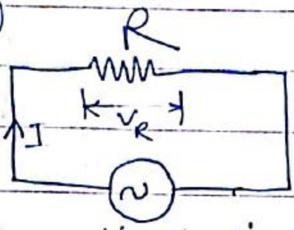
Behaviour of a Pure Resistor in an AC Ckt. :-

Consider a pure resistor R connected across an alternating vtg source v as shown in figure.

Let, the alternating voltage be;

$$v = V_m \sin\omega t \quad \text{--- (1)}$$



$V = V_m \sin\omega t$.

Current:- $I = \frac{v}{R} = \frac{V_m}{R} \sin\omega t$

$$\Rightarrow \boxed{I = I_m \cdot \sin\omega t} \quad \text{--- (2)}$$

$\because V_m/R = I_m$

where; I_m = max^m value of alternating current.

From eqn (1) & (2); it is clear that the current is in phase with vtg. in a purely resistive ckt.

Waveform:-

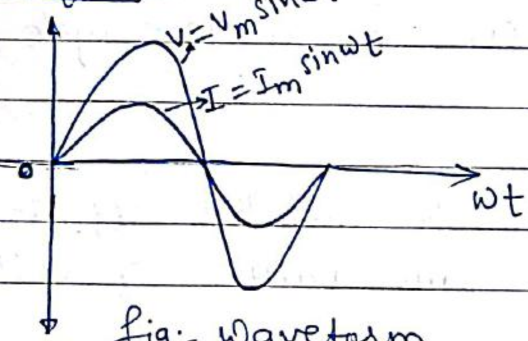


fig:- Waveform

Phasor Diagram:-

fig:- phasor dia.

Impedence:- It is the resistance offered to the flow of current in an ac ckt. In purely resistive circuit;

$$Z = \frac{V}{I} = \frac{V_m \sin\omega t}{I_m \sin\omega t} = \frac{V_m}{I_m} = R.$$

Phase Difference:- from eqn (1) & (2); the vtg & current are in phase with each other, the phase difference is zero.

$$\boxed{\phi = 0^\circ}$$

Powerfactor :- It is defined as the cosine angle betn the vtg & current phasors.

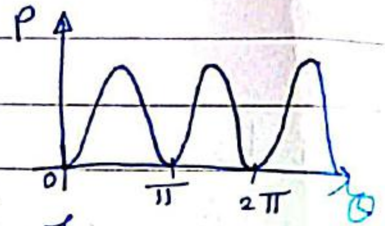
$\therefore$ powerfactor $= \cos\phi = \cos(0^\circ) = 1$

Power :- (1) The instantaneous power is given by;

$$p = v \cdot i$$
$$= V_m \sin\omega t \cdot I_m \sin\omega t$$
$$= V_m I_m \sin^2\omega t$$
$$= V_m I_m \left(\frac{1-\cos 2\omega t}{2}\right) = \frac{V_m I_m}{2}(1-\cos 2\omega t)$$

$$\boxed{p = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t} \quad —$$

fluctuating power



(2) Average power :-

$$P_{avg} = \frac{V_m I_m}{2} \quad — 0$$

$\because$ The freq of fluctuating power is twice the applied vtg freq. & its average value over a complete cycle is zero.

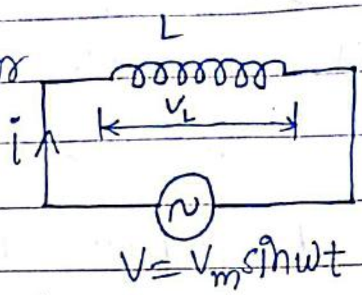
$$P_{avg} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$

$$\boxed{P_{avg} = V_{rms} I_{rms}}$$

$\therefore$ Thus, power in a purely resistive ckt is equal to the product of rms values of voltage and current.

Behaviour of a Pure Inductor in an AC ckt.

consider; a pure inductor L connected across an alternating vtg (v) as shown in fig.



let; the alternating vtg be;

$$v = V_m \sin\omega t \quad \text{---} (1)$$

Current:-

The alternating current is given by;

$$i = \frac{1}{L}\int v \cdot dt = \frac{1}{L}\int V_m \sin\omega t \cdot dt$$

$$= \frac{V_m}{\omega L}(-\cos\omega t)$$

$$= \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\boxed{i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)} \quad \text{---} (2) \quad \because I_m = \frac{V_m}{\omega L}$$

where; I_m = max^m value of an alternating current.

from equ^n (1) & (2); it is clear that the current lags behind the vtg by 90° in a purely inductive ckt.

Waveform:-

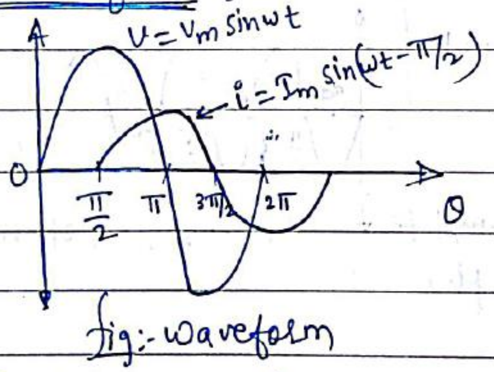


fig:- waveform

Phasor diagram:-

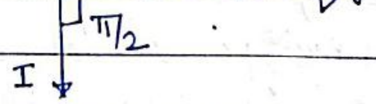


fig:- Phasor dia.

Impedence:- In a purely Inductive ckt;

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{V_m/\omega L} = \omega L \qquad \boxed{Z = \omega L}$$

where; ωL - is called inductive reactance & it is denoted by X_L & its unit is Ω

$$\boxed{X_L = \omega L = 2\pi f L}$$

(a) if dc supply $f=0$ $X_L = 0$

Thus; inductor acts as a short-circuit for a dc supply.

Phase difference:- It is the angle betn the vtg & current phasors.

$$\boxed{\phi = 90^\circ}$$

Powerfactor:- It is defined as the angle betn vtg & current phasors

$$p.f. = \cos\phi = \cos(90^\circ) = 0$$

Power:- (a) Instantaneous power is given by;

$$p = v \cdot i$$

$$p = V_m \sin\omega t \cdot I_m \sin(\omega t - \pi/2)$$

$$p = -V_m I_m \sin\omega t \cdot \cos\omega t \quad \text{--- } \because \sin(\omega t - \pi/2) = -\cos\omega t$$

$$p = -V_m I_m \frac{\sin 2\omega t}{2} \quad \text{--- } \because 2\sin\omega t \cdot \cos\omega t = \sin 2\omega t, \ \sin\omega t \cos\omega t = \frac{\sin 2\omega t}{2}$$

$$\boxed{p = -\frac{V_m I_m}{2} \sin 2\omega t}$$

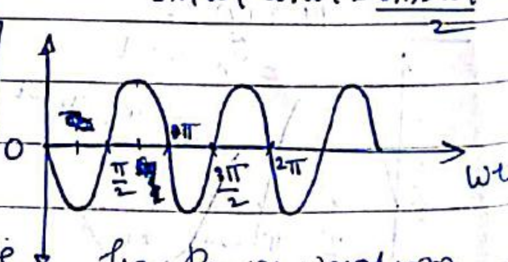


fig: Power waveform

(b) Average power:-

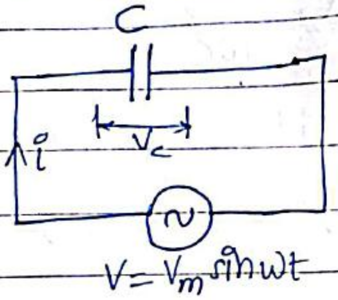
Average power for one complete cycle is zero (becoz, the freq is doubled).

$$\boxed{P_{avg} = 0}$$

Hence; Power consumed by a purely inductive ckt. is zero.

Behaviour of a Pure Capacitor in an AC Ckt:-

Consider; a pure capacitor c connected across an alternating vtg. (v) as shown in figure.



Let; the alternating vtg. is given by; $v = v_m \sin wt$ —— (1)

Current:- The alternating current is given by;

$$i = C \frac{dv}{dt} = C \frac{d}{dt}(V_m \sin wt)$$

$$= C V_m \cos wt \, (w) = wC V_m \cos wt$$

$$= wC V_m \sin(wt + \pi/2)$$

$$\boxed{i = I_m \sin(wt + \pi/2)} \text{ —— (2)}$$

$\because I_m = wC V_m$

where; I_m = max^m value of an alternating current.

From eq^n (1) & (2); it is clear that the current leads the vtg by 90° in a purely capacitive ckt.

Waveform:- | Phasor dia:-

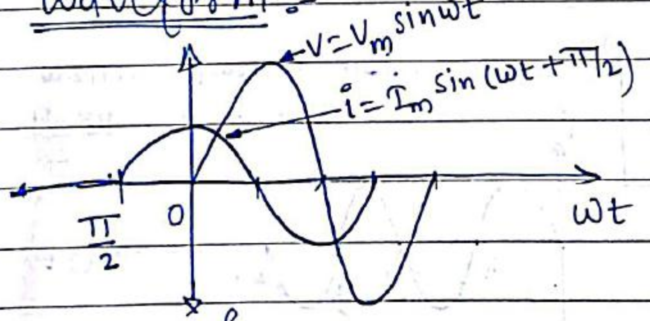


fig:- waveform

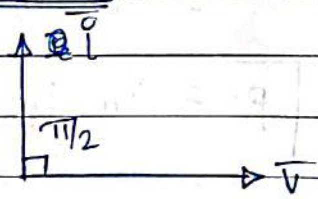


fig:- phasor dia

Impedence:- In a purely capacitive ckt;

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{wC V_m} = \frac{1}{wC} \qquad \boxed{Z = \frac{1}{wC}}$$

where; $\frac{1}{wC}$ is called capacitive reactance.

& it is denoted by X_C & measured in Ω

$$\boxed{Z = X_C = \frac{1}{wC} = \frac{1}{2\pi f C}}$$

(a) for dc supply $f = 0$ $X_C = \infty$
Hence; the capacitor acts as an open ckt for a dc supply

Phase difference :- It is the angle betn the vtg & current phasors.

$$\boxed{\phi = 90^\circ}$$

Powerfactor :- It is defined as the cosine angle betn vtg & current phasors

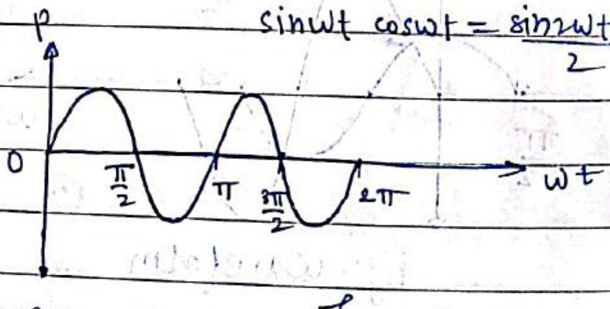
$$P.F. = \cos\phi = \cos(90^\circ) = 0$$

Power :- (a) The instantaneous power is given by;

$$P = v \cdot i$$
$$= V_m \sin wt \cdot I_m \sin(\omega t + \pi/2)$$
$$= V_m I_m \sin wt \cdot \cos wt \quad \because \sin(\omega t + \pi/2) = \cos wt$$

$\because 2\sin wt \cos wt = \sin 2wt$
$\sin wt \cos wt = \frac{\sin 2wt}{2}$

$$\boxed{P = \frac{V_m I_m}{2} \sin 2wt}$$



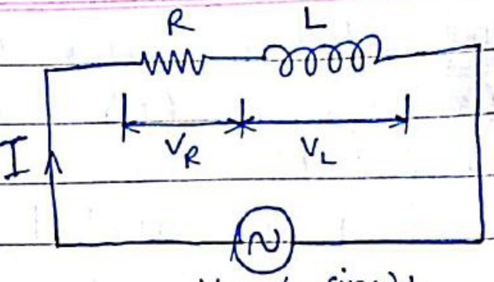
(b) Average Power :- The average power for one complete cycle is zero. (becoz; freq is doubled).

$$\boxed{P_{avg} = 0}$$

Hence; power consumed by purely capacitive ckt. is zero.

Series R-L ckt. :-

Figure shows a pure resistor (R) connected in series with a pure inductor (L) across an alternating vtg. (V).



$V_{rms} = V_m \sin \omega t$

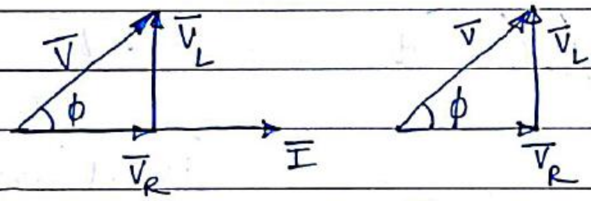
fig: series R-L ckt.

Let; V_{rms} & I_{rms} be the rms value of applied vtg and current.

$\therefore$ Potential difference across R $= V_R = R \cdot I$

$\therefore$ potential difference across L $= V_L = X_L \cdot I$

$\therefore$ The $\bar{V}_R$ is in phase with current I whereas the vtg $\bar{V}_L$ leads the current $\bar{I}$ by $90°$.



$$\boxed{\cos\phi = \frac{V_R}{V}}$$

fig(a):- phasor dia (b) Vtg. triangle

Impedence:-

$$\bar{V} = \bar{V}_R + \bar{V}_L$$

$$= R\bar{I} + jX_L\bar{I}$$

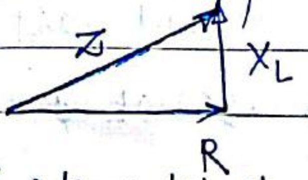
$$\bar{V} = \bar{I}(R + jX_L)$$

$$\Rightarrow \boxed{\frac{\bar{V}}{\bar{I}} = R + jX_L = \bar{Z}}$$

$\bar{Z} = Z \angle \phi$ — where; $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$

$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{\omega L}{R}\right)$

Where; Z - complex impedence of R-L ckt.



$$\boxed{\cos\phi = \frac{R}{Z}}$$

fig:- Impedence triangle

Current :- From phasor diagram, it is clear that the current (I) lags behind the vtg (V) by angle (ϕ).

If the applied vtg. is given by;

$$v = V_m \sin\omega t$$

then the current eqn will be;

$$I = I_m \sin(\omega t - \phi)$$

where; $I_m = \dfrac{V_m}{Z}$

& $\phi = \tan^{-1}\left(\dfrac{\omega L}{R}\right)$

Waveform :-

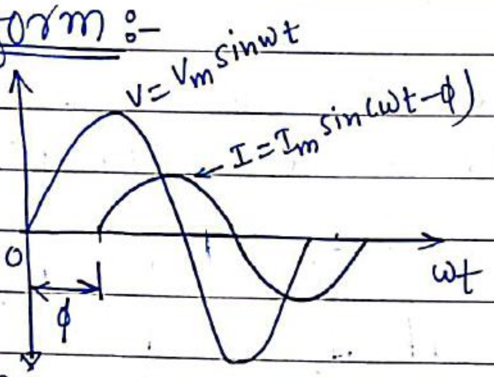


fig:- waveform

Phasor dia :-

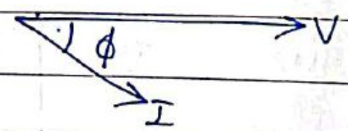


fig:- Phasor dia

Power:-

(a) Instantaneous power

$$p = v \cdot I$$

$$= V_m \sin\omega t \cdot I_m \sin(\omega t - \phi)$$

$$= V_m I_m \sin\omega t \cdot \sin(\omega t - \phi) \quad \text{— } \sin A \cdot \sin B$$

$$P = V_m I_m \frac{1}{2}\{\cos\phi - \cos(2\omega t - \phi)\} \quad = \frac{1}{2}\{\cos(A-B) - \cos(A+B)\}$$

$$\boxed{P = \underbrace{\frac{V_m I_m}{2}\cos\phi}_{\text{Constant Power}} - \underbrace{\frac{V_m I_m}{2}\cos(2\omega t - \phi)}_{\text{fluctuating power}}}$$

Thus; power consists of a constant part $\left(\dfrac{V_m I_m}{2}\cos\phi\right)$ & a fluctuating power $\left(\dfrac{V_m I_m}{2}\cos(2\omega t - \phi)\right)$.

∴ The frequency of the fluctuating part is twice the applied vtg freq & its average value over one complete cycle is zero.

(b) **Average power**:

$$P_{avg.} = \frac{V_m I_m}{2} \cos\phi$$

$$P_{avg} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi$$

$$\boxed{P_{avg.} = V_{rms} I_{rms} \cos\phi} \quad \text{—— (*)}$$

The average power (P) is dependent on the in-phase component of the current.

The average power is also called as <u>Active power</u> & is measured in <u>watt</u>

We know that a pure inductor & capacitor consume no power becoz all the power received from the source in a half cycle is returned to the source in the next half cycle. This circulating power is called <u>reactive power</u>.

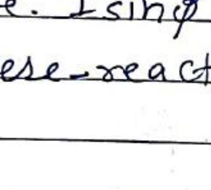
Reactive power is the product of vtg & ~~current~~ reactive component of the c/n i.e. $I\sin\phi$ and is measured in VAR (volt-ampere-reactive).

∴ Reactive Power $Q = VI\sin\phi$ —— (*)

∴ The product of vtg & current is known as apparent power (S) and is measured in VA (Volt-Amp)

$$S = \sqrt{P^2 + Q^2} \quad \text{—— (*)}$$

<u>Power triangle</u>:-



$$\boxed{\cos\phi = \frac{P}{S}}$$

In terms of ckt. components

$\cos\phi = R/Z$

Other Subjects: https://www.studymedia.in/fe/notes and $V = Z \cdot I$

$$\therefore P = VI\cos\phi = \cancel{Z}I \cdot I \cdot \frac{R}{\cancel{Z}} = I^2R$$

$$\therefore Q = VI\sin\phi = \cancel{Z}I \cdot I \cdot \frac{X_L}{\cancel{Z}} = I^2X_L$$

$$\therefore S = VI = ZI \cdot I = I^2Z$$

— ⊛

Powerfactor:- It is defined as the cosine angle betⁿ the vtg & current phasors.

powerfactor = $\cos\phi$

$\therefore$ from vtg triangle $\Rightarrow$ $pf = \frac{V_R}{V}$

$\therefore$ from Impedence triangle $\Rightarrow$ $Pf = \frac{R}{Z}$

$\therefore$ from power triangle $\Rightarrow$ $pf = \frac{P}{S}$

In case of an R-L series ckt., the p.f. is lagging in nature.

Series R-C Ckt. :-

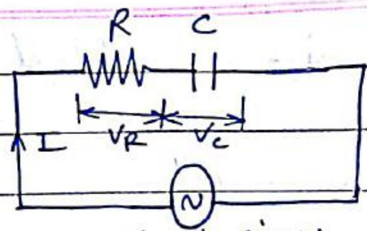


figure shows a pure Resistor (R) connected in series with a pure capacitor (c) across an alternating vtg (V)

let; V_{rms} & I_{rms} - be the rms value of applied vtg & c/n respectively.

$\therefore$ potential diff. across R $= V_R = RI$

$\therefore$ potential diff. across C $= V_C = X_C I$

$\therefore$ The vtg $\bar{V}_R$ is in phase with current $(\bar{I})$ whereas $\bar{V}_C$ lags behind the current $(\bar{I})$ by $90°$.

$$\therefore \bar{V} = \bar{V}_R + \bar{V}_C$$

phasor Dia :- The current (I) flows through the R & C. so, that I is taken as reference

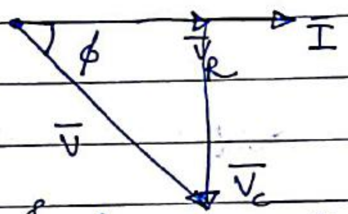


fig (a) :- phasor diagram

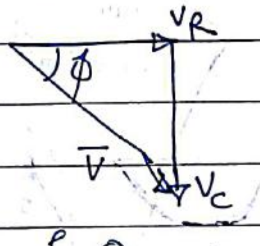


fig (b) :- vtg. Triangle

$$\boxed{\cos\phi = \frac{V_R}{V}}$$

Impedence :-

$$\bar{V} = \bar{V}_R + \bar{V}_C$$

$$= R\bar{I} - jX_C\bar{I}$$

$$\bar{V} = \bar{I}(R - jX_C)$$

$$\Rightarrow \frac{\bar{V}}{\bar{I}} = R - jX_C = \bar{Z}$$

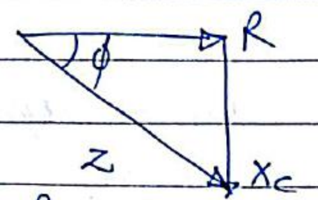
$$\therefore \bar{Z} = Z\angle -\phi$$

$$\therefore Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}} \quad \& \quad \phi = \tan^{-1}\left(\frac{X_C}{R}\right)$$

$$\phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

Where; $\bar{Z}$ - is called complex impedence of series R-C ckt.

Impedence Triangle:-



$$\boxed{\cos\phi = \frac{R}{Z}}$$

fig:- Impedence triangle

Current:-

From the phasor diagram, it is clear that the current I leads the vtg (V) by an angle ϕ. If the applied vtg is given by $V = V_m \sin\omega t$ then the current will be;

$$I = I_m \sin(\omega t + \phi).$$

Where; $I_m = \frac{V_m}{Z}$

$$\therefore \phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

Waveform:-

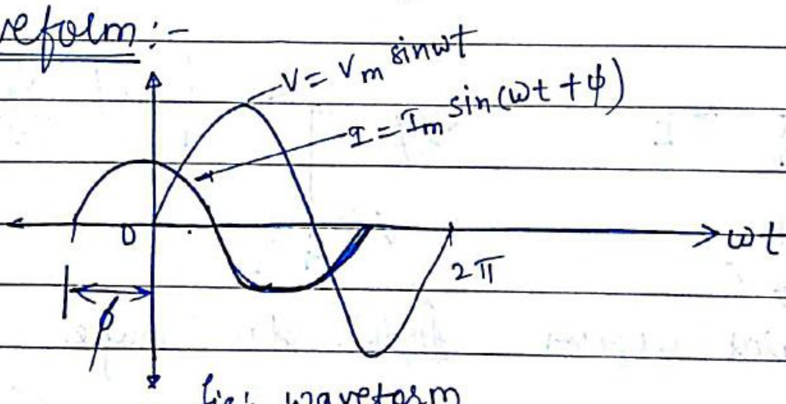


fig:- waveform

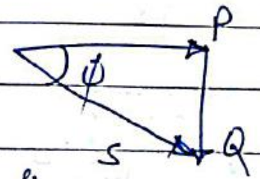
Power:-

Active Power (P) = $VI\cos\phi = I^2R$

Reactive Power (Q) = $VI\sin\phi = I^2X_C$

Apparent Power (S) = $VI = I^2Z$

Power triangle:-



$$\boxed{\cos\phi = \frac{P}{S}}$$

fig:- Power triangle

powerfactor:- It is defined as the angle betn vtg & c/n phasors.

$$pf = \cos\phi$$

∴ from vtg triangle; $pf = \frac{V_R}{V}$

from Impedence triangle; $pf = \frac{R}{Z}$

from power triangle; $pf = \frac{P}{S}$

In case of an R-C series ckt., the powerfactor is leading in nature.

Numericals:-

(1) An ac ckt. consists of a pure resistance of 10Ω & is connected across an ac supply of 230 V, 50 Hz. Calculate; (i) current; (ii) power consumed; (iii) pf; (iv) write down the equns for vtg & current.

Soln:- Givendata:- $R = 10\Omega$

$V = 230\text{ V}$

$f = 50\text{ Hz}$

(i) c/n $I = V/R = \frac{230}{10} = 23\text{ Amp}$

(ii) power consumed $P = VI = 230 \times 23 = 5290\text{ watt}$

(iii) $pf = \cos\phi = \frac{R}{R}$ ∵ In case of pure R → angle betn vtg & c/n is zero $\phi = 0$

$$\boxed{pf = \cos\phi = 1}$$

(iv) $V_{rms} = \frac{V_m}{\sqrt{2}}$ ⇒ $V_m = \sqrt{2}\,V_{rms} = \sqrt{2} \times 230 = 325.27\text{ V}$

$I_{rms} = \frac{I_m}{\sqrt{2}}$ ⇒ $I_m = \sqrt{2}\,I_{rms} = \sqrt{2} \times 23 = 32.53\text{ Amp.}$

$\omega = 2\pi f = 2\pi \times 50 = 100\pi = 314.16\text{ rad/sec}$

∴ $v = V_m \sin\omega t = 325.27 \sin(314.16t)$

$I = I_m \sin\omega t = 32.53 \sin(314.16t)$

② An inductive coil having negligible resistance & 0.1 henry inductance is connected across 200V, 50 Hz supply. Calculate; (i) inductive reactance; (ii) rms value of c/n ; (iii) power ; (iv) p.f ; (v) equ^n for v & I

Sol^n:- Givendata:-

$L = 0.1\ H$

$V = 200\ V$

$f = 50\ Hz$

(i) Inductive reactance $(X_L) = 2\pi f L = 2\pi \times 50 \times 0.1$

$= \underline{31.42\ \Omega}$

(ii) rms value of c/n $\Rightarrow I = \dfrac{V}{X_L} = \dfrac{200}{31.42} = \underline{\underline{6.37}}$ Amp

(iii) power $\Rightarrow$ Since; the current is lags behind the vtg by 90°.

$\therefore \phi = 90° \qquad \cos\phi = \cos(90°) = 0$

$\therefore P = VI\cos\phi = 0 \qquad \boxed{P = 0}$

(iv) powerfactor $\Rightarrow \boxed{\cos\phi = 0}$

(v) Equ^ns of vtg & current :-

$V_m = \sqrt{2}\, V_{rms} = \sqrt{2} \times 200 = 282.84\ V$

$I_m = \sqrt{2}\, I_{rms} = \sqrt{2} \times 6.37 = 9$ Amp.

$\omega = 2\pi f = 2 \times 50 \times 3.1416 = 314.16$ rad/sec

$\therefore V = V_m \sin\omega t = 282.84 \sin(314.16t)$

& $\therefore I = I_m \sin(\omega t \cancel{\phi} - \pi/2) = 9 \sin(314.16t - \pi/2)$

③ The vtg & current through ckt. elements are;

$v = 100 \sin(314t + 45°)$ volt

$i = 10 \sin(314t + 315°)$ Amp.

(i) Identify the ckt. elements

(ii) find the value of elements

(iii) Obtain an expression for power

Sol^n:- **Givendata**:- $v = 100 \sin(314t + 45^\circ)$ — ①

$i = 10 \sin(314t + 315^\circ)$.

$= 10 \sin(314t + 315^\circ - 360^\circ)$

$i = 10 \sin(314t - 45^\circ)$ — ②

(i) from eq^n ① & ②; the angle i.e. phase difference is (-90°) i.e. it is clear that the current (i) is lags behind the vtg by 90°. Hence; ckt. element is an inductor

(ii) Value of Element:-

$$X_L = \frac{V}{I} = \frac{V_m}{I_m} = \frac{100}{10} = 10\ \Omega$$

$$X_L = \omega L$$

$$10 = 2\pi f L$$

$$L = \frac{10}{2\pi f} = \frac{10}{100\pi} = 31.8\ mH$$

$$\boxed{L = 31.8\ mH}$$

(iii) Expression of power:-

$$\therefore P = -\frac{V_m I_m}{2} \sin 2\omega t = -\frac{100 \times 10}{2} \sin(2 \times 314t)$$

$$\Rightarrow P = -500 \sin(2\omega t)$$

$$\Rightarrow \boxed{P = -500 \sin 628t}$$

Series Resonance :-

A circuit containing reactance is said to be in resonance if the vtg. across the circuit is in phase with the current through it. At resonance, the circuit thus behaves as a pure resistor and the net reactance is zero.

Consider the series R-L-C circuit as shown in fig. ①

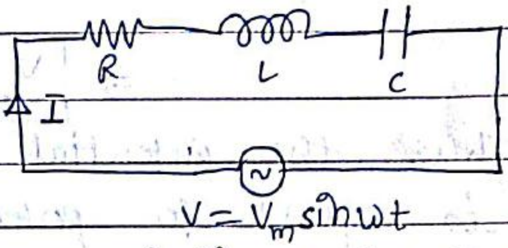


fig ①:- series RLC ckt.

∴ The Impedence of ckt is given by;

$$\therefore \bar{Z} = R + jX_L - jX_C$$

$$\Rightarrow \bar{Z} = R + j\omega L - j\frac{1}{\omega C}$$

$$\Rightarrow \bar{Z} = R + j\left[\omega L - \frac{1}{\omega C}\right]$$

At resonance, $Z_r = R$ i.e. resistive.

Therefore; the condⁿ for resonance is;

$$\omega L - \frac{1}{\omega C} = 0$$

$$\omega_0^2 LC - 1 = 0$$

$$\omega_0^2 = \frac{1}{LC} \Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \quad \text{— } \because \omega_0 = 2\pi f_0$$

$$\Rightarrow 2\pi f_0 = \frac{1}{\sqrt{LC}} \Rightarrow \boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

where; f_0 = Resonant frequency of the ckt.

powerfactor:- powerfactor = $\cos\phi = R/Z$

But; at resonance; $Z_r = R$

$$\therefore \cos\phi = \frac{R}{R} = 1 \Rightarrow \boxed{\cos\phi = 1}$$

Current:- Since; Z is minimum, the current is max^m at resonance. Thus, the ckt accepts more current and as such, an

R-L-C circuit under resonance is called an acceptor circuit.

$$I_0 = \frac{V}{Z} = \frac{V}{R}.$$

__Voltage__:- At Resonance; $\omega_0 L = \frac{1}{\omega_0 C}$

$$\Rightarrow \omega_0 L \cdot I_0 = \frac{1}{\omega_0 \cdot C} I_0$$

$$\Rightarrow \boxed{V_{L_0} = V_{C_0}}$$

Thus; the potential difference across inductor is equal to potential difference across capacitor being equal & opposite cancel each other.

Also; I_0 - is max^m

& hence; V_{L_0} & V_{C_0} - will be max^m

Thus; the voltage magnification takes place during resonance. Hence; it is also referred to as vtg. magnification ckt.

__Phasor diagram:__

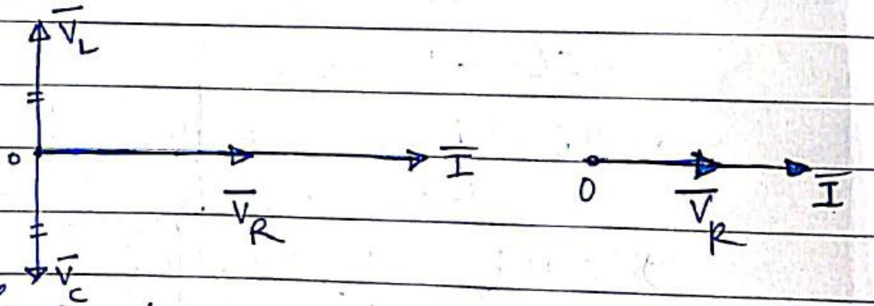


fig ②:- phasor dia

__Behaviour of R, L & C with change in freq:-__

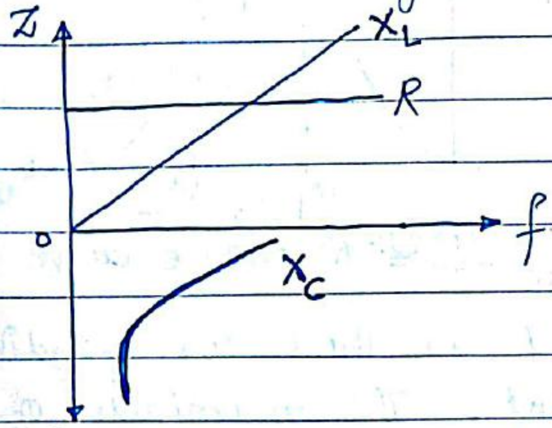
∴ Resistance remains constant with change in frequencies.

∴ Inductive reactance (X_L) is directly proportional to frequency (f) ($\because X_L = 2\pi f L$)

∴ Capacitive reactance (X_C) is inversely proportional to frequency (f) ($\because X_C = \frac{1}{2\pi f C}$).

$\therefore X_L \propto f$ — ∵ It can be drawn as a straight line passing through origin.

$\therefore X_C \propto \frac{1}{f}$ — ∵ It can be drawn as a rectangular hyperbola in fourth quadrant.



∴ Total Impedence;

$$Z = R + j(X_L - X_C)$$

Fig ③ :- Behaviour of R, L & C with change in freq.

ⓐ When $f < f_0$, impedence is capacitive and decreases up to f_0. The powerfactor is leading in nature.

ⓑ At $f = f_0$, impedence is resistive. The powerfactor is unity.

ⓒ When $f > f_0$, impedence is inductive & goes on increasing beyond f_0. The power factor is lagging in nature.

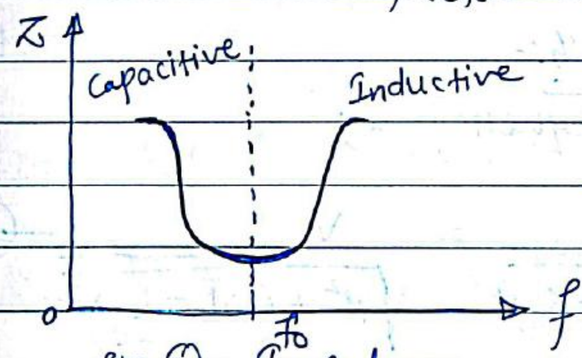
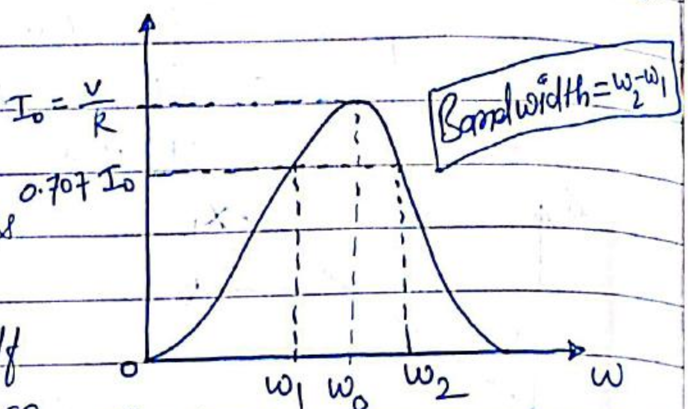


Fig ④ :- Impedence

Bandwidth :- For series R-L-C ckt., bandwidth is defined as the range of frequencies for which the power delivered to R is greater than or equal to $\frac{P_0}{2}$.

where; P_0 = is power delivered to R at resonance.

From the shape of curve; it is clear that there are two frequencies for which the power delivered to R is half the power at resonance.



fig⑤:- Resonance curve

For this reason, these frequencies are referred as those corresponding to the half-power points. The magnitude of current at each half-power point is the same.

∴ Hence;

$$I_1^2 R = \frac{1}{2} I_0^2 R = I_2^2 R$$

Accordingly, the bandwidth may be identified on the resonance curve as the range of frequencies over which the magnitude of the current is equal to or greater than 0.707 of the current at resonance.

From fig. ⑤;

The Bandwith = $\omega_2 - \omega_1$

Expression for Bandwidth:-

Generally; at any frequency ω;

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad \text{—(1)}$$

∴ At half-power points; $I = I_0/\sqrt{2}$

But; $I_0 = \frac{V}{R} \Rightarrow I = \frac{V}{\sqrt{2} \cdot R}$ —(2)

from eqn (1) & (2) $\Rightarrow \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{V}{\sqrt{2} \cdot R}$

$$\Rightarrow \frac{1}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}} = \frac{1}{\sqrt{2}\cdot R}$$

$$\Rightarrow \sqrt{2}R = \sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}.$$

$\therefore$ Squaring on b.s.

$$\therefore R^2+\left(\omega L-\frac{1}{\omega C}\right)^2 = 2R^2$$

$$\Rightarrow \left(\omega L-\frac{1}{\omega C}\right)^2 = R^2$$

$$\Rightarrow \omega L-\frac{1}{\omega C} = \pm R$$

$$\Rightarrow \omega L-\frac{1}{\omega C} \pm R = 0 \quad \text{— Multiply by } (\omega C)$$

$$\Rightarrow \omega^2 LC - 1 \pm R\omega C = 0 \quad \text{— Divide by } \frac{1}{LC}$$

$$\Rightarrow \omega^2 - \frac{1}{LC} \pm \frac{R}{L}\omega = 0$$

$$\Rightarrow \omega^2 \pm \frac{2R}{2L}\omega - \frac{1}{LC} = 0$$

$$\Rightarrow \boxed{\omega = \pm\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2}+\frac{1}{LC}}}$$

For low values of R, the term $\left(\frac{R^2}{4L^2}\right)$ can be neglected in comparison with the term $\left(\frac{1}{LC}\right)$.

$$\therefore \omega = \pm\frac{R}{2L} \pm \sqrt{\frac{1}{LC}}$$

$$\Rightarrow \boxed{\omega = \pm\frac{R}{2L} \pm \frac{1}{\sqrt{LC}}} \quad \text{— (3)}$$

Also;

The resonant freq. of ckt. is given by;

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \quad \text{— (4)}$$

Substitute equ^n (4) in (3)

$$\boxed{\omega = \pm \frac{R}{2L} + \omega_0}$$ —— Considering only +ve sign of ω_0.

$\therefore$ $\boxed{\omega_1 = \omega_0 - \frac{R}{2L}} \Rightarrow \boxed{f_1 = f_0 - \frac{R}{4\pi L}}$

& $\boxed{\omega_2 = \omega_0 + \frac{R}{2L}} \Rightarrow \boxed{f_2 = f_0 + \frac{R}{4\pi L}}$

$\therefore$ Bandwidth $= \omega_2 - \omega_1 = \frac{R}{L}$

(or) Bandwidth $= f_2 - f_1 = \frac{R}{2\pi L}$

Quality-factor :- It is a measure of voltage magnification in series resonant ckt. It is also a measure of selectivity (or) sharpness of the series resonant circuit.

$$Q_0 = \frac{\text{vtg. across inductor (or) capacitor}}{\text{vtg. at resonance}}$$

$$\Rightarrow Q_0 = \frac{V_{L0}}{V} = \frac{V_{C0}}{V}$$

Substituting values of V_{L0} and V;

$$Q_0 = \frac{I_0 X_{L0}}{I_0 R} = \frac{X_{L0}}{R} = \frac{\omega_0 L}{R} \Rightarrow \boxed{Q_0 = \frac{\omega_0 L}{R}}$$

(or) $$Q_0 = \frac{I_0 X_{C0}}{I_0 R} = \frac{X_{C0}}{R} = \frac{1}{\omega_0 RC} \Rightarrow \boxed{Q_0 = \frac{1}{\omega_0 RC}}$$

Substituting values of ω_0;

$$Q_0 = \frac{\frac{1}{\sqrt{LC}} L}{R} \quad \text{(or)} \quad Q_0 = \frac{1}{\frac{1}{\sqrt{LC}} RC} = \frac{1}{\sqrt{\frac{C}{L}}\, R}$$

$$\Rightarrow \boxed{Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}}} \qquad \Rightarrow \boxed{Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}}}$$