Supplementary material for ‘Multi-period equilibrium in coupled transportation system and transactive energy community’

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Appendix A. Proof of Proposition 3

For the non-cooperative game $\Gamma_{\mathrm{TP}}(\cdot)$, the gradients of objective functions of all players $F(\gamma) = [\nabla Y_1(\gamma_1, \gamma_{-1}), \cdots, \nabla Y_Z(\gamma_Z, \gamma_{-Z})]$ are calculated as follows:

$$\frac{\partial Y_1}{\partial p_{i,t}} = \lambda^{\mathrm{p}}_{n,t} + 2\mathrm{m}^{\mathrm{p}} p_{i,t} \quad \forall\, i, t \tag{A.1}$$

$$\frac{\partial Y_1}{\partial q_{i,t}} = \lambda^{\mathrm{q}}_{n,t} + 2\mathrm{m}^{\mathrm{q}} q_{i,t} \quad \forall\, i, t \tag{A.2}$$

$$\frac{\partial Y_1}{\partial p^{\mathrm{g}}_{w,t}} = 2\mathrm{m}^{\mathrm{p}_2}_{w} p^{\mathrm{g}}_{w,t} + \mathrm{m}^{\mathrm{p}_1}_{w} \quad \forall\, w, t \tag{A.3}$$

$$\frac{\partial Y_1}{\partial q^{\mathrm{g}}_{w,t}} = 2\mathrm{m}^{\mathrm{q}_2}_{w} q^{\mathrm{g}}_{w,t} \quad \forall\, w, t \tag{A.4}$$

$$\frac{\partial Y_1}{\partial \hat{q}^{\mathrm{g}}_{w,t}} = \mathrm{m}^{\mathrm{q}_1}_{w} \quad \forall\, w, t \tag{A.5}$$

$$\frac{\partial Y_1}{\partial p^{\mathrm{ch}}_{i,t}} = 0 \quad \forall\, i, t \tag{A.6}$$

$$\frac{\partial Y_1}{\partial p^{\mathrm{dis}}_{i,t}} = 0 \quad \forall\, i, t \tag{A.7}$$

$$\frac{\partial Y_1}{\partial e_{i,t}} = 0 \quad \forall\, i, t \tag{A.8}$$

$$\frac{\partial Y_2}{\partial \lambda^{\mathrm{p}}_{n,t}} = \sum_{i\in\Omega_n} p_{i,t} + \sum_{a\in\Omega_n} \mathrm{E_c} x_{a,t} + \sum_{j\in\Omega_n} p^{\mathrm{f}}_{nj,t} - \sum_{m\in\Omega_n} \left(p^{\mathrm{f}}_{mn,t} - l_{mn,t}\mathrm{R}_{mn}\right) \quad \forall\, n, t \tag{A.9}$$

$$\frac{\partial Y_2}{\partial \lambda^{\mathrm{q}}_{n,t}} = \sum_{i\in\Omega_n} q_{i,t} + \sum_{j\in\Omega_n} q^{\mathrm{f}}_{nj,t} - \sum_{m\in\Omega_n} \left(q^{\mathrm{f}}_{mn,t} - l_{mn,t}\mathrm{X}_{mn}\right) \quad \forall\, n, t \tag{A.10}$$

$$\frac{\partial Y_2}{\partial l_{mn,t}} = R_{mn}\lambda^{\mathrm{p}}_{n,t} + X_{mn}\lambda^{\mathrm{q}}_{n,t} \quad \forall\, (m,n) \in B, t \tag{A.11}$$

$$\frac{\partial Y_2}{\partial u_{n,t}} = 0 \quad \forall\, n, t \tag{A.12}$$

$$\frac{\partial Y_2}{\partial p^{\mathrm{f}}_{mn,t}} = \lambda^{\mathrm{p}}_{m,t} - \lambda^{\mathrm{p}}_{n,t} \quad \forall\, (m,n) \in B, t \tag{A.13}$$

$$\frac{\partial Y_2}{\partial q^{\mathrm{f}}_{mn,t}} = \lambda^{\mathrm{q}}_{m,t} - \lambda^{\mathrm{q}}_{n,t} \quad \forall\, (m,n) \in B, t \tag{A.14}$$

$$\frac{\partial Y_3}{\partial f^{rs,kg}_{t\prime,t}} = \frac{\partial Y_3}{\partial f^{rs,kg}_{t\prime,t}} \frac{\partial f^{rs,kg}_{t\prime,t}}{\partial x_{a,t}} = \sum_{a\in\mathcal{A}} \omega\varphi_a(x_{a,t})\delta^{rs}_{akg} + C_{t',t}\left(f^{rs,kg}_{t',t}\right) = c^{rs,kg}_{t',t} \quad \forall\, rs, kg, t', t \tag{A.15}$$

$$\frac{\partial Y_3}{\partial f^{rs,ke}_{t\prime,t}} = \frac{\partial Y_3}{\partial f^{rs,ke}_{t\prime,t}} \frac{\partial f^{rs,ke}_{t\prime,t}}{\partial x_{a,t}} = \sum_{a\in\mathcal{A}} \omega\varphi_a\left(x_{a,t}\right)\delta^{rs}_{ake} + \sum_{a\in\mathcal{A}^C} \lambda^{\mathrm{p}}_{n(a),t}\mathrm{E_c}\delta^{rs}_{ake} + C_{t',t}\left(f^{rs,ke}_{t',t}\right) = c^{rs,ke}_{t',t} \quad \forall\, rs, ke, t', t \tag{A.16}$$

$$\frac{\partial Y_3}{\partial d_{t',t}^{rs,v}}=0 \quad \forall rs,v,t',t \tag{A.17}$$

The Jacobian matrix $JF(\gamma)$ can be calculated as:

$$JF(\gamma)=$$

$$\begin{bmatrix}
2\mathrm{m}^{\mathrm{p}} & 0 & 0 & 0 & \cdots & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2\mathrm{m}^{\mathrm{q}} & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2\mathrm{m}_w^{\mathrm{p}2} & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2\mathrm{m}_w^{\mathrm{q}2} & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 0 & 0 & 0 & \cdots & 0 & 0 & \mathrm{R}_{mn} & 0 & -1 & 0 & 0 & \mathrm{E_c}\delta_{ake}^{rs} & 0 \\
0 & 1 & 0 & 0 & \cdots & 0 & 0 & \mathrm{X}_{mn} & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & \mathrm{R}_{mn} & \mathrm{X}_{mn} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial\left(\sum_{a\in\mathcal{A}}\varphi_a(x_{a,t})\right)\omega{\delta_{akg}^{rs}}^2}{\partial x_{a,t}}+\rho & \frac{\partial\left(\sum_{a\in\mathcal{A}}\varphi_a(x_{a,t})\right)\omega\delta_{akg}^{rs}\delta_{ake}^{rs}}{\partial x_{a,t}}+\rho & 0 \\
0 & 0 & 0 & 0 & \cdots & \mathrm{E_c}\delta_{ake}^{rs} & 0 & 0 & 0 & 0 & 0 & \frac{\partial\left(\sum_{a\in\mathcal{A}}\varphi_a(x_{a,t})\right)\omega\delta_{akg}^{rs}\delta_{ake}^{rs}}{\partial x_{a,t}}+\rho & \frac{\partial\left(\sum_{a\in\mathcal{A}}\varphi_a(x_{a,t})\right)\omega{\delta_{ake}^{rs}}^2}{\partial x_{a,t}}+\rho & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

where $\rho=\frac{\partial C_{t',t}}{\partial f_{t',t}^{rs,kv}}$. The Jacobian matrix $JF(\gamma)$ is symmetric. Thus, there exists an equivalent optimization problem of the non-cooperative game Γ_{TP},