Total No. of Questions : 9]

SEAT No. :

PA-4292

[Total No. of Pages : 4

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F.E.

ENGINEERING MATHEMATICS - II

(2019 Pattern) (Semester - II) (107008)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Question No. 1 is compulsory.*
2) *Solve Q.No.2 or Q.No.3, Q.No.4 or Q.No.5, Q.No.6 or Q.No.7, Q.No.8 or Q.No.9.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Figures to the right indicate full marks.*
5) *Electronic pocket calculator is allowed.*
6) *Assume suitable data, if necessary.*

Q1) Write the correct option for the following multiple choice questions.

a) $\int_0^{\pi/2} \sin^5 x\,dx =$ **[2]**

i) $\frac{15}{8}$

ii) 0

iii) $\frac{8}{15}$

iv) $\frac{8}{15}\frac{\pi}{2}$

b) To evaluate integration $\int_0^a \int_{\sqrt{a^2-y^2}}^{y+a} f(x,y)\,dxdy$ we should first evaluate the inner integral with respect to **[2]**

i) y

ii) x

iii) xy

iv) y then x

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c) The general form of equation of sphere is **[2]**

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ for which centre and radius are given by

i) $c(u,v,w);\ r = \sqrt{u^2 + v^2 + w^2 - d}$

ii) $c(-u,-v,-w);\ r = \sqrt{u^2 + v^2 + w^2 + d}$

iii) $c(u,v,w);\ r = \sqrt{u^2 + v^2 + w^2 + d}$

iv) $c(-u,-v,-w);\ r = \sqrt{u^2 + v^2 + w^2 - d}$

d) The curve $x = t^2, y = t - \frac{t^3}{3}$ is **[2]**

i) symmetric about X-axis

ii) symmetric about Y-axis

iii) symmetric about both the axes

iv) none of these

e) $\iiint dx\,dy\,dz$ represents **[1]**

i) volume

ii) centre of gravity

iii) Area

iv) Moment of inertia.

f) Total number of loops for the curve $r = a \sin 5\theta$ are **[1]**

i) 2

ii) 3

iii) 4

iv) 5

Q2) a) If $I_n = \int_0^{\pi/4} \tan^n \theta d\theta$ then show that $I_n = \frac{1}{n-1} - I_{n-2}$ **[5]**

b) Evaluate $\int_0^{\infty} \sqrt{x}\, e^{-x^3} dx$. **[5]**

c) Prove that $\int_0^{\infty} \frac{e^{-ax} \sin x}{x} dx = \cot^{-1} a$ **[5]**

OR

Q3) a) If $I_n = \int_0^{\pi/2} x^n \cos x \, dx$, then prove that $I_n = \left(\frac{\pi}{2}\right)^n - n(n-1) I_{n-2}$. **[5]**

b) Evaluate $\int_0^1 x^3 (1-\sqrt{x})^5 \, dx$. **[5]**

c) Prove that $\int_0^{\infty} e^{-x^2-2bx} \, dx = \frac{\sqrt{\pi}}{2} e^{b^2} [1 - erf(b)]$ **[5]**

Q4) a) Trace the curve : $y^3 = x^2 (2a - y)$. **[5]**

b) Trace the curve : $r = a \cos 3\theta$. **[5]**

c) Find the length of the upper arc of one loop of Lemiscale $r^2 = a^2 \cos 2\theta$ **[5]**

OR

Q5) a) Trace the curve : $ay^2 = x^2(a - x)$. **[5]**

b) Trace the curve : $r = a\left(\sqrt{2} + \sin\theta\right)$. **[5]**

c) Trace the curve : $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$. **[5]**

Q6) a) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere **[5]**

$x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$.

Also find the point of contact.

b) Find the equation of right circular cone having its vertex at the origin and passing through the circle : $x^2 + z^2 = 25, y = 4$. **[5]**

c) Find the equation of right circular cylinder of radius 3 whose axis is the line $\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-5}{-1}$. **[5]**

OR

Q7) a) Show that the spheres $x^2 + y^2 + z^2 = 25$ and $x^2 + y^2 + z^2 - 18x - 24y - 40z + 225 = 0$ touch externally and also find their point of contact. **[5]**

b) Find the equation of right circular cone whose vertex is at (0, 0, 10) and whose intersection with the XoY plane is a circle of radius 5. **[5]**

c) Find the equation of right circular cylinder of radius 2 whose axis passes through (1, 2, 3) and has direction ratios 2, –3, 6. **[5]**

Q8) a) Change the order of integration and evaluate $\int_0^\infty \int_y^\infty \frac{e^{-x}}{x} dxdy$. **[5]**

b) Find the area of one loop of the curve $r = a \cos 2\theta$. **[5]**

c) Find the x-co-ordinate of the centre of gravity of the area bounded by $y^2 = x$ and $x + y = 2$. Given that $A = \frac{9}{2}$ is the area of the region bounded by the given curves. **[5]**

OR

Q9) a) Evaluate $\iint x^2 y^2 \, dxdy$ over positive quadrant of $x^2 + y^2 = a^2$, using polar transformations. **[5]**

b) Prove that volume bounded by cylinders $y^2 = x$, $x^2 = y$ and planes $z = 0$, $x + y + z = 2$ is $\frac{11}{30}$. **[5]**

c) Find the x-co-ordinate of the centre of gravity of one loop of $r = a \sin 2\theta$, (in first quadrant). Given that the area of loop is $A = \frac{\pi a^2}{8}$. **[5]**

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