Total No. of Questions : 9]

SEAT No. :

P6485

[Total No. of Pages : 4

[5868]-101

F.E. (Semester- I & II)

ENGINEERING MATHEMATICS - I

(2019 Pattern) (107001)

Time : 2½ Hours] ***[Max. Marks : 70***

Instructions to the candidates:

1) ***Q. 1 is compulsory.***
2) ***Attempt Q2 or Q3, Q4 or Q5,Q6 or Q7, Q8 or Q9.***
3) ***Neat diagrams must be drawn wherever necessary.***
4) ***Figures to the right indicate full marks.***
5) ***Use of electronic pocket calculator is allowed.***
6) ***Assume suitable data, if necessary.***

Q1) Write the correct option for the following multiple choice questions.

a) If eigen value of a square matrix A is zero then. **[1]**

i) A is non-singular ii) A is orthogonal

iii) A is singular iv) None of these

b) If u = y^x then $\frac{\partial u}{\partial x}$ is equal to **[1]**

i) 0 ii) xy^{x-1}

iii) $y^x \log y$ iv) None of these

c) The orthogonal transformation x = py transforms the quadratic form $Q = x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2x_3$ to the canonical form $Q' = y_1^2 + 2y_2^2 + y_3^2$. The rank of quadratic from is **[2]**

i) 2 ii) 3

iii) 1 iv) 0

d) $u = \sec^{-1}\left[\frac{x^2 + y^2}{xy^2}\right]$. Find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ **[2]**

i) –tan u ii) –cot u

iii) tan u iv) cot u

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e) If u = $x^2 – y^2$ and v = $2xy$ then the value of $\dfrac{\partial(u,v)}{\partial(x,y)}$ is **[2]**

i) $4(x^2 + y^2)$ ii) $-4(x^2 + y^2)$

iii) $4(x^2 – y^2)$ iv) 0

f) A system of linear equations Ax = B, where B is a null (zero) matrix is **[2]**

i) Always consistent

ii) Consistent only if |A| = 0

iii) Consistent only if |A| ≠ 0

iv) In consistent if ρ (A) < No. of variables

Q2) a) If $z = \tan(y + ax) + (y - ax)^{3/2}$ find value of $\dfrac{\partial^2 z}{\partial x^2} - a^2 \dfrac{\partial^2 z}{\partial y^2}$. **[5]**

b) If $u = \tan^{-1}\left(\dfrac{x^3 + y^3}{x - y}\right)$ then prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u$$ **[5]**

c) If $u = f\left(x^2 - y^2; y^2 - z^2; z^2 - x^2\right)$ find value of $\dfrac{1}{x}\dfrac{\partial u}{\partial x} + \dfrac{1}{y}\dfrac{\partial u}{\partial y} + \dfrac{1}{z}\dfrac{\partial u}{\partial z}$ **[5]**

OR

Q3) a) If $u = ax + by\,; v = bx - ay$ find value of $\left(\dfrac{\partial u}{\partial x}\right)_y \left(\dfrac{\partial x}{\partial u}\right)_v \left(\dfrac{\partial y}{\partial v}\right)_x \left(\dfrac{\partial v}{\partial y}\right)_u$ **[5]**

b) If $u = \sin^{-1}\left(\sqrt{x^2 + y^2}\right)$ then find value of $x^2 \dfrac{\partial^2 u}{\partial x^2} + 2xy \dfrac{\partial^2 u}{\partial x \partial y} + y^2 \dfrac{\partial^2 u}{\partial y^2}$ **[5]**

c) If $u = f(r,s)$ where $r = x^2 + y^2$; $S = x^2 - y^2$ then show that $y\dfrac{\partial u}{\partial x} + x\dfrac{\partial u}{\partial y} = 4xy\dfrac{\partial u}{\partial r}$. **[5]**

Q4) a) If $x = uv$ and $y = \dfrac{u+v}{u-v}$, find $\dfrac{\partial(u,v)}{x,y}$. **[5]**

b) Examine for functional dependence $u = \dfrac{x-y}{1+xy}, v = \tan^{-1}x - \tan^{-1}y$ and if depedent find the relation between them. **[5]**

c) Discuss maxima and minima of $f(x,y) = x^2 + y^2 + 6x + 12$ **[5]**

OR

Q5) a) Prove $JJ' = 1$ for $x = u \cos v$, $y = u \sin v$. **[5]**

b) In calculating the volume of a right circular cone, errors of 2% and 1% are made in measuring the height and radius of base respectively find the error in the calculated volume. **[5]**

c) Find maximum value of $u = x^2y^3z^4$ such that $2x + 3y + 4z = a$ by Langrange's method. **[5]**

Q6) a) Investigate for what values of μ & λ the equations $x+y+z = 6$, $x+2y+3z = 10$, $x+2y+\lambda z = \mu$ have i) No solution ii) Infinitely many solutions. **[5]**

b) Examine for linear dependence and independence the vectros (1,1,3), (1,2,4), (1,0,2). If dependent, find the relation between them. **[5]**

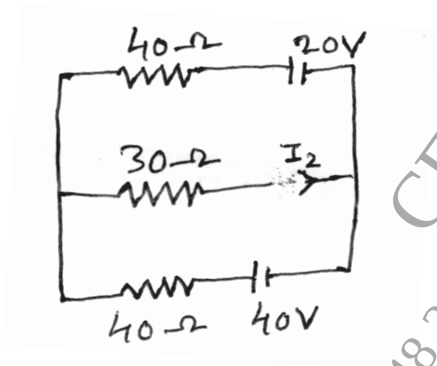
c) Verify whether matrix $A = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$ is orthogonal or not. **[5]**

OR

Q7) a) Solve the system of equations $x+y+2z = 0$, $x+2y+3z=0$, $x+3y+4z=0$. **[5]**

b) Examine following vectors for linear dependence and independence (1,–1,1), (2,1,1), (3,0,2). If dependent, find the relation between them. **[5]**

c) Determine the currents in the network given in the figure . **[5]**



Q8) a) Find the eigen values of the matrix $A=\begin{bmatrix}1 & -2\\-5 & 4\end{bmatrix}$. **[5]**

Find eigen vector corresponding to the highest eigen value.

b) Verify cayley-Hamilton theorem for $A=\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$. Hence find A^{-1} if it exists. **[5]**

c) Find the modal matrix p which diagonalises $A=\begin{bmatrix}5 & 3\\3 & 5\end{bmatrix}$. **[5]**

OR

Q9) a) Find the eigen values of $A=\begin{bmatrix}1 & 2 & 3\\0 & 3 & 2\\0 & 0 & -2\end{bmatrix}$. **[5]**

Find eigen vector corresponding to the highest eigen value.

b) Verify cayley-Hamilton theorem for $A=\begin{bmatrix}1 & 0 & 0\\1 & 0 & 1\\0 & 1 & 0\end{bmatrix}$ **[5]**

c) Reduce the quadratic form $Q=x_1^2+2x_2^2+x_3^2+2x_2x_3-2x_3x_1+2x_1x_2$ to canonical form by congruent transformations. **[5]**

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