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SKN Sinhgad Institute of Technology and Science, Lonavala.

F.E., Semester - I (2020-21)

Engineering Mathematics I

Notes of Unit V (Matrices)

<u>Minor of a matrix</u>: It is a determinant of a matrix obtained by deleting some rows and columns of a given matrix. The order of square matrix so obtained is called order of a minor.

<u>Minor of an element a_{ij}</u>: Let A be any square matrix, then minor of an element a_{ij} is denoted by M_{ij} and is determinant of a matrix obtained by deleting i^{th} row and j^{th} column of A.

<u>Rank of a matrix</u>: A non-zero matrix A is said to be of rank r if there is (i) at least one minor of order r which is not equal to zero and (ii) all the minors of order (r+1) must be equal to zero.
The rank of a matrix A is denoted by $\rho[A]$.

Elementary transformation do not alter the rank of a matrix.

<u>Echelon form</u>: A non-zero matrix A is said to be in echelon form if (i) all the zero rows must be at the bottom of matrix, (ii) first non-zero element of each must be equal to 1 called as leading one, (iii) in any two successive non-zero rows, the leading one in the upper row must be on the left of leading one in the lower row.

$$\begin{bmatrix} 1 & & & \\ \downarrow & 1 & & \\ 0 & \downarrow & 1 & \\ 0 & 0 & \downarrow & \\ 0 & 0 & 0 & \end{bmatrix}$$

The number of non-zero rows in an echelon form is equal to rank of a matrix.

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Normal form : By performing elementary transformation any non-zero matrix can be reduce to one of the following four forms called as normal forms $[I_r]$, $\begin{bmatrix} I_r \\ 0 \end{bmatrix}$, $[I_r \quad 0]$, $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ where I_r is identity matrix of order r, and 0 denotes null or zero matrix.

$$\begin{bmatrix} 1 & \rightarrow & 0 & 0 & 0 \\ \downarrow & 1 & \rightarrow & 0 & 0 \\ 0 & \downarrow & 1 & \rightarrow & 0 \\ 0 & 0 & \downarrow & & \\ 0 & 0 & 0 & & \end{bmatrix}$$

Finding non-singular matrices P and Q such that PAQ is in normal form:

If A is any $m \times n$ matrix then it can be written as $A = I_m A_{m\times n} I_n$.

Apply elementary transformation to reduce matrix A to normal form. Apply the same row transformation on a matrix which is on the left of A and the same column transformation on the matrix which is on right of A. When A get reduce to normal the corresponding matrices get reduce to P and Q.

$$\begin{array}{cccccc} A & = & I_m & A & I_n \\ \downarrow & & row\downarrow & \downarrow & \downarrow column \\ normal\ form & = & P & A & Q \end{array}$$

If A is non-singular matrix then normal form of A is $[I_n]$ i.e. $I_n = PAQ$ and then $A^{-1} = QP$.

System of linear equations:

Consider the system of m equations in n unknowns

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$

The above system of equation can be written in matrix form as

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

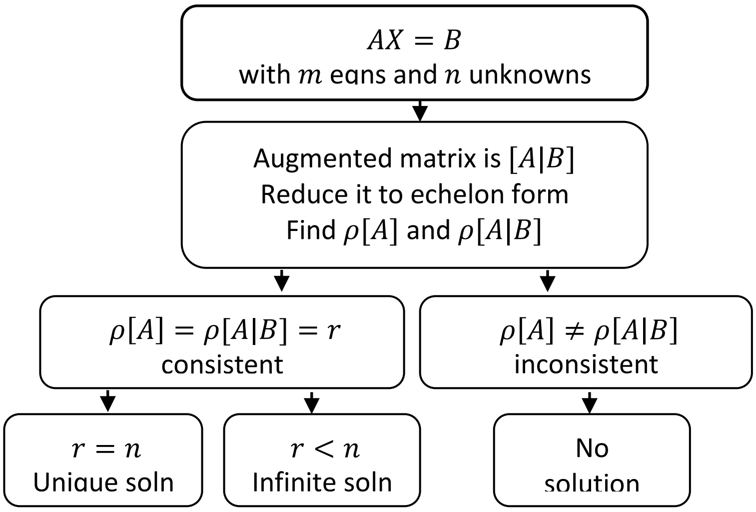
i. e. $AX = B$

The set of values of $x_1, x_2, \cdots, x_n$ which satisfies all m equations simultaneously is called solution of the system $AX = B$ and such system is called consistent system, otherwise it is called inconsistent system.

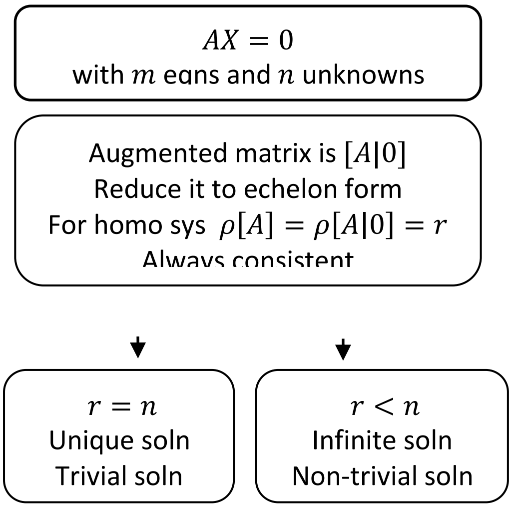
For the system of equation $AX = B$ if $B \neq 0$ then it is called non-homogeneous system and if $B = 0$ then is called homogeneous system.

The matrix $\left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array}\right]$ is called augmented matrix and is denoted by $[A|B]$.

Conditions for consistency of non-homogeneous system.



For finding infinite number of solutions, put arbitrary value for $n - r$ variables and find remaining variables in terms of these arbitrary constants.
Conditions for consistency of non-homogeneous system.



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Linear dependent and independent vectors

The vectors $X_1, X_2, \ldots, X_m$ are said to be linearly dependent if there exist m scalars $c_1, c_2, \ldots, c_m$, not all zero, such that $c_1X_1 + c_2X_2 + \ldots + c_m X_m = 0$

The vectors $X_1, X_2, \ldots, X_m$ are said to be linearly independent if every relation of the type
$c_1X_1 + c_2X_2 + \ldots + c_m X_m = 0$ implies $c_1 = c_2 = \ldots = c_m = 0$.

Consider $c_1X_1 + c_2X_2 + \ldots + c_m X_m = 0$, which gives homogeneous system of equation in $c_1, c_2, \ldots, c_m$. If this homogeneous system possesses non-trivial solutions then vectors are linearly dependent and if trivial solution then vectors are linearly independent.

Linear Transformation: $Y = AX$ is called linear transformation. If $|A| = 0$ it is called non-singular transformation and if $|A| \neq 0$ it is called singular or regular transformation, in this case A^{-1} exists and inverse transformation is given by $X = A^{-1}Y$.

Orthogonal Matrix: A square matrix A is said to be orthogonal if $AA^T = I = A^TA$. For orthogonal matrix $A^{-1} = A^T$.

1) Reduce the following matrix A to its normal form and hence find its rank,

i) $A = \begin{bmatrix} 2 & -1 & -1 & -3 \\ 2 & 4 & -1 & 0 \\ 4 & -3 & 2 & -1 \end{bmatrix}$

ii) $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & 7 \end{bmatrix}$

iii) $A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ -2 & -5 & 3 & 0 \\ 1 & 0 & 1 & 10 \end{bmatrix}$

iv) $A = \begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & -4 & -2 & 1 \\ 5 & 2 & 4 & 3 \end{bmatrix}$

v) $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

vi) $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 1 & -1 & 4 & 0 \\ -2 & 3 & 8 & 0 \end{bmatrix}$

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2) Examine the consistency of the system of equations and solve if consistent:

i) $x+y+z=6,\ 2x-2y+3z=7,\ x-y+2z=5,\quad 3x+y+z=8,$

ii) $x+y-z+t=2,\ \ 2x+3y+4t=9,\ \ y-2z+3t=2,$

iii) $2x-y-z=2,\ x+2y+z=2,\ \ 4x-7y-5z=2,$

iv) $10x+4y-2z=-4,\ -17x+y+2z-3w=2,$

$x+y+w=6,\ -34x+16y-10z+8w=4,$

v) $2x+y-z+3w=8,\ x+y+z-w=-2,\ \ 3x+2y-z=6,\ 4y+3z+2w=-8,$

vi) $x+2y+3z=0, 2x+3y+z=0, 4x+5y+4z=0,\ \ x+2y-2z=0$

vii) $x+y-z+w=0, x-y+2z-w=0,\ \ 3x+y+w=0$

3) Examine whether following vectors are linearly dependent. If so, find relation between them

i) $X_1=(-4,1,0), X_2=(3,1,2), X_3=(1,1,1)$

ii) $X_1=(2,-2,4), X_2=(-1,3,-3), X_3=(1,1,1)$

iii) $X_1=(3,1,-4), X_2=(2,2,-3), X_3=(0,-4,1)$

iv) $X_1=(-1,5,0), X_2=(16,8,-3), X_3=(-64,56,9)$

v) $X_1=(1,2,-1,0), X_2=(1,3,1,2), X_3=(4,2,1,0),\ \ X_4=(6,1,0,1)$

vi) $X_1=(2,-1,3,2), X_2=(1,3,4,2), X_3=(3,-5,2,2)$

4) Show that the matrix $A=\begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{bmatrix}$ is orthogonal matrix, hence find A^{-1}.

5) Show that the transformation $y_1=2x_1+x_2+x_3,\ \ y_2=x_1+x_2+2x_3,\ \ y_3=x_1-2x_3$ is non-singular. Also find the values of x_1, x_2, x_3 if $y_1=1, \quad y_2=2, y_3=-1$ by using inverse transformation.

6) Find non-singular matrices P and Q such that PAQ is in the normal form and hence find A^{-1} if it exists.

$$A = \begin{bmatrix} 4 & -1 & -5 \\ 15 & 1 & -5 \\ 5 & 4 & 9 \end{bmatrix}$$

7) Find the values of a, b, c if the matrix A is orthogonal where $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & b \\ 2 & -2 & c \end{bmatrix}$

8) Find the non-singular matrices P and Q such that PAQ is in normal form.
Hence find rank of A, where $A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 3 & -3 & 1 & 2 \\ 2 & 1 & -3 & -6 \end{bmatrix}$

9) Determine the values of λ for which the equations $x + 2y + z = 3,\ x + y + z = \lambda,$ $3x + y + 3z = \lambda^2$ are consistent and solve them for these values of λ.

10) If A is orthogonal, find a, b, c.

(i) $A = \begin{bmatrix} 1/3 & 2/3 & a \\ 2/3 & 1/3 & b \\ 2/3 & -2/3 & c \end{bmatrix}$ (ii) $A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$

11) For what values of k the equation $x + y + z = 1,\ x + 2y + 4z = k,\ x + 4y + 10z = k^2$ have infinite number of solutions? Hence find the solutions.

12) For what values of λ does the following system of equations possess a non-trivial solution/ Obtain the solution for these values of λ.

$$3x + y - \lambda z = 0, 4x - 2y - 3z = 0, 2\lambda x + 4y - \lambda z = 0$$

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