4	327.7	297.5	237.0
23	2.1	2.1	193.3	189.5	4.0
24	2.5	2.5	221.6	221.4	0.0
25	1.1	1.1	68.3	70.0	34.0
26	1.1	1.1	46.5	48.6	55.0
27	1.3	1.4	106.3	69.6	125.0
28	1.1	1.1	32.6	61.7	43.0
29	3.7	3.9	386.1	385.9	226.0
30	3.0	3.1	149.6	166.1	82.0
31	1.6	1.6	104.1	106.2	71.0
32	1.3	1.3	1817.5	1864.9	63.0
33	0.5	0.5	31.0	22.9	41.0
34	0.6	0.6	22.4	18.0	19.0
35	0.8	0.8	25.1	25.0	28.0
36	0.7	0.7	33.7	31.4	16.0
37	1.4	1.4	179.2	182.5	72.0
38	1.6	1.6	19.4	17.6	0.0
39	2.9	3.0	112.7	106.4	55.0
40	0.9	0.9	35.6	37.1	43.0
41	2.8	2.8	166.0	159.7	92.0
42	0.7	0.7	10.6	10.1	17.0
43	1.2	1.2	81.2	83.7	35.0
44	0.7	0.7	14.9	16.0	7.0

Table B.7: BECA, Heat: RMSE, forecasts using optimized ϕ and actual consumption for comparison.

B.2.5 Heat, Optimize phi: Forecast statistics

B.2.6 Water: Models (Exp.), 1-step

ID	intercept	sin1	cos1	sin2	cos2	sin3	cos3
1	1	1	1	0	0	0	0
2	1	1	1	0	0	0	0
3	1	1	1	0	0	0	0
4	1	1	1	0	0	0	0
5	1	0	0	0	0	0	0
6	1	1	1	0	0	0	0
7	1	1	1	0	0	0	0
8	1	0	0	0	0	0	0
9	1	0	0	0	0	0	0
10	1	1	1	0	0	0	0
11	1	0	0	0	0	0	0
12	1	1	1	0	0	0	0
13	1	1	1	0	0	0	0
14	1	0	0	0	0	0	0
15	1	1	1	0	0	0	0
16	1	0	0	0	0	0	0
17	1	0	0	0	0	0	0
18	1	0	0	0	0	0	0
19	1	1	1	0	0	0	0
20	1	0	0	0	0	0	0
21	1	1	1	0	0	0	0
22	1	0	0	0	0	0	0
23	1	0	0	0	0	0	0
24	1	0	0	0	0	0	0
25	1	1	1	0	0	0	0
26	1	1	1	0	0	0	0
27	1	1	1	0	0	0	0
28	1	0	0	0	0	0	0
29	1	0	0	0	0	0	0
30	1	0	0	0	0	0	0
31	1	1	1	0	0	0	0
32	1	0	0	0	0	0	0
33	1	0	0	0	0	0	0
34	1	0	0	0	0	0	0
35	1	1	1	0	0	0	0
36	1	0	0	0	0	0	0
37	1	1	1	0	0	0	0
38	1	1	1	0	0	0	0
39	1	1	1	0	0	0	0
40	1	1	1	0	0	0	0
41	1	0	0	0	0	0	0
42	1	1	1	0	0	0	0
43	1	1	1	0	0	0	0
44	1	1	1	0	0	0	0

B.2.7 Water: Models (Var.), 1-step

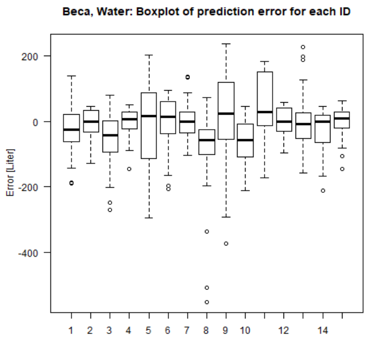
ID	intercept	sin1	cos1	sin2	cos2	sin3	cos3
1	1	1	1	0	0	0	0
2	1	1	1	0	0	0	0
3	1	1	1	0	0	0	0
4	1	1	1	0	0	0	0
5	1	0	0	0	0	0	0
6	1	1	1	0	0	0	0
7	1	1	1	0	0	0	0
8	1	0	0	0	0	0	0
9	1	0	0	0	0	0	0
10	1	1	1	0	0	0	0
11	1	0	0	0	0	0	0
12	1	1	1	0	0	0	0
13	1	1	1	0	0	0	0
14	1	0	0	0	0	0	0
15	1	1	1	0	0	0	0
16	1	0	0	0	0	0	0
17	1	0	0	0	0	0	0
18	1	0	0	0	0	0	0
19	1	1	1	0	0	0	0
20	1	0	0	0	0	0	0
21	1	1	1	0	0	0	0
22	1	0	0	0	0	0	0
23	1	0	0	0	0	0	0
24	1	0	0	0	0	0	0
25	1	1	1	0	0	0	0
26	1	1	1	0	0	0	0
27	1	1	1	0	0	0	0
28	1	0	0	0	0	0	0
29	1	0	0	0	0	0	0
30	1	0	0	0	0	0	0
31	1	1	1	0	0	0	0
32	1	0	0	0	0	0	0
33	1	0	0	0	0	0	0
34	1	0	0	0	0	0	0
35	1	1	1	0	0	0	0
36	1	0	0	0	0	0	0
37	1	1	1	0	0	0	0
38	1	1	1	0	0	0	0
39	1	1	1	0	0	0	0
40	1	1	1	0	0	0	0
41	1	0	0	0	0	0	0
42	1	1	1	0	0	0	0
43	1	1	1	0	0	0	0
44	1	1	1	0	0	0	0

B.2.8 Water: Forecast statistics

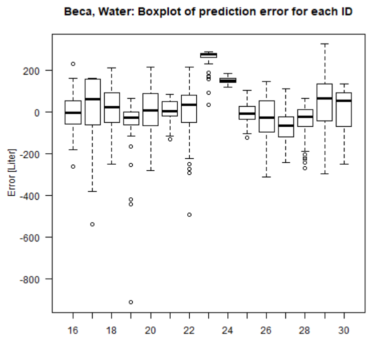
	ID	Mean RMSE(Exp.)	Mean RMSE(Var.)	Accu. forecast(Exp.)	Accu. forecast(Var.)	Accu. consumption
1	1	67.6	69.1	12758.2	15565.2	13882.0
2	2	47.3	51.8	4601.7	4488.4	5283.0
3	3	72.8	78.9	11211.8	12729.0	15361.0
4	4	41.7	43.8	4712.7	5605.6	4718.0
5	5	121.9	147.4	19971.1	22687.3	20684.0
6	6	95.9	110.0	9422.6	14538.0	9322.0
7	7	51.8	55.7	12362.5	12823.6	12301.0
8	8	100.3	75.9	11215.9	15466.5	18079.0
9	9	143.1	178.1	18878.4	21931.1	17214.0
10	10	173.7	175.8	5209.4	8355.6	10681.0
11	11	98.6	118.2	14371.0	15127.5	10608.0
12	12	50.5	57.1	5194.0	5334.1	5236.0
13	13	70.1	75.4	17452.0	17200.2	17766.0
14	14	59.2	55.7	3648.3	2810.5	5621.0
15	15	53.0	51.0	6050.1	7992.4	5587.0
16	16	91.4	105.3	21167.4	21368.5	21653.0
17	17	124.7	151.0	15166.3	18072.2	12766.0
18	18	144.4	174.7	20157.4	14581.3	18663.0
19	19	60.9	49.5	5919.2	9078.1	10318.0
20	20	128.7	145.9	18015.9	17243.5	17129.0
21	21	44.9	49.1	6119.6	6190.3	5515.0
22	22	138.8	147.6	26491.6	26557.6	25313.0
23	23	136.6	141.4	25394.2	18276.8	1096.0
24	24	94.8	87.5	14123.2	4610.9	0.0
25	25	52.4	68.3	9661.3	9450.4	10175.0
26	26	90.7	93.5	13446.4	11631.6	16063.0
27	27	64.2	59.6	9528.0	14066.8	15642.0
28	28	60.1	63.2	3951.1	7159.3	7520.0
29	29	144.4	181.0	29992.1	27508.8	24017.0
30	30	115.6	162.4	9159.3	10499.4	8071.0
31	31	76.2	82.1	10505.1	8944.1	10155.0
32	32	95.8	98.2	10240.0	3456.0	10440.0
33	33	27.9	24.2	1558.9	3428.2	6148.0
34	34	38.4	34.5	1073.9	3170.5	4709.0
35	35	51.8	52.9	7568.0	9040.8	8061.0
36	36	66.4	75.1	10036.0	9066.1	7389.0
37	37	85.3	107.5	28558.6	28839.7	20580.0
38	38	90.8	84.1	4272.3	341.2	7.0
39	39	119.7	122.8	11358.3	12431.1	12319.0
40	40	41.0	43.7	4342.3	5102.8	4646.0
41	41	145.0	122.9	14660.5	15312.7	14655.0
42	42	49.3	49.8	5922.8	2971.4	3171.0
43	43	52.5	61.0	8491.5	8911.9	8911.0
44	44	53.8	60.4	6087.2	6600.8	6273.0

Table B.8: BECA, Water: Mean RMSE, accumulated forecast for both methods and accumulated consumption.

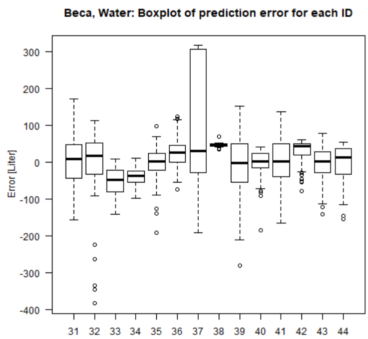
B.2.9 Water: Boxplots of prediction error



(a) IDs 1 to 15.

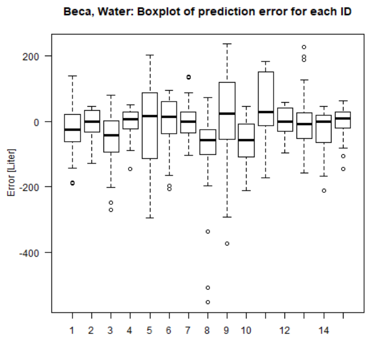


(b) IDs 16 to 30.

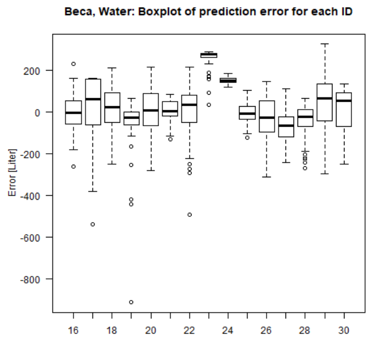


(c) IDs 31 to 45.

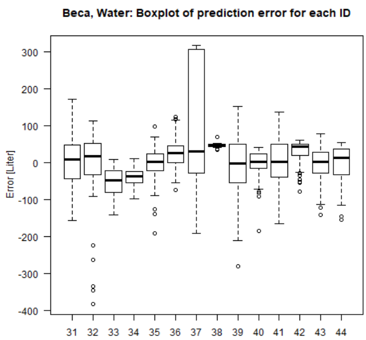
Figure B.3: BECA, Water: Boxplots of prediction error. Exponential forgetting.



(a) IDs 1 to 15.



(b) IDs 16 to 30.



(c) IDs 31 to 45.

Figure B.4: BECA, Water: Boxplots of prediction error. Variable forgetting.

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