

APPENDIX A. PROOF OF PROPOSITION 3

For the non-cooperative game $\Gamma_{\text{TP}}(\cdot)$, the gradients of objective functions of all players $F(\gamma) = [\nabla Y_1(\gamma_1, \gamma_{-1}), \dots, \nabla Y_Z(\gamma_Z, \gamma_{-Z})]$ are calculated as follows:

$$\frac{\partial Y_1}{\partial p_{i,t}} = \lambda_{n,t}^p + 2m^p p_{i,t} \quad \forall i, t \quad (\text{A.1})$$

$$\frac{\partial Y_1}{\partial q_{i,t}} = \lambda_{n,t}^q + 2m^q q_{i,t} \quad \forall i, t \quad (\text{A.2})$$

$$\frac{\partial Y_1}{\partial p_{w,t}^g} = 2m_w^{p2} p_{w,t}^g + m_w^{p1} \quad \forall w, t \quad (\text{A.3})$$

$$\frac{\partial Y_1}{\partial q_{w,t}^g} = 2m_w^{q2} q_{w,t}^g \quad \forall w, t \quad (\text{A.4})$$

$$\frac{\partial Y_1}{\partial \hat{q}_{w,t}^g} = m_w^{q1} \quad \forall w, t \quad (\text{A.5})$$

$$\frac{\partial Y_1}{\partial p_{i,t}^{\text{ch}}} = 0 \quad \forall i, t \quad (\text{A.6})$$

$$\frac{\partial Y_1}{\partial p_{i,t}^{\text{dis}}} = 0 \quad \forall i, t \quad (\text{A.7})$$

$$\frac{\partial Y_1}{\partial e_{i,t}} = 0 \quad \forall i, t \quad (\text{A.8})$$

$$\frac{\partial Y_2}{\partial \lambda_{n,t}^p} = \sum_{i \in \Omega_n} p_{i,t} + \sum_{a \in \Omega_n} E_c x_{a,t} + \sum_{j \in \Omega_n} p_{nj,t}^f - \sum_{m \in \Omega_n} (p_{mn,t}^f - l_{mn,t} R_{mn}) \quad \forall n, t \quad (\text{A.9})$$

$$\frac{\partial Y_2}{\partial \lambda_{n,t}^q} = \sum_{i \in \Omega_n} q_{i,t} + \sum_{j \in \Omega_n} q_{nj,t}^f - \sum_{m \in \Omega_n} (q_{mn,t}^f - l_{mn,t} X_{mn}) \quad \forall n, t \quad (\text{A.10})$$

$$\frac{\partial Y_2}{\partial l_{mn,t}} = R_{mn} \lambda_{n,t}^p + X_{mn} \lambda_{n,t}^q \quad \forall (m, n) \in B, t \quad (\text{A.11})$$

$$\frac{\partial Y_2}{\partial u_{n,t}} = 0 \quad \forall n, t \quad (\text{A.12})$$

$$\frac{\partial Y_2}{\partial p_{mn,t}^f} = \lambda_{m,t}^p - \lambda_{n,t}^p \quad \forall (m, n) \in B, t \quad (\text{A.13})$$

$$\frac{\partial Y_2}{\partial q_{mn,t}^f} = \lambda_{m,t}^q - \lambda_{n,t}^q \quad \forall (m, n) \in B, t \quad (\text{A.14})$$

$$\frac{\partial Y_3}{\partial f_{t',t}^{rs,kg}} = \frac{\partial Y_3}{\partial f_{t',t}^{rs,kg}} \frac{\partial f_{t',t}^{rs,kg}}{\partial x_{a,t}} = \sum_{a \in \mathcal{A}} \omega \varphi_a(x_{a,t}) \delta_{akg}^{rs} + C_{t',t} \left(f_{t',t}^{rs,kg} \right) = c_{t',t}^{rs,kg} \quad \forall rs, kg, t', t \quad (\text{A.15})$$

$$\frac{\partial Y_3}{\partial f_{t',t}^{rs,ke}} = \frac{\partial Y_3}{\partial f_{t',t}^{rs,ke}} \frac{\partial f_{t',t}^{rs,ke}}{\partial x_{a,t}} = \sum_{a \in \mathcal{A}} \omega \varphi_a(x_{a,t}) \delta_{ake}^{rs} + \sum_{a \in \mathcal{A}^C} \lambda_{n(a),t}^p E_c \delta_{ake}^{rs} + C_{t',t} \left(f_{t',t}^{rs,ke} \right) = c_{t',t}^{rs,ke} \quad \forall rs, ke, t', t \quad (\text{A.16})$$

$$\frac{\partial Y_3}{\partial d_{t',t}^{rs,v}} = 0 \quad \forall rs, v, t', t \quad (\text{A.17})$$

The Jacobian matrix $JF(\gamma)$ can be calculated as:

$$JF(\gamma) =$$

$$\left[\begin{array}{cccccccccccc} 2\mathbf{m}^p & 0 & 0 & 0 & \cdots & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\mathbf{m}^q & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\mathbf{m}_w^{p2} & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\mathbf{m}_w^{q2} & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 & \cdots & \mathbf{0} & 0 & \mathbf{R}_{mn} & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{E}_c \delta_{ake}^{rs} & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 & \mathbf{0} & \mathbf{X}_{mn} & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & \mathbf{R}_{mn} & \mathbf{X}_{mn} & \mathbf{0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & -1 & 0 & 0 & 0 & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & -1 & 0 & 0 & 0 & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & \mathbf{E}_c \delta_{ake}^{rs} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{0} \end{array} \right] \begin{array}{l} \\ \\ \\ \\ \\ \\ \frac{\partial(\sum_{a \in \mathcal{A}} \varphi_a(x_{a,t})) \omega \delta_{ake}^{rs}{}^2}{\partial x_{a,t}} + \rho \\ \frac{\partial(\sum_{a \in \mathcal{A}} \varphi_a(x_{a,t})) \omega \delta_{ake}^{rs} \delta_{ake}^{rs}}{\partial x_{a,t}} + \rho \\ 0 \end{array}$$

where $\rho = \frac{\partial C_{t',t}}{\partial f_{t',t}^{rs,kv}}$. The Jacobian matrix $JF(\gamma)$ is symmetric. Thus, there exists an equivalent optimization problem of the non-cooperative game Γ_{TP} ,