Total No. of Questions : 9]

SEAT No. :

PA-4296

[Total No. of Pages : 4

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F.E.

ENGINEERING MATHEMATICS - I

(2019 Pattern) (107001) (Semester - I) (End - Sem)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt Q.1 compulsory, Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.*
2) *Use of electronic pocket calculator is allowed.*
3) *Assume suitable data, if necessary.*
4) *Figures to the right indicate full marks.*

Q1) Write the correct option for the following multiple choice questions. **[10]**

a) If $u = x^3 + y^3$ then $\dfrac{\partial^2 u}{\partial x \partial y}$ is equal to **[2]**

i) 3 ii) –3

iii) 2 iv) 0

b) If $x = u^2 - v^2$, $y = 2uv$ and $\dfrac{\partial(x,y)}{\partial(u,v)} = 4(u^2 + \text{v}^2)$ then $\dfrac{\partial(u,v)}{\partial(x,y)}$ is equal to **[2]**

i) $4\ (x^2 + y^2)$ ii) $4\ (u^2 + v^2)$

iii) $\dfrac{1}{4(u^2+v^2)}$ iv) 1

c) For $c_1x_1 + c_2x_2 = 0$ where, x_1, x_2 are non-zero vectors and c_1, c_2 are constants then x_1, x_2 are linearly independent if **[2]**

i) $c_1 \neq 0, c_2 \neq 0$ ii) $c_1 \neq 0, c_2 = 0$

iii) $c_1 = 0, c_2 \neq 0$ iv) $c_1 = 0, c_2 = 0$

P.T.O.

d) The quadratic form corresponding to the matrix $M=\begin{bmatrix} 1 & 2 & 3 \\ 2 & -4 & 6 \\ 3 & 6 & 5 \end{bmatrix}$ is **[2]**

i) $Q(x) = x_1^2 - 4x_2^2 + 5x_3^2 + 4x_1x_2 + 6x_1x_3 + 12x_2x_3$

ii) $Q(x) = x_1^2 + 2x_2^2 + 3x_3^2$

iii) $Q(x) = x_1^2 - 4x_2^2 + 5x_3^2 + 2x_1x_2 + 3x_1x_3 + 6x_2x_3$

iv) $Q(x) = x_1^2 - 4x_2 + 5x_3^2$

e) If $u = x^2 + y^2 + 2x$, $\frac{\partial u}{\partial y}$ is equal to **[1]**

i) $2x + 2$ ii) $2y$

iii) $2x + 2y + 2$ iv) 2

f) If for a square matrix M of order 2, sum of diagonal elements = 4 and |M|=3 then. Characteristic equation of A is **[1]**

i) $\lambda^2 - 3\lambda + 4 = 0$ ii) $\lambda^2 - 4\lambda + 3 = 0$

iii) $\lambda^2 + 3\lambda + 4 = 0$ iv) $\lambda^2 + 4\lambda + 3 = 0$

Q2) a) If $u = 2x + 3y$, $v = 3x - 2y$ find value of $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v \left(\frac{\partial y}{\partial v}\right)_x \left(\frac{\partial v}{\partial y}\right)_u$. **[5]**

b) If $u = \operatorname{cosec}^{-1}\sqrt{\frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}}$ then prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan 4}{144}\left[\tan^2 u + 13\right].$$ **[5]**

c) If $x = u + v + w$, $y = uv + vw + uw$, $z = uvw$ and ϕ is function of x, y, z then prove that $u \cdot \frac{\partial \phi}{\partial u} + v\frac{\partial \phi}{\partial v} + w\frac{\partial \phi}{\partial w} = x \cdot \frac{\partial \phi}{\partial x} + 2y\frac{\partial \phi}{\partial y} + 3z\frac{\partial \phi}{\partial z}$. **[5]**

OR

Q3) a) If $z = \tan(y + ax) - (y - ax)^{3/2}$ then find value of $\frac{\partial^2 z}{\partial x^2} - a^2 \frac{\partial^2 z}{\partial y^2}$. **[5]**

b) If $u = \log(x^3 + y^3 - x^2y - xy^2)$ then find value of $x^2 \cdot \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \cdot \frac{\partial^2 u}{\partial y^2}$ **[5]**

c) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ then find value of $x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} + z \cdot \frac{\partial u}{\partial z}$ **[5]**

Q4) a) If $x = v^2 + w^2$, $y = w^2 + u^2$, $z = u^2 + v^2$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ **[5]**

b) In calculating the volume of a right circular cylinder, using the formula : $V = \pi r^2 h$,
errors of 2% and 1% are made in measuring the height and radius of base respectively. Find the error in the calculated volume. **[5]**

c) Find stationary points of :
$f(x, y) = 3x^2 - y^2 + x^3$ and find f_{max} where if exists. **[5]**

OR

Q5) a) If $x = u + v$, $y = v^2 + w^2$, $z = u^3 + w^3$ then find $\frac{\partial u}{\partial x}$, using jacobian. **[5]**

b) Examine for functional dependence :

$$u = \frac{x + y}{1 - xy}, v = \tan^{-1} x + \tan^{-1} y$$ **[5]**

c) Find stationary value of $u = x^2 + y^2 + z^2$ under the condition $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ using Lagrange's method. **[5]**

Q6) a) Solve the following system of linear equations. $4x + 2y + z + 3w = 0$, $6x + 3y + 4z + 7w = 5$, $2x + y + w = -1$. **[5]**

b) Examine whether the vectors $x_1 = (2, 2, 1)$, $x_2 = (1, 3, 1)$, $x_3 = (1, 2, 2)$ are linearly independent or dependent. If dependent, find the relation between them. **[5]**

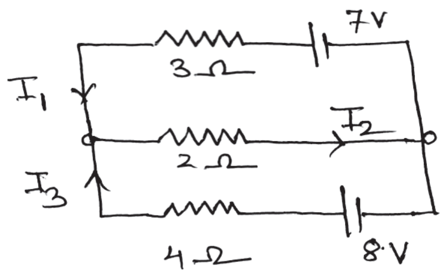
c) Find the values of a, b, c if A is orthogonal, where $A = \frac{1}{9}\begin{bmatrix} -8 & 4 & a \\ 1 & 4 & b \\ 4 & 7 & c \end{bmatrix}$. **[5]**

OR

Q7) a) Determine values of K for which the equations $x+y+z=1$, $2x+y+4z=k$, $4x+y+10z=k^2$ are inconsistent. **[5]**

b) Examine whether the vectors $x_1=(3, 1, -4)$, $x_2=(2, 2, -3)$, $x_3=(0, -4, 1)$ are linearly independent or dependent. If dependent, find the relation between them. **[5]**

c) Determine the currents in the following network. **[5]**



Q8) a) Find the eigen values and eigen vectors of the matrix $A=\begin{bmatrix} 14 & -10 \\ 5 & -1 \end{bmatrix}$. **[5]**

b) By using cayley Hamilton theorem, find the inverse of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$, if it exists. **[5]**

c) Reduce the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ to its diagonal form by finding modal matrix P. **[5]**

OR

Q9) a) Find the eigen values of $A=\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. Also find eigen vector corresponding to the largest eigen value of A. **[5]**

b) Verify cayley Hamilton theorem for $A=\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$. Hence find A^4. **[5]**

c) Find the transformation which reduces the quadratic form $x_1^2+2x_2^2+3x_3^2+2x_1x_2-2x_1x_3+2x_2x_3$ to the canonical form by using congruent transformations. Also write the canonical form. **[5]**

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