Fundamental Concepts:

① Particle

It is a material point which contains definite qty. of matter without dimension.

② Rigid Body :-

It is a body which does not undergo any deformation (change in shape and size) under the action of external force.

Combination of large No. of particles which remains at fixed distance from each other when external force is applied or removed.

③ Force

An action which tends to change the state of rest of body or motion of body on which it acts.

principles of statics:-

① Newton's 1^{st} law:-

Everybody continues to be in its state of rest or in the state of uniform motion along straight line unless it is acted upon by external unbalanced force.

② Newton's 2^{nd} law:-

when unbalanced force acts on particle, it will have an acceleration which is proportional to the magnitude of force. this accl^n will be in the direction of force along straight line.

Rate of change of momentum of body is directly proportional to the force acting on it & it is in the direction of force.

③ Newton's 3^{rd} law:-

Forces of action & Reaction bet^n bodies in contact will have same magnitude, line of action but opp. direction

* Mechanics :-

The branch of physical science that deals with state of the rest or state of the motion is called as Mechanics.

* Engg. Mechanics :-

The branch of Applied science, which deals with laws ~~of~~ & Principals of Mechanics, alongwith their applications to the real-life engineering problems.

Engg. Mechanics is essential for an engineer in
- Planning, designing, & construction of various structures & machines.

* Classification of Engg. Mechanics :-

- Engg. Mechanics
  - Mechanics of Solids
    - Mechanics of Rigid Bodies
      - Statics
      - Dynamics.
        - Kinetics
        - Kinematics.
    - Mechanics of Deformable Bodies
  - Mechanics of Fluids.

In our syllabus, we are going to study the mechanics of Rigid Bodies i.e. Statics & Dynamics.

* **Statics :-**

It is the branch of Engg. Mechanics which deals with the forces and their effects (acting) upon the bodies which are at rest.

* **Dynamics :-**

It is the branch of engg. mechanics which deals with forces & their effects (acting) upon the bodies which are in motion.

Dynamics is again divided into two categories:

i) Kinetics ii) Kinematics :

Kinetics :-

It is the branch of Dynamics which deals with the bodies in motion by considering the forces which causes the motion.

Kinematics :-

It is the branch of dynamics which deals with the bodies in motion without considering the force which is responsible for the motion.

+ Mass -: (m)

The quantity of matter possessed by a body.

S.I. unit :- gram (g)
kilogram (kg)

$\therefore$ 1 kg = 1000 g

+ Weight :- (W)

- Force acting on the object due to gravity.
- It is the product of mass & gravitational Accelaration.

$W = m \times g$

S.I. Unit :- Newton (N)
KiloNewton (KN)

1 KN = 1000 N.

* **Force :- [(F) or (P)]**

It is an external agency which produces or tends to produce, destroy or tends to destroy motion.

OR

It is an external agent which changes or tends to change the state of rest or state of motion of body.

It is a vector quantity.

Force is also known as rate of change of momentum.

∴ As momentum = mass × velocity

but mass will never change, then

Force = mass × rate of change of velocity

= mass × Acceleration

$F = P = m \cdot a$

S.I. Unit :- Newton (N)

KiloNewton (KN) $= 10^3$ N

MegaNewton (MN) $= 10^6$ N

Giganewton (GN) $= 10^9$ N

TeraNewton (TN) $= 10^{12}$ N.

1 kg Force :-

Force required to produce unit gravitational acceleration on unit mass.

∴ 01 kg force = mass × Accl^n (gravity)

$= 1\ kg \times 9.81\ m/s^2$

$= 9.81\ kg \cdot \frac{m}{s^2}$

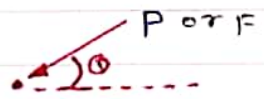
∴ 1 kg-F $= 9.81\ kg \cdot \frac{m}{s^2}$

1 kg-F = 9.81 N.

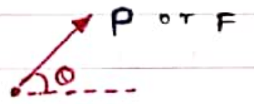
i.e. $\boxed{1\ kg = 9.81\ N.}$

• Charactoristics of force :-

1) Magnitude :- The value of force i.e. 10 N, 2 KN etc.
2) Direction :- Line of action & angle formed with fixed axis.
3) Nature of force or sense :-
It means whether the force is push or pull.



Push type force



Pull type force

Push :- Force acting towards the point
Pull :- Force acting away from the point.

4) point of application:-
The point at which or through which the force acts.

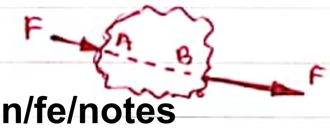
• Effects of force:-

Force may produce following effects on the body:
1) It may change the state of body.
i.e. if body is at rest, force may bring it in motion or if body is in motion, force may accelerate it or force may stop it or force may retard it.

2) It may produce internal stress in the body.
3) It may produce deformation in non-rigid body.
4) It may produce rotational effect in body.
5) It may keep the body in stable state (Equillibrium).

* Principle of Transmissibility of force.

The state of the rigid body will not change if ~~we~~ force acting on a body is replaced by another force of same magnitude & direction acting anywhere on the body along the line of action of replaced force.



∴ The value of force will not change along the line of action of force within the body.

- System of force :-

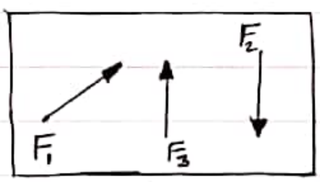
When a single agency is acting on body then it is known as force. But
when no. of forces are acting on the body simultaneously, then it is known "system of force."

- Types of Force system :-

1) Coplaner forces 2) Non co-planer forces
3) Co-linear forces 4) Non- Co-linear forces.
5) Concurrent forces 6) Non concurrent forces
7) Parallel forces. (Like & unlike).
8) coplaner concurrent forces
9) Coplaner Non concurrent forces
10) Non coplaner cocurrent forces
11) Non coplaner non concurrent forces.

1) Co-planer Forcesystem /Forces :-

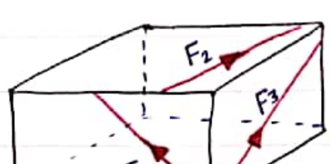
The Forces whose line of action lie on the same plane are called as Co-planer Forces.



The forces which are acting in the same plane are known as co-planer forces.

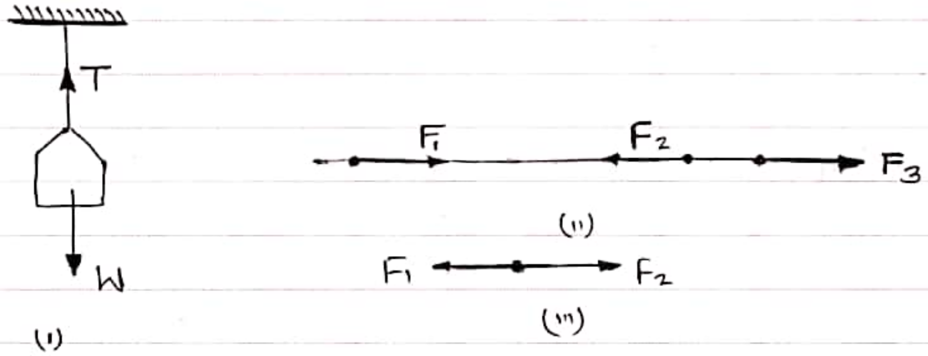
2) Non- Co-planer force system/forces :-

- A Forces ~~system~~ whose line of action does not lie in the same plane (i.e. lie in differnt planes)
- Forces which are acting in the different planes known as non co-planer forces(system).



3) Co-linear force system/Forces :-

- The forces whose line of action lies on the same line are called as co-linear forces.
- The Forces which are acting along the same straight line are called as co-linear forces.

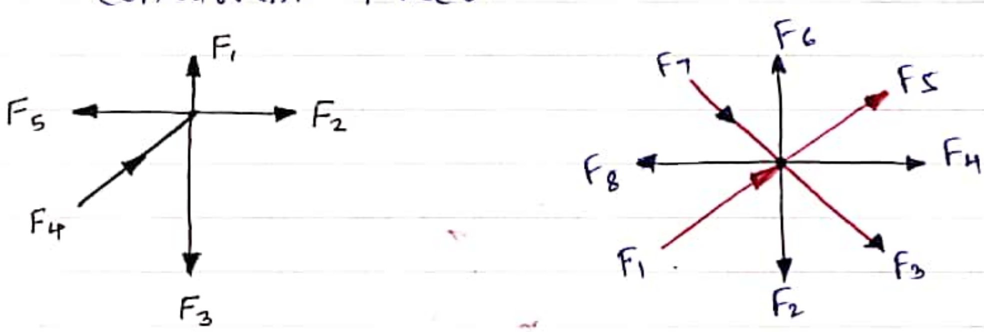


4) Non-colinear Forces / force system :-

- The forces which are not acting along the same straight line are known as non co-linear forces.
- The forces whose line of action doesn't lie on the same line.

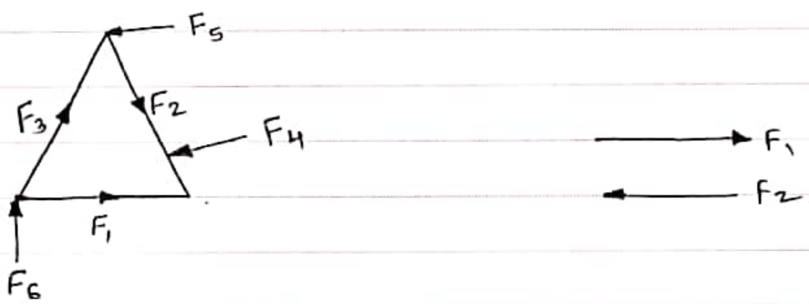
5) Concurrent forces / Force system :-

- The Forces whose line of action meets at one common point are called concurrent forces.
- The forces which are passing through a common point are concurrent forces
- The forces which meets at one point are concurrent forces.



6) Non- concurrent force system/ forces :-

- The Forces which are not passing through common point OR
- The forces whose line of action does not meet at common point OR
- The forces which doesn't meet at one point are called as non- concurrent forces.



7) Parallel forces / Force system:-

The forces whose line of actions are parallel to each other are called as parallel forces.

a) Like parallel Forces:-

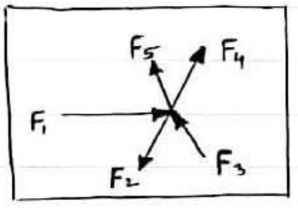
- The Forces whose line of actions are parallel to each other and having same direction are called like parallel forces.
- Forces which are parallel to each other & acting in same direction are called as Like parallel forces.

b) Unlike parallel Forces:-

- The forces which are parallel to each other but having different directions or
- The forces which are parallel to each other & acting in opposite direction are called as unlike parallel forces.

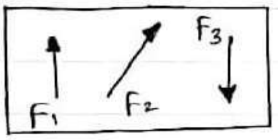
8) Coplaner concurrent forces :-

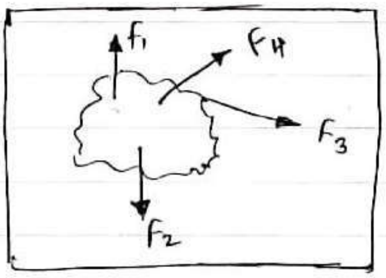
The forces which meets at one point & their lines of action also lie on the same plane are called as coplaner concurrent force system.



9) Coplaner nonconcurrent forces.

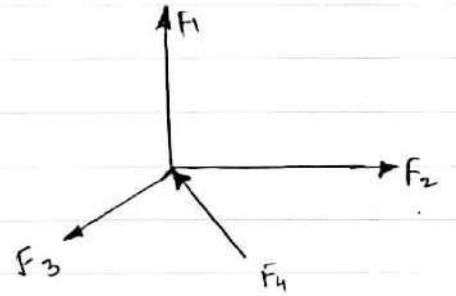
The forces which do not meet at one point but their line of action lies on the same plane are known as coplaner non-concurrent system of force.

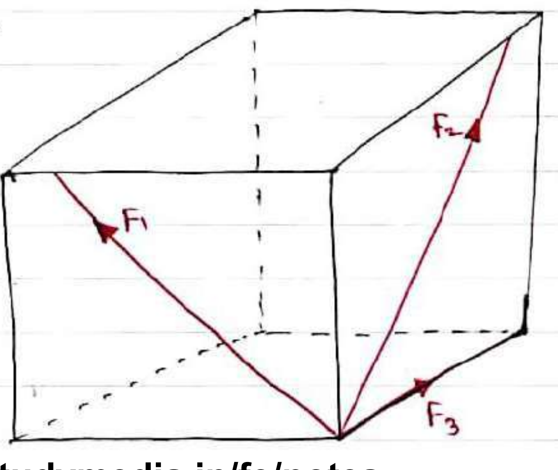




10) Non coplaner concurrent forces:

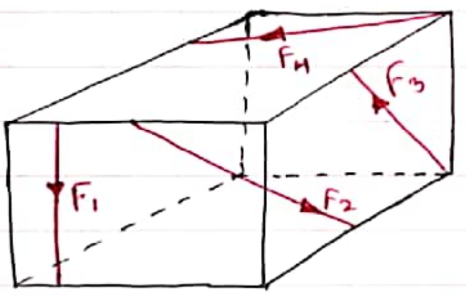
The forces which meets at one point but their line of action do not lie on the same plane are known as non coplane concurrent forces.





III) Non coplaner non concurrent forces:-

The forces which do not meet at one point & their lines of actions do not lie on the same plane are called as non coplaner non concurrent forces.



* Resultant force:-

- If number of forces are acting simultaneously on a body, then it is possible to find out single force which could replace them, i.e. which would produce the same effect as produced by all given forces. This single force is called as Resultant force.

OR

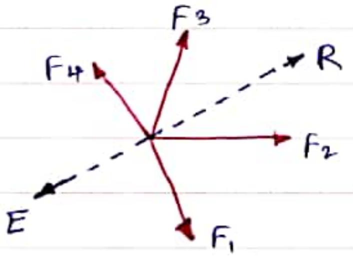
- It is a single force which produces the same effect that is produced by number of forces when acting together.

Resultant is denoted by (R).

* Equillibrant :-

It is the single force which when acting with all other forces keeps the body at rest or in equillibrium.

It is denoted by (E).



The Resultant & Equillibrant are equal in magnitude but opposite in the direction.

* Composition and Resolution of force :-

Composition :-

The process of finding out the resultant force of a given force system.

Resolution :-

It is the procedure of splitting up a single force into number of components without changing the effect of same on the body.

Forces are generally resolved into two components along two mutually $\perp^{ar}$ directions.

- **Methods of composition / methods for the Resultant Force.**

A] Analytical method B] Graphical Method.

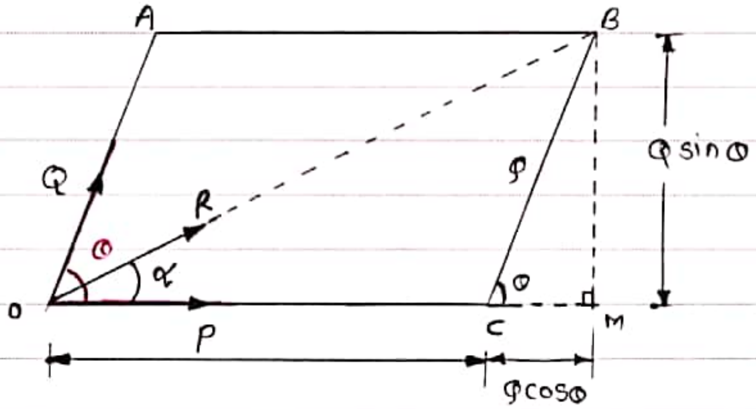
① → Resultant of two concurrent forces i.e. Law of parallelogram of forces.

② → Resultant of more than two forces (concurrent & non concurrent) Method of Resolution.

- **Resultant of two concurrent forces :-**

Law of parallelogram of forces :-

If two forces acting simultaniously on a partical, be represented in magnitude & direction by two adjecent sides of parallelogram, then the diagonal passing through the point of intersection of two forces will represent the resultant in magnitude & direction.



Consider two forces P & Q acting at a point represented by two sides OA & OC of a parallelogram OABC.

Let, θ is the angle between two forces P & Q.
α be the angle between Force P & Resultant R.

Lets draw a line $BM \perp OC$ which intersects OC at M.

In Δ CMB

$$\sin\theta = \frac{BM}{BC} = \frac{BM}{Q}.$$

$$\therefore BM = Q\sin\theta$$

Also, $$\cos\theta = \frac{CM}{BC} = \frac{CM}{Q}$$

$$\therefore CM = Q\cos\theta.$$

Now in Δ OMB,

we have, $$(OB)^2 = (OM)^2 + (BM)^2$$

$$\therefore R^2 = (OC + CM)^2 + (BM)^2$$

$$\therefore R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$$

$$\therefore = P^2 + 2PQ\cos\theta + Q^2\cos^2\theta + Q^2\sin^2\theta$$

$$\therefore R^2 = P^2 + 2PQ\cos\theta + Q^2(\cos^2\theta + \sin^2\theta)$$

$$R^2 = P^2 + 2PQ\cos\theta + Q^2$$

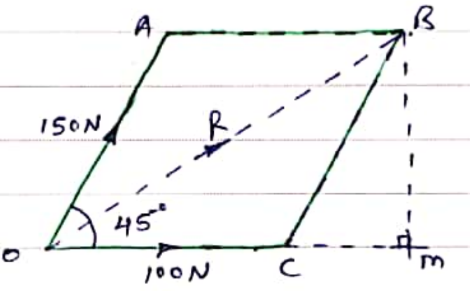
Taking Sq. Root,

$$\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$$ ... magnitude.

$$\boxed{\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}}$$ ...... direction.

Examples Based on parallelogram Law of forces.

① Two forces of 100 N & 150 N are acting simultaniously at a point. What is the resultant of these two forces, If the angle betn them is 45°.



Given: $P = 100\,N$

$Q = 150\,N$.

$\theta = 45°$

$R = ?$, $\alpha = ?$

solution:

Accr to law of parallelogram of force,

$\therefore R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$\therefore R = \sqrt{100^2 + 150^2 + 2 \times 100 \times 150 \times \cos 45°}$

$R = 231.76\,N.$

$\therefore R = 232\ N.$ — Ans. ..... magnitude

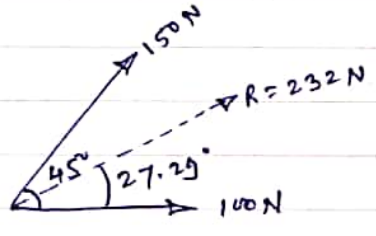
$\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

$= \dfrac{150\sin 45°}{100 + 150\cos 45°}$

$\tan\alpha = \dfrac{106.066}{206.066}$

$\therefore \boxed{\alpha = 27.29°}$ ..... Direction.

Ans:



② The sum of two forces is 270 N & their resultant is 180 N. if Resultant is Perpendicular to P, Find the two forces P & Q.

⇒

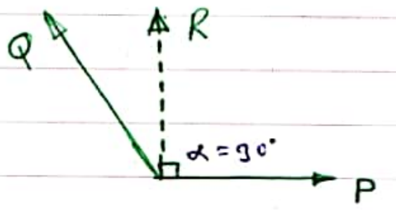
<u>Given</u> :-

$P + Q = 270$ N

$R = 180$ N

$\alpha = 90°$

∴



<u>Sol^n</u> : Acc to law of parallelogram,

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$\tan 90 = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$\therefore \infty = \frac{Q\sin\theta}{P + Q\cos\theta}.$$

Here if $P + Q\cos\theta = 0$, then only above term will become infinity.

Thus, $P + Q\cos\theta = 0$

$\therefore P = -Q\cos\theta$

$\therefore -P = Q\cos\theta.$ .... ①

Now, ∴ $R^2 = P^2 + Q^2 + 2PQ\cos\theta.$

$\therefore 180^2 = P^2 + Q^2 + 2P(-P).$ .... from eq^n ①

$\therefore 180^2 = P^2 + Q^2 + (-2P^2)$

$\therefore 180^2 = P^2 - 2P^2 + Q^2$

$\therefore 180^2 = -P^2 + Q^2$

$180^2 = Q^2 - P^2$

$180^2 = (Q-P)(Q+P)$

$= (Q-P) \times 270$

$$\therefore Q - P = \frac{180^2}{270}$$

$\therefore \; Q - P = 120 \quad -(ii)$

& we know that

$Q + P = 270 \cdot \quad -(iii)$

solving (ii) & (iii) i.e. add eqⁿ (ii) & (iii)

$$\begin{array}{rl} & Q - P = 120 \\ + & Q + P = 270 \\ \hline & 2Q + 0 = 390 \end{array}$$

$\therefore \; 2Q = 390$

$\boxed{Q = 195\ N}$

& $\boxed{P = 75\ N}$ } ANS.

(3) For two forces P & Q acting at a point. maximum resultant is 2000N and minimum magnitude of resultant is 800 N. find values of P & Q.

⇒ we know that,

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta.$$

for max value of R, cosθ must be 1 i.e. $\cos\theta = 1$

i.e. ∴ $\theta = 0$, then only $\cos\theta = 1$

$\therefore \; R_{max} = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$R_{max} = \sqrt{P^2 + Q^2 + 2PQ\cos 0}$

$R_{max} = \sqrt{P^2 + Q^2 + 2PQ}$

$2000 = \sqrt{P^2 + 2PQ + Q^2}$

$2000 = \sqrt{(P+Q)^2}$

$\boxed{2000 = P + Q.}$ ---- eqⁿ (1)

Now for minimum value of R,

$\therefore \; \theta = 180°.$

$R_{min} = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$= \sqrt{P^2 + Q^2 + 2PQ\cos 180}$

$\therefore R_{min} = \sqrt{P^2 + Q^2 - 2PQ}$

$= \sqrt{P^2 - 2PQ + Q^2}$

$R_{min} = \sqrt{(P-Q)^2}$

$\boxed{800 = P - Q}$ ..... eq - (II)

Solving eqn (I) & (II) we get,

i.e. Add eqn (I) & (II)

$\therefore \quad P + Q = 2000$

$+ \quad P - Q = 800$

$2P + 0 = 2800$

$P = \frac{2800}{2}$

$\boxed{P = 1400\,N}$ ... Ans

& $\quad P + Q = 2000$

$\therefore 1400 + Q = 2000$

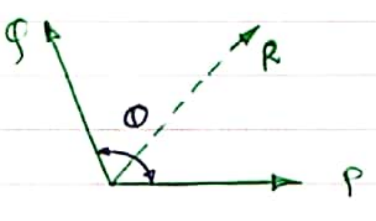
$\therefore Q = 2000 - 1400$

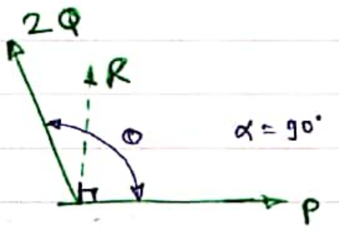
$\boxed{Q = 600\,N}$ .... Ans.

Ans $\quad P = 1400\,N$

$\quad Q = 600\,N$

(4) The angle betn the two forces P & Q is θ. If Q is doubled then new resultant is perpendicular to P. Proove that Q = R.





$R^2 = P^2 + Q^2 + 2PQ\cos\theta$ —(I)

Also $\tan 90 = \frac{Q\sin\theta}{P + Q\cos\theta}$

$\infty = \frac{2Q\sin\theta}{P + (2Q)\cos\theta} = \infty$

if $P + (2Q)\cos\theta = 0$ then only above term will be infinity.

$\therefore \quad P + [2Q \cdot \cos\theta] = 0$

$\therefore \quad -P = 2Q\cos\theta$. — put in eqⁿ ①

$\therefore \quad R^2 = P^2 + Q^2 + (P \times (-P))$

$= P^2 + Q^2 - P^2$

$R^2 = Q^2$

$\therefore$ Taking sq. root.

$R = Q$ — <u>proved.</u>

⑤ The angle betⁿ the two concurrent forces is 90° & their resultant is 2500N. The resultant makes an angle of 46° with one force. Determine magnitude of each force.

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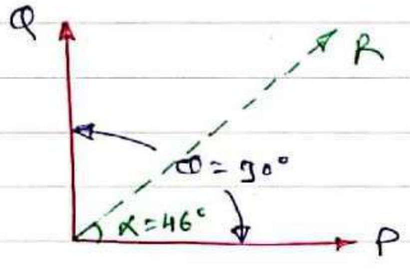


$\theta = 90°$
$\alpha = 46°$
$R = 2500\,N.$

by using, $R^2 = P^2 + Q^2 + 2PQ\cos\theta$.

$2500^2 = P^2 + Q^2 + 2PQ\cos 90$

As $\cos 90° = 0$ then

$\therefore 2500^2 = P^2 + Q^2$ ...①

Now,

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} = \frac{Q\sin 90}{P + Q\cos 90}$$

$$\therefore \tan\alpha = \frac{Q}{P}$$

$$\therefore \tan 46 = \frac{Q}{P}$$

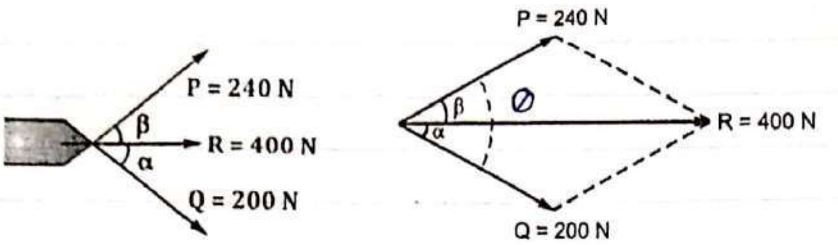
$\therefore$ <u>$Q = 1.0355\,P$.</u> — substitute in eqⁿ -①

$\therefore \quad 2500^2 = P^2 + [(1.0355)^2 P^2]$

$\therefore \quad P^2 = 3017537.93$

$\therefore$ $\boxed{P = 1737\ N \quad Q = 1798\ N}$

Resultant force R = 400 N has two component forces P = 240 N and Q = 200 N as shown. Determine direction of component forces i.e. α and β w.r.t. resultant force.



let $\theta = (\alpha + \beta)$

using law of parallelogram,

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$400^2 = 240^2 + 200^2 + 2 \times 240 \times 200 \cos\theta$$

$$62400 = 96000 \cos\theta$$

$$\therefore \cos\theta = \frac{62400}{96000} = 0.65$$

$$\therefore \theta = \cos^{-1}(0.65) = 49.46^\circ$$

$$\therefore \boxed{\theta = 49.46^\circ}$$

we know that

$$\tan\beta = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$\tan\beta = \frac{200\sin\theta}{240 + (200\cos\theta)}$$

$$= \frac{200\sin 49.46}{240 + (200\cos 49.46)}$$

$$\tan\beta = 0.411$$

$$\therefore \beta = \tan^{-1}(0.411)$$

$\boxed{\beta = 22.33^\circ}$ – Direction of P

$$\therefore \theta = \alpha + \beta$$

$$49.46 = \alpha + 22.33$$

$\therefore \boxed{\alpha = 27.13^\circ}$ ... Direction of Q.

• Resultant of more than two Forces:-

Method of Resolution.

When two or more coplaner concurrent or non concurrent forces are acting on a body, then the resultant can be found out by using resolution of forces. A resolution procedure is described below.

1) Resolve all the forces Horizontally & find the algebric sum of all the horizontal components.

If a force is denoted by F, then

F_x = Horizontal component of Force F (along x dirⁿ).

F_y = Vertical component of force F. (along y dirⁿ).

Thus, in this step find ΣF_x.

2) Resolve all the forces vertically & find the algebric sum of all vertical components.

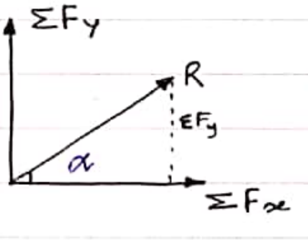
i.e. Find ΣF_y.

3) The resultant R of the given forces will be given by the equation:

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

4) The resultant force will be inclined at an angle α with horizontal, such that

$$\tan\alpha = \frac{\Sigma F_y}{\Sigma F_x}$$



* Resolution of Force:-

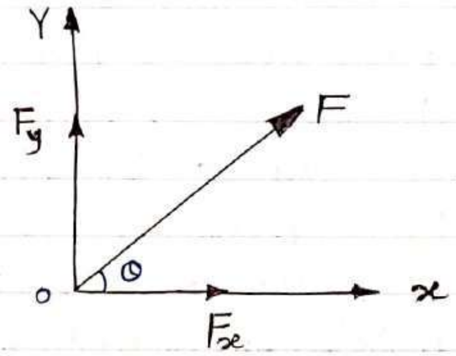
Following 3 methods are used to resolve the force

1) Orthogonal Resolution (Perpendicular resolution)
2) Non- perpendicular / Non-orthogonal resolution.
3) Resolution into two Parallel components.

① Orthogonal or perpendicular Resolution :-

In this method, single Force is split up into two components which are perpendicular to each other along x direction & y direction.

Consider force F as shown below, which makes an angle θ with x-axis.



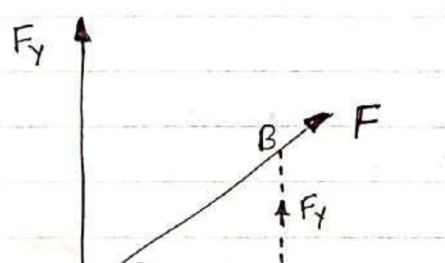
Then, F_x = Horizontal component of force F along x dirn

F_y = vertical component of force F along y dirn.

θ = angle made by force F with x-axis. &

angle betn X & Y axis is 90°.

Now, lets deaw a perpendicular to x axis at point A. This perpendicular will intersect F at point B as shown below.



∴ In right angle Δ OAB,

$$\cancel{\cos\theta = \frac{AB}{OB}}$$

$$\cancel{\cos\theta = \frac{AB}{F} = \frac{F_Y}{F}}$$

$$\cancel{\therefore F_Y = F\cos\theta}$$

In Δ OAB,

$$\sin\theta = \frac{AB}{OB} = \frac{F_Y}{F}$$

$$\therefore \sin\theta = \frac{F_Y}{F}$$

$$\therefore \boxed{F_Y = F\sin\theta}$$

---- y component of force

&

$$\cos\theta = \frac{OA}{OB}$$

$$\cos\theta = \frac{F_x}{F}$$

$$\therefore \boxed{F_x = F\cos\theta}$$

---- x component of force.

Following are the different cases of resolution of Force into two perpendicular components.

<u>Sign conventions:</u>

1) component acting towards Right are positive & towards left are considered as negative.

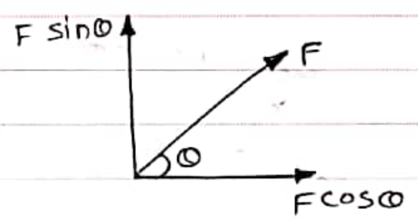
$\longrightarrow$ +ve $\qquad$ $\longleftarrow$ −ve.

2) Component acting upward are positive & acting downward are considered as negative.

$\uparrow$ +ve $\qquad$ $\downarrow$ −ve.

<u>Cases</u> :- (Force acting away from point).

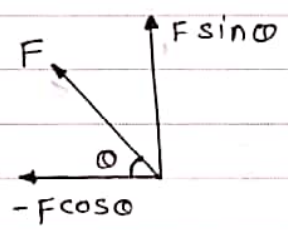
ⓐ Force in 1st quadrant



$F_x = F\cos\theta$

$F_y = F\sin\theta$

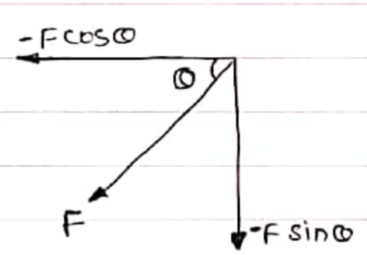
ⓑ Force in 2nd quadrant.



$\therefore\ F_x = -F\cos\theta$

$F_y = F\sin\theta$.

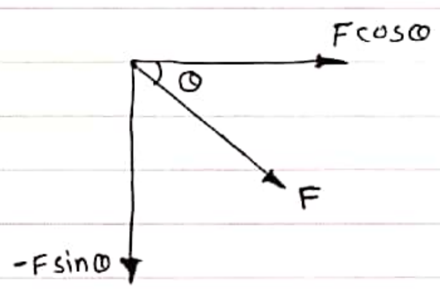
ⓒ Force in IIIrd quadrant



$\therefore\ F_x = -F\cos\theta$

$\therefore\ F_y = -F\sin\theta$

ⓓ Force in IVth quadrant

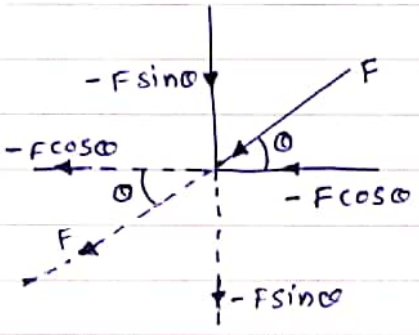


$\therefore\ F_x = F\cos\theta$

$F_y = -F\sin\theta$

Cases of Resolution: (force acting towards the point).

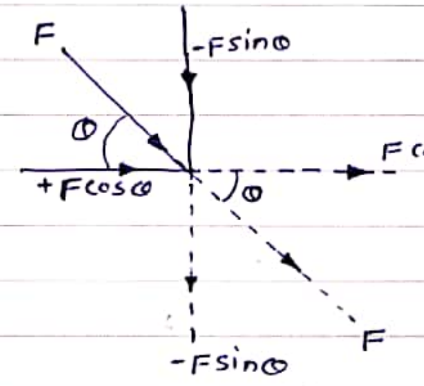
a) force in 1st quadrant



$\therefore F_x = -F\cos\theta$

$\therefore F_y = -F\sin\theta$

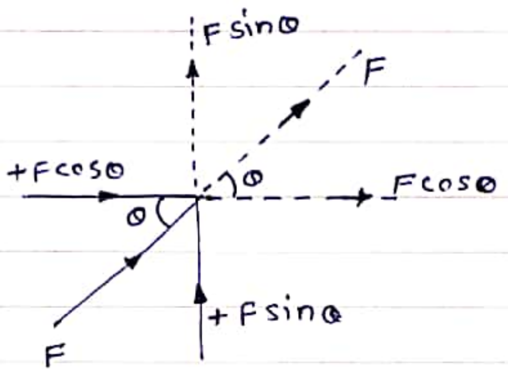
b) Force in 2nd quadrant



$\therefore F_x = F\cos\theta$

$\therefore F_y = -F\sin\theta$

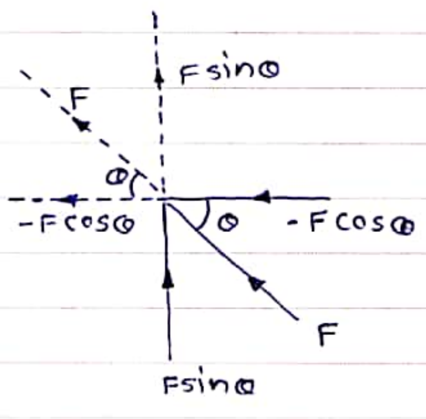
c) force in 3rd quadrant



$\therefore F_x = F\cos\theta$

$\therefore F_y = F\sin\theta$

d) Force in 4th quadrant

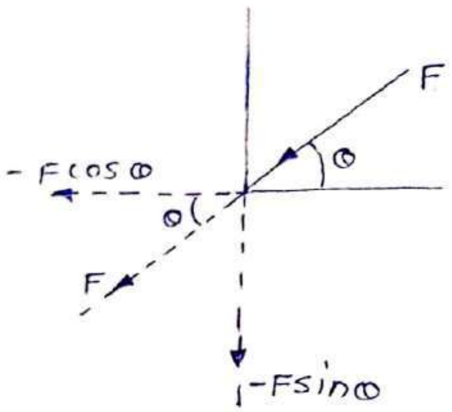


$\therefore F_x = -F\cos\theta$

$\therefore F_y = F\sin\theta$.

Another method of Resolution of force acting towards the point in different quadrants.

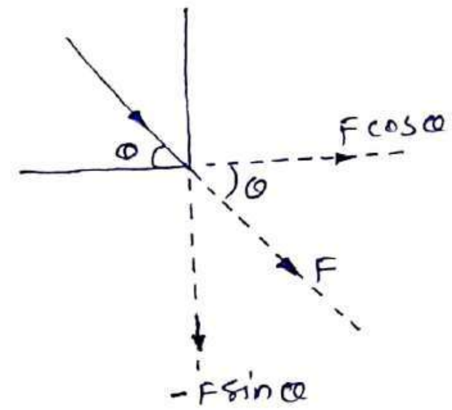
(a) Force acting towards the point in 1st quadrant



$\therefore F_x = -F\cos\theta$

$\therefore F_y = -F\sin\theta$

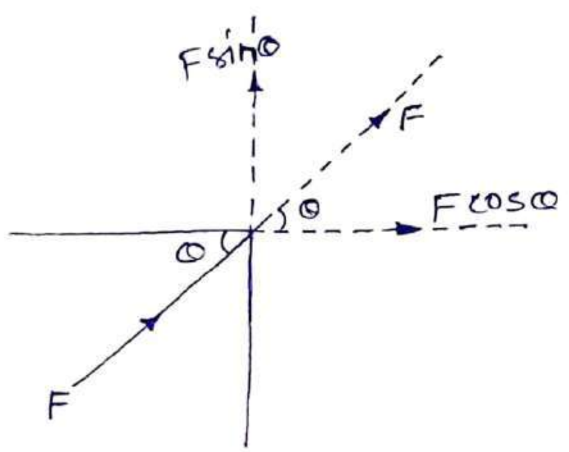
(b) Force acting towards point in the 2nd quadrant.



$\therefore F_x = F\cos\theta$

$\therefore F_y = -F\sin\theta$

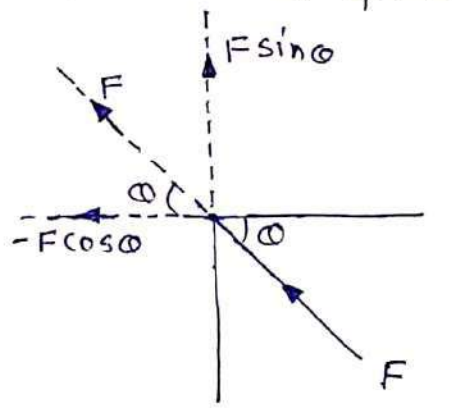
(c) Force acting towards the point in 3rd quadrant.



$\therefore F_x = F\cos\theta$

$\therefore F_y = F\sin\theta$

(d) Force acting towards the point in 4th quadrant.



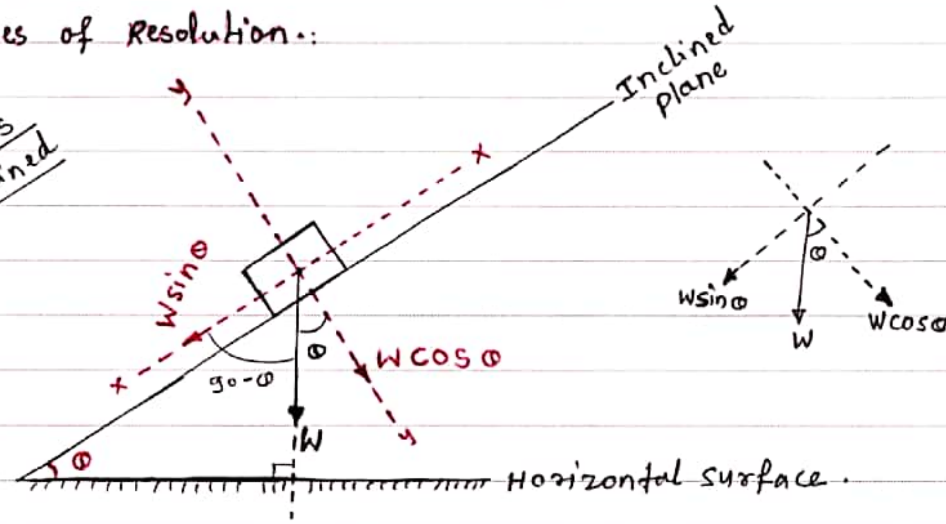
$\therefore F_x = -F\cos\theta$

$\therefore F_y = F\sin\theta$.

- Special Cases of Resolution :

(1)

* selecting x-axis parallel to inclined plane *

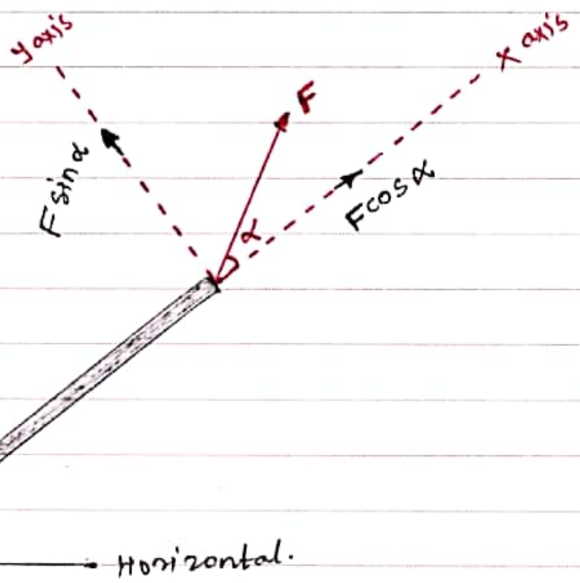


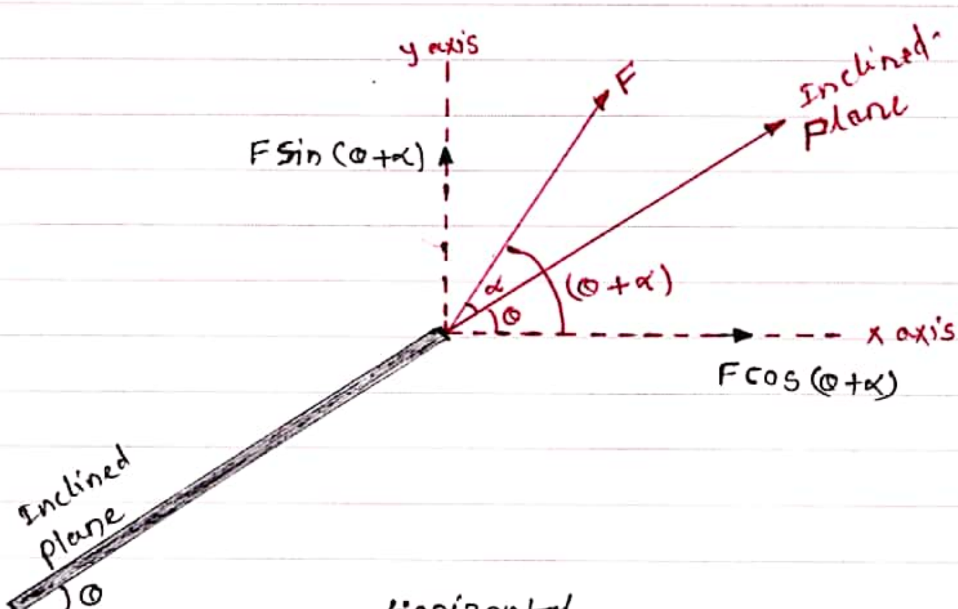
component along the plane = W sin θ

component perpendicular to plane = W cos θ.

(2)

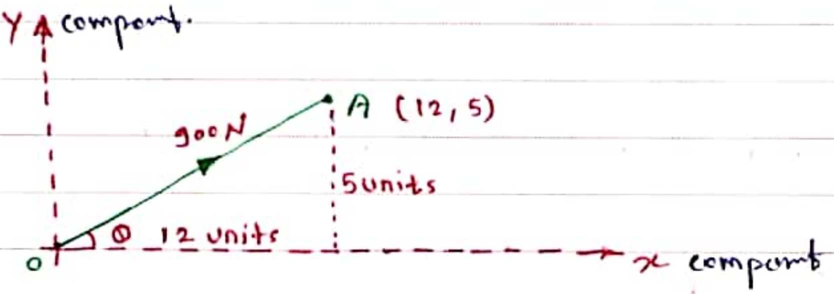
* selecting x-axis along the plane *





Examples based on orthogonal Resolution (Perpendicular components).

① A force of 900 N acts from origin to a point A(12,5). find two orthogonal components.



The angle of force with Horizontal is,

$\tan\theta = \frac{5}{12}$

$\therefore \theta = \tan^{-1}\left(\frac{5}{12}\right)$ force is in 1st quadrant.

$\boxed{\theta = 22.62^\circ}$

$\therefore$ x - component of force,

$F_x = F\cos\theta$

$= 900\cos 22.62$

$\boxed{F_x = 830.77\ N}$ $\rightarrow$

Y - component of force,

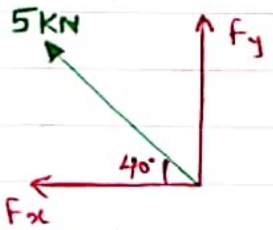
$F_y = F\sin\theta$

$= 900\sin 22.62$

$\boxed{F_y = 346.16\ N}$ $\uparrow$

② find x & Y components for the following force systems.

ⓐ



$\boxed{\theta = 40^\circ}$

Here force is in 2nd quadrant.

x component, $F_x = -F\cos\theta$

$= -5\cos 40$

$= -3.83$ KN.

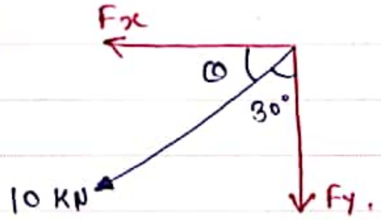
$= 3.83$ KN (left)

Y component, $F_y = F\sin\theta$

$= 5\sin 40$

$= 3.21$ KN Upwards.

(b)



x component, $F_x = -F\cos\theta$

$F_x = -10\cos 60$

$= -5\ KN$

$\boxed{F_x = 5\ KN \text{ towards left}}$

θ = angle ~~fo~~ of force with x axis.

$= 90 - 30$

$\theta = 60°.$

Here, force is in 3rd quadrant.

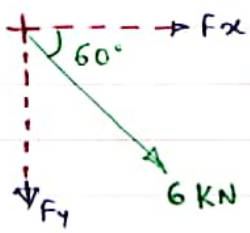
y component, $F_y = -F\sin\theta.$

$\therefore F_y = -10\sin 60$

$= -8.66\ KN$

$\therefore \boxed{F_y = 8.66\ KN \text{ Downward}}$

(c)



Here, force is in 4th quadrant.

&, $\theta = 60°$.

x component,

$F_x = F\cos\theta$

$= 6\cos 60$

$\boxed{F_x = 3\ KN \text{ towards right}}$

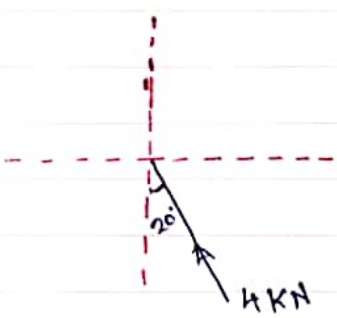
y component,

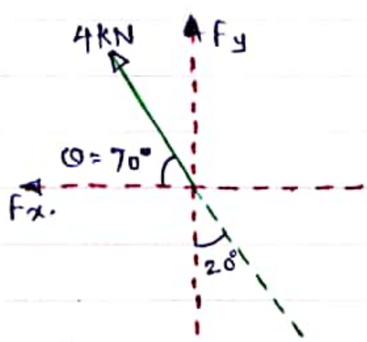
$F_y = -F\sin\theta$

$= -6\sin 60 = -5.2\ KN.$

$\boxed{F_y = 5.2\ KN.\ \text{Downward}}$

(d)





* convert push type force into pull. Thus force is in 2nd quadrant & θ = 70° *

x component, $f_x = -F\cos\theta$

$= -4\cos 70°$

$= -1.37\ KN.$

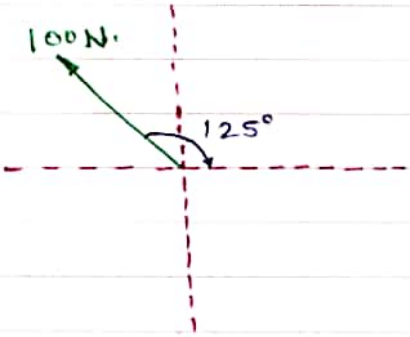
$\boxed{F_x = 1.37\ KN \text{ towards left}}$

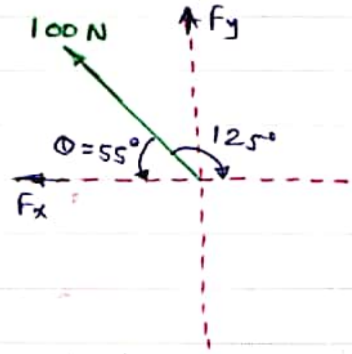
y component, $F_y = F\sin\theta$

$= 4\sin 70°$

$\boxed{F_y = 3.76\ KN \text{ upward}}$

(e)





Here, $\theta = 55°$, force is in 2nd quadrant.

x-component; $F_x = -F\cos\theta$.

$= -100\cos 55°$

$= -57.36$ N.

$\boxed{F_x = 57.36 \text{ N towards left}}$

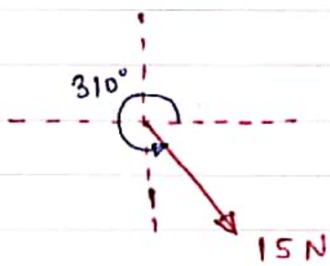
y-component,

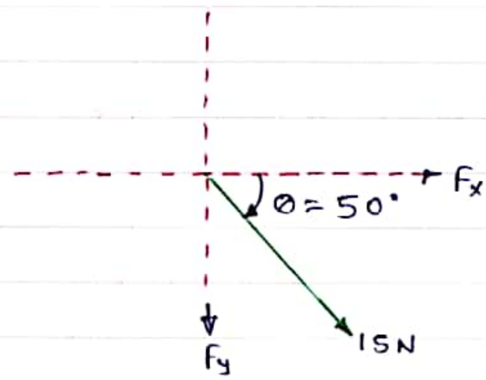
$F_y = F\sin\theta$

$= 100\sin 55°$

$\boxed{F_y = 81.91 \text{ N upward}}$

(f)





Here force is in 4th quadrant,

$\therefore$ $\theta = 50°$.

x component, $F_x = F\cos\theta$

$= 15\cos 50°$

$\boxed{F_x = 9.64 \text{ N}}$ towards Right

y component, $F_y = -F\sin\theta$

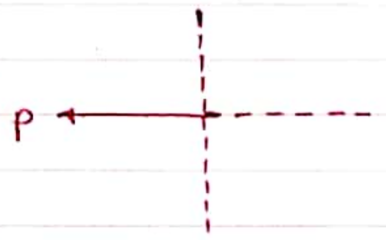
$= -15\sin 50°$

$F_y = -11.49$ N

$\boxed{F_y = 11.49 \text{ N.}}$ Downwards

③ Find x & y components for the following force system.

ⓐ

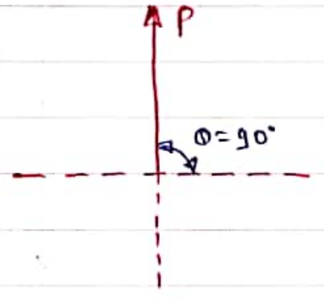


Here force is on negative x axis.
$\theta = 0°$

x component, $F_x = -F \cos\theta$
$= -p \cos 0 = -p$
$\therefore \boxed{F_x = p}$ toward left

y component, $F_y = F \sin\theta$
$= p \sin 0$
$\boxed{F_y = 0}$

ⓑ



Here, force is on y axis.
$\therefore \theta = 90°$.

x component, $F_x = -F\cos\theta = -p \cos 90 = 0$
$\therefore \boxed{F_x = 0}$

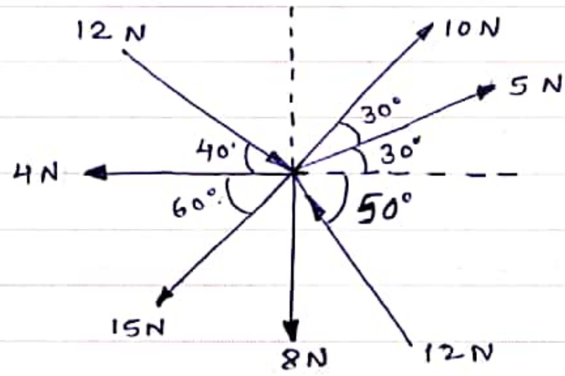
y component, $F_y = F\sin\theta = p \sin 90 = p$
$\therefore \boxed{F_y = p}$

Thus, we can say that,

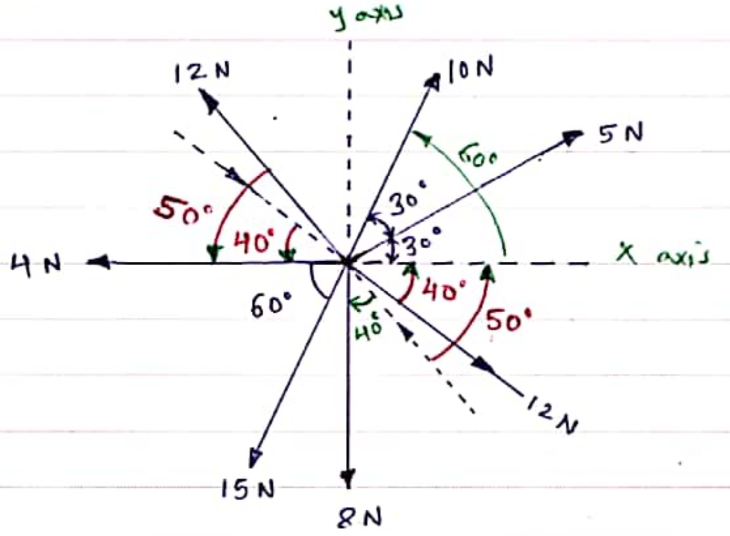
* if a force is on x axis, then its x components is the force itself & y component will be zero.

* if force is on y axis, then its y component is the force itself & x component will be zero.

③ Determine the resultant of following force system as shown in figure.



⇒ Solution :-



Convert all the push type forces into pull type.
Now Resolving all the forces Horizontally along x axis, we have

$$\Sigma F_x = 5\cos 30^\circ + 10\cos 60^\circ - 12\cos 50^\circ - 4 - 15\cos 60 + 12\cos 40^\circ$$

$$= -0.69\ N$$

$\therefore \boxed{\Sigma F_x = 0.69 N}$ towards left (due to -ve sign)

Now resolving all the forces vertically along y axis, we have,

$$\Sigma F_y = 5\sin 30^\circ + 10\sin 60^\circ + 12\sin 50^\circ - 15\sin 60^\circ - 8 - 12\sin 40^\circ$$

$\therefore \Sigma F_y = -8.35\ N.$

$\boxed{\Sigma F_y = 8.35 N}$ Downwards.

The Resultant is given by,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$R = \sqrt{(0.69)^2 + (8.35)^2}$$

$\boxed{R = 8.38\ N}$ - magnitude of Resultant.

Direction of Resultant is given by,

$$\tan\alpha = \frac{\Sigma F_y}{\Sigma F_x}$$

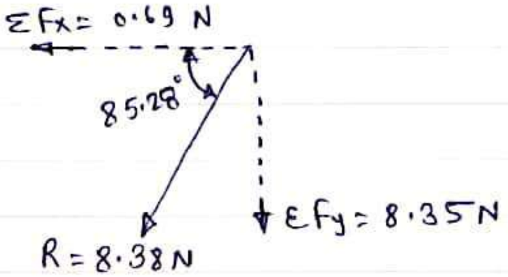
$$\tan\alpha = \frac{8.35}{0.69}$$

$$\tan\alpha = 12.1014$$

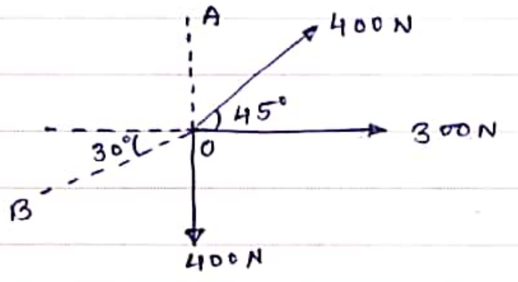
$$\therefore \alpha = \tan^{-1}(12.1014)$$

$$\boxed{\alpha = 85.28^\circ}$$

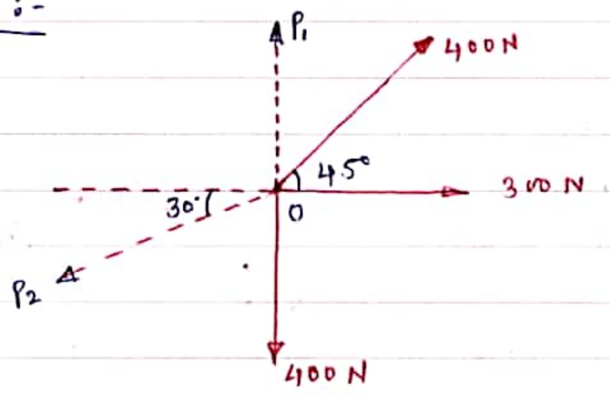
As ΣF_x & ΣF_y are negative, thus, Resultant lie in 3rd quadrant.



④ Three concurrent co-planer forces acts on a body at point O. Determine two additional forces along OA & OB such that Resultant of five forces is zero. Refer the given figure.



✧ Solution :-



Let force along OA $= P_1$ &

Force along OB $= P_2$

As it is given that Resultant $R = 0$, thus

$\Sigma F_x = 0$

$\Sigma F_y = 0$

Now let us resolve the forces Horizontally,

using, $\Sigma F_x = 0$

$\therefore \quad 300 + 400 \cos 45° - P_2 \cos 30° = 0.$

$\therefore \quad 300 + 282.843 - 0.866\, P_2 = 0.$

$\therefore \quad 0.866\, P_2 = 582.843$

$\therefore \quad \boxed{P_2 = 673.03\ N}$

Resolving forces vertically, using $\Sigma F_y = 0$,

$\therefore\ 400 \sin 45 + P_1 - P_2 \sin 30 - 400 = 0$

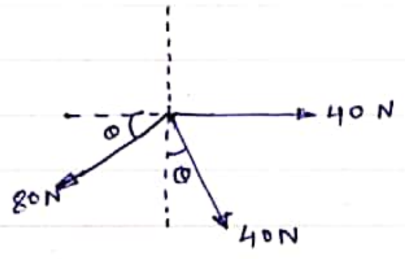
$282.843 + P_1 - 673.03 \sin 30° - 400 = 0.$

$\therefore \quad -453.672 + P_1 = 0$

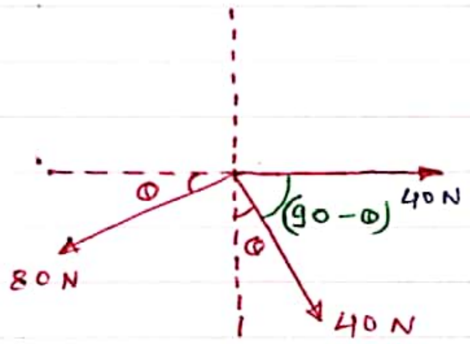
$\boxed{P_1 = 453.672\ N}$ — Ans.

Ans — $P_1 = 453.672\ N$ & $P_2 = 673.03\ N$.

⑤ Determine the angle θ for which the resultant of the three forces is vertical. Also find corresponding magnitude of Resultant by reffering the fig. given below.



<u>Solution:-</u>



The Resultant R is vertical, but direction is unknown.
Assuming R to be downwards,

$\Sigma F_x = 0$ — because R is vertical
$\Sigma F_y = -R$
$\therefore$

Resolving forces Horizontally.

$\therefore \Sigma F_x = 0$

$\therefore 40 - 80\cos\theta + 40\cos(90-\theta) = 0$

$\therefore 40 - 80\cos\theta + 40\sin\theta = 0$

Divide both sides by 40

$\therefore 1 - 2\cos\theta + \sin\theta = 0$

$\therefore 1 + \sin\theta = 2\cos\theta$

squaring both sides.

$\therefore (1+\sin\theta)^2 = (2\cos\theta)^2$

$\therefore 1 + 2\sin\theta + \sin^2\theta = 4\cos^2\theta$

$\therefore 1 + 2\sin\theta + \sin^2\theta = 4(1-\sin^2\theta)$

$\therefore 1 + 2\sin\theta + \sin^2\theta = 4 - 4\sin^2\theta$

$\therefore 5\sin^2\theta + 2\sin\theta - 3 = 0.$

Solving the eqⁿ

$\therefore \sin\theta = 0.6$ OR $\sin\theta = -1$

$\therefore \theta = \sin^{-1} 0.6$ OR $\theta = \sin^{-1}(-1)$

$\theta = 36.87°$ OR $\theta = -90°$ — neglecting this.

$\boxed{\theta = 36.87°}$

Now Resolving forces vertically

$\therefore \quad \Sigma F_y = -R$

$\therefore \quad -80 \sin \theta - 40 \sin(90 - \theta) = -R$

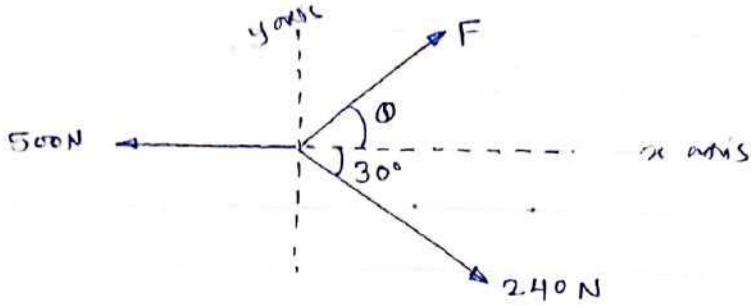
$\therefore \quad -80 \sin 36.87° - 40 \sin(90 - 36.87) = -R$

$\therefore \quad -R = -80.00$ N

$\therefore \quad \boxed{R = 80 N}$

As the answer is positive, our assumption of R being downward is correct.

(7) The force system as shown in figure have a resultant of 200 N along positive Y axis, determine the magnitude & direction (θ) of Force F.



→ As Resultant of 200 N lie on positive Y axis i.e. upwards, it means,

$\Sigma F_x = 0$ & $\Sigma F_y = R$

Now, Resolving forces Horizontally,

$$\Sigma F_x = F\cos\theta - 500 + 240\cos 30 = 0$$

$$\therefore F\cos\theta = 500 - 240\cos 30$$

$$F\cos\theta = 292.154 \quad -(1)$$

Resolving forces vertically,

$$\Sigma F_y = R$$

$$\therefore F\sin\theta - 240\sin 30 = 200$$

$$F\sin\theta = 200 + 240\sin 30$$

$$F\sin\theta = 320 \quad -(11)$$

Now Divide eqn (11) by eqn (1)

$$\therefore \frac{\cancel{F}\sin\theta}{\cancel{F}\cos\theta} = \frac{320}{292.154}$$

$$\tan\theta = 1.0953$$

$$\therefore \theta = \tan^{-1}(1.0953)$$

$$\boxed{\theta = 47.6^\circ}$$

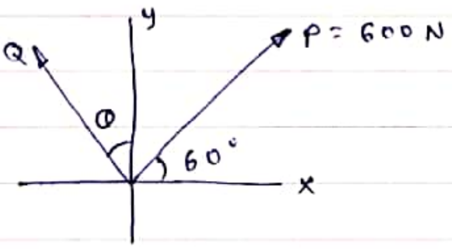
From eqn (1)

$$F\cos\theta = 292.154$$

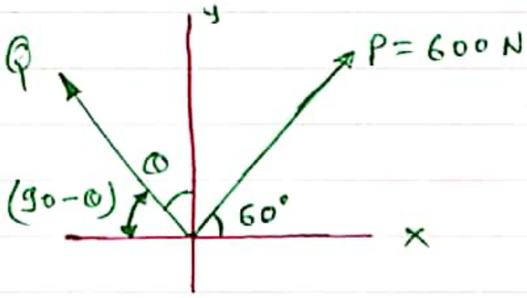
$$F\cos 47.6 = 292.154$$

$$\therefore \boxed{F = 433.31\ N}$$

The resultant of two forces P & Q is 1200 N vertical. Determine force Q & corresponding angle θ for the system of force as shown in figure.



⇒



As Resultant is vertical, (Assuming it be upward)

$\Sigma f_x = 0$ & $\Sigma F_y = R$

Resolving forces Horizontally,

$\therefore \Sigma f_x = P\cos 60 - Q\cos(90-\theta) = 0$

$\therefore 600\cos 60 - Q\sin\theta = 0$

$\therefore 300 = Q\sin\theta$ — ①

Now resolving forces vertically,

$\Sigma f_y = R$

$\therefore P\sin 60 + Q\sin(90-\theta) = R$

$\therefore 600\sin 60 + Q\cos\theta = 1200$

$\therefore 519.61 + Q\cos\theta = 1200$

$\therefore Q\cos\theta = 680.38$ — ⑪

Divide eqⁿ ① by eqⁿ ⑪, we get

$$\frac{Q\sin\theta}{Q\cos\theta} = \frac{300}{680.38}$$

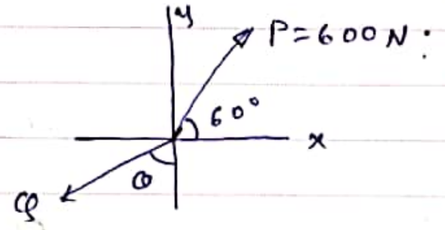
$\therefore \tan\theta = 0.441$

$\therefore \boxed{\theta = 23.79^\circ}$ — substitute in eqⁿ ①

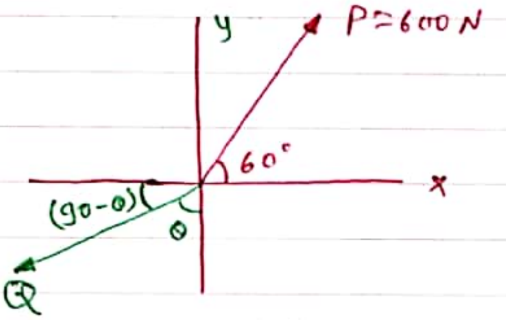
$\therefore Q\sin\theta = 300$

$Q = \frac{300}{\sin 23.79}$ $\therefore \boxed{Q = 743.59\ N}$

The resultant of two forces P & Q is 1200 N horizontally leftward. Determine force Q & corresponding angle θ for the system of force shown in figure.



$\Rightarrow$



As resultant is Horizontal towards left; its y-component will be zero.

$\therefore \Sigma F_x = -R$ & $\Sigma F_y = 0$.

Now Resolving forces Horizontally,

$\therefore \Sigma F_x = -R$

$\therefore P\cos 60 - Q\cos(90-\theta) = -1200$

$\therefore 600\cos 60 - Q\sin\theta = -1200$

$\therefore 300 - Q\sin\theta = -1200$

$\therefore -Q\sin\theta = -1200 - 300 = -1500$

$\therefore Q\sin\theta = 1500$ — (I)

Now resolving forces vertically.

$\therefore \Sigma F_y = 0$

$\therefore 600\sin 60 - Q\sin(90-\theta) = 0$

$\therefore 519.61 - Q\cos\theta = 0$

$\therefore Q\cos\theta = 519.61$ — (II)

from eqn (I) & (II) [Divide eqn (I) by eqn (II)]

$$\therefore \frac{Q\sin\theta}{Q\cos\theta} = \frac{1500}{519.61}$$

$\tan\theta = 2.886$

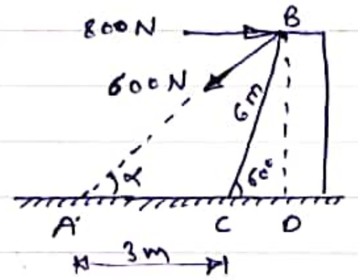
$\therefore \boxed{\theta = 70.89^\circ}$ — put in eqn (I)

$\therefore Q\sin\theta = 1500$

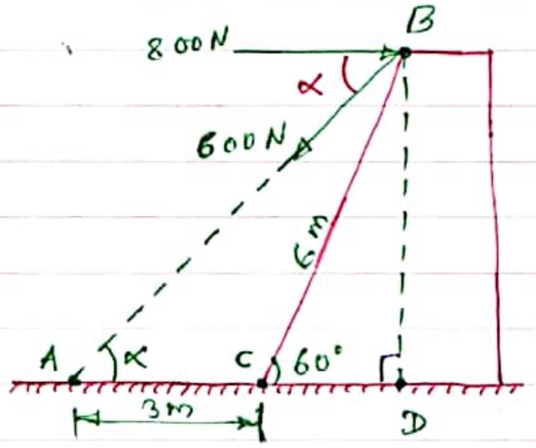
$Q\sin 70.89 = 1500$

$\therefore Q = \frac{1500}{\sin 70.89}$ $\therefore \boxed{Q = 1587.45\ N}$

combine two forces 800 N & 600 N, which act on the fixed dam structure at B into a single equivalent force R, if AC = 3 m, BC = 6 m & angle BCD = 60°. Refer. fig.



⇒ Solution: ⇒



In Δ BDC, Angle BDC = 90°.

$\therefore \sin 60 = \dfrac{BD}{BC} = \dfrac{BD}{6}$

$\therefore BD = 6 \sin 60 = \underline{\underline{5.2\ m}}$

similarly, $\cos 60 = \dfrac{DC}{BC} = \dfrac{DC}{6}$

$\therefore DC = 6 \cos 60 = \underline{\underline{3\ m}}$

Now in Δ BDA, Angle BDA = 90°

$\therefore \sin \alpha = \dfrac{BD}{AD}$

$\therefore \tan \alpha = \dfrac{BD}{AC + DC} = \dfrac{5.2}{3+3} = \dfrac{5.2}{6}$

$\therefore \tan \alpha = 0.866$

$\boxed{\alpha = 40.91°}$

Thus force diagram will be as follows.

Now Resolving forces acting at B in Horizontal dirn, we have,

$$\Sigma F_x = 800 - 600 \cos 40.91$$

$$\Sigma F_x = 346.59 \text{ N.} \quad (\text{toward right})$$

Resolving forces vertically,

$$\Sigma F_y = -600 \sin 40.91$$

$$= -392.92 \text{ N}$$

$$\therefore \Sigma F_y = 392.92 \text{ N} \quad (\text{Downward})$$

Resultant R is given by,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(346.59)^2 + (392.92)^2}$$

$$\boxed{R = 523.94 \text{ N}}$$

Direction of Resultant,

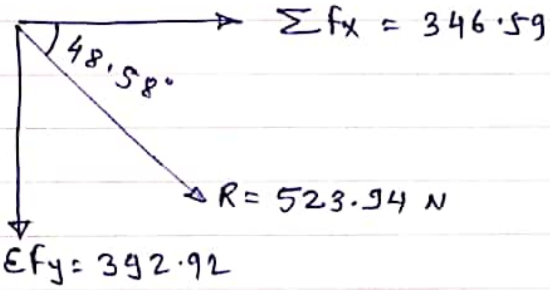
$$\tan \theta = \frac{\Sigma F_y}{\Sigma F_x} = \frac{392.92}{346.59}$$

$$\tan \theta = 1.134$$

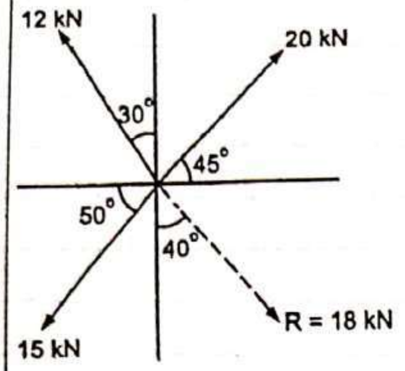
$$\therefore \boxed{\theta = 48.58^\circ}$$

As ΣF_x is positive & ΣF_y is negative, Resultant will lies in 4th quadrant.

$\therefore$ Ans



Q. R = 18 kN is the resultant of four concurrent forces out of which only three are known. Find the fourth force in magnitude and direction.



Here Resultant R is given in figure.

* R = 18 kN
* Angle of Resultant with x-axis = 90 - 40 = 50°

- We know that,
  R_x = X component of Resultant = $\Sigma F_x = 18 \cos 50°$
  R_y = Y component of Resultant = $\Sigma F_y = -18 \sin 50°$

- Assume that fourth force is F which makes θ angle with x-axis

∴ X component of force F will be, = $F \cos\theta$
Y component of force F will be = $F \sin\theta$

- ∴ Resolving forces along x & y dirn,

* ∴ $\Sigma F_x = 20 \cos 45 - 12 \cos 60 - 15 \cos 50° + F \cos\theta$

∴ $18 \cos 50 = -1.49 + F \cos\theta$.

∴ $F \cos\theta = 13.06$ -------- (1)

* ∴ $\Sigma F_y = 20 \sin 45 + 12 \sin 60 - 15 \sin 50 + F \sin\theta$

$-18 \sin 50 = 13.04 + F \sin\theta$

∴ $F \sin\theta = -26.83$ ------ (2)

From eqn (1) & (11)

* As x component of force F is $F \cos\theta$ which is positive & y component of force F is $F \sin\theta$ which is negative it means force F is in fourth quadrant.

Divide eqn (2) by eqn (1)

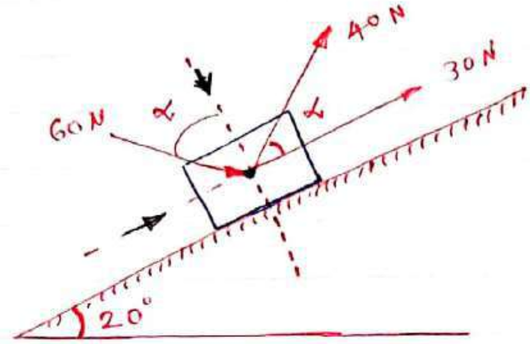
∴ $\frac{\cancel{F} \sin\theta}{\cancel{F} \cos\theta} = \frac{-26.83}{13.06}$ ∴ $\tan\theta = (-2.054)$

∴ $\theta = \tan^{-1}(-2.054) = -64.04°$

Ans $\boxed{\theta = 64.04°}$ - Negative sign indicates 4th quadrant. (2)

∴ From eqn (1), ∴ $F \cos 64.04 = 13.06$ ∴ $\boxed{F = 29.83 N}$

(n) Determine the required value of α if the resultant of three forces is to be parallel to the inclined surface. Find the corresponding magnitude of the Resultant.



let us select x axis parallel to the inclined plane. As resultant is acting parallel to the plane, lets assume that it is acting along positive x dirn i.e. acting up the plane.

As Resultant is acting along +ve x axis, its y component will be zero. $\Sigma F_y = 0$.

$\therefore \Sigma F_x = R$

$30 + 40\cos\alpha + 60\sin\alpha = R \quad -①$

Also,

$\therefore \Sigma F_y = 0$

$\therefore 40\sin\alpha - 60\cos\alpha = 0$

$\therefore 40\sin\alpha = 60\cos\alpha$

$\therefore \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{60}{40}$

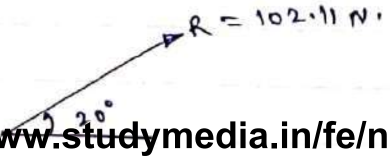
$\therefore \tan\alpha = \dfrac{6}{4}$

$\therefore \boxed{\alpha = 56.3^\circ}$

$\therefore R = 30 + 40\cos\alpha + 60\sin\alpha$

$= 30 + 40\cos 56.3 + 60\sin 56.3$

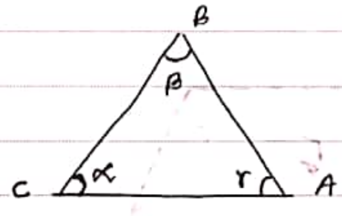
$\boxed{R = 102.11 \text{ N.}}$



② Non-Perpendicular / Non-Orthogonal Resolution :-

When a force is resolved or split into two directions which are not perpendicular to each other, then the reso--lution is called as Non-perpendicular / Non-orthogonal.

Sine Rule



It states that, the ratio of two sides of an oblique triangle is in same proportion as that of the ratio of sines of their opposite angle.

OR

The Ratio of two sides of Δle will be equal to the ratio of sine of their opposite angle.

$\therefore \dfrac{AB}{BC} = \dfrac{\sin\alpha}{\sin\gamma}$ —(i) , $\dfrac{BC}{AC} = \dfrac{\sin\gamma}{\sin\beta}$ —(ii)

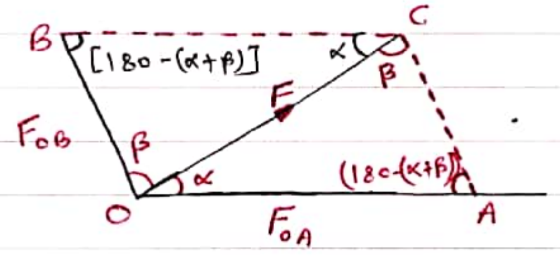
$\dfrac{AC}{AB} = \dfrac{\sin\beta}{\sin\alpha}$ —(iii)

Thus we can write

$$\frac{AB}{\sin\alpha} = \frac{BC}{\sin\gamma} = \frac{AC}{\sin\beta}$$ — sine rule.

Thus to resolve the force into two non-perpendicular components,

construct the parallalogram by keeping original given force along diagonal & two components along two sides of parallelogram passing through point of application of force.

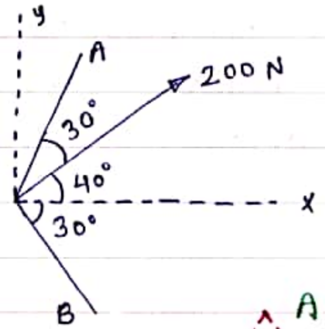


In Δ OAC, By sine rule,

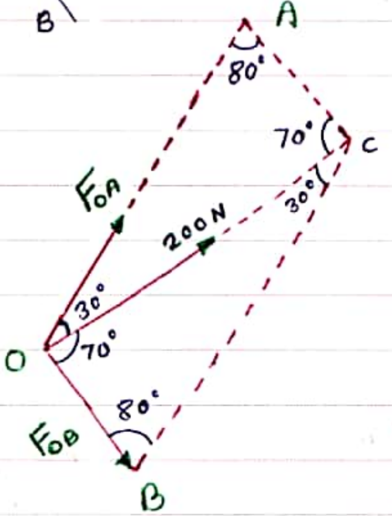
$$\frac{F_{OB}}{\sin\alpha} = \frac{F_{OA}}{\sin\beta} = \frac{F}{\sin[180-(\alpha+\beta)]}$$

Using above equation, two components i.e. F_{OB} & F_{OA} can be calculated.

Resolve 200 N force into two components along A & B directions. Refer the given figure.



⇒ Solution :-



As angle betn OA & OB is 100°, resolution is non perpendicular. Thus in Δ OBC, by sine rule,

$$\frac{200}{\sin 80} = \frac{F_{OB}}{\sin 30} = \frac{F_{OA}}{\sin 70°}$$

$$\therefore \quad F_{OB} = \frac{200}{\sin 80} \times \sin 30.$$

$$\therefore \quad \boxed{F_{OB} = 190.84 \text{ N}}$$

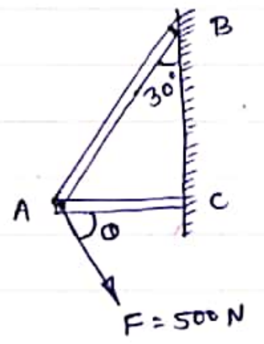
Simillarly,

$$\therefore \quad \frac{200}{\sin 80} = \frac{F_{OA}}{\sin 70}$$

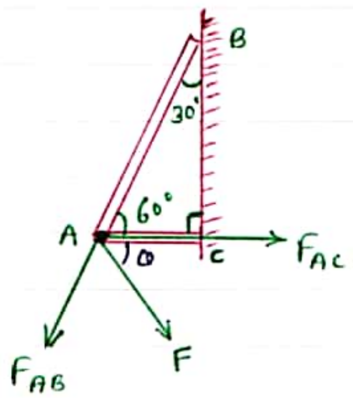
$$\therefore \quad F_{OA} = \frac{200 \sin 70}{\sin 80}$$

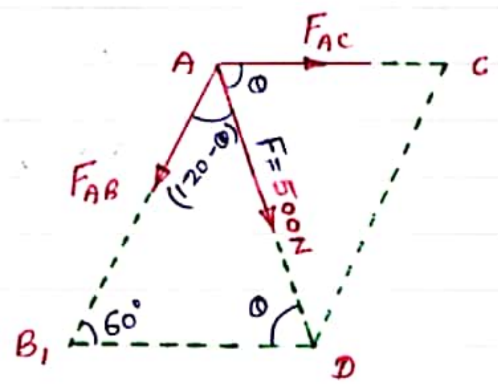
$$\therefore \quad \boxed{F_{OA} = 101.5 \text{ N}}$$

The force F acting on the frame has a magnitude of 500 N & it is to be resolved into two components acting along strut AB & AC. Determine the angle θ so that the component F_{AC} is directed from A towards C & has magnitude of 400 N. Refer the given figure.



⇒ Solution.





Because AC & AB are not perpendicular, lets use Non-perpendicular resolution.

Thus in ΔADB_1, by sine rule,

$$\frac{F_{AB}}{\sin\theta} = \frac{F_{AC}}{\sin(120-\theta)} = \frac{F}{\sin 60}$$

As $F_{AC} = 400$ N.

$$\therefore \frac{400}{\sin(120-\theta)} = \frac{500}{\sin 60}$$

$$\therefore \frac{400 \sin 60}{500} = \sin(120-\theta)$$

$$\therefore 0.693 = \sin(120-\theta).$$

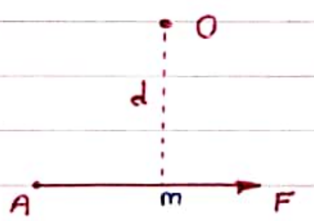
$$\therefore 120 - \theta = 43.87^\circ$$

$$\therefore \boxed{\theta = 76.13^\circ}$$ .... Ans.

* Moment :-

The turning effect produced by a force on the body, on which it acts is called as Moment.

Moment of force about any point is the product of magnitude of force and perpendicular distance of force from the point about which moment is to be taken.



Clockwise ↻ → +ve

Anticlockwise ↺ → −ve.

Let force F is applied at point A as shown. & O is any point about which we want to take moment.

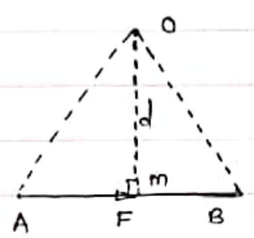
Thus from point O, Draw OM line perpendicular to the line of action of force.

OM = d = Perpendicular distance betn Force & point O.

$\therefore$ Moment = M = $\underline{F \times d}$.

- Graphical representation of Moment :-

- Select suitable scale to locate the force graphically.
- Draw a line parallel to line of action of force. length of this line can be calculated from selected scale.
- Length of a line will be equal to magnitude of force.
- In the figure, Force F is repersented by line AB.
- Now let O is the moment centre. &



OM = d = perpendicular distance

$\therefore$ by defination, $M_o = F \times d$.

Now join OA & OB. Then, In Δ OAB

Area $(\Delta OAB) = \frac{1}{2} \times AB \times OM$.

2 (Area of ΔOAB) $= F \times d. = M_o$

Thus the moment of any force about any point is numerically equal to twice Area of triangle in which base represents force & height represents the perpendicular distance.

* Varignon's Theorem [Law of Moments].

It states that if number of forces are acting simultaniously on a body, the algebric sum of moments of all the forces about any point is equal to moment of their resultant about same point.

Mathematically,

$$\Sigma(F \times d) = (R \times x)$$

where F = All forces acting on a body

d = $\perp^{lar}$ distance.

R = Resultant of all forces.

x = $\perp^{lar}$ distance of Resultant force about a point where moment is taken.

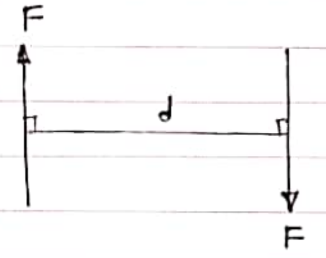
Use :- This theorm is useful to calculate or find the position or location of Resultant of non-concurrent force system.

* Couple *

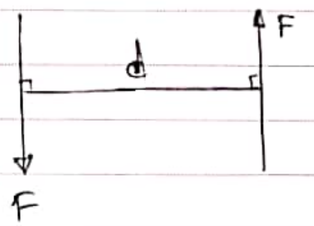
A pair of two equal and opposite (unlike) parallel forces (of same magnitude) is known as couple.

Properties of couple :-

1) Two unlike parallel, non-colinear forces of same magnitude will form couple.
2) Resultant of couple is always zero.
3) Moment of couple is product of one of the force & lever arm of couple. $\therefore M = F \times d$.
   lever arm of couple = $\perp^{lar}$ distance betⁿ couple forces.
4) Couple can not be balanced by single force.
5) Couple can be balanced ~~by~~ only by another couple of opposite nature.
6) moment of couple is independant of moment centre.
   i.e. Ref. point is not required to take moment of couple.

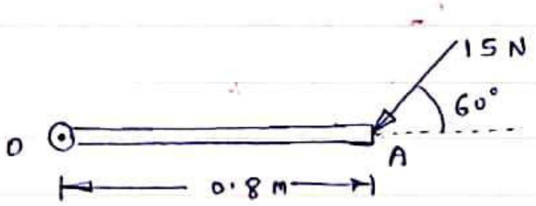
① 

Clockwise couple.

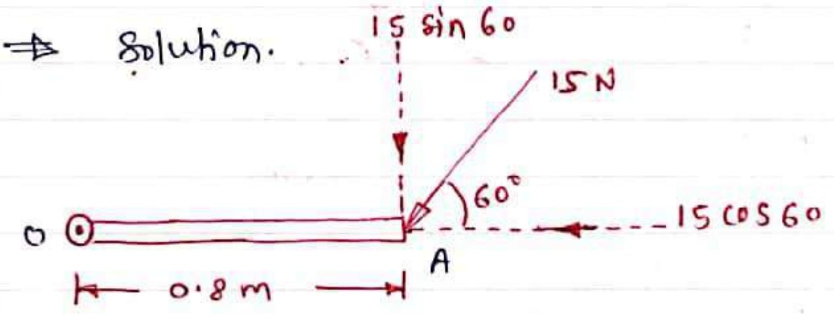
⑪ 

Anticlockwise couple.

A force of 15 N is applied at an angle of 60° to the edge of door as shown in figure. Find the moment of this force.



Solution.



Resolving force at point B & then taking the moment about point O,

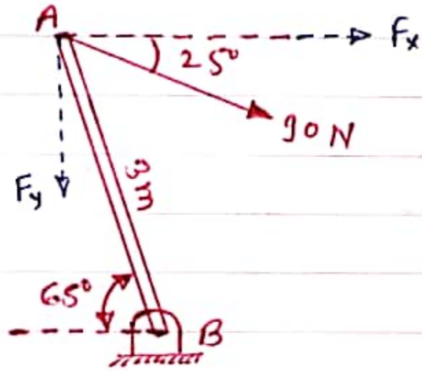
$M_o = (15 \cos 60^\circ \times 0) + (15 \sin 60^\circ \times 0.8)$

$= 0 + 10.39$

$\boxed{M_o = 10.39 \text{ N-m}}$.

A 90 N force is applied to the control rod AB. Determine moment of this force about point B.

⇒ Solution :-



As we don't know perpendicular distance betⁿ force & point 'B'; let us resolve the force into two components.

∴ X-component of force

$F_x = 90 \cos 25 = 81.567$ N.

Y-component of force,

$F_y = 90 \sin 25 = 38.036$

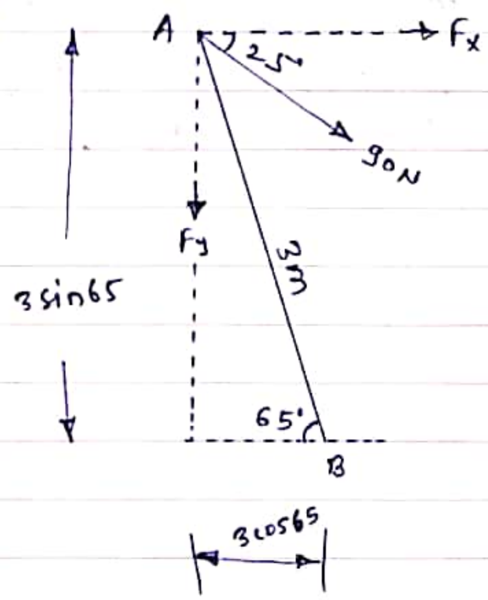
Now taking moments about point B.

$M_B = [F_x \times (3 \sin 65)] - [F_y \times (3 \cos 65)]$

$M_B = (81.56 \times 2.72) - (38.036 \times 1.27)$

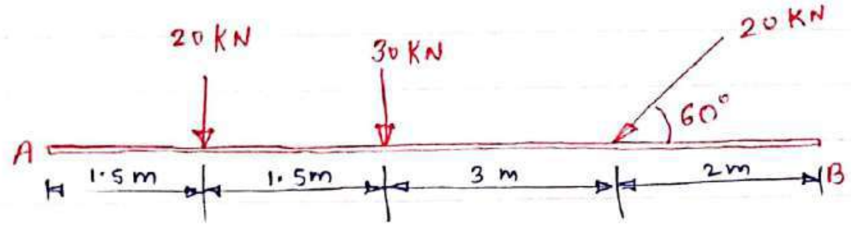
$= 221.843 - 48.31$

$\boxed{M_B = 173.53 \text{ N·m}}$



Examples on Non concurrent forces & use of Varignon's theorem

① Determine the resultant of the system of forces acting on a beam as shown in Figure.



⇒

Above force system is Non concurrent force system.

Resolving the forces Horizontally,

$\Sigma f_x = -20 \cos 60 = -10$ KN $= 10$ KN (toward left)

Resolving forces vertically,

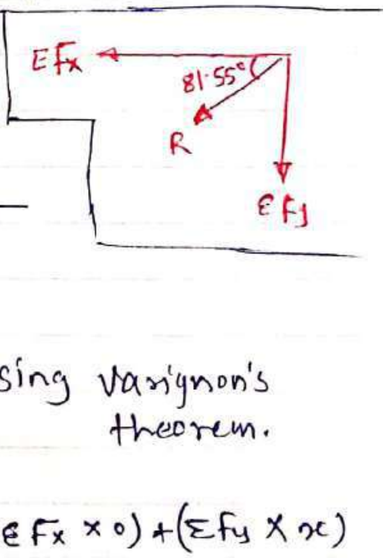
$\Sigma F_y = -20 - 30 - 20 \sin 60$

$= -67.32$ KN.

$\Sigma F_y = 67.32$ KN (downwards)

Resultant, $R = \sqrt{(\Sigma F_x)^2 + (\Sigma f_y)^2} = \sqrt{10^2 + 67.32^2}$

$\therefore \boxed{R = 68.06 \text{ KN}}$



Direction of Resultant, $\tan\alpha = \dfrac{\Sigma f_y}{\Sigma f_x} = \dfrac{67.32}{10.00}$

$\therefore \boxed{\alpha = 81.55°}$

Now taking moment about point A & using Varignon's theorem.

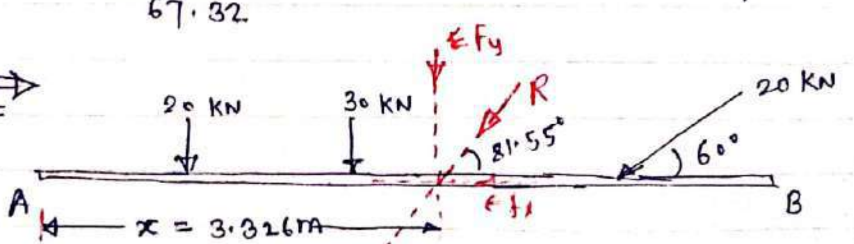
ΣM_A = Moment of Resultant @ A.

$\therefore (20 \times 1.5) + (30 \times 3) + (20 \sin 60 \times 6) = (\Sigma F_x \times 0) + (\Sigma F_y \times x)$

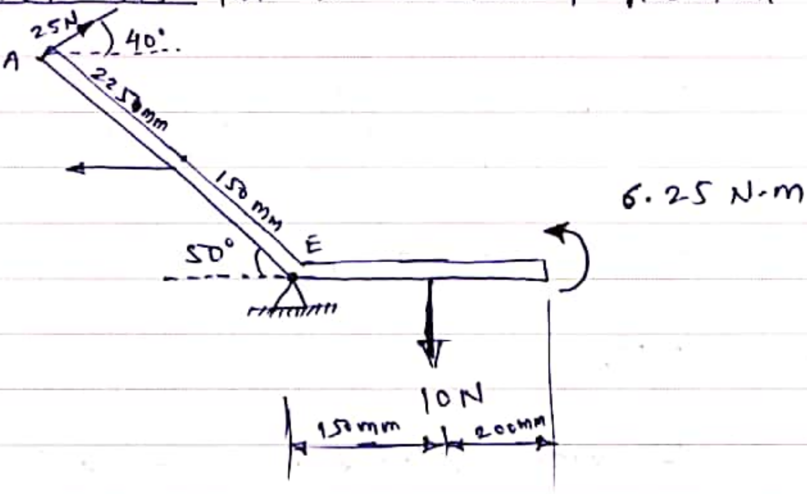
$\therefore 223.92 = 67.32 \times x$

$\therefore x = \dfrac{223.92}{67.32} = 3.326$ m from point A.

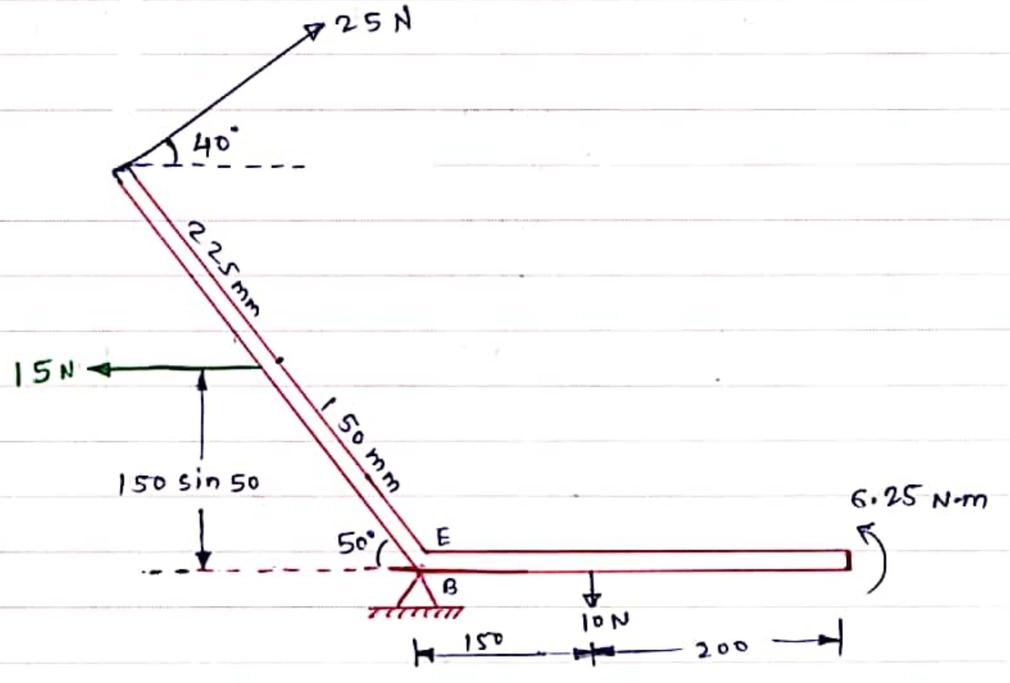
$\therefore$ **Ans ⇒**



For a given force system, find the resultant in magnitude & direction. Also find the location of Resultant.



⇒ Solution



Resolving forces horizontally.

$\Sigma f_x = -15 + 25 \cos 40 = 4.15 \ N$ (toward right)

Resolving forces vertically,

$\Sigma f_y = 25 \sin 40 - 10$

$= 6.07 \ N$ (upward).

$\therefore R = \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2}$

$\therefore R = \sqrt{4.15^2 + 6.07^2}$ $\therefore \boxed{R = 7.35 N}$

$\tan\alpha = \frac{\Sigma f_y}{\Sigma f_x} = \frac{6.07}{4.15}$

$\therefore$ $\boxed{\alpha = 55.64°}$ - Angle made by resultant

To find exact position of R,

let x is the perpendicular distance betn R & point B.

using varignons theorem at point B.

$\therefore R \times x = \Sigma M_B$

$\therefore \quad = (10 \times 150) - (15 \times 150 \sin 50) + (25 \times 375) - (6.25 \times 1000)$

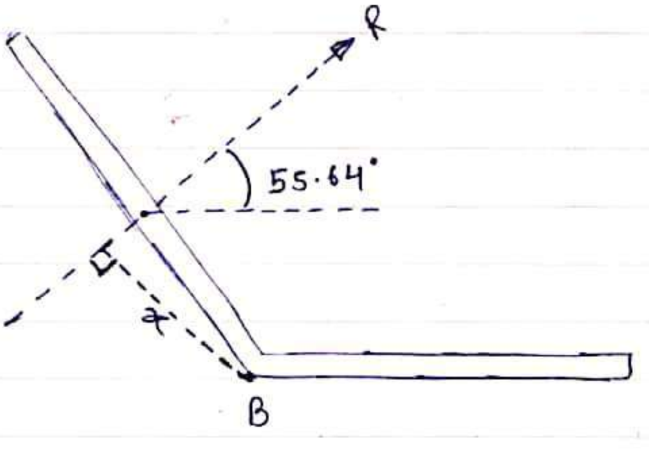
$\therefore R \times x = 1500 - 1723.6 + 9375 - 6250$

$R \times x = 2901.4$ N·mm (clockwise).

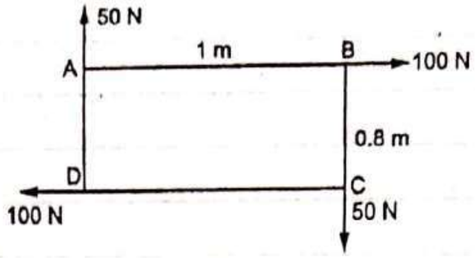
$\therefore x = \frac{2901.4}{R}$

$= \frac{2901.4}{7.35}$

$\boxed{x = 394.75 \text{ mm}}$ from Point B.



Find resultant moment of two couples for the loading as shown in Fig



Moment of 50 N couple, is clockwise,

$= 50 \times 1$

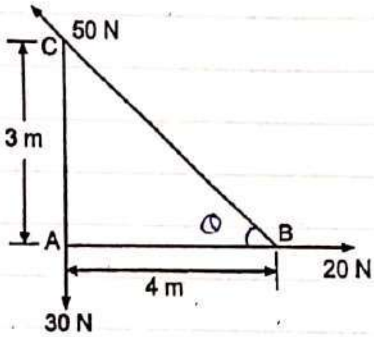
$= 50$ N·m

Moment of 100 N couple is also clock wise,

$= 100 \times 0.8 = 80$ N·m.

Resultant moment $= 50 + 80 = 130$ N·m

Find the Resultant and it's point of application on y- axis for the force system acting on Triangular plate as shown in Fig.



$\theta = \tan^{-1}\left(\frac{3}{4}\right)$

$\theta = 36.86°$ with Horizontal

Resolving forces in x & y dirn

$\therefore \Sigma F_x = 20 - 50\cos\theta$

$= 20 - 50\cos 36.86$

$\Sigma F_x = -20$ N

$= 20$ N (←)

$\Sigma F_y = -30 + 50\sin 36.86$

$= 0.00007$

$\simeq 0$

$\therefore R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \underset{\leftarrow}{20 \text{ N.}}$ toward left (as $\Sigma F_y = 0$)

$\therefore$ let us apply varignons theorem at point B.

$\therefore \Sigma M_B = R \cdot x$.

$\therefore -(30 \times 4) = R x$.

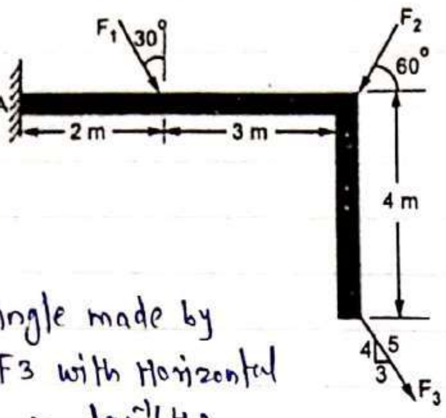
$\therefore R \cdot x = -120$ N·m. $= 120$ N·m

As moment of Resultant is negative i.e. it is anticlockwise.

$\therefore -R \cdot x = -120 \quad \therefore x = \frac{-120}{-20} = \underline{\underline{6 \text{ m}}}$

$\therefore x = 6$ m from point B and it will be above point B to create anticlockwise moment.

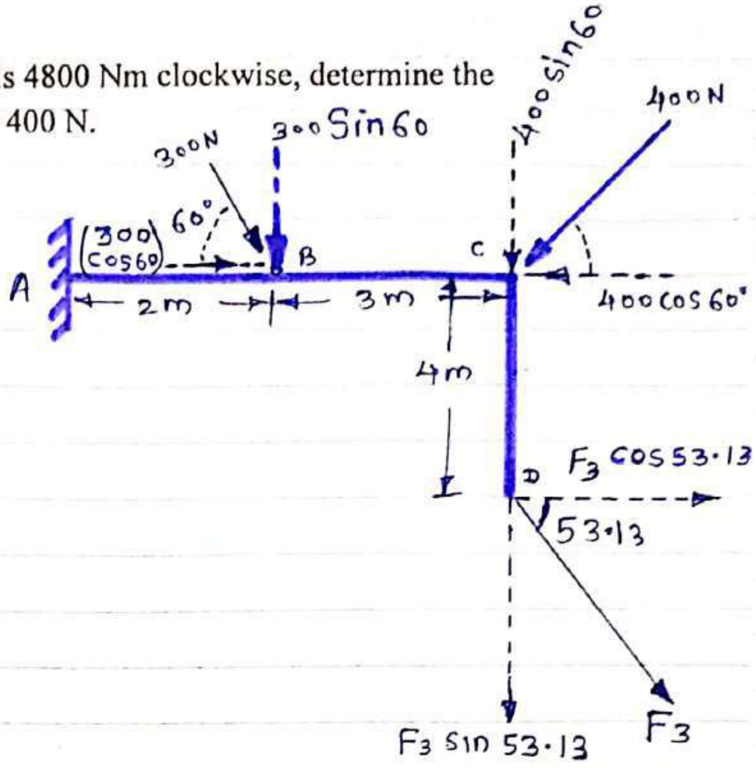
If the resultant moment about point A is 4800 Nm clockwise, determine the magnitude of F_3 if $F_1 = 300$ N and $F_2 = 400$ N.



Angle made by F3 with Horizontal

$= \tan^{-1}\left(\frac{4}{3}\right)$

$= \underline{53.13^\circ}$



Given: Resultant Moment = 4800 N·m ↻

let us apply varignon's theorem at point A,

$\therefore \Sigma M_A = (R \cdot x)$

$\therefore [(300 \quad 60) \times 2] + [(400 \sin 60) \times 5] + [(F_3 \sin 53.13) \times 5] - [(F_3 \cos 53.13) \times 4] = 4800$

$\therefore 519.62 + 1732.05 + 3.99 F_3 - 2.4 F_3 = 4800$

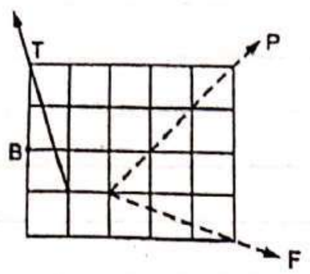
$\therefore 2251.67 + 1.59 F_3 = 4800$

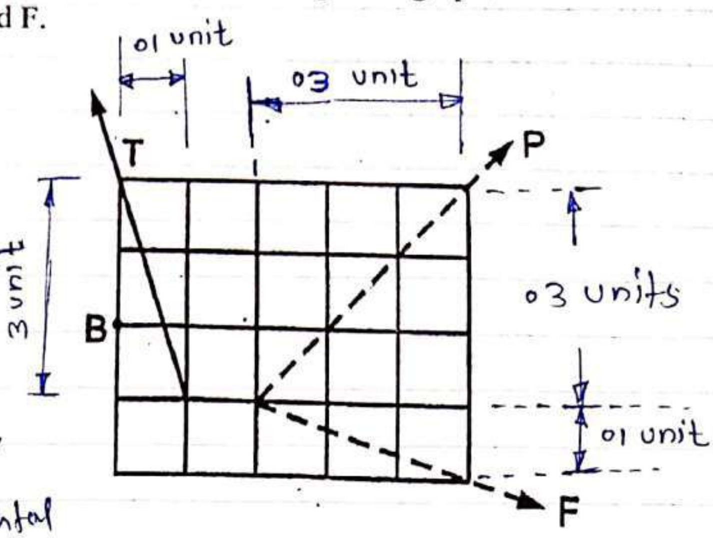
$\therefore 1.59 F_3 = 2548.33$

$\therefore F_3 = \frac{2548.33}{1.59}$ OR $F_3 = \frac{2548.33}{1.6}$

$\therefore \boxed{F_3 = 1602.72 \text{ N}}$ $\boxed{F_3 = 1592.7 \text{ N}}$

The three forces shown in Fig. create a vertical resultant acting through point B. If P = 361 N, compute the values of T and F.





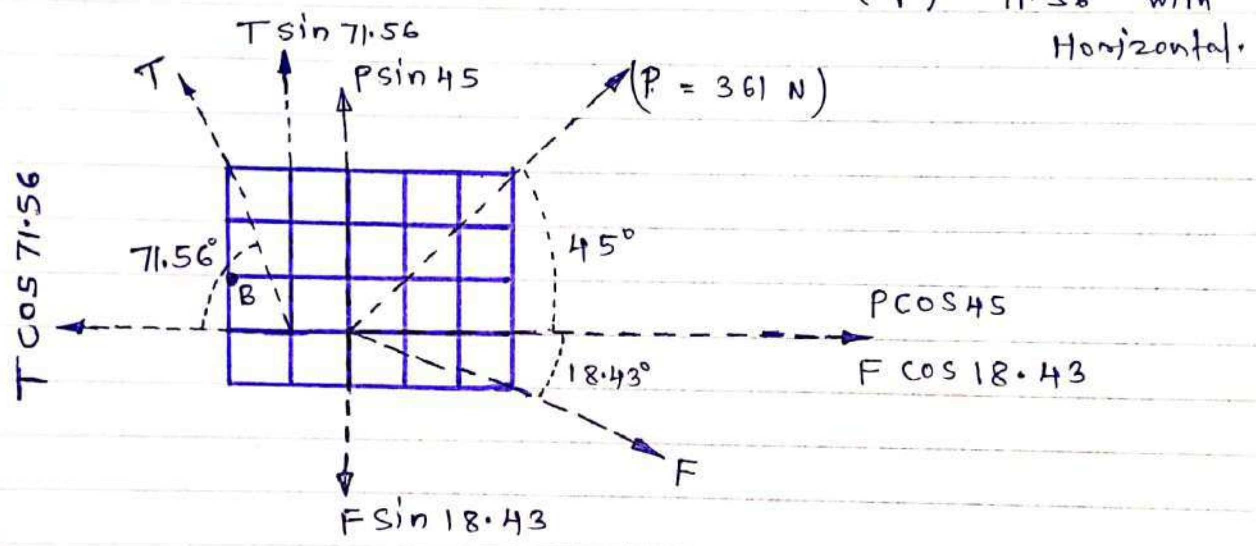
From given figure,

1] Angle made by Force P is,

$= \tan^{-1}\left(\frac{3}{3}\right) = 45^\circ$ with Horizontal

2] Angle made by Force F is, $= \tan^{-1}\left(\frac{1}{3}\right) = 18.43^\circ$ with Horizontal.

3) Angle made by Force T is $= \tan^{-1}\left(\frac{3}{1}\right) = 71.56^\circ$ with Horizontal.



Resolving forces along x- & y dirn,
As Resultant is vertical, let it is upward, then
$\Sigma F_x = 0$ & $\Sigma F_y = R$.

$\therefore \Sigma F_x = P\cos 45 + F\cos 18.43 - T\cos 71.56$

$0 = 361\cos 45 + F\cos 18.43 - T\cos 71.56$

$0 = 255.26 + 0.95F - 0.32T$

$\therefore 0.95F - 0.32T = -255.26$ ------- (1)

$\therefore \Sigma F_y = P\sin 45 - F\sin 18.43 + T\sin 71.56$

$R = 361\sin 45 - F\sin 18.43 + T\sin 71.56$

$R = 255.26 - 0.32F + 0.95T$ ---- (2)

Now, As it is given that Resultant acts at point B, let us apply Varignon's theorem at point B.

$\therefore \ \Sigma M_B = R \times x.$

But As R is acting at B point, its perpendicular distance from point B itself will be zero.

$\therefore \ \Sigma M_B = R \times x$

$\therefore \ -(P\cos 45 \times 1) - (P\sin 45 \times 2) + (T\cos 71.56 \times 1) - (T\sin 71.56 \times 1) - (F\cos 18.43 \times 1) + (F\sin 18.43 \times 2) = R \times 0$

As $P = 361$ N

$\therefore \ -255.26 - (255.26 \times 2) + 0.32T - 0.95T - 0.95F + (0.32F \times 2) = 0$

$\therefore \ -765.78 + 0.32T - 0.95T - 0.95F + 0.64F = 0$

$\therefore \ -765.78 - 0.63T - 0.31F = 0$

$\therefore \ -0.31F - 0.63T = 765.78 \quad --- (3)$

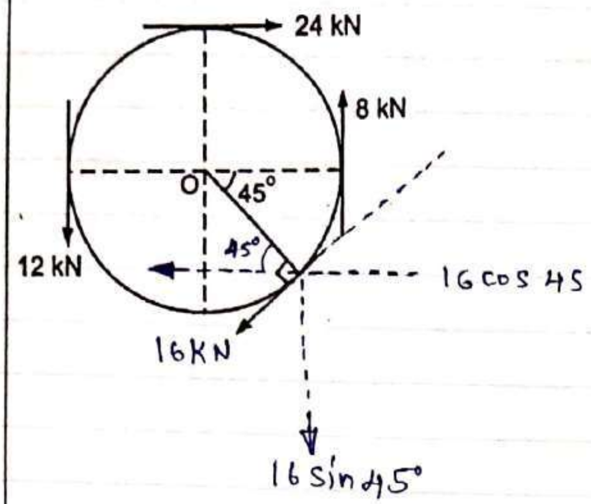
Solving eqn (1) and (3) simultaneously.

$$\left.\begin{aligned} 0.95F - 0.32T &= -255.26 \\ -0.31F - 0.63T &= 765.78 \end{aligned}\right\} \text{solving these eqn.}$$

$\boxed{F = -581.716 \text{ N}}$ &

$\boxed{T = -929.282 \text{ N}}$

Determine the resultant of four forces tangent to the circle of radius 1.5 m as shown. Determine its location w.r.t. 'O'.



Resolving forces in x and y direction,

$\therefore \Sigma F_x = 24 - 16\cos 45$
$= 12.69$ KN (→)

$\therefore \Sigma F_y = -12 + 8 - 16\sin 45$
$= -15.31$ kN
$= 15.31$ KN (↓)

Resultant R,

$R = \sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2}$

$R = \sqrt{12.69^2 + (-15.31)^2}$

$\boxed{R = 19.88 \text{ kN}}$

Direction of Resultant,

$\alpha = \tan^{-1}\left(\frac{\Sigma Fy}{\Sigma Fx}\right)$

$\alpha = \tan^{-1}\left(\frac{15.31}{12.69}\right)$

$\boxed{\alpha = 50.35°}$ in 4th quadrant.

To know exact location of Resultant, let us apply varignon's theorem at point O.

$\Sigma M_o = R \cdot x.$

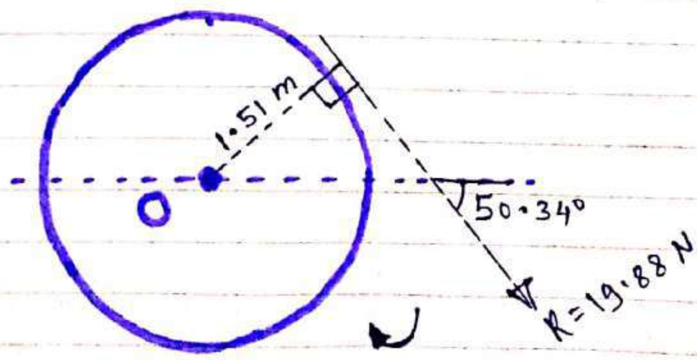
$-(12 \times 1.5) + (24 \times 1.5) - (8 \times 1.5) + (16 \times 1.5) = (19.88 \times x)$

$30 = (R \cdot x)$

As product of R·x is positive it means moment of Resultant is positive. To create positive clockwise moment of Resultant it must be acting on Right side of O (in 4th quadrant).

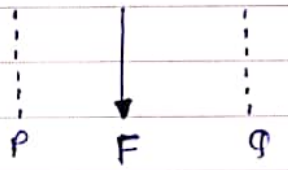
$\therefore \quad 30 = 19.88 \times x.$

$x = 1.51$ m from O on the right hand side.



③ Resolution into two Parallel Components.

Case (a) Two parallel components lies on either sides of force



For this case,

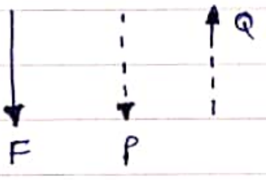
- both components must have same direction as that of the original force.
- let P & Q are the two parallel component of force F, then

$$P + Q = F$$

- Using varignon's theorem we can find the magnitude of other force.

Case (b)

when Two components lies on one side of resultant



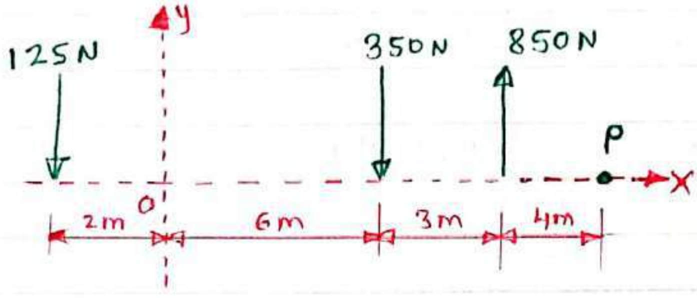
- For this Both components must have opposite direction.
- Component near the force will have same direction as that of force (F).
- let P & Q are the components, then

$$-P + Q = F$$

- Using Varignon's theorem we can find magnitude of other force.

Examples on Parallel forces & use of Varignon's Theorem

① Replace the force system by single force resultant & specify its point of application measured along x-axis from point P. Refer the fig.



As forces are parallel & vertical, thus their x-components will be zero.

$\therefore$ To find resultant, $\Sigma F_x = 0$

$\therefore \Sigma F_y = -125 - 350 + 850$
$= 375$ N (upward)

$\therefore R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{0 + 375^2}$

$\boxed{R = 375 \text{ N.}}$

As All forces are vertical, Resultant will also be vertical.
i.e. $\alpha = 90°$

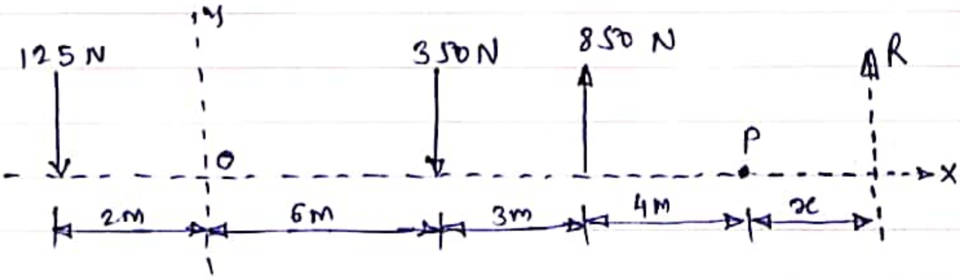
$\tan\alpha = \frac{\Sigma F_y}{\Sigma F_x} = \frac{375}{0}$

$\therefore \alpha = \tan^{-1}(\infty)$
$\boxed{\alpha = 90°}$

Thus R is vertical acting upward.
Now using varignon's theorem about point P,

$R \cdot x = (850 \times 4) - (350 \times 7) - (125 \times 15)$
$(R \cdot x) = -925$ N·m.

* As moment of Resultant is −ve, it is anticlockwise. To produce anticlockwise moment about P, Resultant must be placed to the right of P as shown in fig.



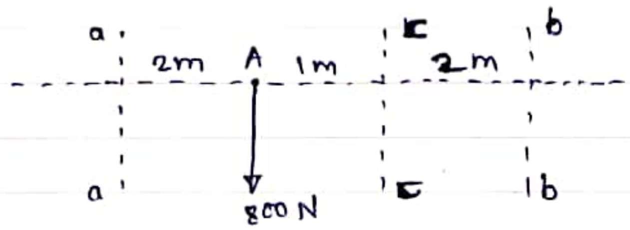
$\therefore \quad R \times x = -925$ N·m.

$-(375 \times x) = -925$ N·m.

$x = \dfrac{-925}{-375}$

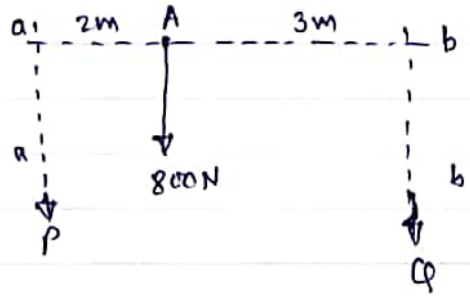
$\boxed{x = 2.467 \text{ m}}$ from point p toward right.

Resolve 800 N force at A into two parallel compo-
-nents P & Q acting respectively along ① a-a & b-b.
② b-b & c-c



Case ① Resolution along a-a & b-b.

Here force is betn two components, thus both components must have same direction as that of force.



$\therefore \quad R = \Sigma F_y \qquad \therefore \quad -800 = -P - Q$

$800 = P + Q \quad -①$

Now taking moment at b-b; & using varignon theorem.

$-(800 \times 3) = -(P \times 5)$

$\therefore \quad P = \dfrac{800 \times 3}{5}$

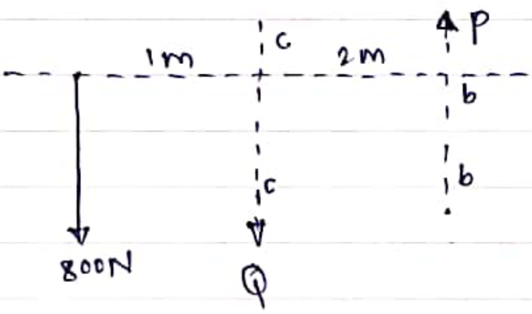
$\therefore \boxed{P = 480\,N}$ downward

$\therefore \quad P + Q = 800$

$\therefore \quad Q = 800 - 480$

$\boxed{Q = 320\,N}$ downward.

Case - (ii) Resolution along b-b & c-c.



Now two component lie on one side of resultant force (800 N). Thus they must have opposite direction. & the component near the 800 N will have same direction as that of 800 N force.

$\therefore \quad R = \Sigma F_y$

$\quad = -Q + P$

$\therefore -800 = -Q + P \quad$ (ii)

Now taking moment at b-b & using varignon's theorem.

$-(800 \times 3) = -(Q \times 2)$

$\therefore \dfrac{-2400}{-2} = Q.$

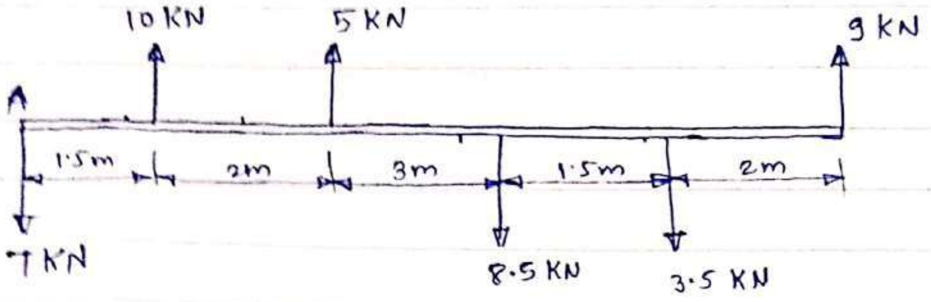
$\therefore \boxed{Q = 1200}\,N$ - downward.

$\therefore \quad -800 = -Q + P$

$\therefore \quad -800 = -1200 + P$

$\therefore \boxed{P = 400\,N}$ - upward

Find the resultant of following force system and also locate it from point A.



As all forces are vertical, $\Sigma F_x = 0$

Resolving forces vertically,

$$\Sigma F_y = -7 + 10 + 5 - 8.5 - 3.5 + 9$$

$$\Sigma F_y = 5 \text{ KN} \quad \text{(upward)}$$

Resultant, $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{(\Sigma F_y)^2}$ ...... As $\Sigma F_x = 0$.

$\therefore R = \Sigma F_y$

$\therefore R = 5$ KN (upward)

& $\boxed{\alpha = 90^\circ}$

Now using varignon's theorem at point A

$\therefore \Sigma M_A = R \cdot x$

$\therefore -(10 \times 1.5) - (5 \times 3.5) + (8.5 \times 6.5) + (3.5 \times 8) - (9 \times 10) = R \cdot x$

$\therefore -39.25$ KN·m $= R \cdot x$.

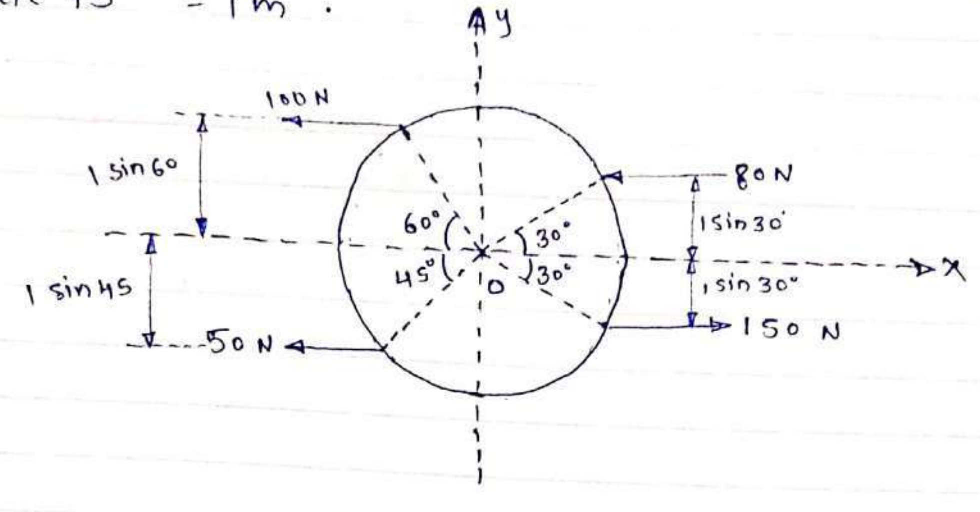
$\therefore R \cdot x = 39.25$ KN·m (Anticlockwise)

$\therefore 5 \times x = 39.25$

$x = \frac{39.25}{5} = 7.85$ m from point A.

$\boxed{x = 7.85 \text{ m from A}}$

(4) Determine the resultant of the parallel force system as shown in figure & locate it w.r.t. 'O'. Radius of circle is = 1 m.



As all forces are Horizontal, $\Sigma F_y = 0$.

∴ Resolving forces Horizontally,

$$\Sigma F_x = -100 - 50 - 80 + 150$$

$$\Sigma F_x = -80 \text{ N}$$

$$\Sigma F_x = 80 \text{ N} \quad (\text{towards left})$$

As $\Sigma F_y = 0$ thus,

$$R = \Sigma F_x$$

∴ Resultant = $R = 80$ N (toward left)

To find exact location of R, use varignon's theorem at point 'O'.

$$\therefore \quad \Sigma M_o = R \cdot x.$$

$$\therefore -(100 \times \sin 60) + (50 \times \sin 45) - (150 \times \sin 30) - (80 \times \sin 30) = R \cdot x.$$

$$\therefore \quad -166.25 \text{ N·m} \circlearrowleft \text{ (Anticlockwise)} = R \cdot x.$$

$$\therefore \quad R \cdot x = 166.25$$

$$\therefore \quad 80 \times x = 166.25$$

$$x = \frac{166.25}{80}$$

$$\boxed{x = 2.08 \text{ m}}$$