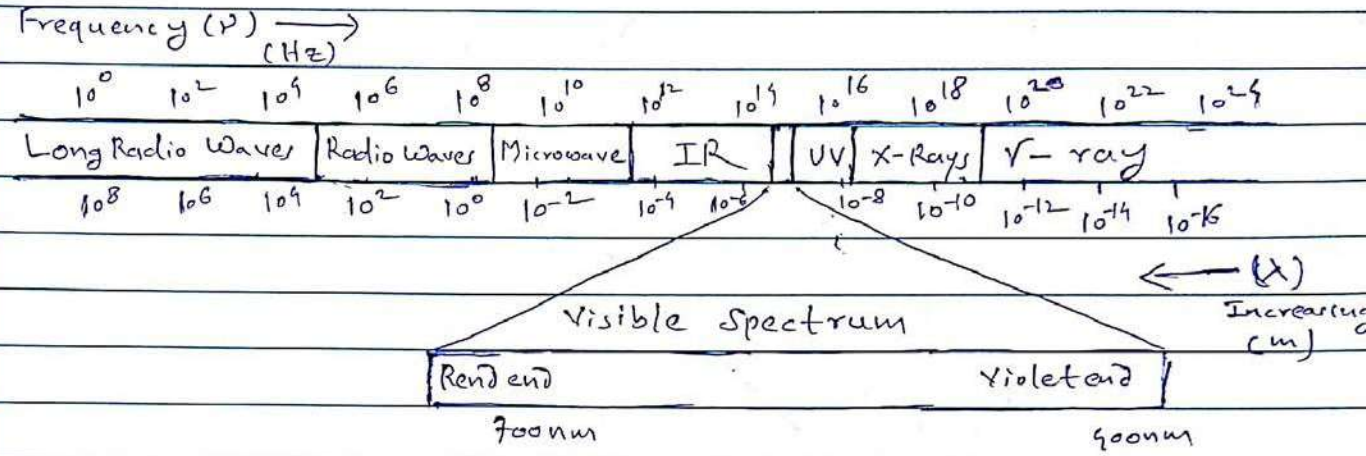
Interference

* Pre-requisites

(1) Electromagnetic Spectrum and Light:



Electromagnetic spectrum is an arrangement of the various electromagnetic waves in a continuous sequence of frequencies and wavelengths. At one end, it starts with the gigantic radio waves having wavelengths of the order of few hundred kilometers to the at the other end with the γ-rays of shortest wavelengths of the order of 10^{-12}m. In this range, it forms the various regions, known as radio waves microwaves, infrared (IR), visible light, ultraviolet (UV), x-rays and γ-rays including cosmic rays.

Among all these regions of electromagnetic spectrum, the shortest region, ranging from 0.4 um to 0.7 um wavelength is known as visible portion of the spectrum i.e; light. Therefor light is that part of an electromagnetic spectrum, when it falls on the retina of eye, it produces the sensation of vision.

Interference:

When two or more waves having constantant phase difference (coherent), same frequency (monochromatic) and nearly equal amplitudes are superimposes then the net intensity of these waves gets redistributed in alternate regions of maximum and minimum intensity. This intensity redistribution is known as interference.

Conditions for Sustained/stable Interference:

1. The sources of light must emit coherent waves.
2. The sources of light must emit monochromatic waves.
3. The interfering waves must have nearly same amplitudes.
4. The waves coming from the sources must travel along the same path.
5. The separation between the sources of waves must be as small as possible.
6. The sources of waves must be narrow sources, as broad sources are equivalent to many fine sources.

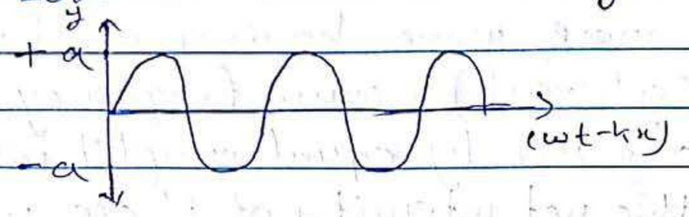
Classification:

1. Constructive Interference

When the crest of one wave overlaps with the crest of other and the trough of one overlaps with the trough of others, then the interference obtained is known as Constructive Interference.

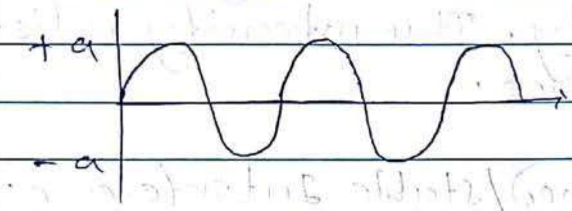
This requires, all the interfering waves must be in phase.

Let's consider a wave $y = a\sin(\omega t - kx)$

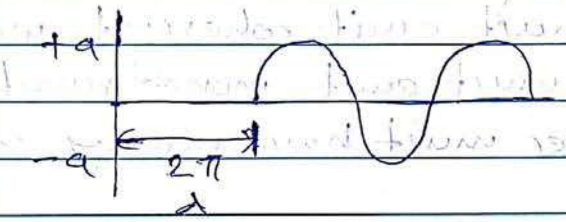


Other waves With respect to above wave

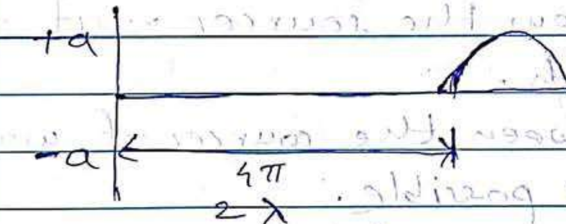
Phase Difference Path Diffrence



$\phi = 0$ $\Delta = 0$



$\phi = 2\pi$ $\Delta = \lambda$



$\phi = 4\pi$ $\Delta = 2\cdot\lambda$

$\phi = 2n\pi$ $\Delta = n\lambda$.

where, $n = 0, 1, 2, 3, \ldots$

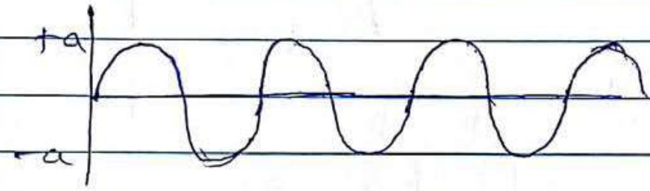
Thus, for constructive interference, phase difference between interfering waves must be even multiple of π and path difference must be integral multiple of wavelength.

Destructive Interference:

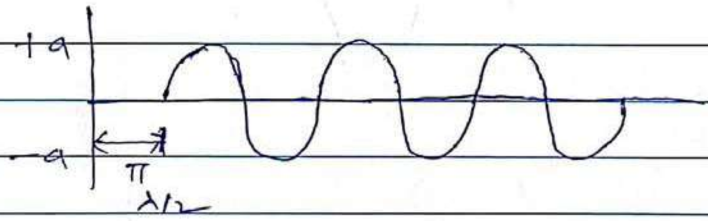
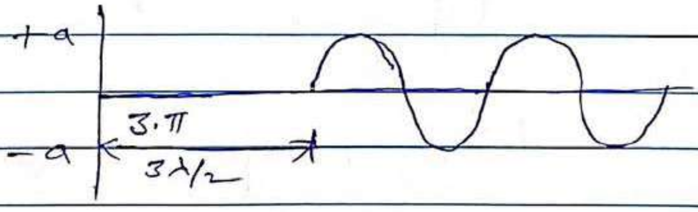
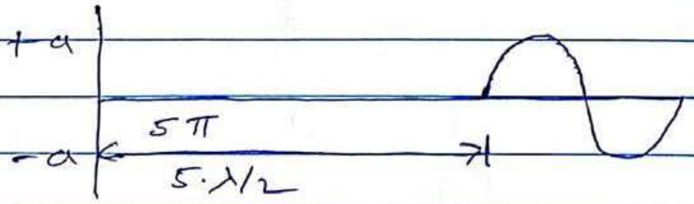
In this case, when the crest of one wave overlaps with the trough of the other and vice-versa. the interference obtained is known as Destructive Interference.

For this interference to takes place, wave must be out of phase while interfering.

Let us consider the same wave, $y = a\sin(\omega t - kx)$



Other waves | with respect to above wave

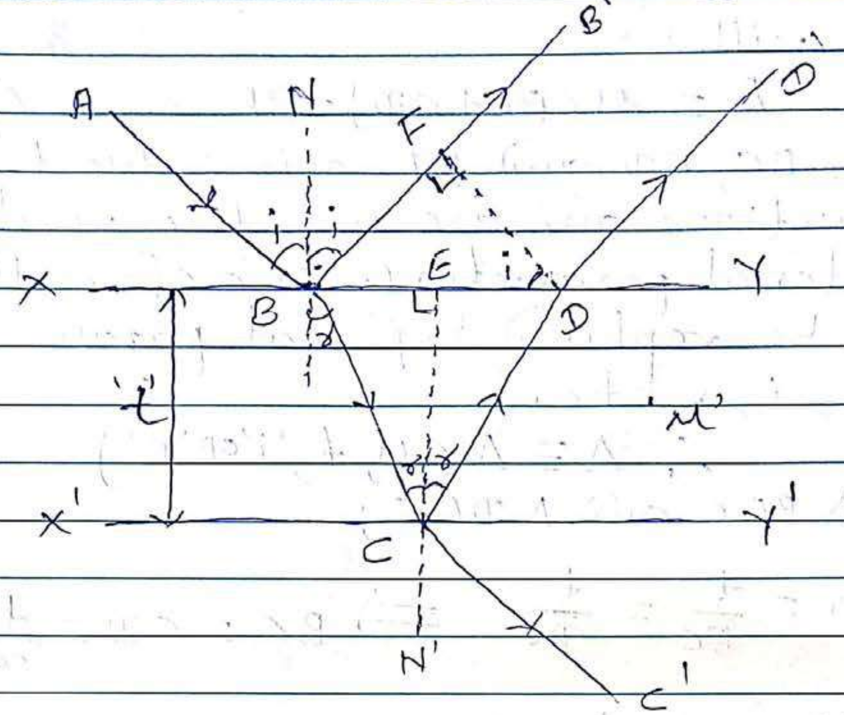
	Phase difference	Path Difference
	$\phi = \pi$	$\Delta = 1 \cdot \frac{\lambda}{2}$
	$\phi = 3 \cdot \pi$	$\Delta = 3 \cdot \frac{\lambda}{2}$
	$\phi = 5 \cdot \pi$	$\Delta = 5 \cdot \frac{\lambda}{2}$

$\therefore \phi = (2n+1)\pi \qquad \Delta = (2n+1)\frac{\lambda}{2}$

where, $n = 0, 1, 2, 3, \ldots$

Thus, for destructive interference, phase difference must be odd multiple of π and the path difference must be odd multiple of half of the wavelength

* Interference in Uniform Thickness Thin Film



Consider a thin film of uniform thickness 't' and refractive index 'μ'.

XY is a top surface of the film and X'Y' is a bottom surface.

At point B, AB is a incident ray at an angle of incidence 'i'.

At point B, BB' is reflected part and BC is a refracted one (at an angle of refraction 'r')

At point C, CD is reflected part.

At point D, DD' is refracted part.

DF is a normal drawn on BB'.

The two outgoing rays BB' and DD' are very close to each other and undergoes interference.

The geometrical path difference between these reflected light rays is

$$(BC + CD) - BF.$$

Therefore optical path difference between them will be

$$\Delta = \mu(BC+CD) - BF \quad \text{--- (1)}$$

Here, BC, CD and BF arises due to geometrical constructions and are not the real optical or material parameters. Therefore, these terms must be replaced by real parameters like μ, t, i, r etc.

$$\therefore \Delta \equiv \Delta(\mu, t, \text{'i' OR 'r'})$$

In ΔBEC and ΔDEC;

$$\cos r = \frac{t}{BC} = \frac{t}{DC} \Rightarrow BC = CD = \frac{t}{\cos r} \quad \text{--- (2)}$$

For, BF, consider ΔBFD;

$$\sin i = \frac{BF}{BD} = \frac{BF}{BE+ED}$$

$$\therefore BF = (BE+ED)\cdot \sin i \quad \text{--- (3)}$$

Again from ΔBEC and ΔDEC

$$\sin r = \frac{BE}{BC} = \frac{ED}{CD}$$

$$\therefore BE = BC \cdot \sin r = \frac{t}{\cos r} \cdot \sin r \quad \text{--- (4)}$$

$$ED = CD \cdot \sin r = \frac{t}{\cos r} \cdot \sin r \quad \text{--- (5)}$$

Substitute the values of BE and ED from equations (4) and (5) into equation (3)

$$\therefore BF = \frac{2t}{\cos r} \cdot \sin r \cdot \sin i \quad \text{--- (6)}$$

Now, lets subutitute the values of BC, CD and BF from equations (2) and (6) in equation (1).

$$\therefore \Delta = \frac{2\mu t}{\cos r} - \frac{2t}{\cos r} \cdot \sin r \sin i$$

$$= \frac{2\mu t}{\cos r} - \frac{2t}{\cos r} \cdot \sin r \cdot \sin i \cdot \frac{\sin r}{\sin r}$$

$$= \frac{2\mu t}{\cos r} - \frac{2\mu t}{\cos r} \cdot \sin^2 r$$

$$= \frac{2\mu t}{\cos r} (1 - \sin^2 r)$$

$$= \frac{2\mu t}{\cos r} \cdot \cos^2 r$$

$$\therefore \Delta = 2\mu t \cos r \quad \text{—— (7)}$$

Here, BB' and DD' are the reflections from the surface of denser medium.
Therefore, due to stoke's law, an addional path difference of $\frac{\lambda}{2}$ must be consider in the path difference equation.

$$\therefore \Delta = 2\mu t \cos r + \frac{\lambda}{2} \quad \text{—— (8)}$$

The condition for constructive interference is

$$\Delta = n \cdot \lambda$$

$$\therefore 2\mu t \cos r + \frac{\lambda}{2} = n\lambda$$

$$\therefore 2\mu t \cos r = \frac{2n\lambda}{2} - \frac{\lambda}{2}$$

$$\therefore 2\mu t \cos r = (2n-1)\frac{\lambda}{2} \quad \text{—— (9)}$$

where, $n = 1, 2, 3, 4, \ldots$

Condition for destructive interference is

$$\Delta = (2n+1)\frac{\lambda}{2}$$

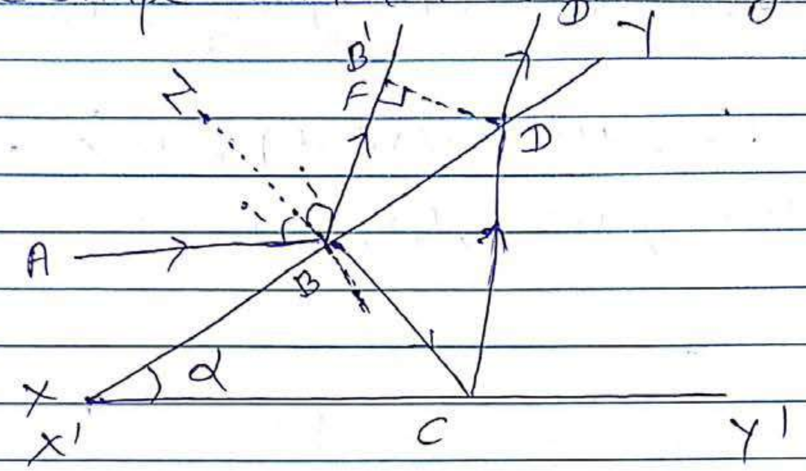
$$\therefore \Delta\, 2\mu t \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\therefore 2\mu t \cos r = 2n \cdot \frac{\lambda}{2} + \frac{\lambda}{2} - \frac{\lambda}{2}$$

$$\therefore 2\mu t \cos r = n\lambda \quad \text{——— (10)}$$

where, $n = 0, 1, 2, 3, 4, \ldots$

* Wedge shape Thin Film : (Only conditions)



The final equation for optical path difference is

$$\Delta = 2\mu t \cos(r+\alpha) \quad \text{—— (1)}$$

For the system of reflected ray's, due to stoke's law

$$\Delta = 2\mu t \cos(r+\alpha) + \frac{\lambda}{2} \quad \text{—— (2)}$$

Condition for constructive interference is

$$\Delta = n\lambda$$

$$\therefore 2\mu t \cos(r+\alpha) + \frac{\lambda}{2} = n\lambda$$

$$\therefore 2\mu t \cos(r+\alpha) = (2n-1)\frac{\lambda}{2} \quad \text{—— (3)}$$

where, $n = 1, 2, 3, 4, \ldots$

Condition for destructive interference is

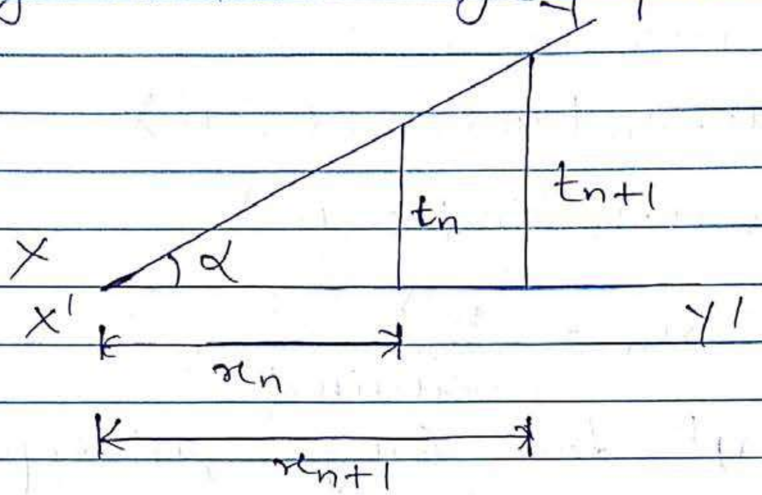
$$\Delta = (2n+1) \cdot \frac{\lambda}{2}$$

$$\therefore 2\mu t \cos(r+\alpha) + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\therefore 2\mu t \cos(r+\alpha) = n\lambda \quad \text{--- (4)}$$

where, $n = 0, 1, 2, 3, 4, \ldots$

* Fringe width for Wedge shaped Thin Film :



The center to center separation between any two consecutive fringes is known as fringe width.
Here for n^{th} and ~~nt~~ $(n+1)^{th}$ consecutive fringes fringe width 'β' is

$$\beta = x_{n+1} - x_n \quad \text{——} \ (1)$$

Let's assume that these fringes are dark fringes. Therefore, for n^{th} order dark fringe the condition for destructive interference is

$$2\mu t_n \cos(r+\alpha) = n\lambda \quad \text{——} \ (2)$$

From, the geometry of wedge shaped thin film

$$\tan\alpha = \frac{t_n}{x_n} = \frac{t_{n+1}}{x_{n+1}} \quad \text{——} \ (3)$$

$$\therefore t_n = x_n \cdot \tan\alpha = x \quad \text{——} \ (4)$$

substitute the value of t_n from eq^n (4) into eq^n (2)

$$\therefore 2\mu x_n \cdot \tan\alpha \cdot \cos(r+\alpha) = n\lambda \quad \text{——} \ (5)$$

Now, for normal incidence

$i = r = 0$.

$\therefore 2\mu \cdot x_n \tan\alpha \cdot \cos\alpha = n\lambda$

$\therefore 2\mu x_n \sin\alpha = n\lambda$.

$$\therefore x_n = \frac{n\lambda}{2\mu \sin\alpha} \quad \text{--- (6)}$$

For $(n+1)^{th}$ dark fringe, just replace n by (n+1)

$$\therefore x_{n+1} = \frac{(n+1)\lambda}{2\mu \sin\alpha} \quad \text{--- (7)}$$

Substitute the values of equations (6) and (7) in equation (1).

$$\therefore \beta = \frac{(n+1)\lambda}{2\mu \sin\alpha} - \frac{n\lambda}{2\mu \sin\alpha}$$

$$= \frac{n\lambda}{2\mu \sin\alpha} + \frac{\lambda}{2\mu \sin\alpha} - \frac{n\lambda}{2\mu \sin\alpha}$$

$$\therefore \beta = \frac{\lambda}{2\mu \sin\alpha} \quad \text{--- (8)}$$

For thin wedge α is very very small.

$\therefore \sin\alpha = \alpha$.

$$\therefore \beta = \frac{\lambda}{2\mu\alpha} \quad \text{--- (9)}$$

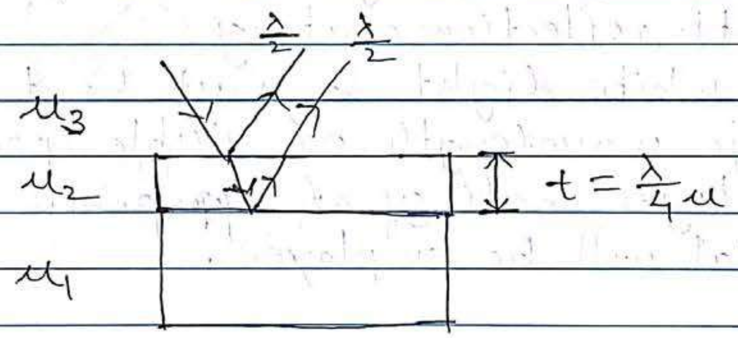
Thus, $\beta = \beta(\lambda, \mu, \alpha)$.

Applications of Interference

* Anti-Reflection Coatings:

It is a kind of uniform thickness thin film coatings on the surfaces of optical components to reduce the reflections of light. In most of the optical instruments like telescopes, cameras etc. uses the lenses provided with anti-reflection coatings.

For anti-reflection coatings, the materials of the coating must be selecting in such a way that it's refractive index must be less than that of the optical components and also the thickness of the coating is apropriate to fulfill the necessary condition of destructive interference for reflected light.



Here, refractive index follow the trends of $\mu_1 > \mu_2 > \mu_3$ because of this both the reflected light rays originate from top and bottom surfaces of the coatings will aquire an additional path of $\frac{\lambda}{2}$ due to stoke's law. Therefore, an additional path difference becomes zero $\left(\frac{\lambda}{2} - \frac{\lambda}{2}\right)$.

Therefore, optical path difference for these two reflected rays will be

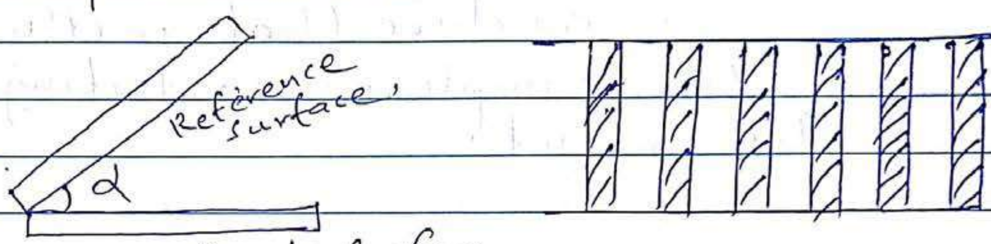
$$\Delta = 2\mu t \cos r$$

And $\Delta = 2\mu t$ for normal incidence, as $r = 0$.

* Testing Of Optical Flatness of Surfaces:

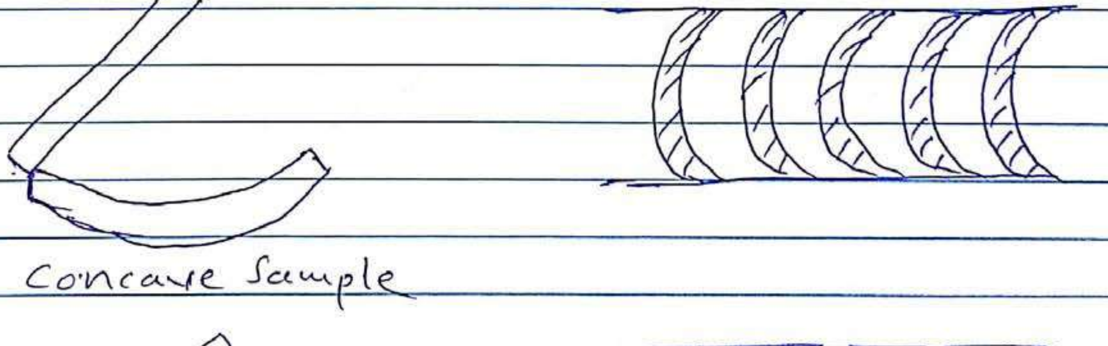
Wedge shaped thin film interference pattern can be commonly used to test the optical flatness of surfaces of the optical components.

If both the surfaces of wedge shaped thin films are perfectly smoth and optically flat then it always produces vertical straight fringes of equal widths.

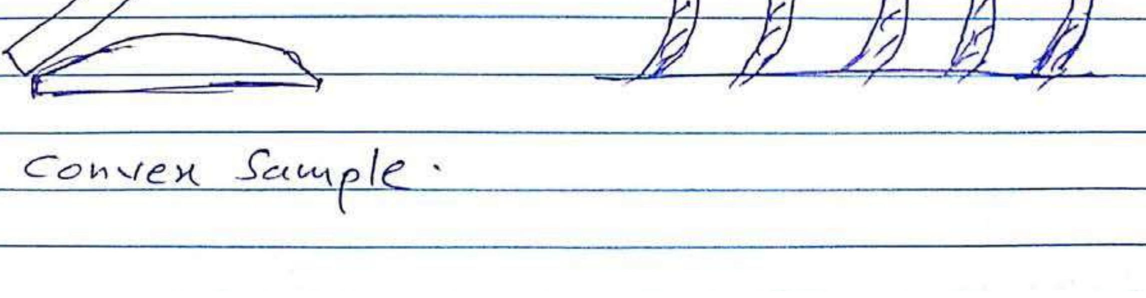


Sample Surface

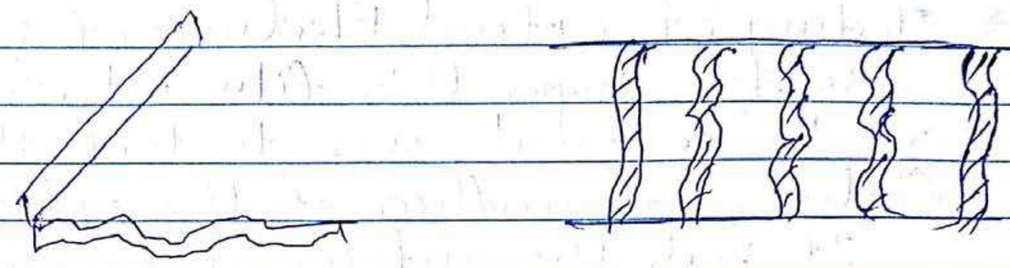
And if the sample under testing is not optically flat then depending upon the geometry of the surface of the sample we get different fringes. Lets consider some typical cases as follows.



Concave Sample



Convex Sample.



Irregular surface

Depending upon the nature of fringes we can conclude that wether the surface of the sample under testing is optically flat or not.

* Numericals based on Unit-I :

(A) Interference

(1) A parallel beam of light of wavelength 622 nm incident on a glass plate of refractive index 1.5. The angle of refraction into the plate is 60°. Calculate the smallest thickness of the plate which will appear dark by reflection.

Solⁿ: Given: $\mu = 1.5$, $r = 60°$, $\lambda = 622\text{ nm} = 622 \times 10^{-7}\text{ cm}$

For smallest thickness, $n = 1$.

And for dark band / fringe, condition for destructive interference for uniform thickness thin film is (In Reflected light)

$$2\mu t \cos r = n\lambda$$

$$\therefore t = \frac{n\lambda}{2\mu \cos r}$$

$$\therefore t = \frac{1 \times 622 \times 10^{-7}}{2 \times 1.5 \times \cos 60}$$

$$\therefore t = 4.15 \times 10^{-5}\text{ cm}$$

(2) A wedge shaped air film having an angle of 40 seconds is illuminated by mono-chromatic light and fringes are observed vertically through a microscope. The distance measured between consecutive bright fringes is 0.12 cm. Calculate the wavelength of light used.

Solⁿ: Given: For air film, $\mu = 1$

$\beta = 0.12\text{ cm}$

And $\alpha = 40$ seconds.

<u>Important</u> <u>Note</u> – For any problems based on the concept of fringe width of wedge shaped thin film, the value of <u>wedge angle</u> 'α' must be substituted in '<u>radians</u>'.

As

$$\alpha = 40 \text{ second} = 40 \times \frac{1}{60} \text{ minutes}$$

$$\alpha = 40 \times \frac{1}{60} \times \frac{1}{60} \text{ degree}$$

$$\therefore \alpha = 40 \times \frac{1}{60} \times \frac{1}{60} \times \frac{\pi}{180} \text{ radians}$$

Formula for fringe width 'β' is

$$\beta = \frac{\lambda}{2\mu\alpha}$$

$$\therefore \lambda = 2\mu\alpha\beta$$

$$\therefore \lambda = 2 \times 1 \times \frac{40\pi}{3600 \times 180} \times 0.12 \text{ cm}$$

$$\therefore \lambda = 4.654 \times 10^{-5} \text{ cm}$$

$$\therefore \lambda = 4654 \, A^{0}$$

(3) Fringes of equal thickness are observed in a thin glass wedge of refractive index 1.52. The fringe spacing is 1 mm and the wavelength of light is 5893 A°. Calculate the angle of wedge in seconds of an arc.

Solⁿ: Given:

$\lambda = 5893\,A^\circ = 5893 \times 10^{-8}$ cm

$\beta = 1$ mm $= 0.1$ cm

$\mu = 1.52$

Formula: $$\beta = \frac{\lambda}{2\mu\alpha}$$

$$\therefore \alpha = \frac{\lambda}{2\mu\beta} = \frac{5893 \times 10^{-8}}{2 \times 1.52 \times 0.1}$$

$$\therefore \alpha = 1.9385 \times 10^{-4} \text{ radians.}$$

As π radians $= 180$ degree

$$\therefore 1 \text{ radians} = \frac{180}{\pi} \text{ degrees}$$

$$= \frac{180}{\pi} \times 60 \text{ minutes}$$

$$= \frac{180}{\pi} \times 60 \times 60 \text{ seconds}$$

$$\therefore \alpha = 1.9385 \times 10^{-4} \times \frac{180 \times 3600}{\pi} \text{ seconds}$$

$$\therefore \alpha = 39.98 \text{ seconds of an arc.}$$

(4) Two optically plane glass strips of length 10 cm are placed one over the other. A thin foil of thickness 0.010 mm is introduced between the plates at one end to form an air film. If the light used has wavelength 5900 A°, find the separation between consecutive bright fringes.

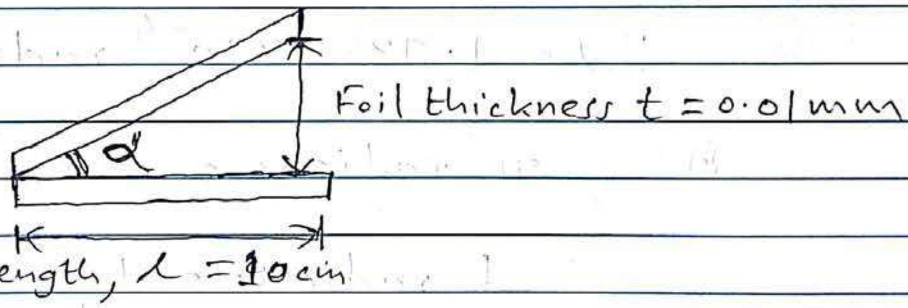
Sol^n: Given: for air film $\mu = 1$

$$\lambda = 5900\ A^\circ = 5900 \times 10^{-10}\ m$$

Length of the glass strip, $l = 10\ cm$

Thickness of the foil, $t = 0.010\ mm = 0.01 \times 10^{-1}\ cm$

The diagram of wedge forming due to glass strips and thin foil is as follows



From the geometry of the wedge,

$$\tan\alpha = \frac{t}{l} \simeq \alpha$$

As 'α' is very very small for thin wedge

$$\therefore \alpha = \frac{0.01 \times 10^{-1}}{10} = 10^{-4}\ \text{radians}$$

Formula is

$$\beta = \frac{\lambda}{2\mu\alpha} = \frac{5900 \times 10^{-10}}{2 \times 1 \times 10^{-4}} = 2.95 \times 10^{-3}\ m.$$

$$\therefore \beta = 2.95\ mm.$$

(5) A glass of refractive index 1.5 is to be coated with a transparent material of refractive index 1.2, so that the reflection of light of wavelength 6000 A° is eliminated by interference. What is the required thickness of the coating?

Solⁿ: Given:

$$\lambda = 6000\ A^\circ = 6000 \times 10^{-8}\ \text{cm}$$

Refractive index of glass is

$$\mu = 1.5$$

And of the coating material is

$$\mu' = 1.2$$

Formula for thickness in anti-reflection coating is

$$t = \frac{\lambda}{4\mu'}$$

$$\therefore t = \frac{6000 \times 10^{-8}}{4 \times 1.2}\ \text{cm}$$

$$\therefore t = 1.25 \times 10^{-5}\ \text{cm}$$

Diffraction:

Enchroachment of light into the geometrical shadow of an obstacle OR its bending around the corner of an obstacle when the size of an obstacle is comparable to the wavelength of light is known as diffraction.

Like interference, in diffraction also the intensity is also getting distributed in fringes of alternate bright and dark intensity. The central fringe of the diffraction pattern is having highest intensity as the order of fringe increases its intensity decreases drastically.

To observe a diffraction pattern a source of light, an obstacle and a screen is necessary. depending upon the relative separation between these three elements, diffraction can be classified as eighther Fresnel diffraction or Fraunhofer diffraction.

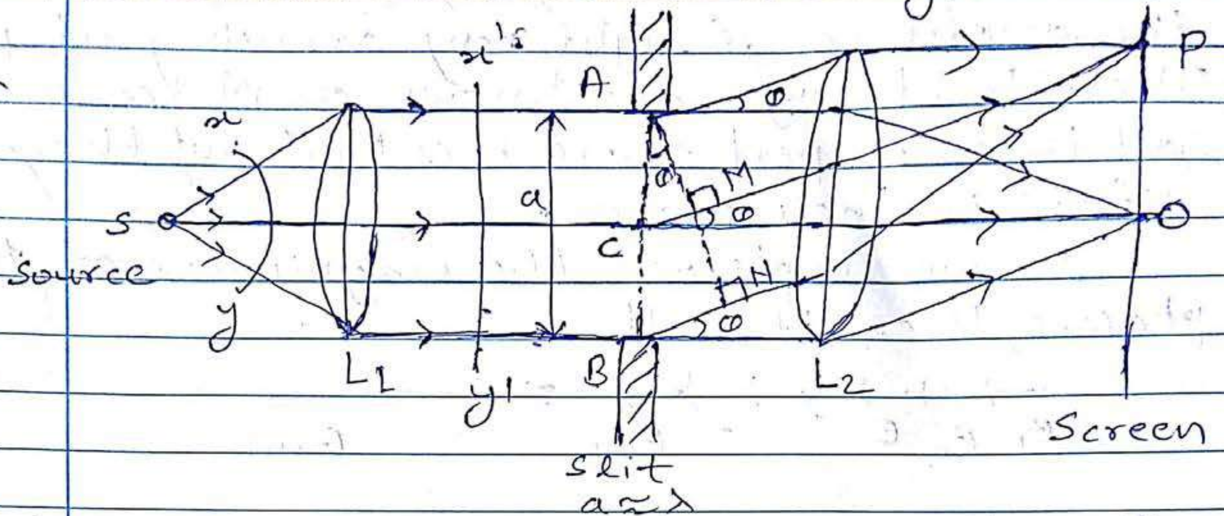
If source of light as well as screen is at finite distances from an obstacle like slit then the diffraction pattern obtained is known as Fresnel diffraction.

And if the source of light and the screen is at infinite distances from the obstacle then the diffraction pattern obtained is known as Fraunhofer diffraction.

Comparision of Fresnel and Fraunhofer Diffraction.

Fresnel Diffraction	Fraunhofer Diffraction
(1) Distances are finite	Here distances are infinite
(2) No biconvex lenses are needed	Biconvex lenses are needed.
(3) Wavefronts are spherical or cylindrical.	Wavefronts are plane wavefronts
(4) Angle of diffraction will be different for different light rays.	All light rays are having same angle of diffraction
(5) Theoretical and mathematical formulation is complicated	Theoretical and mathematical formulation is simple.

* Fraunhofer Diffraction at Single Slit :



To observe a Fraunhofer diffraction, lets place two biconvex lenses L_1 and L_2 in between source S and slit AB & in between slit AB and the screen respectively. Here slit width 'a' is comparable to the wavelength of the light source. Therefore, when a parallel beam of ligh rays (plane wavefront x'y') strikes the slit, it undergoes diffraction and along with the light rays traveling along same direction, some of the rays (intensity) will also travel along the bended direction by an amount of bending 'θ' known as angle of diffraction.

To formulate the equation for diffraction, lets consider some point 'P' on the screen. If we know the intensity (amplitude) at this representative point we can analyse the diffraction pattern. The resultant amplitude at point 'p' can be findout from the vector addition of amplitudes of light rays which are simultaneously arriving at point p.

Lets consider that $E_1, E_2, E_3, \ldots, E_N$ are the amplitudes of light ray arriving at point P. These light rays are known as phasors as all their properties are identical but they are differe in their phases.

If we simple add the magnitudes of these phasors, then it will

But these phasors are vectors. Therefore we have to add them vectorially.

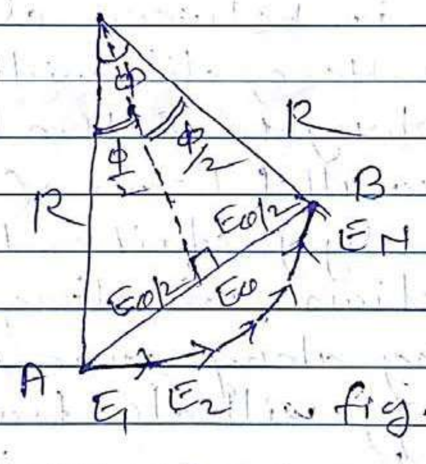


fig. Phasor Diagram.

The path difference between extreeme diffracted light rays from the start and the end of the slit is

$$\Delta = BN = a \sin\theta \quad \text{--- (1)}$$

$$\therefore \phi = \frac{2\pi}{\lambda} a \sin\theta \quad \text{--- (2)}$$

where ϕ is known as the total phase difference. The phase difference between any two adjacent phasors (light rays) is

$$\Delta\phi = \frac{2\pi}{\lambda} \frac{a}{N} \sin\theta \quad \text{--- (3)}$$

From phasor diagram,

$$\sin\left(\frac{\phi}{2}\right) = \frac{E_0/2}{R}$$

$$\therefore \sin\frac{\phi}{2} = \frac{E_0}{2R}$$

$$\therefore E_0 = 2R \cdot \sin\left(\frac{\phi}{2}\right) \quad \text{——— (4)}$$

Here, 'R' arises due to geometrical construction. It must be replaced in terms of real parameter. From the property of a circular arc.

$$\left.\begin{array}{c}\text{Angle subtended} \\ \text{by arc}\end{array}\right\} = \frac{\text{length of arc}}{\text{radius}}.$$

$$\therefore \phi = \frac{E_m}{R} \quad \text{——— (5)}$$

$$\therefore R = \frac{E_m}{\phi} \quad \text{——— (6)}$$

Substitute this value of R in equation (4)

$$\therefore E_0 = 2 \cdot \frac{E_m}{\phi} \cdot \sin\left(\frac{\phi}{2}\right) \quad \text{——— (7)}$$

Lets substitute,

$$\frac{\phi}{2} = \alpha \Rightarrow \phi = 2\alpha \quad \text{——— (8)}$$

$$\therefore E_0 = 2 \cdot \frac{E_m}{2\alpha} \cdot \sin\alpha$$

$$\therefore E_0 = E_m\left(\frac{\sin\alpha}{\alpha}\right) \quad \text{——— (9)}$$

This is the equation of resultant amplitude.

As intensity is proportional to square of the amplitude. For unity constant of proportionality

$$I_\theta = I_m \cdot \left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{—— (10)}$$

where I_θ is resultant intensity

I_m is maximum intensity.

α is the half of the total phase difference

And $\alpha = \frac{\phi}{2} = \frac{\pi}{\lambda} \cdot a\sin\theta$ —— (11)

<u>Condition for Central Maxima:</u>

As $E_\theta = E_m\left(\frac{\sin\alpha}{\alpha}\right)$

And for maxima, it is expected that $E_\theta = E_m$.

$$\therefore E_\theta = \frac{E_m}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \cdots\right]$$

$$\therefore E_\theta = E_m\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{6!} + \cdots\right]$$

In this equation if $\alpha = 0 \Rightarrow E_\theta = E_m$.

Therefore for maxima.

$\alpha = 0$ mean $\frac{\pi}{\lambda} \cdot a\sin\theta = 0$

i.e; $\left(\frac{\pi}{\lambda} \cdot a\right) \cdot \sin\theta = 0$.

But $\left(\frac{\pi}{\lambda} \cdot a\right) \neq 0 \quad \therefore \sin\theta = 0$.

$\therefore \theta = 0$.

The condition for central maxima is

$\alpha = 0, \ \phi = 0 \Rightarrow \theta = 0$.

Condition for minima:

As $E_{\theta} = E_m \cdot \left(\frac{\sin\alpha}{\alpha}\right)$

For minima $E_{\theta} = 0$.

As $E_m \neq 0$;

$$\therefore \left(\frac{\sin\alpha}{\alpha}\right) = 0.$$

$\therefore \alpha \neq 0$ and $\sin\alpha = 0$.

$\therefore \alpha = n\pi$

i.e, $\frac{\pi}{\lambda} a \sin\theta = n\pi$

$$\therefore a\sin\theta = n\lambda$$

i.e; $\alpha = n\pi$

is known as condition for minima.

Condition for Higher Order Maximas:

As higher order maximas are normally situated at the middle betn two adjancent minimas

$$\therefore \alpha = (n+\tfrac{1}{2})\pi$$

is the condition for higher order maxima.

Relative Intensity of higher order maximas with respect to central maxima is given by.

As $I_{\theta} = I_m \cdot \left(\frac{\sin\alpha}{\alpha}\right)^2$

Replace α by $(n+\frac{1}{2})\pi$

$$\therefore I_{\theta} = I_m \cdot \frac{\sin^2[(n+\frac{1}{2})\pi]}{(n+\frac{1}{2})^2\pi^2}$$

i.e.

$$I_{r_n} = \frac{I_0}{I_m} = \frac{\sin^2\left[(n+\frac{1}{2})\pi\right]}{(n+\frac{1}{2})^2\pi^2}$$

For 1^{st} order maxima, $n=1$

$$\therefore I_{r_1} = \frac{\sin^2(3\pi/2)}{(3\pi/2)^2} = 0.0456 = 4.56\%$$

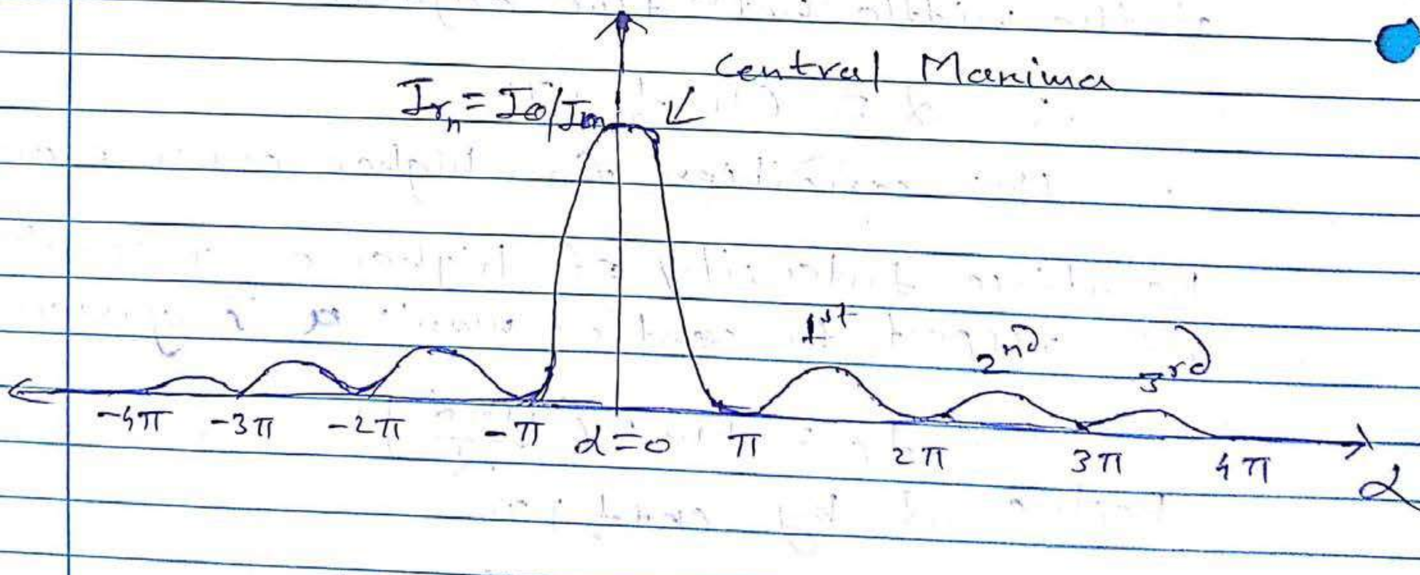
For 2^{nd} order maxima, $n=2$

$$I_{r_2} = \frac{\sin^2(5\pi/2)}{(5\pi/2)^2} = 0.016 = 1.6\%$$

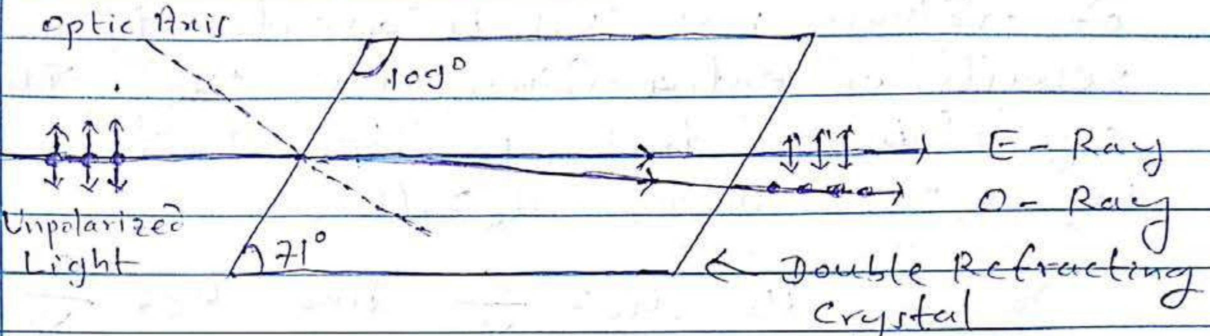
And for 3^{rd} order maxima, $n=3$

$$I_{r_3} = \frac{\sin^2(7\pi/2)}{(7\pi/2)^2} = 0.008 = 0.8\%$$

Intensity Distribution Plot:



Phenomena of Double Refraction



When a narrow beam of unpolarized light is incident on a doubly refracting crystal, it produces two wavefronts corresponding to two polarized refracted rays. This is known as double refraction.

Huygen's Theory of Double Refraction:

Postulates

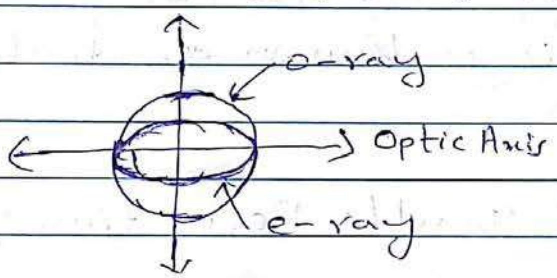
1. Every point in double refracting crystal produces two wavefronts, one is spherical wavefront and the other is elliptical wavefront.
2. The light ray which is formed due to spherical wavefront is known as ordinary ray (O-ray) which obeys laws of optics. whereas the light ray which is formed due to elliptical wavefront is known as extra ordinary ray (e-ray) which does not obeys the laws of optics.
3. Inside double refracting crystal the velocity of ordinary ray is same in all direction whereas extra ordinary ray travels with different speed in different direction
4. Along optic axis both the rays travels with same speed.

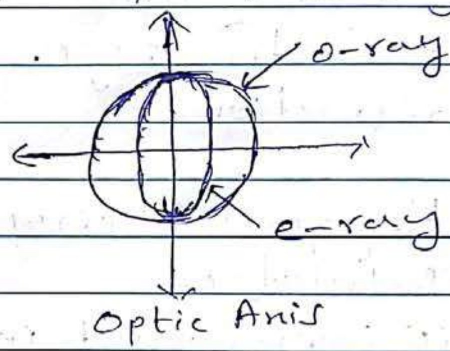
(5) In given double refracting crystal, if velocity of ordinary ray (V_o) is greater than the velocity of extra ordinary ray (V_e). Then, the crystal is said to be positive crystal.

i.e; $V_o > V_e$ means $\mu_e > \mu_o$

$$\text{As} \quad \mu_o = \frac{c}{V_o} \quad \text{And} \quad \mu_e = \frac{c}{V_e}$$

where μ_o is refractive index of o-ray and μ_e is refractive index of e-ray.

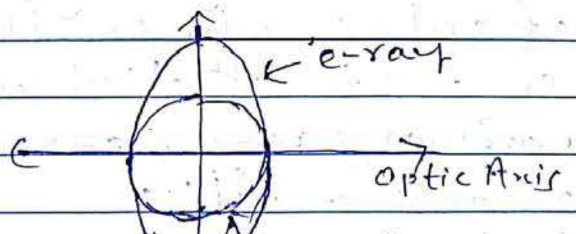


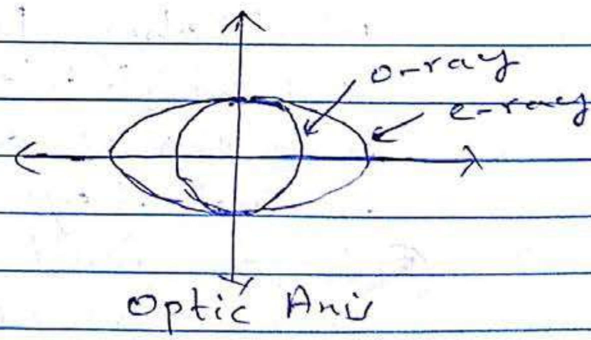


e.g. Quartz, Ice etc.

(6) If the velocity of extra ordinary ray is greater than the velocity of ordinary ray then it is said to be negative crystal.

i.e; $V_e > V_o$ mean $\mu_o > \mu_e$





e.g. Calcite, Tourmaline etc.

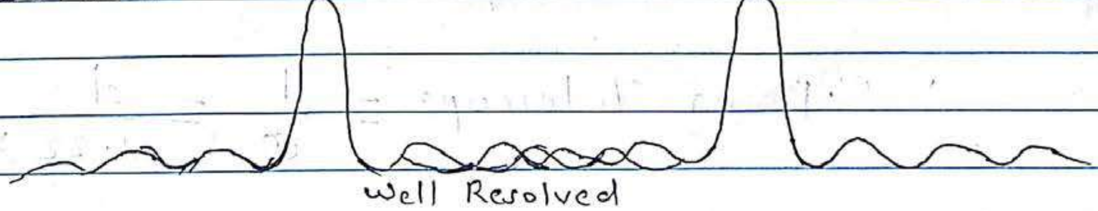
* Resolving Power:

The ability of an optical instrument to produce distinctly separate images of two objects located very close to each other is called its resolving power. It is defined as the reciprocal of the smallest angle subtended at the objective by two point objects, which can just be distinguished as separate.

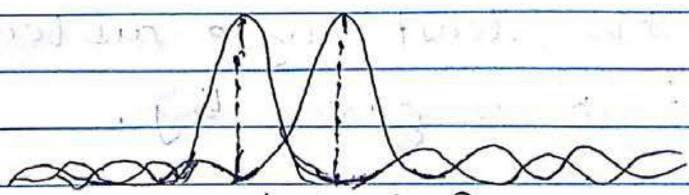
Rayleigh's Criterion of Resolution:

According to Rayleigh's criterion, two closely spaced point sources of light are said to be just resolved if the central maximum of the diffraction pattern due one falls on the first minimum of the diffraction pattern of the other by an optical instrument.

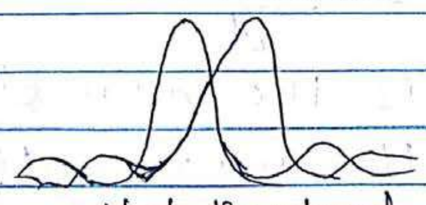
If the separation between the central maximums of the diffraction patterns of these two sources is further smaller than that of the Rayleigh's criterion then it is said to be unresolved. And if the separation is large enough then it is said to well resolved.



Well Resolved



Just Resolved



Not Resolved

Resolving Power of Grating:

Resolving power of a grating (R.P.) is defined as the ratio of the wavelength (λ) of a line to the wavelength difference ($d\lambda$) with the next line which is seen to be just separated from the first.

For nth order principal maxima and N number of slits. R.P. can be expressed as

$$\text{R.P. of Grating} = \frac{\lambda}{d\lambda} = n \cdot N$$

Resolving Power of a Telescope:

The resolving power (R.P.) of a telescope is difined as the reciprocal of the least angle subtended at the objective by two distinct point objects which can be distinguished just as separate in the focal plane of the telescope.

$$\therefore \text{R.P. of Telescope} = \frac{1}{\theta} = \frac{d}{1.22\lambda}$$

As the least angle subtended at the object 'θ' is given by,

$$\theta = \frac{1.22\lambda}{d}$$

where 'd' is the diameter of the aperture and 'λ' is the wavelength of light source.

(B) Diffraction

(1) A monochromatic light of wavelength 5500 A° is incident normally on a slit of width 2×10^{-4} cm. Calculate the angular positions of first and second minima?

Sol^n: Given:

Slit width, $a = 2\times10^{-4}$ cm

$\lambda = 5500\ A^\circ = 5500\times10^{-8}$ cm

For 1st minima and 2nd minima, n=1 and n=2 respectively.

Formula:

Condition for minimum intensity for the case of single slit Fraunhofer diffraction is

$$a \sin\theta = n\lambda .$$

For, 1st order minima

$$a \sin\theta_1 = 1\cdot\lambda$$

$$\therefore \sin\theta_1 = \frac{\lambda}{a} = \frac{5500\times10^{-8}}{2\times10^{-4}}$$

$$\therefore \theta_1 = \sin^{-1}\left(\frac{5500\times10^{-8}}{2\times10^{-4}}\right)$$

$$\therefore \theta_1 = 15.96^\circ .$$

For, 2nd order minima

$$a \sin\theta_2 = 2\cdot\lambda$$

$$\therefore \theta_2 = \sin^{-1}\left(\frac{2\times5500\times10^{-8}}{2\times10^{-4}}\right)$$

$$\therefore \theta_2 = 33.37^\circ .$$

(2) Find the half angular width of the central maxima in the Fraunhofer diffraction pattern due to a single slit having a width of 7.07×10^{-5} cm, when illuminated by light having wavelength of 5000 A°.

Soln: Given:

$$a = 7.07 \times 10^{-5} \text{ cm}$$

$$\lambda = 5000 \text{ A}^\circ = 5000 \times 10^{-8} \text{ cm}$$

Formula:

As half angular width of the central maxima corresponds to the position of 1st minima

$$\therefore a \sin\theta = 1 \cdot \lambda$$

$$\therefore \theta = \sin^{-1}\left(\frac{\lambda}{a}\right)$$

$$\therefore \theta = \sin^{-1}\left(\frac{5 \times 10^{-5}}{7.07 \times 10^{-5}}\right) = 45^\circ.$$

(3) A slit of width 'a' is illuminated by white light. For what slit width, the first minimum for red light falls at an angle of 30°? For red light $\lambda = 6500$ A°

Soln: Given: $\theta = 30^\circ$, $\lambda = 6500 \text{ A}^\circ = 6500 \times 10^{-8}$ cm

for 1st minimum, $n = 1$.

Formula:

Condition for 1st minimum is

$$a \sin\theta = 1 \cdot \lambda$$

$$\therefore a = \frac{1 \cdot \lambda}{\sin\theta} = \frac{1 \times 6500 \times 10^{-8}}{\sin 30^\circ} = 1.3 \times 10^{-4} \text{ cm}$$

$$\therefore a = 1.3 \times 10^{-4} \text{ cm}.$$

(2) Find the half angular width of the central maxima in the Fraunhofer diffraction pattern due to a single slit having a width of 7.07×10^{-5} cm, when illuminated by light having wavelength of 5000 A°.

Sol^n: Given:

$$a = 7.07 \times 10^{-5}\ cm$$

$$\lambda = 5000\ A^\circ = 5000 \times 10^{-8}\ cm$$

Formula:

As half angular width of the central maxima corresponds to the position of 1st minima

$$\therefore a \sin\theta = 1 \cdot \lambda$$

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Sol^n: Given: $\theta = 30^\circ$, $\lambda = 6500 A^\circ = 6500 \times 10^{-8}$ cm

for 1st minimum, $n = 1$.

Formula:

Condition for 1st minimum is

$$a \sin\theta = 1 \cdot \lambda$$

$$\therefore a = \frac{1 \cdot \lambda}{\sin\theta} = \frac{1 \times 6500 \times 10^{-8}}{\sin 30^\circ} = 1.3 \times 10^{-4}\ cm$$

$$\therefore a = 1.3 \times 10^{-4}\ cm.$$

(4) In a plane transmission grating, the angle of diffraction for the second order principal maximum for the wavelength of 5×10^{-5} cm is 30°. Calculate the number of lines/cm of the grating surface.

Sol^n: Given: $\lambda = 5 \times 10^{-5}$ cm, $\theta = 30^\circ$

and for 2^{nd} maximum $n = 2$

Formula for Maxima.

$$(a+b) \sin\theta = n\lambda$$

$$\therefore \frac{1}{(a+b)} = \frac{\sin\theta}{n\lambda}$$

$\frac{1}{(a+b)}$ is known as number of lines/cm.

For 2^{nd} Maximum,

$$\frac{1}{(a+b)} = \frac{\sin 30^\circ}{2 \times 5 \times 10^{-5}} = \frac{1}{2 \times 2 \times 5 \times 10^{-5}}$$

$$\therefore \frac{1}{(a+b)} = \frac{1}{20 \times 10^{-5}} = 5000 \text{ lines/cm.}$$

There are 5000 lines/cm.

(5) What is the highest order spectrum that is visible with light of wavelength 6000 A° by means of a grating having 5000 lines per cm?

Sol^n: Given: $\lambda = 6000$ A° $= 6000 \times 10^{-8}$ cm

$$\frac{1}{(a+b)} = 5000 \text{ lines/cm}$$

Formula for maximum is

$$(a+b) \sin\theta = n\lambda.$$

$$\therefore n = \frac{(a+b)\sin\theta}{\lambda}$$

For n to be maximum, $\sin\theta$ must be maximum

$$\therefore \sin\theta = 1$$

$$\therefore n_{max.} = \frac{(a+b)}{\lambda} = \frac{1}{5000 \times 6000 \times 10^{-8}}$$

$$\therefore n_{max.} = 3.333$$

But 'n' is an integer. Therefore $n_{max.} = 3$.

(6) Monochromatic light from laser of wavelength 6238 A° is incident normally on a diffraction grating containing 6000 lines/cm. Find the angles at which the 1st and 2nd order maximum are obtained.

Sol^n: Given: $(a+b) = 1/6000$ cm

$\lambda = 6238$ A° $= 6238 \times 10^{-8}$ cm

Formula for maximum is $(a+b)\sin\theta = n\lambda$

For 1st order, $(a+b)\sin\theta_1 = 1 \cdot \lambda$

$$\therefore \theta_1 = \sin^{-1}\left[\frac{\lambda}{(a+b)}\right] = \sin^{-1}\left(6238 \times 10^{-8} \times 6000\right)$$

$$\therefore \theta_1 = 21.98^\circ$$

For 2nd order $(a+b)\sin\theta_2 = 2 \cdot \lambda$

$$\therefore \theta_2 = \sin^{-1}\left(\frac{2\lambda}{a+b}\right) = \sin^{-1}\left(2 \times 6238 \times 10^{-8} \times 6000\right)$$

$$\therefore \theta_2 = 48.47^\circ.$$

(7) A grating has 620 rulings/mm and is 5.05 mm wide. What is the smallest wavelength interval that can be resolved in the third order at 481 nm wavelength.

Sol^n: Given: $\lambda = 481\,nm$; $n = 3$

$$\frac{1}{(a+b)} = 620 \text{ rulings/m}$$

$$\text{width} = 5.05\,mm$$

Number of lines 'N' is

$$N = 620 \times 5.05 = 3131.$$

Formula: $$R.P. = \frac{\lambda}{d\lambda} = nN$$

$$\therefore d\lambda = \frac{\lambda}{nN} = \frac{481}{3 \times 3131}\,nm$$

$$\therefore d\lambda = 0.0512\,nm.$$

(8) A plane grating just resolves two lines in the second order. Calculate the grating element if $d\lambda$ is $6A^o$, λ is 6×10^{-5} cm and the width of the ruled surface is 2cm.

Sol^n: Given: $\lambda = 6 \times 10^{-5}\,cm$

$$d\lambda = 6 \times 10^{-8}\,cm$$

width of grating $= 2\,cm$, $n = 2$.

Formula: $$\frac{\lambda}{d\lambda} = nN$$

$$\therefore N = \frac{\lambda}{n\,d\lambda} = \frac{6 \times 10^{-5}}{2 \times 6 \times 10^{-8}} = 500.$$

$$N = \frac{1}{(a+b)} \times \text{width}$$

$$\therefore \frac{1}{(a+b)} = \frac{N}{\text{width}} = \frac{500}{2} = 250 \text{ lines/cm}$$

∴ Grating element $(a+b)$ is

$$(a+b) = \frac{1}{250}\ cm = 0.004\ cm$$

$$(a+b) = 0.004\ cm.$$

(9) The anglular separation of two stars is 1.5 seconds. Find the minimum aperture of a telescope objective if the two stars are to be distinguished as separate. Wavevent λ is 5700 A^o.

Sol^n: Given: $\lambda = 5700 \times 10^{-8}$ cm

$\theta = 1.5$ seconds

$$\therefore \theta = 1.5 \times \frac{1}{60 \times 60} \times \frac{\pi}{180}\ radians = 0.72 \times 10^{-5}\ radians$$

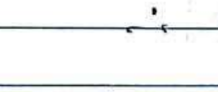
Formula: $\theta = 1.22 \frac{\lambda}{d}$

$$\therefore d = 1.22 \frac{\lambda}{\theta} = \frac{1.22 \times 5700 \times 10^{-8}}{0.72 \times 10^{-5}} = 9.65\ cm$$

$\therefore d = 9.65$ cm

(10) Find the separation of two points on the moon that can be resolved by a 5cm telescope. The distance of the moon from the earth is 3.8×10^5 km. The human eye is most sensitive to light of wavelength 5500 A^o.

Sol.^{n}: Given: $d = 500$ cm $= 5$ m, $\lambda = 5500 A^o = 5500 \times 10^{-10}$ m



$\tan\theta \simeq \theta = \frac{x}{r}$ 'x' is separation of two points on moon.

And 'r' is distance of moon from earth

$r = 3.8 \times 10^5$ kg $= 3.8 \times 10^8$ m.

As $\theta = \frac{1.22\lambda}{d} = \frac{x}{r}$ $\therefore x = \frac{1.22\lambda}{d} \times r$

$$\therefore x = \frac{1.22 \times 5500 \times 10^{-10} \times 3.8 \times 10^8}{5} = 50.996\ m.$$

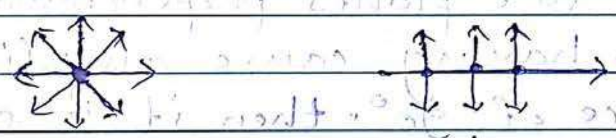
$\therefore x = 50.996$ m.

* Polarization :-

Light is a transverse wave, consisting of electric and magnetic fields vibrating perpendicular to each other and to the direction of propagation.

The polarization of an electromagnetic wave like light referes to the orientation of its electric field vector 'E' with respect to its direction of propagation.

If the oscillations of electric field vector 'E' in a light occurs in all the possible planes perpendicular to its direction of propagation, then the light is said to be unpolarized light. e.g. all ordinary light sources.



Head on view. Side view.

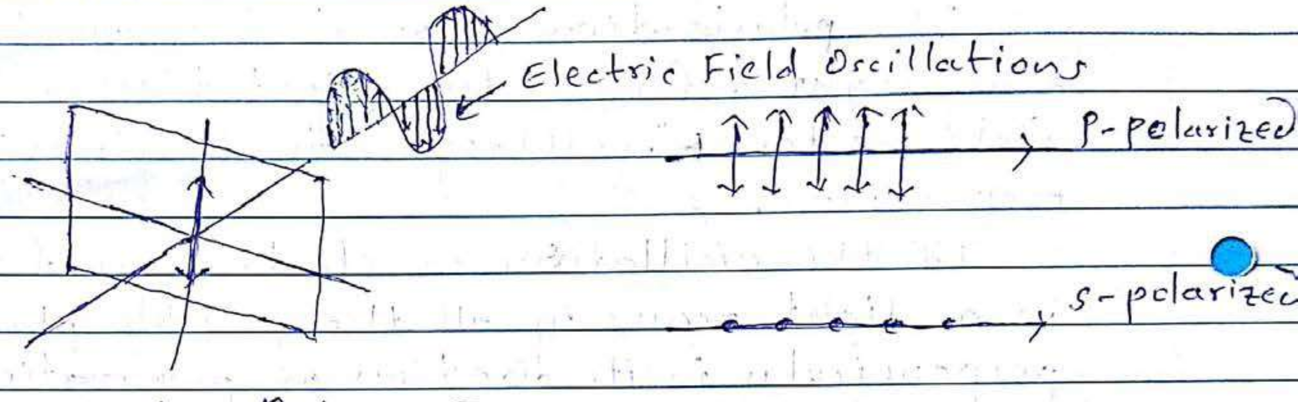
The process of transforming unpolarized light into polarized light is known as polarization. Therefore, polarized light is the light containing oscillations of electric field vectors in certain prefered directions. Usually light will shows three different states of polarization which are

(1) Plane OR Linear Polarization
(2) Circular Polarization
(3) Elliptical Polarization

Plane OR Linear Polarization:

A light containing oscillations of electric field vector in a plane perpendicular to its direction of propagation is known as plane polarized

OR linearly polarized light.
If we observe a plane or linearly polarized light along its direction of propagation then tip of electric field vector describes a line.



Circularly Polarized Light:
If light is composed of oscillations of electric field vectors in two planes perpendicular to each other and having same amplitude and phase difference of 90° then it is said to be circularly polarized light.
If we observe this circularly polarized light along its direction of propagation then tip of the electric field would appear to be moving in a circle as it approaches the observer. If the sense of rotation is clockwise or along right hand then it is known as Dextro-rotatory OR Right handed circularly polarized light. And if it is opposite then it is said to be Leavo-rotatory of Left handed circularly polarized light.

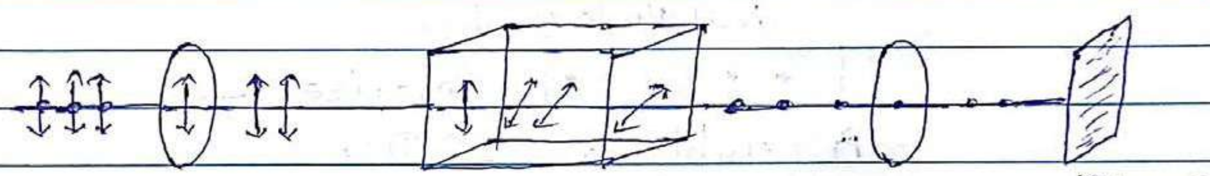
Elliptically Polarized Light:
This is exactly identical to the case of circularly polarized light, only the amplitude of oscillations of electric field vector is different.

Application:

Liquid Crystal Display as an application of Polarization:

The phenomena of polarization is smartly used the Liquid Crystal Display's (LCD) in many devices and gadgets like wristwatches, computer screens, timers, clocks and TVs etc.

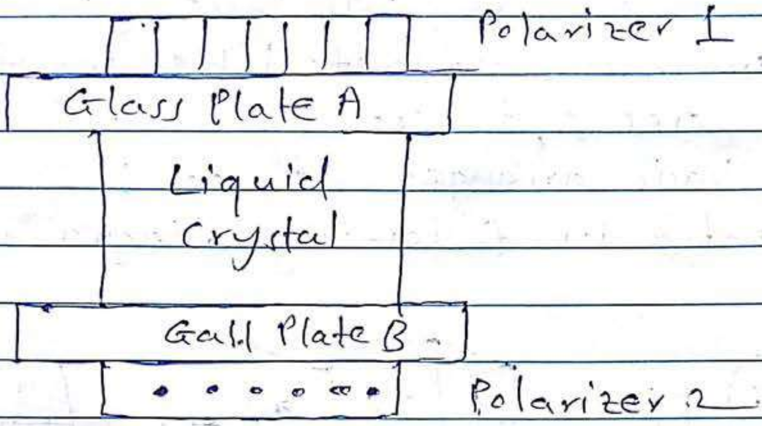
The basic arrangement of LCD structure is represented in following diagram



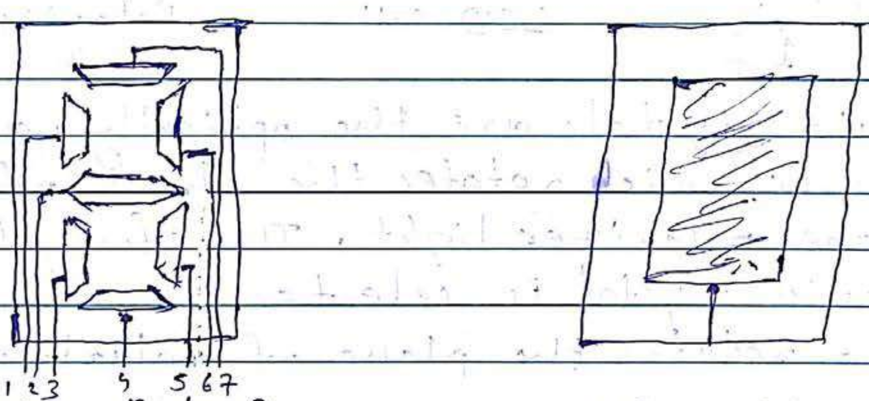
Liquid crystals are the optically active substances, which rotates the plane of polarization of plane polarized light. Therefore, the thickness of liquid crystal is selected in such a way that, it rotates the plane of polarization by 90°.

When an unpolarized light passes through polarizer 1, it gets linearly polarized. This linearly polarized light comes out from LCD it gets rotated by 90° and then passes through polarizer 2 whose transmission axis is perpendicular to that of polarizer 1. The reflecting mirror sends back the light which emerges unobstructed by the polarizer 1. This cause uniform illumination in the display. When the external voltage is applied across the liquid crystal these molecules get aligned along the field direction

Therefore the plane of polarization will not change and polarizer 2 blocks the light. This will produce a dark digit or alphabet.



Assembly of LCD.

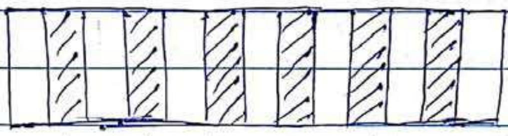


Glass Pate A　　　　Glass Plate B

It consists of a liquid crystal sandwiched between two thin glass plates with transparent conducting coating on the inner faces. The conducting plate (Glass plate A) is etched in the form of 7-cegment display. This whole assembly is then placed between two palarizing sheets with crossed plane of polarization.

* Diffraction Grating: (only conditions)

Grating is a multi-slit optical component consisting of large number of parallel slits of same width and separated by equal opaque spaces.
It is obtained by ruling equidistant parallel lines on a glass plate with the help of a fine diamond pointer. The lines acts as an opaque spaces and the space between two lines acts as a transparant slit. of respective widths 'b' and 'a'



The number of parallel slits on the surfaces of plane transmission gratings can be of the order of 15,000, 20,000, 30,000 etc and known a number of lines per inch (N).
The center to center distance between adjacent slits can be given by (a+b) and known as grating element.
The resultant amplitude in the diffraction pattern from grating can be expressed as

$$E_\theta = E_m \left(\frac{\sin\alpha}{\alpha}\right)\left(\frac{\sin N\beta}{\sin\beta}\right)$$

Arises due to single slit diffraction

represents the interference of all these single slit diffraction patterns.

And

$$\beta = \frac{\Delta\phi}{2} = \frac{\pi}{\lambda}(a+b)\sin\theta$$

And
$$\Delta\phi = \frac{2\pi}{\lambda}(a+b)\sin\theta$$
Represents phase difference between adjacent slits.

Corresponding intensity equation is

$$I_\theta = I_m \left(\frac{\sin\alpha}{\alpha}\right)^2 \cdot \left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

Condition for central maxima:

For central maxima

E_θ must be qual to E_m.

As resultant intensity distribution in this diffraction pattern arises due to interference of single slit diffraction patterns from the real slits of grating. Therefore, ~~the~~ we have to use condition of constructive interference, i.e.

$$\Delta = n\lambda$$

Here $\Delta = (a+b)\sin\theta$.

$$\therefore (a+b)\sin\theta = n\lambda$$

$$\therefore \beta = \frac{\pi}{\lambda}\cdot(a+b)\sin\theta = \frac{\pi}{\lambda}\cdot n\lambda$$

$$\boxed{\therefore \beta = n\pi}$$

The term

$\frac{\sin N\beta}{\sin\beta}$ takes $\frac{0}{0}$ form as $\beta = n\pi$

Therefore to overcome such indeterminancy, we have to use 'L' Hospital's Rule. i.e;

$$\frac{\sin N\beta}{\sin\beta} = \lim_{\beta \to n\pi} \frac{d(\sin N\beta)}{d(\sin\beta)}$$

$$= \lim_{\beta \to n\pi} \frac{N\cos N\beta}{\cos\beta}$$

$$\therefore \frac{\sin N\beta}{\sin\beta} = N(\pm 1)^n$$

Therefore intensity of central maxima will be

$$I_0 = N^2 E_m \left(\frac{\sin\alpha}{\alpha}\right)^2 \propto N^2$$

And $\beta = n\pi$ OR $(a+b)\sin\theta = n\lambda$ is known as condition for central maxima.

<u>Condition for Minima</u>:

For minima, E_θ must be equal to 0.

$$\therefore \left(\frac{\sin N\beta}{\sin\beta}\right) = 0$$

$$\therefore \sin\beta \neq 0 \text{ And } \sin N\beta = 0$$

$$\therefore N\beta = n\pi$$

$$\therefore \beta = \frac{n\pi}{N} \text{ And } n \neq m \cdot N.$$

where $m = 1, 2, 3, 4, \ldots$

As $\beta = \frac{n\pi}{N}$

$\therefore \frac{\pi}{\lambda}(a+b)\sin\theta = \frac{n\pi}{N}$

$\therefore (a+b)\sin\theta = \frac{n\lambda}{N}$ And $n \neq mN$

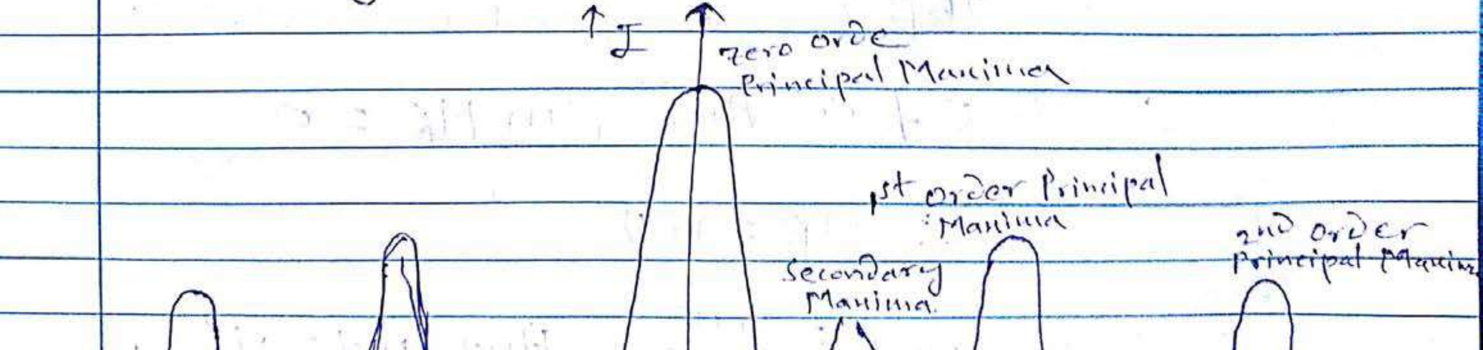
There are $(N-1)$ minimas will be obrerved in between any two consecutive principal maximas.

Secondary Maxima :

As there are $(N-1)$ minima between two consecutive principal maxima there must be $(N-2)$ other maxima coming alternatively with the minima between two concecutive principal maxima. These maxima are known as secondary maxima.

The positions of these maxima can be calculated as by taking the derivative of intensity with respect to β and equate to zero.

Intensity Distribution Curve

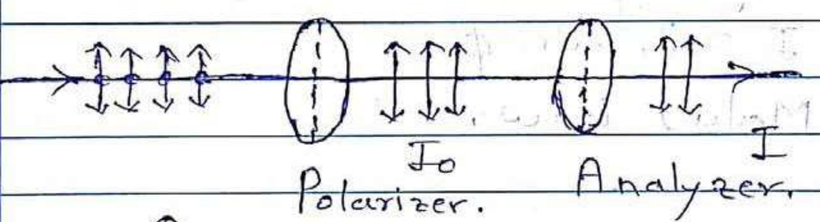


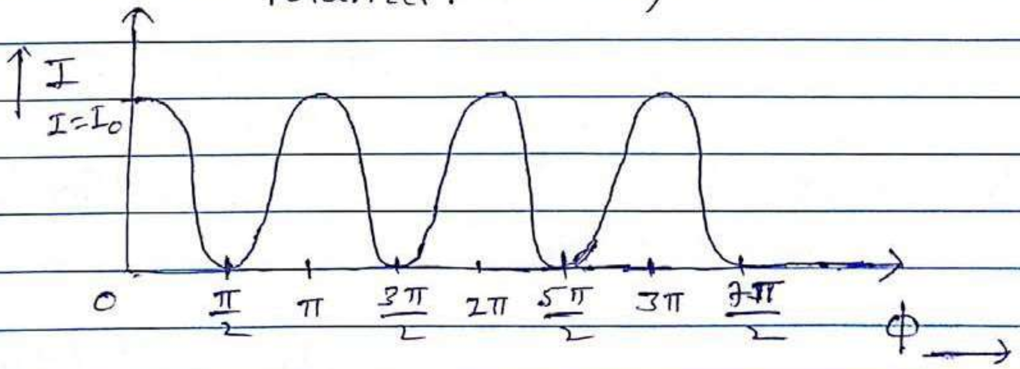
Malu's Law:

When an unpolarized light is incident on the polarizer, and then passed through the analyzer, the intensity (I) of light transmitted by analyzer depends upon the intencity (I_0) transmitted by polarizer and the square of the cosine of the angle between the axes of polarizer and analyzer.
i.e.

$$I = I_0 \cos^2(\phi) \quad — (1)$$

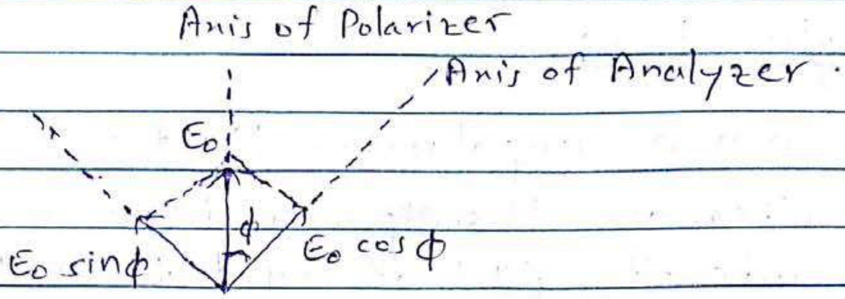
where ϕ is the angle between the axes of polarizer and the axes of analyzer.





Same enplaination interms of amplitude and its components:

If 'ϕ' is the angle between the axis of polarizer and analyzer. And 'E_0' is the amplitude of light transmitted by polarizer and 'E' is the amplitude of light transmitted by analyzer.



Then, $E = E_0 \cos\phi$ ——— (2)

As $E_0 \sin\phi$ component is perpendicular to the axis of analyzer and hence it is blocked.

Squaring equation (2).

$$\therefore\ E^2 = E_0^2 \cos^2\phi$$

$$\Rightarrow\ I = I_0 \cos^2\phi$$

This is Malus Law.