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UNIT-II

ELECTROSTATICS & AC FUNDAMENTALS

A) ELECTROSTATICS - 2A

Electrostatics is that branch of science which deals with the phenomenon associated with charge at rest.

COLOUMB'S LAWS OF ELECTROSTATICS

First Law · like charge repel eachother & unlike charges attract each other.

Second Law - The magnitude of force of attraction or repulsion between any two charged bodies

(i) is directly propotional to the product of two charges
(ii) is inversely propotional to square of the distance between them
(iii) Depends on medium between the charges.

$$F \propto \frac{Q_1 Q_2}{d^2}$$

$$F = K\frac{Q_1 Q_2}{d^2} \qquad \text{here } K = \frac{1}{4\pi\varepsilon_0\varepsilon_r}$$

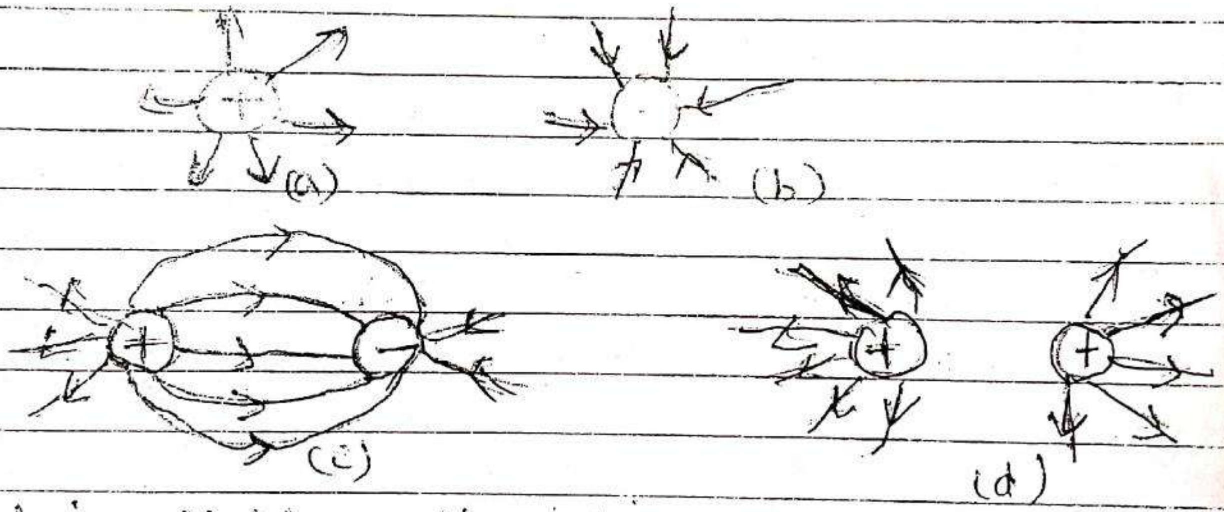
$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m.

ELECTRIC FIELD -

When a charged body is placed near another charge it experiences a force of attraction or repulsion depending upon the kind of charges.

DIFFERENT QUANTITIES RELATING TO ELECTRO STATICS

(ii) Electric field -
It is defined as the space in which an electric charge experiences a force



Electric field configuration
(a) isolated positively charged conductor
(b) isolated negatively charged conductor
(c) two equal unlike charges.
(d) two equal like charges.

Electric flux -
It is defined as the ~~flux~~ total number of electric field lines. It is represented by the symbol ψ (Psi) The unit of electric flux is coulomb

$$\psi = Q$$

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Electric flux density

It is defined as the flux passing through unit area at right angle to the direction of field. Denoted by 'D' measured in coulomb /m^2

$$D = \frac{\psi}{A} \ C/m^2$$

Electric field strength

It is defined as the force experienced by unit positive charges at a particular point in a given electric field. denoted by E measured in Newton/ coulomb.

$$E = F/Q = N/C \text{ or } V/m$$

It is also called electric field intensity. It is different at different point in a non uniform electric field & same at all points in uniform electric field.

Permittivity (ε)

It is defined as the ability of the medium to allow passage of flux to pass through it.

The ratio of electric flux density in a vacuum or free space to the corresponding electric ~~flux~~ field strength is known as the permittivity of free space. It is denoted by ε_0 measured in F/m

$$\varepsilon_0 = 8.854 \times 10^{-12} \ F/m$$

(a) Absolute Permittivity -

$$\varepsilon = \frac{D}{E} \ F/m$$

(b) Relative Permittivity -

$$\varepsilon_r = \frac{D}{D_0}$$

Potential Gradient -

$$dV = -E\,dx$$

$$\boxed{E = -\frac{dV}{dx}}$$

Defined as the rate of change of potenteal with distance in the direction of electric field.

CAPACITOR -

Two conductors separated from each other by an insulating material form capacitor. The insulating material is called as dielectric.

CAPACITANCE

It is the property of capacitor to store the electric charge

Action of capacitor -

Consider the conducting plates P & Q separated by an air shown in figure. Each plate has large number of free electrons. As soon as plates are connected to DC supply, e^- are attracted from plate P to the positive side of the battery. This causes the deficiency of e^- on the plate 'P', resulting into ⊕ve charge on plate 'P'. The e^- from the ⊖ve side of the battery are repelled & collected on plate Q. Thus charge on plate 'P' is positive & 'Q' is negative.

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Relation between Voltage, capacitance & charge

let V = applied voltage in volts
Q = Charge stored by capacitor in coulomb.

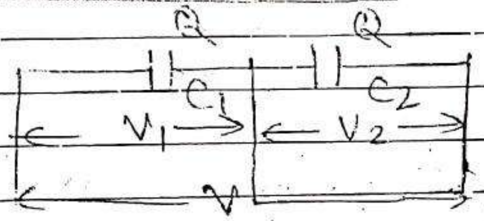
$$Q \propto V$$

$$Q = CV$$

$$\boxed{C = \frac{Q}{V}}$$

Hence capacitance can be defined as the ratio of stored ~~charge~~ charge to the applied voltage. It is measured in Farad.

CAPACITORS IN SERIES



consider the two capacitors in series connected across the supply voltage 'V'

Let Q be the charge on each capacitor.

$$Q = C_1V_1$$

$$Q = C_2V_2$$

$$V_1 = \frac{Q}{C_1} \quad \& \quad V_2 = \frac{Q}{C_2}$$

Also the charge stored by equivalent capacitor

$$Q = CV$$

$$V = \frac{Q}{C}$$

As the charge on each capacitor & on equivalent capacitor is same we have.

$$Q = CV = C_1V_1 = C_2V_2$$

Now the total voltage. $V = V_1 + V_2$

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

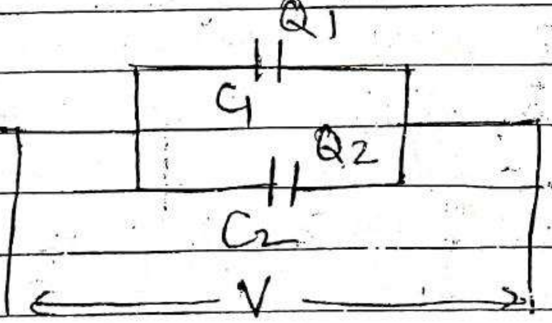
- so if number of capacitors are connected in series then equivalent capacitance

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots \frac{1}{C_n}}$$

CAPACITORS IN PARALLEL

consider the two capacitors in parallel connected across the supply voltage 'V'

let Q_1 be the charge on capacitor C_1
Q_2 be the charge on capacitor C_2



$Q_1 = C_1V$
$Q_2 = C_2V$

Now Q be the total charge in ~~tot~~ coulomb
$Q = CV$

$$Q = Q_1 + Q_2$$

$$CV = C_1V + C_2V$$

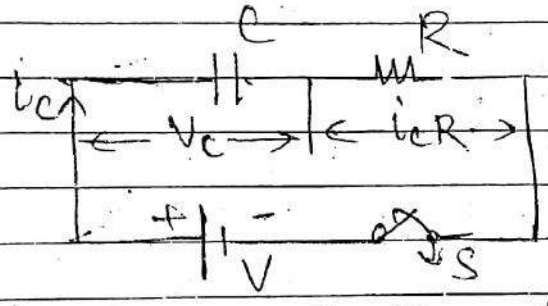
$$C = C_1 + C_2$$

If number of capacitance are connected in parallel

$$\boxed{C = C_1 + C_2 + \cdots C_n}$$

Charging of capacitor

consider a circuit as shown in figure, where capacitor C is connected in series with a resistance R across a battery having V & switch 's'.



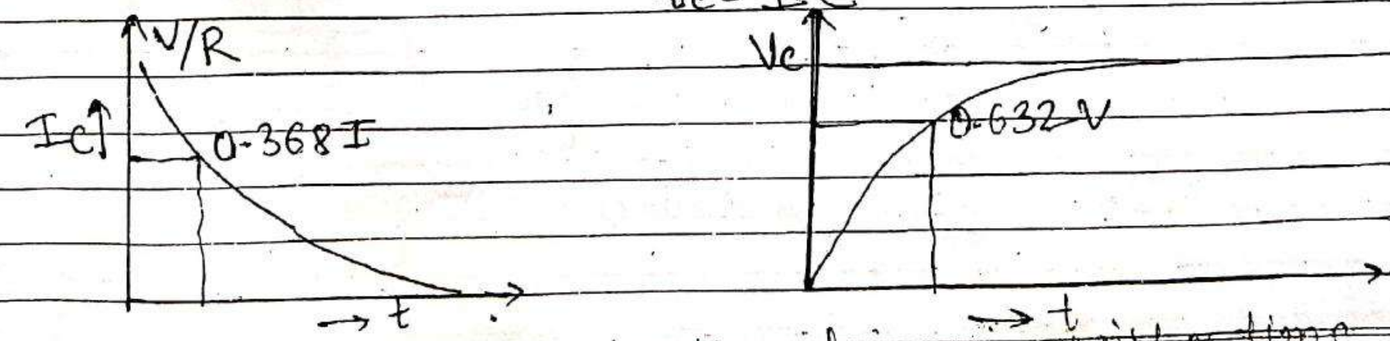
Charging of capacitor through Resistor.

During charging: The voltage across capacitor

$$V_C = V(1 - e^{-t/RC})$$

current through capacitor

$$i_C = I e^{-t/RC}$$



Voltage & current variation ~~in~~ with the time during charging of C

Time constant (charging)

Time constant may be defined as the time during which the capacitor voltage actually reaches to its 63.2% of its final value.

i.e.

$$V_c = V\left(1-e^{-t/RC}\right)$$

if $t = RC$

$$V_c = V(1-e^{-1})$$

$$\boxed{V_c = 0.632V}$$

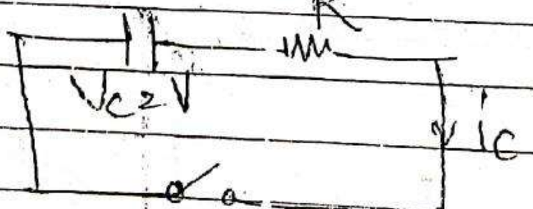
Similarly

$$i_c = Ie^{-1}$$

$$\boxed{i_c = 0.368I}$$

DISCHARGING OF CAPACITOR

Consider a circuit shown where capacitor 'C' is being discharged through a resistor 'R'. Capacitor is fully charged to voltage V volts & discharges through resistor R. The current flowing through the circuit is in the opposite direction to that of charging. The discharging process is controlled by a switch 's'



Discharging of a capacitor through resistor.

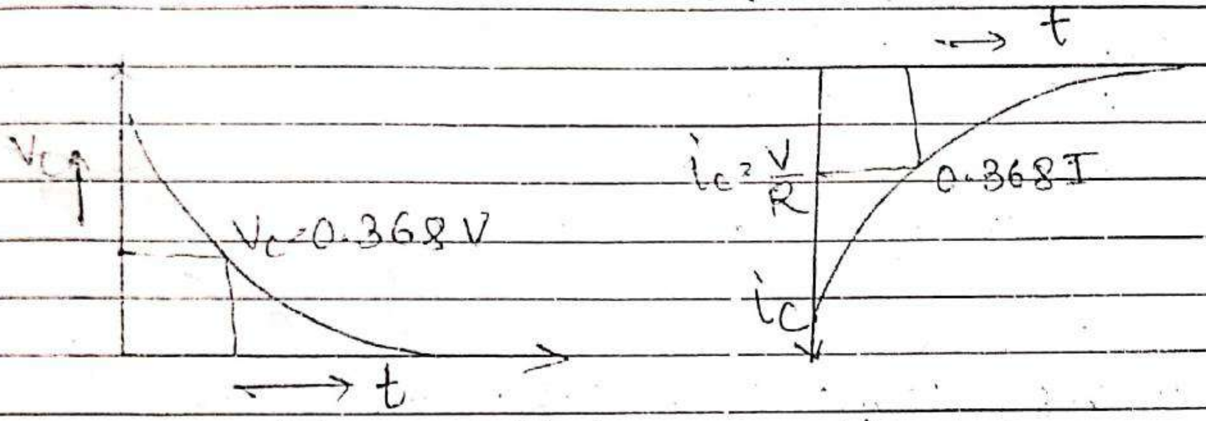
During discharging

The voltage across capacitor

$$V_c = V e^{-t/RC}$$

The current through capacitor.

$$i_c = -\frac{V}{R} e^{-t/RC}$$



Variation of discharge voltage & current.

Time constant

As $V_c = V e^{-t/RC}$

If $t = RC$

then $V_c = V e^{-1}$

$V_c = 0.368V$

Hence, time constant can be defined as time required for the capacitor voltage to fall 36.8% of its max^{m} value.

Similarly for current,

$$i_c = -I e^{-t/RC}$$

$$t = RC$$

$$i_c = -I e^{-1} = 0.368I.$$

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ENERGY STORED IN A CAPACITOR

Let us consider a capacitor having capacitance 'C' Farad & charge to voltage 'V' volts.

Let 'q' be the charge on capacitor in coulomb. Then potential difference across the capacitor.

$$V = \frac{q}{C}$$

Now work done ~~is~~ to move the charge of one coulomb from one plate to another

$$dw = v\,dq = \frac{q}{C}\,dq \text{ Joule}$$

This work done is stored in the form of potential energy in the electric field. Now total energy stored in the capacitor when it is charged to 'Q' coulomb.

$$\int dw = \int_0^Q \frac{q}{C}\,dq$$

$$W = \frac{1}{C}\int_0^Q q\,dq$$

$$W = \frac{1}{C} \times \frac{Q^2}{2}$$

$$W = \frac{1}{2}\frac{Q^2}{C}$$

$Q = CV$

$$\boxed{W = \frac{1}{2} CV^2 \text{ Joule}}$$

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Ques -1) A capacitor of 50 μF is uniformaly charged with a current of 10 mA for 10 sec. Calculate (i) Charge (ii) ~~capacitor~~ voltage (iii) stored energy.

Sol^n - Given - $C = 50\,\mu F$

$I = 10\,mA$

$t = 10\,sec.$

(i) charge

$$Q = It = 10 \times 10^{-3} \times 10 = \underline{0.1\,C}$$

(ii) capacitor voltage

$$V = \frac{Q}{C} = \frac{0.1}{50 \times 10^{-6}} = \underline{2000\,V}$$

(iii) Energy stored.

$$E = \frac{1}{2} CV^2 = \frac{1}{2} \times 50 \times 10^{-6} \times (2000)^2$$

$$\boxed{E = 100\,J.}$$

Ques-3) A capacitor consist of 2 parallel plates ea 120 mm^2 separated by 1mm in air. when a voltage of 1000V is applied between the plates calculate (i) the charge on the capacitor
(ii) Electric flux density
(iii) the electric field strength in the dielectric

Soln - Given

$A = 120 mm^2 = 120 \times 10^{-6} m^2$
$d = 1 mm = 1 \times 10^{-3} m$
$V = 1000 V$
$\varepsilon_r = 1$

Solution -
(i) Charge on the capacitor.

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

$$C = \frac{8.854 \times 10^{-12} \times 1 \times 120 \times 10^{-6}}{1 \times 10^{-3}}$$

$= 1.06$ PF

$Q = CV = 1.06 \times 10^{-12} \times 1000 = 1.06 nC$

(ii) Electric flux density

$$D = \frac{Q}{A} = \frac{1.06 \times 10^{-9}}{120 \times 10^{-6}}$$

$= 8.854 \times 10^{-6} C/m^2$

(iii) Electric field strength.

$$E = \frac{D}{\varepsilon_0} = \frac{8.854 \times 10^{-6}}{8.854 \times 10^{-12}}$$

$E = 1 \times 10^{6} V/m$

(B) AC FUNDAMENTALS

Sinusoids

A Sinusoid is a signal has the form of Sine or cosine function. These signals reverses at regular time intervals & has alternative negative & positive values.

The general representation of sinusoidal quantity (voltage or current) is

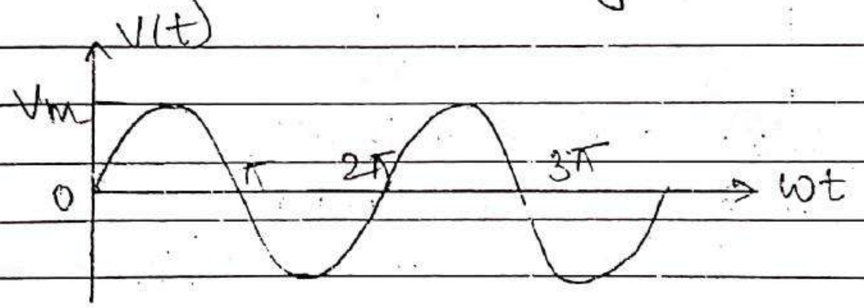
$x(t) = A \sin \omega t$ or $V(t) = V_m \sin \omega t$

Where A = Amplitude of the sinusoid

ω = angular freq in rad/sec $= 2\pi f = 2\pi/T$

Where f = freq in Hz $= \frac{1}{T}$ in cycle/sec

T = Time period = Time for completing one cycle.



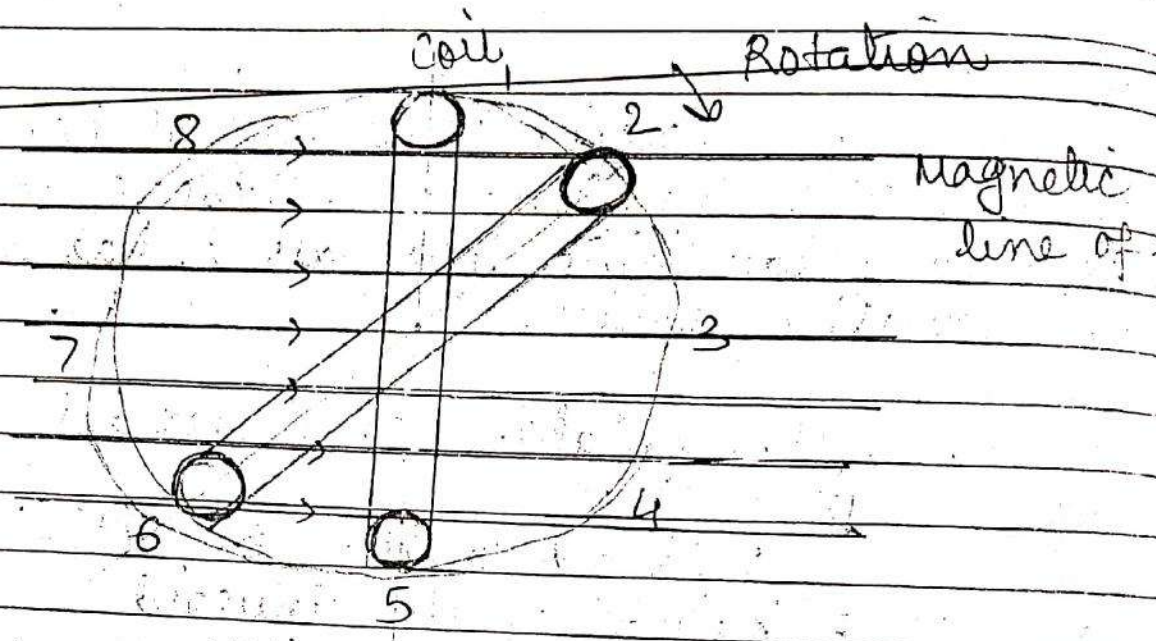
GENERATION OF SINUSOIDAL VOLTAGE

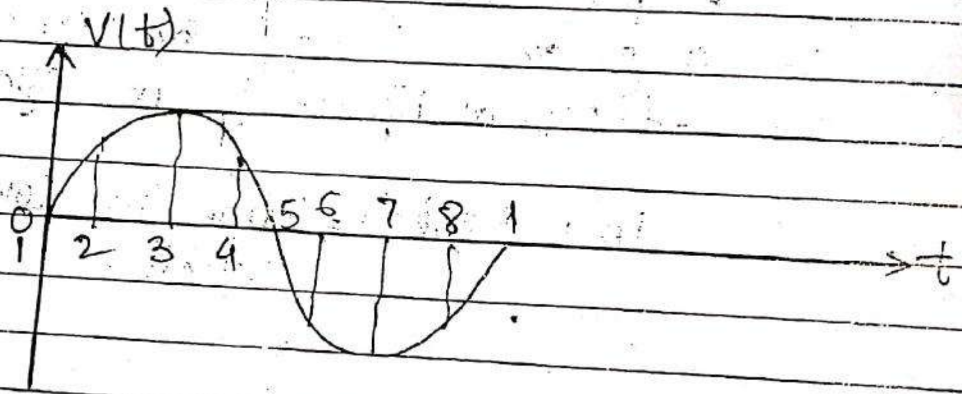
→ AC generators (or Alternator) generates AC or sinusoid voltage based on the principle of faraday's law of electromagnetic induction.

→ It states whenever flux is cut by the conductor, then an EMF is induced across the conductor.

→ Lets a coil having length 'l' is placed in a magnetic field as directed in the figure.

→ Now the coil is rotated by means of some mechanical force, then the flux linkage changes with respect to time & the voltage induced is given by the formula $v = Blv\sin\theta$





Where B = Magnetic flux density
l = length of conductor
v = Velocity of the conductor.

Terms related to alternating quantity

1) Instantaneous value - The value of the alternating quantity at a given instant (time) is called instantaneous value

2) Amplitude (Peak value or Max^m value) - It is the Max^m value of an alternating quantity attained by it in a cycle. It is representated as Vm or Im

3) cycle - one complete set of positive and negative values of an alternating quantity is called as cycle

4) Time period - Time taken by an alternating quantity to complete one cycle is known as time period.

$$T = \frac{1}{f} \text{ (sec)}$$

f in Hz

5) Frequency - No. of cycles per second is called freq

6) Phase difference - It is the fractional part of a period by which one has advanced (lead) over or (lag) behind the other.

$$V(t) = V_m \sin(\omega t + \phi)$$

ϕ = phase

The following condition must satisfy to compare two sinusoidal signals.

Note - To find the phase difference betn two sinusoid those to have same freq. with positive amplitude

Values of Alternating voltage & current

(i) Peak value or Max^m value - It is the max^m value of an alternating quantity Reached in one cycle.

(ii) Average value - It is determined by averaging all the instantaneous values over a period of half cycle.

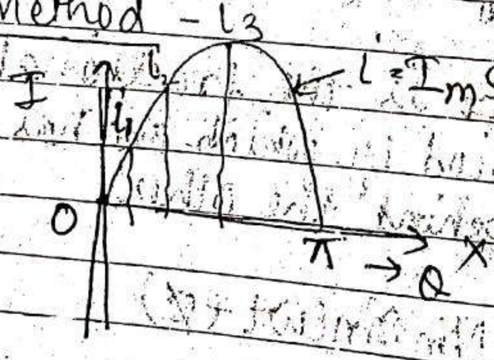
In case of a symmetrical alternating wave form the average value over a complete cycle is zero.

$$I_{avg} = \frac{i_1 + i_2 + \ldots\ldots i_m}{m}$$

$$I_{avg} = \frac{1}{t_2 - t_2}\int_{t_1}^{t_2} i(t)\,dt.$$

+ avg value can be obtained by either graphical or analytical method.

Graphical Method -



$$I_{av} = \frac{i_1 + i_2 + i_3 + \ldots\ldots i_n}{n}$$

Similarly

$$V_{av} = \frac{V_1 + V_2 + V_3 + \ldots\ldots V_n}{n}$$

Analytical method..

Alternating current can be expressed as

$$i = I_m \sin\theta$$

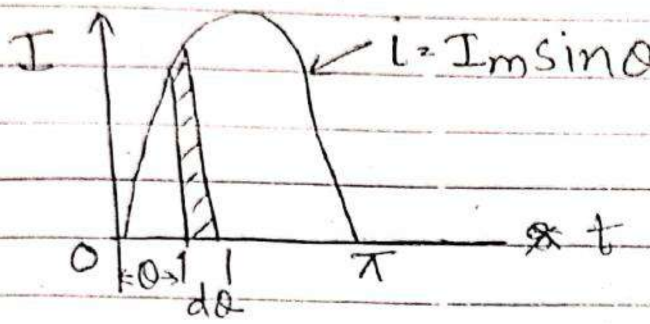


Figure shows one half cycle for instanteneous current i. Here to find are under the curve i, consider an interval of dθ at a distance θ from the origin. Then total area under the curve over half cycle $= \int_0^{\pi} i \, d\theta$.

∴ Avg value of current over a half cycle

$$I_{av} = \frac{\text{Area under the curve over half } \cancel{\text{curve}} \text{ cycle}}{\text{Length of base over half cycle}}$$

$$= \frac{\int_0^{\pi} i \, d\theta}{\pi}$$

$$= \frac{1}{\pi} \int_0^{\pi} I_m \sin\theta \, d\theta$$

$$= \frac{I_m}{\pi} \left[-\cos\theta \right]_0^{\pi}$$

$$I_{av} = \frac{2 I_m}{\pi} = 0.637 I_m$$

$$\boxed{I_{av} = 0.637 I_m}$$

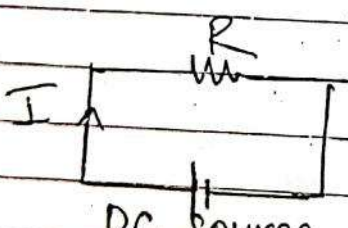
$$V_{av} = \frac{2V_m}{\pi}$$

ROOT MEAN SQUARE (RMS) value OR EFFECTIVE VALUE OF AC CURRENT OR VOLTAGE

From the definition of AC current we know that it varies from instant to instant, whereas the magnitude of DC current remains constant with time. For comparing the relative effectiveness of above two, the effect produced by two currents are compared and one such common effect is heating of resistance by the currents.

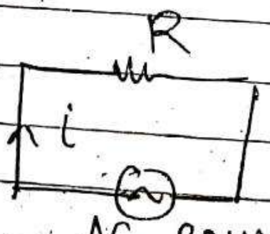
Based on the comparision, rms value of current is defined as follows

The rms value of an AC is given by that of DC which when flowing through a given circuit for a given time produces the same amount of heat as produced by ~~at~~ AC when flowing through the same circuit for same time.



applied for time t

(a)



AC source applied for same t

(b)

RMS value can be found by the following two methods.

(i) Graphical method (ii) Analytical method.

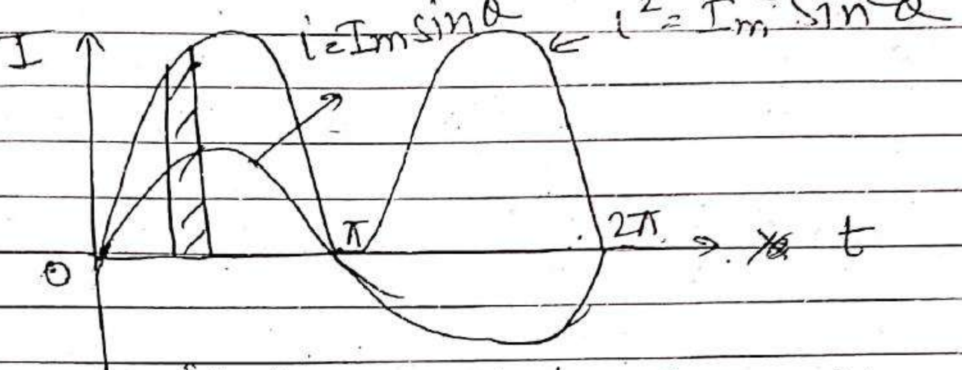
Analytical method

AC can be expressed analytically as

$$I = I_m \sin \theta$$

$$\theta = \omega t$$

taking square of current $i^2 = I_m^2 \sin^2 \theta$.



Wave form for i^2 is plotted in above figure for an half cycle. To find the area under the curve (i^2), an interval of $d\theta$ is consider at a distance θ from origin.

$\therefore$ Area of squared curve over half cycle $= \int_0^{\pi} i^2 d\theta$

The length of base is π, therefore mean value of Square of current over half cycle;

$$= \frac{\text{Area of squared curve over half cycle}}{\text{Length of base over half cycle}}$$

$$= \frac{\int_0^{\pi} i^2 d\theta}{\pi}$$

$$= \frac{1}{\pi} \int_0^{\pi} I_m^2 \sin^2 \theta \, d\theta$$

$$= \frac{I_m^2}{2\pi} \int_0^{\pi} \left[\frac{1 - \cos 2\theta}{2} \right] d\theta$$

$$= \frac{I_m^2}{2\pi} \int_0^{\pi} (1 - \cos 2\theta)\, d\theta$$

$$= \frac{I_m^2}{2\pi} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{\pi}$$

$$= \frac{I_m^2}{2\pi} [\pi]$$

$$= \frac{I_m^2}{2}$$

Hence root mean square i.e. rms value c be calculated as.

$$I_{rms} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

$$\boxed{I_{rms} = 0.707\, I_m}$$

$$\boxed{V_{rms} = 0.707\, V_m}$$

Peak Factor and Form Factor

Peak Factor (PF)

The Peak factor of an ~~Aa~~ alternating quantity is defined as ratio of max^m value to the rms value. This factor may also be called as crest factor or amplitude factor.

$$\text{Peak Factor } K_p = \frac{\text{Max}^m \text{ Value}}{\text{RMS value}}$$

$$= \frac{I_m}{0.707 I_m}$$

$$= \frac{1}{0.707}$$

$$\boxed{K_p = \sqrt{2} = 1.414}$$

Form Factor

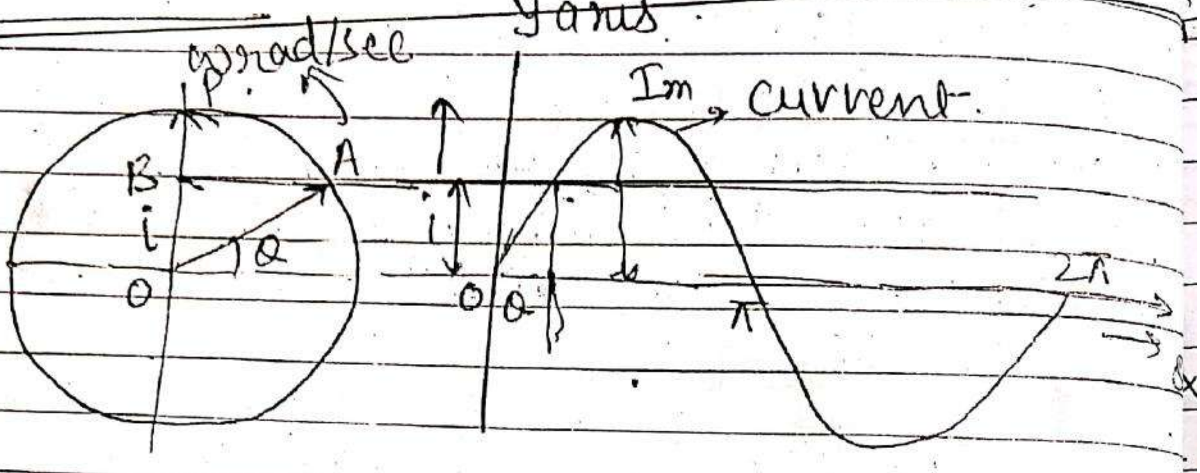
The ratio of rms value to average value of an alternating quantity is called as form factor.

$$K_f = \frac{\text{RMS value}}{\text{Avg value}}$$

$$K_f = \frac{0.707 I_m}{0.637 I_m} = 1.11$$

$$\boxed{K_f = 1.11}$$

PHASOR REPRESENTATION OF ALTERNATIN QUANTITIES



The Alternating quantitees are represented by phas
A phasor is a line of definite length rotating i
an anticlockwise direction at a constant angular
velocity ω. The length of a phasor is equal to the
max^{m} value of the alternating quantity & the ang
velocity is equal to the angular velocity of an
alternating quantity.

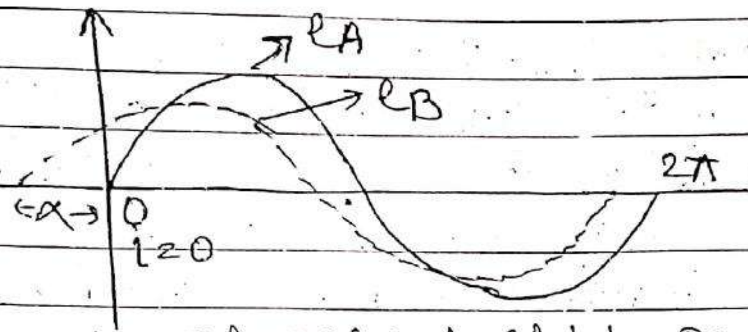
In above fig. consider a phasor
OP = Im is the ~~ve~~ max^{m} value of an alternating
quantity. Let this phasor rotates in an anticlock
wise dir^{n} at a uniform angular velocity of ωrad
The projection of the phasor OP on the
Y axis at any instant gives the instantaneous
Value ~~of the~~

~~OM = OA~~

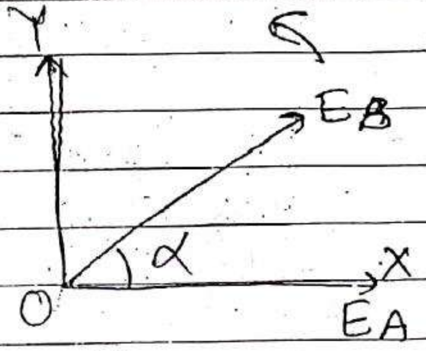
$$OA = OP \sin\omega t$$

$$i = I_m \sin\omega t.$$

PHASOR DIAGRAM



waveforms out of phase



Phasor diagram

Mathematical Representation of phasors

A Phasor can be represented in four forms

$$V = V_m(\sin\omega t \pm \phi)$$

(i) Rectangular form

$$V = X + jY$$

Magnitude of V = $|V| = \sqrt{X^2+Y^2}$

angle of V = $\angle V = \tan^{-1}\left(\frac{Y}{X}\right)$

(ii) Trigonometric form

$$\bar{V} = V(\cos\phi \pm j\sin\phi)$$

(iii) Exponential form $\bar{V} = Ve^{\pm j\phi}$

(iv) Polar form $\bar{V} = V\angle\pm\phi$

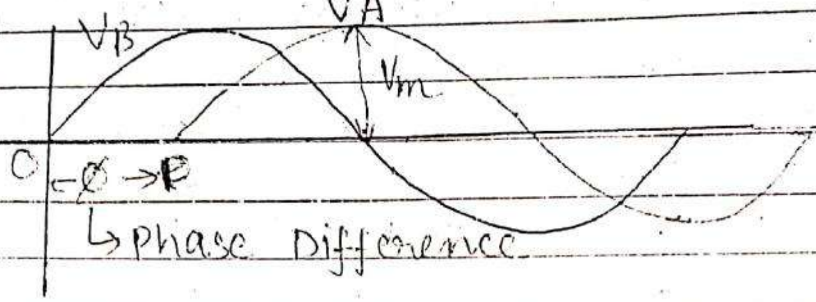
here, $j = \sqrt{-1}$

Phase DIFFERENCE

→ It is not necessary that two voltages or current waves originates at the same instant of time

→ The voltages V_A & V_B donot have the same zero crossover point so we say that ~~three~~ there is a Phase difference between them. Denoted by $\emptyset$

→ Both V_A & V_B have the same freq. and same peak voltage V_m



Concept of Phase difference.

→ We can represent the two voltages mathematically as follows.

$$\left.\begin{aligned} V_A &= V_m \sin\theta \\ V_B &= V_m \sin(\theta + \emptyset) \end{aligned}\right\} \quad -(1)$$

→ The angle $\emptyset$ is known as the Phase difference between V_A & V_B. It is measured in "radians" or degree.

→ If measured in radians the phase difference can take any value between 0 & 2π. but if expressed in degree it can take any value between 0 & 360°.

LEADING & LAGGING PHASE DIFFERENCE

Leading Phase difference.

→ If the phase angle $\emptyset$ in Equation (1) above is positive then the phase difference ϕ is said to be a leading phase difference. In ~~order~~ other words we say that voltage V_B leads the voltage V_A.

→ Mathematically

$$V_A = V_m \sin\theta$$

$$V_B = V_m \sin(\theta + \phi)$$

↳ Phase difference

Positive sign indicates V_B leads

Lagging Phase difference

→ If the phase angle ϕ in Eq (1) is negative, then the phase difference is said to be a lagging phase difference. That means V_B lags behind V_A by ϕ

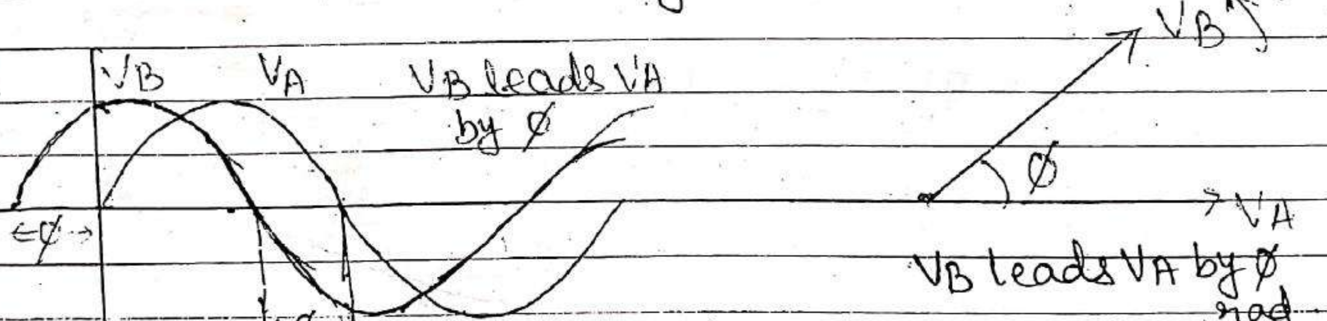
→ Mathematically

$$V_A = V_m \sin\theta$$

$$V_B = V_m \sin(\theta - \phi)$$

↳ Phase Difference

negative sign indicates that V_B lags behind V_A



concept of leading Phase difference