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SCAN ME

Unit II
Applications of Differential Equations

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- Orthogonal trajectory
- Newton’s law of Cooling
- Simple Electrical Circuits
- Heat Flow
- Rectilinear Motion
- S.H.M.

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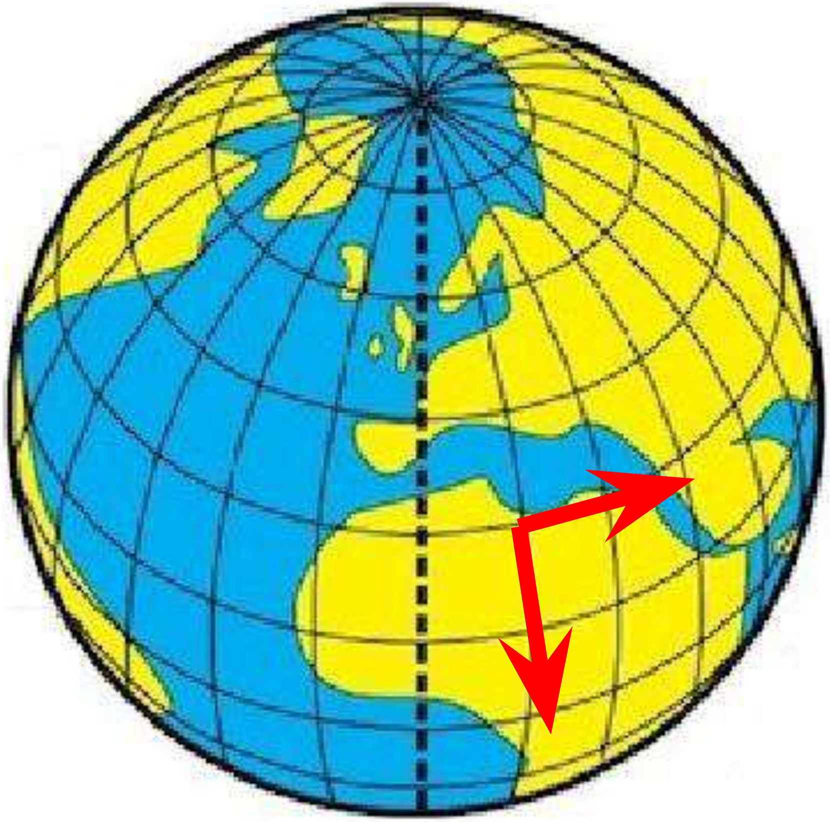
Orthogonal Trajectory



- Method of finding the orthogonal trajectory of family of curves $F(x,y,c)=0 \quad (1)$

Obtain D.E. of (1) by eliminating the arbitrary constant c, resulting in

$$\frac{dy}{dx}=f(x,y) \qquad (2)$$

Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$ in (2) we get

$$-\frac{dx}{dy}=f(x,y) \quad (3)$$

Solving (3) gives $\mathrm{G}(x,y,k)=0$ which is the required orthogonal trajectory of (1)

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Find orthogonal trajectories of parabola y = a x^2 - - (1)

Differentiating (1) w.r.t. x,

$$\frac{dy}{dx} = 2\,a\,x \text{ ------------------------- } (2)$$

Eliminating 'a' from (1) and (2) ,we get D.E. for the family (1).

$$\frac{dy}{dx} = \frac{2y}{x} - - - - - - - - - (3)$$

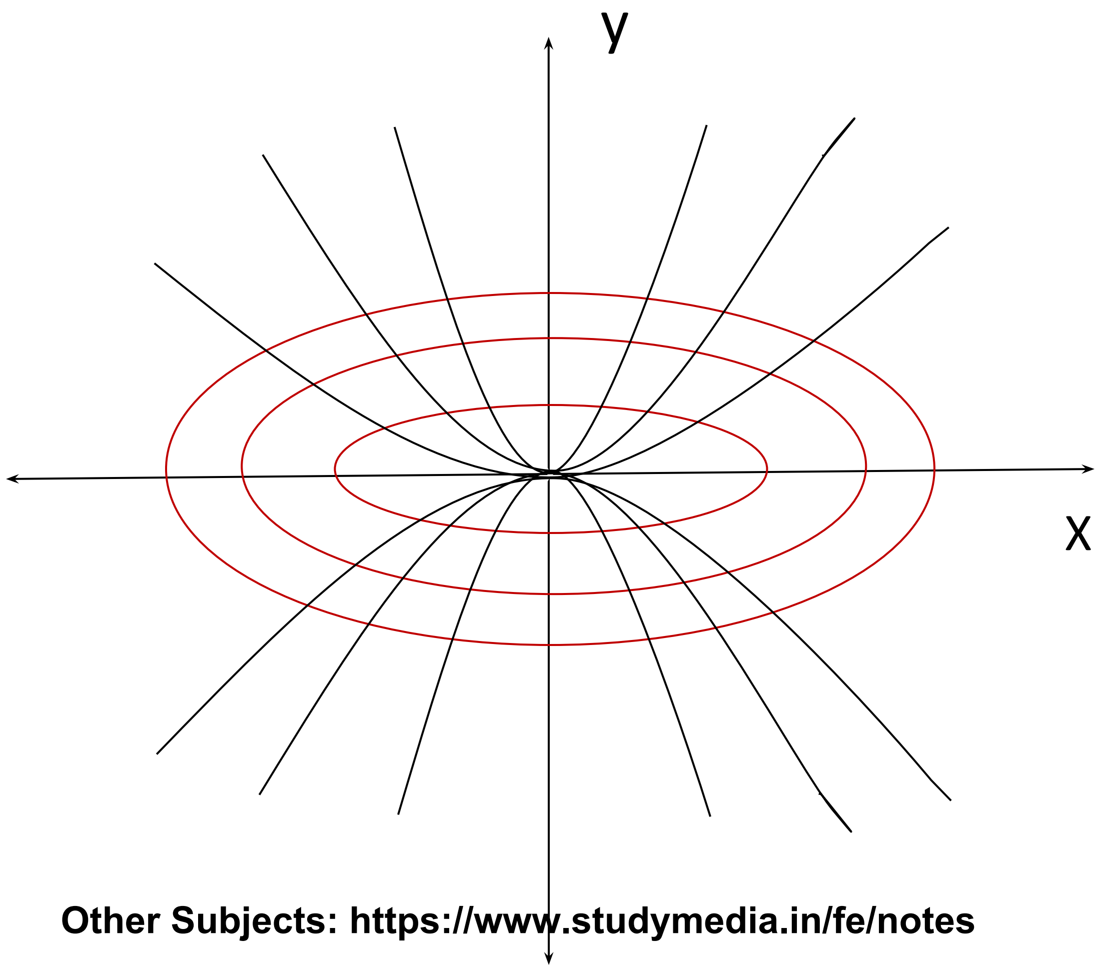
Replacing $\frac{dy}{dx}$ $by\ (-\frac{dx}{dy})$ in (3) , we get

$$-\frac{dx}{dy} = \frac{2y}{x} \Longrightarrow x\,dx + 2\,y\,dy - - - - - - - - - (4)$$

$$This\ is\ D.E.of\ orthogonal\ trajectories\ of\ (1)$$

Solving eqn (4), $\frac{x^2}{2} + y^2 = c^2 \implies \frac{x^2}{(\sqrt{2}\,c)^2} + \frac{y^2}{c^2} = 1$

Family of ellipse. This is the required orthogonal trajectories.-



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Find the orthogonal trajectory of $xy = c$.

Let $xy = c \quad (1)$

Differentiating (1) w.r.t. x

$$\therefore x\frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$

$$\therefore -\frac{dx}{dy} = -\frac{y}{x} \qquad \frac{dx}{dy} = \frac{y}{x}$$

$$xdx = ydy$$

$$\int xdx = \int ydy$$

$$\frac{x^2}{2} = \frac{y^2}{2} + k$$

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- **Find the orthogonal trajectory of $x^2 + y^2 = cx$.**

Let $x^2 + y^2 = cx \quad (1)$

Differentiating (1) w.r.t x

$$\therefore 2x + 2y\frac{dy}{dx} = c \qquad \therefore 2x + 2y\frac{dy}{dx} = \frac{x^2 + y^2}{x}$$

$$2y\frac{dy}{dx} = \frac{x^2 + y^2}{x} - 2x \qquad \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$

Find the orthogonal trajectory of $x^2 + y^2 = a^2$.

Let $x^2 + y^2 = a^2 \quad (1)$

Differentiating (1) w.r.t x

$$\therefore 2x + 2y\frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = -\frac{x}{y}$$

Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$

$$\therefore -\frac{dx}{dy} = -\frac{x}{y} \qquad \therefore \frac{dx}{x} = \frac{dy}{y}$$

$$\therefore \log x = \log y + log\,k \qquad \therefore x = yk$$

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- Find the orthogonal trajectory of $y = mx$.

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- Method of finding orthogonal trajectory of family of curves $F(r,\theta,c)=0$ (1)

Obtain D.E. of (1) by eliminating arb. const. c.

$$\frac{dr}{d\theta}=f(r,\theta) \quad (2)$$

Replace $\frac{dr}{d\theta}$ by $-r^2\frac{d\theta}{dr}$ in (2)

$$\therefore -r^2\frac{d\theta}{dr}=f(r,\theta) \quad (3)$$

Solving (3) gives $G(r,\theta,k)=0$ which is the required orthogonal trajectory.

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- **Find the orthogonal trajectory of $r = a(1 - \cos\theta)$.**

Let $r = a(1 - \cos\theta) \quad (1)$

Differentiating (1) w.r.t. θ

$$\therefore \frac{dr}{d\theta} = a\sin\theta \qquad \therefore \frac{dr}{d\theta} = \frac{r\sin\theta}{1 - \cos\theta}$$

Replacing $\frac{dr}{d\theta}$ by $-r^2 \frac{d\theta}{dr}$

$$\therefore -r^2 \frac{d\theta}{dr} = \frac{r\sin\theta}{1 - \cos\theta} \qquad \therefore \frac{1 - \cos\theta}{\sin\theta} d\theta = -\frac{dr}{r}$$

$$\therefore \int (cosec\theta - \cot\theta)\, d\theta = -\int \frac{dr}{r}$$

$$\therefore \log(cosec\theta - \cot\theta) - \log\sin\theta = -\log r + \log c$$

$$\therefore \frac{1 - \cos\theta}{\sin^2\theta} = \frac{c}{r} \qquad \therefore r = c(1 + \cos\theta)$$

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- Find the orthogonal trajectory of $r^2 = a^2 \cos 2\theta$

Newton's law of Cooling

The rate at which the temperature of a body θ changes is proportional to the difference between the temperature of body and the temperature of the surrounding medium θ_0

$$\frac{d\theta}{dt} \propto \theta - \theta_0$$

$$\therefore \frac{d\theta}{dt} = -k(\theta - \theta_0)$$

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A body at temp 100°C is placed in a room whose temp is 20°C and cools to 60°C in 5 min. Find its temp after a further interval of 3 min.

Given: θ_0 =20, For t=0, θ=100 and for t=5, θ=60

To find: θ=? at t=5+3=8

By Newton's law of cooling, $$\frac{d\theta}{dt} = -k(\theta - \theta_0) = -k(\theta - 20)$$

$$\frac{d\theta}{\theta - 20} = -kdt \qquad \int \frac{d\theta}{\theta - 20} = \int -kdt \qquad (1)$$

For t = 0, $\theta = 100$ and for t = 5, $\theta = 60$ $$\therefore \int_{100}^{60} \frac{d\theta}{\theta - 20} = \int_0^5 -kdt$$

$$[\log(\theta - 20)]_{100}^{60} = -k[t]_0^5 \qquad k = -\frac{1}{5}\log\left(\frac{1}{2}\right)$$

For t = 0, $\theta = 100$ and for t = 8, $\theta = ?$ $$\therefore \int_{100}^{\theta} \frac{d\theta}{\theta - 20} = \int_0^8 -kdt$$

$$[\log(\theta - 20)]_{100}^{\theta} = \frac{1}{5}\log\left(\frac{1}{2}\right)[t]_0^8 \qquad \log\left(\frac{\theta - 20}{80}\right) = \frac{8}{5}\log\left(\frac{1}{2}\right) \qquad \therefore \theta = 46.39$$

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When a thermometer is placed in a hot liquid bath at temp T the temp ϴ indicated by the thermometer rises at the rate of T- ϴ. For a liquid bath at 95°C, the temp reads 15°C at a certain instant t=0 and 35°C at t=10 sec, what will be its temp at t=20 sec.

By Newton's law of cooling, $\frac{d\theta}{dt}\alpha(T-\theta) \quad \therefore \frac{d\theta}{dt}=k(T-\theta)$

Given: T =95, For t=0, ϴ=15 and for t=10, ϴ=35

To find: ϴ=? at t=20 $\quad \frac{d\theta}{95-\theta}=kdt \quad \int\frac{d\theta}{95-\theta}=\int kdt \quad$ (1)

For $t=0,\ \theta=15$ and for $t=10,\ \theta=35 \therefore \int_{15}^{35}\frac{d\theta}{95-\theta}=\int_0^{10}kdt$

$$-\left[\log(95-\theta)\right]_{15}^{35}=k\left[t\right]_0^{10} \qquad k=\frac{1}{10}\log\left(\frac{4}{3}\right)$$

For $t=0,\ \theta=15$ and for $t=20,\ \theta=? \quad \therefore \int_{15}^{\theta}\frac{d\theta}{95-\theta}=\int_0^{20}kdt$

$$-\left[\log(95-\theta)\right]_{15}^{\theta}=\frac{1}{10}\log\left(\frac{4}{3}\right)\left[t\right]_0^{20} \quad \log\left(\frac{80}{95-\theta}\right)=2\log\left(\frac{4}{3}\right) \quad \therefore \theta=50$$

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Water at temperature 100°C cools in 10 min to 80°C in a room of temperature 25°C.

a) Find the temp. of water after 20 min.

b) When is the temp. i) 40°C, ii) 26°C.

Given: θ_0 =25, For t=0, θ=100 and for t=10, θ=80

To find: θ=? at t=20 and t=? when θ=40, 26.

By Newton's law of cooling, $$\frac{d\theta}{dt}=-k(\theta-\theta_0)=-k(\theta-25)$$

$$\frac{d\theta}{\theta-25}=-kdt \qquad \int\frac{d\theta}{\theta-25}=\int -kdt \qquad (1)$$

For $t=0,\ \theta=100$ and for $t=10,\ \theta=80$ $\therefore \int_{100}^{80}\frac{d\theta}{\theta-25}=\int_0^{10}-kdt$

$$[\log(\theta-25)]_{100}^{80}=-k[t]_0^{10} \qquad [\log(80-25)-\log(100-25)]=-k[10-0]$$

$$\log\left(\frac{55}{75}\right)=-10k \qquad k=-\frac{1}{10}\log\left(\frac{55}{75}\right)$$

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For t = 0, $\theta = 100$ and for t = 20, $\theta = ?$ $\therefore \int_{100}^{\theta} \frac{d\theta}{\theta - 25} = \int_{0}^{20} -kdt$

$$[\log(\theta - 25)]_{100}^{\theta} = \frac{1}{10}\log\left(\frac{55}{75}\right)[t]_{0}^{20}$$

$$[\log(\theta - 25) - \log(100 - 25)] = \frac{1}{10}\log\left(\frac{55}{75}\right)[20]$$

$$\log\left(\frac{\theta - 25}{75}\right) = 2\log\left(\frac{55}{75}\right) \qquad \left(\frac{\theta - 25}{75}\right) = \left(\frac{55}{75}\right)^2 \qquad \therefore \theta = 65.33$$

For t = 0, $\theta = 100$ and for $\theta = 40$, $t = ?$ $\therefore \int_{100}^{40} \frac{d\theta}{\theta - 25} = \int_{0}^{t} -kdt$

$$[\log(\theta - 25)]_{100}^{40} = \frac{1}{10}\log\left(\frac{55}{75}\right)[t]_{0}^{t} \qquad \left[\log\left(\frac{15}{75}\right)\right] = \frac{1}{10}\log\left(\frac{55}{75}\right)t \quad \therefore t = 51.89$$

For t = 0, $\theta = 100$ and for $\theta = 26$, $t = ?$ $\therefore \int_{100}^{26} \frac{d\theta}{\theta - 25} = \int_{0}^{t} -kdt$

$$[\log(\theta - 25)]_{100}^{26} = \frac{1}{10}\log\left(\frac{55}{75}\right)[t]_{0}^{t} \qquad \left[\log\left(\frac{1}{75}\right)\right] = \frac{1}{10}\log\left(\frac{55}{75}\right)t \quad \therefore t = 139.20$$

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A body of temperature 80°C is placed in a room of constant temperature 50°C at time t=0. At the end of 5 min the body has cooled to a temp of 70°C.

(a) Find the temp of the body at the end of 10 min,

(b) when will the temp of body be 60°C.

By Newton's law of cooling,

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

$$k = -\frac{1}{5}\log\left(\frac{2}{3}\right)$$

For $t=0,\ \theta = 80$ and for $t = 10,\ \theta = ?$ $\quad\therefore \theta = 63.33$

For $t=0,\ \theta = 80$ and for $\theta = 60,\ t = ?$ $\quad\therefore t = 13.547$

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- A metal ball is heated to a temperature of $100°c$ at time $t = 0$. It is placed in water which is maintained at $40°c$. If the temperature of the ball is reduced to $60°c$ in 4 minutes, find the time at which the temperature of the ball is $50°c$.

Ans $=$ 6.5 minutes

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Simple Electrical Circuits

- If q is charge and $i = \frac{dq}{dt}$ the current in a circuit at any time t then

Voltage drop across a **resistor** of resistance R is Ri

Voltage drop across a **capacitor** of capacitance C is $\frac{q}{C}$

and

Voltage drop across an **inductor** of inductance L is

$$L\frac{di}{dt} = L\frac{d^2q}{dt^2}$$

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Kirchhoff's Voltage law

The algebraic **sum** of all the **voltage drops** across the components of an electrical circuit is **equal to e.m.f.**

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$$\square\, e^{at}\cos bt\,dt = \frac{\square^{\square}}{\square^2 + \square^2}\left[\square + \square\right]$$

$$\square\, e^{at}\sin bt\,dt = \frac{\square^{\square}}{\square^2 + \square^2}\left[\square - \square\right]$$

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A resistance of 100 ohms, an inductance of 0.5 Henry are connected in series with battery of 20 volts. Find the current in a circuit as a function

Equation of Circuit is $L\frac{di}{dt} + Ri = E$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$$ which is linear in i, where $P = \frac{R}{L}$, $Q = \frac{E}{L}$

G.S. Is $ie^{\int R/Ldt} = \int \frac{E}{L}e^{\int R/Ldt}dt + c$

$$ie^{Rt/L} = \int \frac{E}{L}e^{\frac{Rt}{L}}dt + c$$

$$ie^{Rt/L} = \frac{Ee^{\frac{Rt}{L}}}{LR/L} + c$$

$$ie^{Rt/L} = \frac{Ee^{\frac{Rt}{L}}}{R} + c$$

$$i = \frac{E}{R} + ce^{-Rt/L}$$

At $t = 0,\ i = 0$ $\quad 0 = \frac{E}{R} + c \quad c = -\frac{E}{R}$

$$i = \frac{E}{R} - \frac{E}{R}e^{-\frac{Rt}{L}} = \frac{1}{5}(1 - e^{-200t})$$

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A constant emf E volts is applied to a circuit containing a constant resistant R ohms in series with a constant inductance L henries. If the initial current is zero, show that the current builds up to half its theoretical maximum in $\frac{L\log 2}{R}$ sec.

Equation of Circuit is $L\frac{di}{dt} + Ri = E$

$\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$ which is linear in i, where $P = \frac{R}{L}$, $Q = \frac{E}{L}$

G.S. Is $ie^{\int R/Ldt} = \int \frac{E}{L}\mathrm{e}^{\int R/Ldt}dt + c$

$ie^{Rt/L} = \int \frac{E}{L}\mathrm{e}^{\frac{Rt}{L}}dt + c \qquad ie^{Rt/L} = \frac{E\mathrm{e}^{\frac{Rt}{L}}}{LR/L} + c$

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$$ie^{Rt/L} = \frac{E\mathrm{e}^{\frac{Rt}{L}}}{R} + c \qquad i = \frac{E}{R} + ce^{-Rt/L}$$

$$At \quad t = 0,\ i = 0 \qquad 0 = \frac{E}{R} + c \qquad c = -\frac{E}{R}$$

$$\therefore i = \frac{E}{R} - \frac{E}{R}e^{-\frac{Rt}{L}}$$

$i = i_{max}$, if $\frac{\mathrm{E}}{\mathrm{R}}e^{-Rt/L}$ is minimum i.e. if $e^{-Rt/L} = 0$

$\therefore i_{max} = \frac{E}{R}$ For $i = \frac{i_{max}}{2} = \frac{E}{2R} = \frac{E}{R} - \frac{E}{R}e^{-Rt/L}$

$$\frac{E}{R}e^{-Rt/L} = \frac{E}{R} - \frac{E}{2R} = \frac{E}{2R} \qquad e^{-Rt/L} = \frac{1}{2}$$

$$-\frac{Rt}{L} = log\frac{1}{2} \qquad \frac{Rt}{L} = log2 \qquad \therefore t = \frac{Llog2}{R}$$

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In a circuit containing inductance of 640 H, resistance 250 ohms and voltage 500 volts. If the initial current being zero then find the time that elapses before the current reaches 80% of its maximum value.

Equation of Circuit is $L\frac{di}{dt} + Ri = E$

$\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$ which is linear in i, where $P = \frac{R}{L}$, $Q = \frac{E}{L}$

G.S. Is $ie^{\int R/Ldt} = \int \frac{E}{L}\mathrm{e}^{\int R/Ldt}dt + c$

$ie^{Rt/L} = \int \frac{E}{L}\mathrm{e}^{\frac{Rt}{L}}dt + c$ $ie^{Rt/L} = \frac{E\mathrm{e}^{\frac{Rt}{L}}}{LR/L} + c$

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$$ie^{Rt/L} = \frac{Ee^{\frac{Rt}{L}}}{R} + c$$

$$i = \frac{E}{R} + ce^{-Rt/L}$$

$$At \quad t = 0, \; i = 0$$

$$0 = \frac{E}{R} + c$$

$$c = -\frac{E}{R}$$

$$\therefore i = \frac{500}{250} - \frac{500}{250}e^{-250t/640}$$

$$i_{max} = \frac{500}{250}$$

For $i = \frac{80 i_{max}}{100} = \frac{80\times500}{100\times250} = \frac{500}{250} - \frac{500}{250}e^{-250t/640}$

$$1.6 = 2 - 2e^{-250t/640}$$

$$e^{-\frac{250t}{640}} = 0.2$$

$$\therefore t = -\frac{64}{25}\log_e 0.2 \quad = 4.12\ sec.$$

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- In a circuit containing inductance L,Resistance R and voltage E

  Given $L = 640\ H, R = 250\ ohm,$
  $E = 500\ volts$ Find the time that elapses, before it reaches 90% of its maximum value

T= 5.89 sec.

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- An electric circuit contains a resistance of R ohms condenser of capacity C in series with emf E volts. Find the current i at any time t, when $E = E_0 \sin wt$

Equation of Circuit is $R\frac{dq}{dt} + \frac{q}{C} = E$

$$\therefore R\frac{dq}{dt} + \frac{q}{C} = E_0 sinwt \qquad \therefore \frac{dq}{dt} + \frac{q}{RC} = \frac{E_0}{R} sinwt$$

which is linear in q, where $P = \frac{1}{RC}$, $Q = \frac{E_0}{R} sinwt$

G.S. is $qe^{\int dt/RC} = \int \frac{E_0}{R} sinwt\, e^{\int dt/RC} dt + k$

$$qe^{t/RC} = \int \frac{E_0}{R} sinwt\, e^{t/RC} dt + \text{k}$$

$$qe^{t/RC} = \int \frac{E_0}{R} sinwt \, e^{t/RC} dt + \mathrm{k}$$

$$qe^{t/RC} = \left(\frac{E_0}{R}\right) \frac{e^{\frac{t}{RC}}}{\left(\frac{1}{RC}\right)^2 + w^2} \left(\frac{1}{RC} sinw\mathrm{t} - wcoswt\right) + k$$

$$qe^{t/RC} = (E_0 RC^2) \frac{e^{\frac{t}{RC}}}{1+(RCw)^2} \left(\frac{1}{RC} sinw\mathrm{t} - wcoswt\right) + k$$

$$q = \frac{E_0 C}{1+(RCw)^2} (sinw\mathrm{t} - RCwcoswt) + ke^{-t/RC}$$

$$\therefore i = \frac{dq}{dt} = \frac{E_0 C}{1+(RCw)^2} (wcosw\mathrm{t} + RCw^2 sinwt) - \frac{k}{RC} e^{-t/RC}$$

Heat Flow

Fourier's law of Heat conduction

The heat flowing across a surface is proportional to its surface area and to the rate of change of temp w.r.t. its distance normal to the surface.

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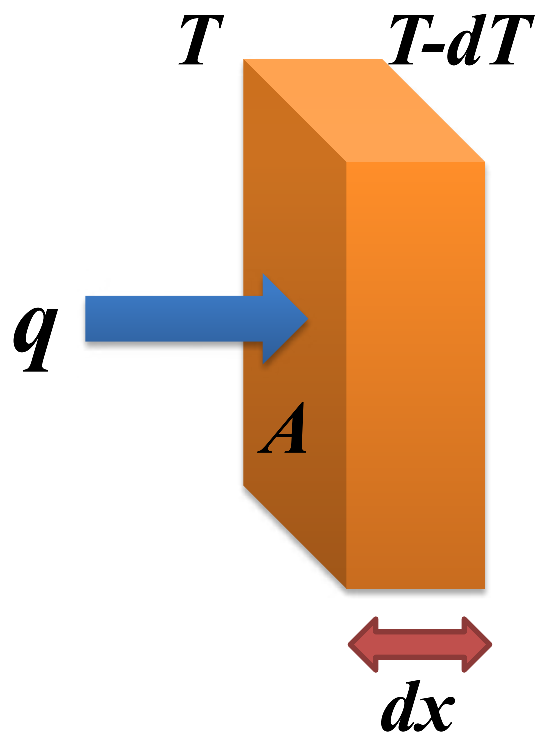
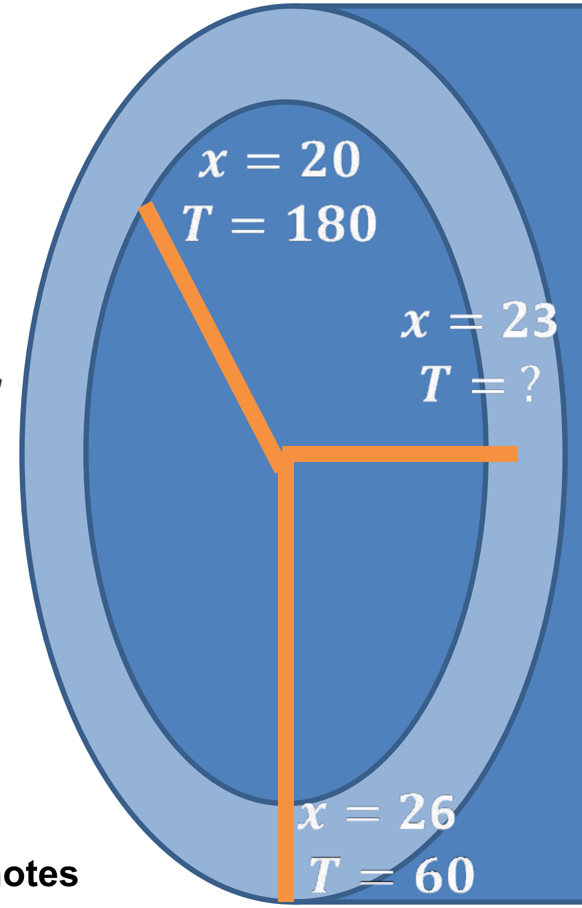
If q **(cal/sec)** be the quantity of heat that flows across a slab of surface area A $\mathbf{cm^2}$ and thickness dx in 1 sec where the difference of temp at the faces of

the slab is dT and k coefficient of thermal conductivity

Then $q = -kA\frac{dT}{dx}$



A pipe 40cm in diameter contains a steam at 180°C and is protected with a covering 6 cm thick for which k=0.0025. If the temp of outer surface of the covering is 60°C, find the temp half way through the covering under steady state condition.



Let q **cal/sec** be the quantity of heat flowing through a surface of pipe having radius x cm and length 1 cm.

Then surface area $A = 2\pi x(1) = 2\pi x$

Given: $x = 20, \quad T = 180$

$x = 26, \quad T = 60$

To find $T = ?, x = 20 + 3 = 23$

By Fourier's law $q = -kA\frac{dT}{dx} = -k2\pi x\frac{dT}{dx}$

$$\therefore dT = -\frac{q}{2k\pi}\frac{dx}{x} \qquad \therefore \int dT = -\frac{q}{2k\pi}\int\frac{dx}{x} \quad (1)$$

At $x = 20,\ T = 180$ and at $x = 26,\ T = 60$

$$\therefore \int_{180}^{60} dT = -\frac{q}{2k\pi}\int_{20}^{26}\frac{dx}{x}$$

$$\therefore [T]_{180}^{60} = -\frac{q}{2k\pi}[log x]_{20}^{26}$$

$$\therefore -120 = -\frac{q}{2k\pi}\left[\log\left(\frac{26}{20}\right)\right]$$

$$\therefore -\frac{q}{2k\pi} = -\frac{120}{\log\left(\frac{26}{20}\right)}$$

At $x = 20,\ T = 180$, at $x = 23,\ T = ?$

$$\therefore \int_{180}^{T} dT = -\frac{q}{2k\pi}\int_{20}^{23}\frac{dx}{x}$$

$$\therefore [T]_{180}^{T} = -\frac{q}{2k\pi}[log x]_{20}^{23}$$

$$\therefore T - 180 = -\frac{120}{\log\left(\frac{26}{20}\right)}\left[log\left(\frac{23}{20}\right)\right]$$

$$\therefore T = 116.08$$

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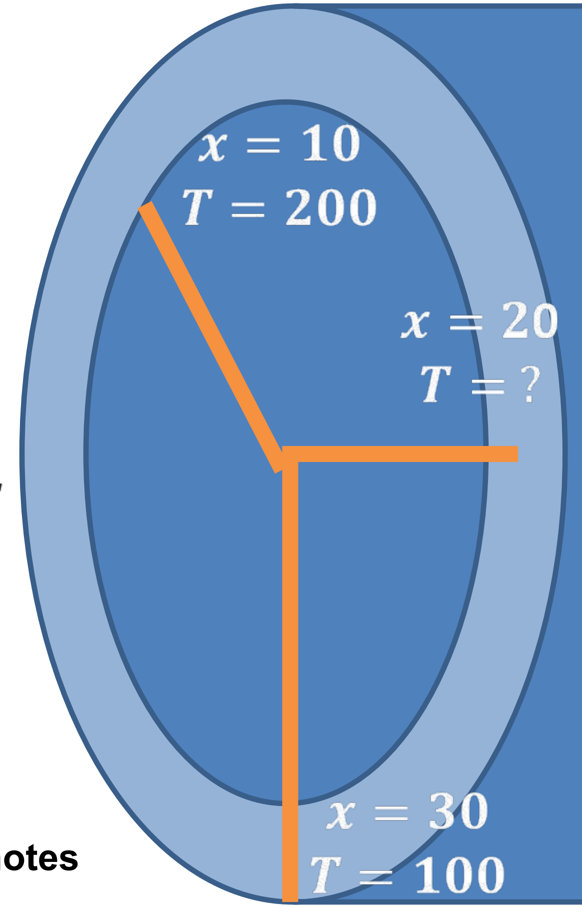
A long hollow pipe has an inner diameter 20 cm and outer diameter of 60 cm. The inner surface is kept at 200°C and the outer surface at 100°C. How much heat is lost per minute from a portion of the pipe 20m long? Find the temp at a distance 20 cm from the centre of the pipe. (k=0.005)



Let q **cal/sec** be the quantity of heat flowing through a surface of pipe having radius x cm and length 1 cm.

Then surface area $A = 2\pi x(1) = 2\pi x$

Given: $x = 10, \quad T = 200$

$x = 30, \quad T = 100$

To find $q = ?$ & $T = ?$ $at \quad x = 20$

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By Fourier's law $q = -kA\frac{dT}{dx} = -k2\pi x\frac{dT}{dx}$

$$\therefore dT = -\frac{q}{2k\pi}\frac{dx}{x} \qquad \therefore \int dT = -\frac{q}{2k\pi}\int\frac{dx}{x} \qquad (1)$$

At $x = 10,\ T = 200$ and at $x = 30,\ T = 100$

$$\therefore \int_{200}^{100} dT = -\frac{q}{2k\pi}\int_{10}^{30}\frac{dx}{x} \qquad \therefore [T]_{200}^{100} = -\frac{q}{2k\pi}[logx]_{10}^{30}$$

$$\therefore -100 = -\frac{q}{2k\pi}[\log(3)] \qquad \therefore -\frac{q}{2k\pi} = -\frac{100}{\log(3)}$$

$$\therefore q = \frac{(2k\pi)100}{\log(3)} = 2.8596\ cal/\text{sec} \quad \text{(for 1cm pipe)}$$

$$= 2.8596 \times 60 \times 2000 = 343152$$

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At $x = 10,\ T = 200$, at $x = 20,\ T = ?$

$$\therefore \int_{200}^{T} dT = -\frac{q}{2k\pi}\int_{10}^{20}\frac{dx}{x}$$

$$\therefore [T]_{200}^{T} = -\frac{q}{2k\pi}[log x]_{10}^{20}$$

$$\therefore T - 200 = -\frac{100}{\log(3)}[log(2)]$$

$$\therefore T = 136.91$$

Other Subjects: https://www.studymedia.in/fe/notes

- One dimensional steady-state heat condition for a hollow cylinder with constant thermal conductivity k in region $a \leq r \leq b,$ the temperature T_r at a distance r,is given by $\frac{d}{dr}\left[r\frac{dT_r}{dr}\right] = 0$ with $T_r = T_1\ when\ r = a, \& \ T_r = T_2\ when\ r = b.$ Use this to determine steady-state temperature distribution T_r in the cylinder in terms of r

Rectilinear Motion

- Rectilinear motion (also called as linear motion) is **motion along a straight line.**

If x is displacement of a particle at time t then its

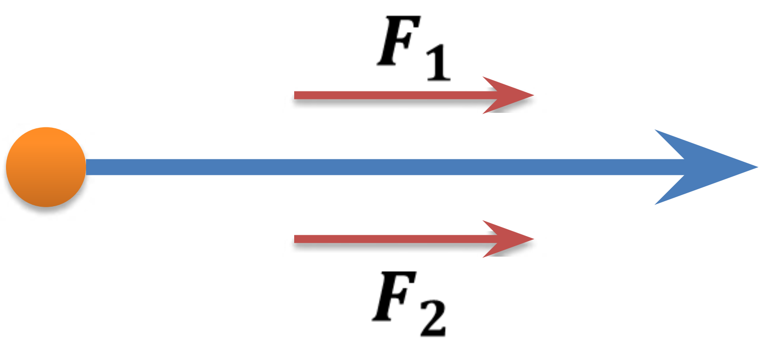
Velocity $v = \dfrac{dx}{dt}$

Acceleration $a = \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2} = v\dfrac{dv}{dx}$

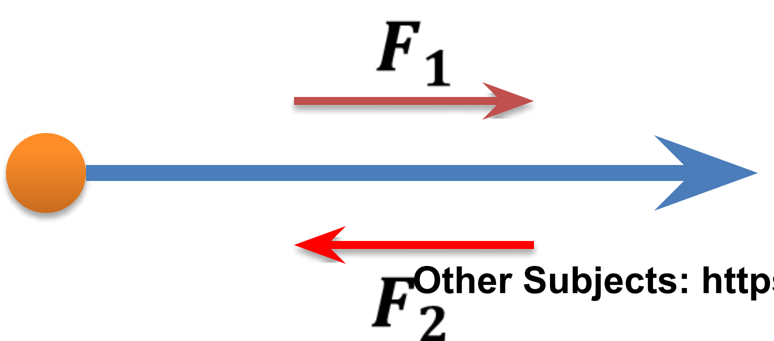
D’Alembert’s principle

Algebraic sum of the forces acting on a body along a given direction is equal to the product of mass and acceleration in that direction.

Net force = Mass X Acceleration



Net force = $F_1 + F_2$



Net force = $F_1 - F_2$

Other Subjects: https://www.studymedia.in/fe/notes

- A body of mass m, falling from rest is subjected to the force of gravity and an air resistance proportional to the square of velocity (kv^2). If it falls through a distance x and possesses a velocity v at that instant then prove that $\frac{2kx}{m} = \log\left(\frac{p^2}{p^2-v^2}\right)$ where $mg = kp^2$.

The forces acting on body are

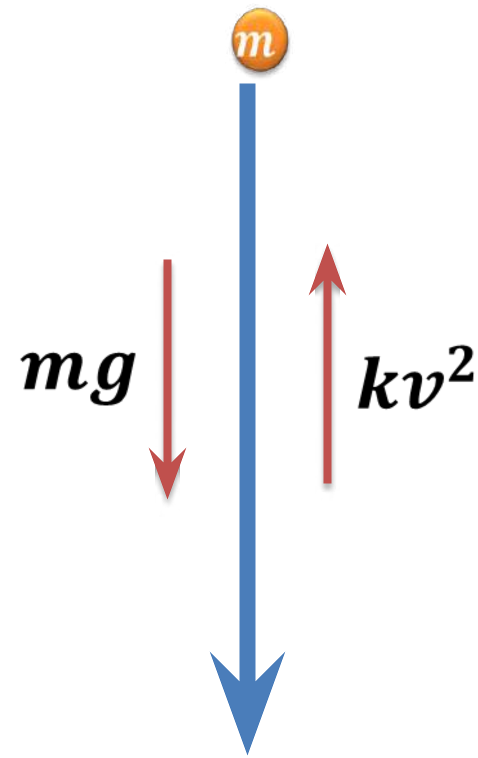
i) Its weight mg

ii) Air resistance kv^2

$\therefore$ Equation of motion is

$$Net\ force = mass \times acceleration$$

$$\therefore mg - kv^2 = mv\frac{dv}{dx}$$



Other Subjects: https://www.studymedia.in/fe/notes

$$\therefore mg - kv^2 = mv\frac{dv}{dx} \qquad \therefore kp^2 - kv^2 = mv\frac{dv}{dx}$$

$$\therefore \frac{k}{m}dx = \frac{vdv}{p^2 - v^2} \qquad \therefore \int \frac{k}{m}dx = \int \frac{vdv}{p^2 - v^2}$$

$$\therefore \frac{kx}{m} = -\frac{1}{2}\log(p^2 - v^2) + c$$

At $x = 0,\ v = 0$ $\therefore 0 = -\frac{1}{2}\log(p^2) + c \quad \therefore c = \frac{1}{2}\log(p^2)$

$$\therefore \frac{kx}{m} = -\frac{1}{2}\log(p^2 - v^2) + \frac{1}{2}\log(p^2) = \frac{1}{2}log\left(\frac{p^2}{p^2 - v^2}\right)$$

$$\therefore \frac{2kx}{m} = log\left(\frac{p^2}{p^2 - v^2}\right)$$

Other Subjects: https://www.studymedia.in/fe/notes

- A body starts moving from rest is opposed by a force per unit mass of value cx and resistance per unit mass of value bv^2, where x and v are the displacement and velocity of the particle at that instant. Show that $v^2 = \frac{c}{2b^2}\left(1 - e^{-2bx}\right) - \frac{cx}{b}$.

The forces acting on body are

i) cx (per unit mass)

ii) Resistance bv^2 (per unit mass)

$\therefore$ Equation of motion is

$$Net\ force = mass \times acceleration$$

$$\therefore -mcx - mbv^2 = mv\frac{dv}{dx}$$

$$\therefore -cx - bv^2 = v\frac{dv}{dx} \qquad \therefore v\frac{dv}{dx} + bv^2 = -cx$$

Putting $v^2 = u, \therefore 2v\frac{dv}{dx} = \frac{du}{dx}$ $\quad \therefore \frac{1}{2}\frac{du}{dx} + bu = -cx$

$$\therefore \frac{du}{dx} + 2bu = -2cx$$

which is linear in u,

where $\mathrm{P} = 2b, Q = -2cx$

G. S. is $\mathrm{u}e^{\int 2bdx} = \int -2cxe^{\int 2bdx}dx + k$

$$\mathrm{u}e^{2bx} = -2c\int xe^{2bx}dx + k \qquad \mathrm{u}e^{2bx} = -2c\left(\frac{xe^{2bx}}{2b} - \frac{e^{2bx}}{4b^2}\right) + k$$

$$v^2 = -2c\left(\frac{x}{2b} - \frac{1}{4b^2}\right) + ke^{-2bx}$$

At $x = 0,\ v = 0$

$$\therefore k = -\frac{c}{2b^2}$$

$$v^2 = -\frac{xc}{b} + \frac{c}{2b^2} - \frac{ce^{-2bx}}{2b^2} = \frac{c}{2b^2}\left(1 - e^{-2bx}\right) - \frac{cx}{b}$$

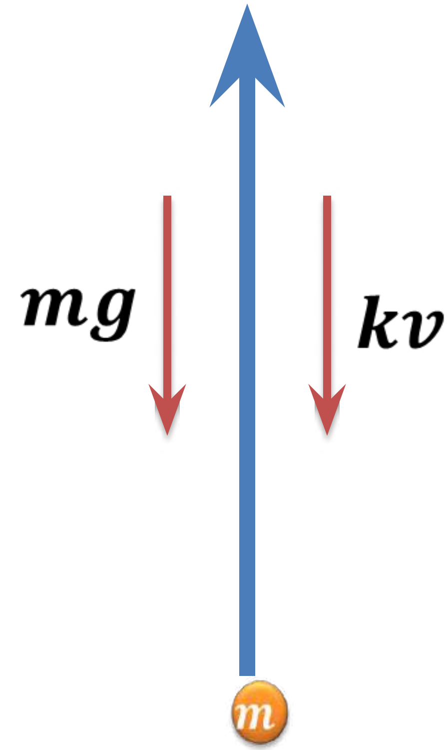
Other Subjects: https://www.studymedia.in/fe/notes

- A particle of mass m moves under gravity in a medium whose resistance is k times its velocity where k is a constant. If the particle is projected vertically upwards with initial velocity V, find the time taken by the particle to reach maximum height.

The forces acting on body are

i) Its weight mg

ii) Air resistance kv



$\therefore$ Equation of motion is

$$Net\ force = mass \times acceleration$$

$$\therefore -mg - kv = m\frac{dv}{dt}$$

Other Subjects: https://www.studymedia.in/fe/notes

$$\therefore -mg - kv = m\frac{dv}{dt} \qquad \therefore -dt = m\frac{dv}{mg + kv}$$

$$\therefore -\int dt = m\int \frac{dv}{mg + kv} \quad \therefore -t = \frac{m}{k}log(mg + kv) + c$$

At $t = 0,\ v = V \therefore 0 = \frac{m}{k}log(mg + kV) + c$

$$\therefore c = -\frac{m}{k}log(mg + kV)$$

$$\therefore -t = \frac{m}{k}log(mg + kv) - \frac{m}{k}log(mg + kV)$$

$$\therefore t = \frac{m}{k}log\left(\frac{mg + kV}{mg + kv}\right)$$

At maximum height

$v = 0, t = t_1$

$$\therefore t_1 = \frac{m}{k}log\left(\frac{mg + kV}{mg}\right)$$

Other Subjects: https://www.studymedia.in/fe/notes

- A particle is moving in a straight line with an acceleration $k\left[x+\frac{a^4}{x^3}\right]$ directed towards origin. If it starts from the rest at a distance 'a' from the origin, Prove that it will arrive at origin at the end of time $\frac{\pi}{4\sqrt{k}}$

Equation of motion is $$v\frac{dv}{dx} = -k\left(x+\frac{a^4}{x^3}\right)$$

$$v\,dv = -k\left(x+\frac{a^4}{x^3}\right)\,dx$$

$$\int v\,dv = -k\int\left(x+\frac{a^4}{x^3}\right)\,dx \text{ ------ }(1)$$

$$\frac{v^2}{2} = -k\left[\frac{x^2}{2}-\frac{a^4}{2x^2}\right]+C$$

Other Subjects: https://www.studymedia.in/fe/notes

- When $x = a, v = 0 \quad \therefore C = 0$

$$\frac{v^2}{2} = -\frac{k}{2}\left[x^2 - \frac{a^4}{x^2}\right]$$

$$v^2 = k\left[\frac{a^4 - x^4}{x^2}\right]$$

$$\therefore v = \pm\sqrt{k}\,\frac{\sqrt{a^4 - x^4}}{x}$$

Since the acceleration is directed towards origin

$$\therefore v = -\sqrt{k}\,\frac{\sqrt{a^4 - x^4}}{x}$$

$$\therefore \frac{dx}{dt} = -\sqrt{k}\,\frac{\sqrt{a^4 - x^4}}{x}$$

Other Subjects: https://www.studymedia.in/fe/notes

- $$\int \frac{x}{\sqrt{a^4-x^4}}\,dx = -\sqrt{k}\int dt + C$$

Put $x^2 = u \quad \therefore x\,dx = \frac{du}{2}$

$$\frac{1}{2}\int \frac{du}{\sqrt{(a^2)^2-u^2}} = -\sqrt{k}\,t + C$$

$$\sin^{-1}\left(\frac{u}{a^2}\right) = -2\sqrt{k}\,t + C_1$$

Or

$$\sin^{-1}\left(\frac{x^2}{a^2}\right) = -2\sqrt{k}\,t + C_1$$

When $t = 0, x = a \;\therefore C_1 = \sin^{-1} 1 = \pi/2$

$$\sin^{-1}\left(\frac{x^2}{a^2}\right) = -2\sqrt{k}\,t + \frac{\pi}{2}$$

At $x = 0 \qquad t = \frac{\pi}{4\sqrt{k}}$

Other Subjects: https://www.studymedia.in/fe/notes

Simple harmonic motion

If a particle moves on a straight line so that the force acting on it always directed towards a fixed point on the line and proportional to its distance from the point the particle is said to move in simple harmonic motion.

The equation of S.H.M. is $$\frac{d^2x}{dt^2} = -w^2x$$

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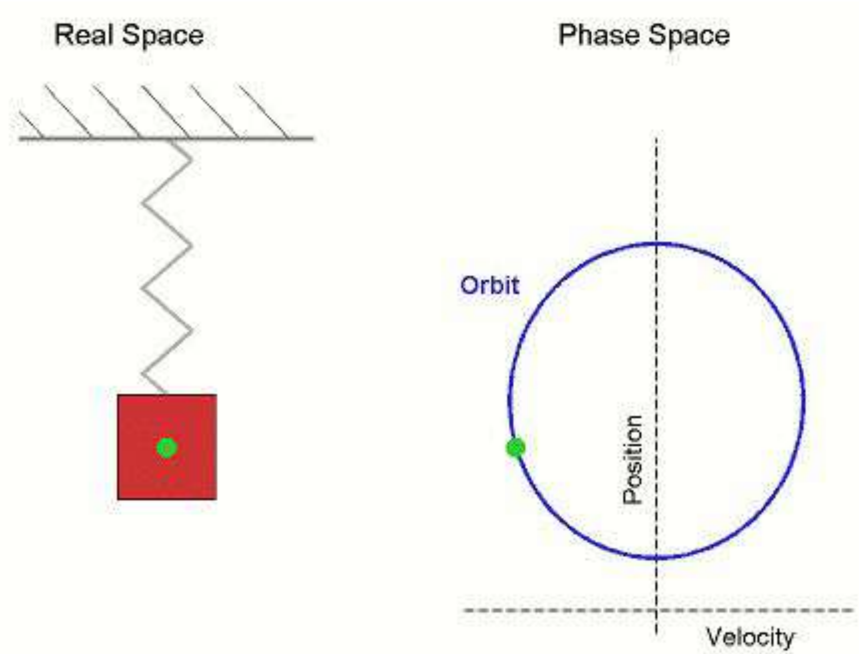
Simple harmonic motion shown both in real space and phase space. The orbit is periodic. (Here the velocity and position axes have been reversed from the standard convention in order to align the two diagrams)



A point executing simple harmonic motion has velocities v_1 and v_2 and accelerations a_1 and a_2 in two positions respectively. Show that the distance between the two positions is $\left|\frac{v_1^2-v_2^2}{a_1+a_2}\right|$.

Let v_1 and v_2 be the velocities, a_1 and a_2 be the accelerations in two positions x_1 and x_2 respectively.

Then equation of S.H.M. is $$\frac{d^2x}{dt^2} = -w^2x$$

At $x = x_1,\ a = a_1$ and at $x = x_2, a = a_2$

$\therefore a_1 = -w^2x_1$ and $a_2 = -w^2x_2$ $\quad \therefore a_1 + a_2 = -w^2(x_1 + x_2)$

$$\text{As } \frac{d^2x}{dt^2} = -w^2x = v\frac{dv}{dx} \qquad -w^2xdx = vdv$$

Other Subjects: https://www.studymedia.in/fe/notes

$$-w^2\int xdx = \int vdv$$

At $x = x_1,\ v = v_1$ and at $x = x_2, v = v_2$

$$-w^2\int_{x_1}^{x_2} xdx = \int_{v_1}^{v_2} vdv \qquad -w^2\left[\frac{x^2}{2}\right]_{x_1}^{x_2} = \left[\frac{v^2}{2}\right]_{v_1}^{v_2}$$

$$-w^2[x_2^2 - x_1^2] = [v_2^2 - v_1^2] \qquad -w^2[x_1^2 - x_2^2] = [v_1^2 - v_2^2]$$

$$\frac{v_1^2 - v_2^2}{a_1 + a_2} = \frac{-w^2[x_1^2 - x_2^2]}{-w^2(x_1 + x_2)} = x_1 - x_2$$

$$|x_1 - x_2| = \left|\frac{v_1^2 - v_2^2}{a_1 + a_2}\right|$$

Other Subjects: https://www.studymedia.in/fe/notes

A particle executes S.H.M. When it is 2 cm from mid path, its velocity is 10 cm/sec and when it is 6 cm from center of its path its velocity is 2 cm/sec. Find its period and its greatest acceleration.

Given: for $x = 2,\ v = 10$ and for $x = 6,\ v = 2$

Then equation of S.H.M. is $\dfrac{d^2x}{dt^2} = -w^2x = v\dfrac{dv}{dx}$

$$-w^2xdx = vdv \quad -w^2\int_2^6 xdx = \int_{10}^2 vdv \qquad -w^2\left[\frac{x^2}{2}\right]_2^6 = \left[\frac{v^2}{2}\right]_{10}^2$$

$$-w^2[36-4] = [4-100] \ \therefore\ w^2 = 3 \therefore w = \sqrt{3}$$

$$\therefore Period\ \ T = \frac{2\pi}{w} = \frac{2\pi}{\sqrt{3}}$$

Given: for $x = 2$, $v = 10$ and for $x = ?$, $v = 0$

$$-w^2 \int_{2}^{x} x\,dx = \int_{10}^{0} v\,dv$$

$$-w^2 \left[\frac{x^2}{2}\right]_2^x = \left[\frac{v^2}{2}\right]_{10}^0$$

$$-3[x^2 - 4] = [0 - 100]$$

$$x^2 = \frac{112}{3} \quad \therefore x = \pm\sqrt{\frac{112}{3}}$$

$$\therefore accel^n = -w^2 x = (3)\sqrt{\frac{112}{3}} = \sqrt{336}$$

Other Subjects: https://www.studymedia.in/fe/notes

Assignments

1. The inner and outer surfaces of a spherical shell are maintained at T_0 and T_1 temp. resp. If the inner and outer radii of the shell are r_0 and $r_1$resp. and thermal conductivity of shell is k find the amount of heat loss from the shell per unit time. Find also the temp. distribution through the shell.

2. A bullet is fired into sand tank , its retardation is proportional to square root of its velocity . Show that the bullet will come to rest in time $\frac{2\sqrt{v}}{k}$, where v is initial velocity.

Other Subjects: https://www.studymedia.in/fe/notes

3. An e.m.f. 200 e–5t is applied to a series circuit consisting 20Ω resistor and 0.01 F capacitor. Find the charge and current at any time , assuming that there is no initial charge on capacitor.

4. A body originally at 85°C cools to 65°C in 25 minutes, the temperature of air being 40°, what will be the temperature of the body after 40 minutes.

5. A pipe 10cm in diameter contains steam at 1000*C* . It is covered with asbestos,5cm thick , for which k=0.0006 and the outside surface is at 300*C* .Find the amount of heat lost per hour from a meter long pipe.

Other Subjects: https://www.studymedia.in/fe/notes