# Amplitude analysis of $\Lambda_b \to J/\psi\, Kp$ decays

Iraq Rabadan
for the Cinvestav group

BPH Spectroscopy Subgroup meeting

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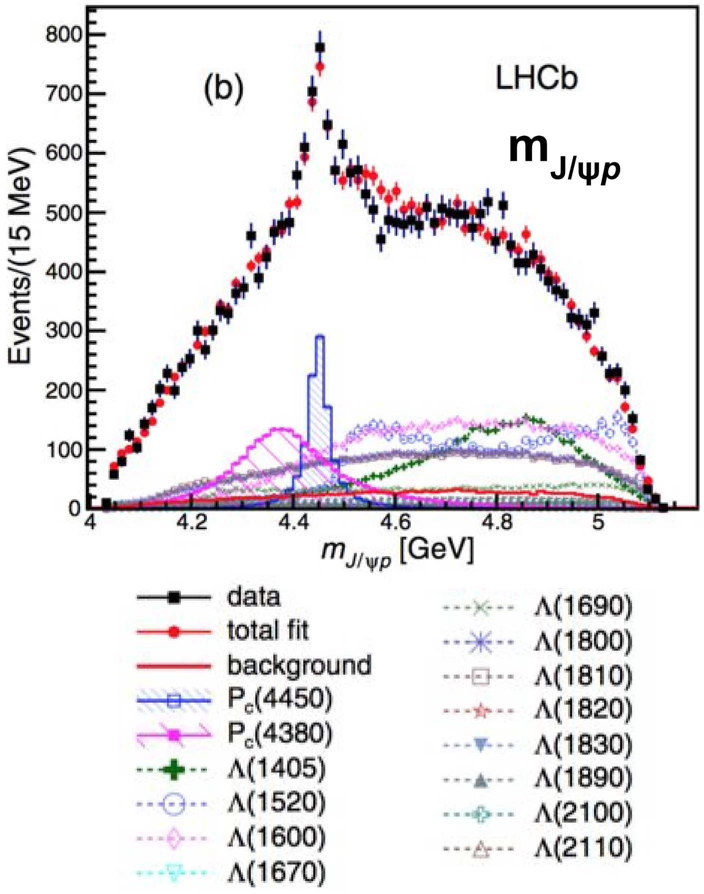
# Introduction

Last summer LHCb surprised us with the discovery of J/ψ p resonances consistent with pentaquarks states.

A 6-D analysis of mass ($m_{Kp}$) and 5 angles was used to properly describe the three body final state of $\Lambda_b \rightarrow$ J/ψ Kp decays, and to discover the nature of the unexpected peaks.

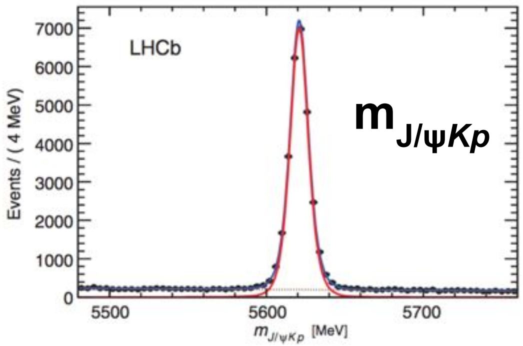
A satisfactory fit in the J/ψ p mass distribution is found only by including 2 additional resonances of masses ~4380 and ~4449 MeV, widths ~205 and ~39 MeV, and spins 3/2 and 5/2, respectively.

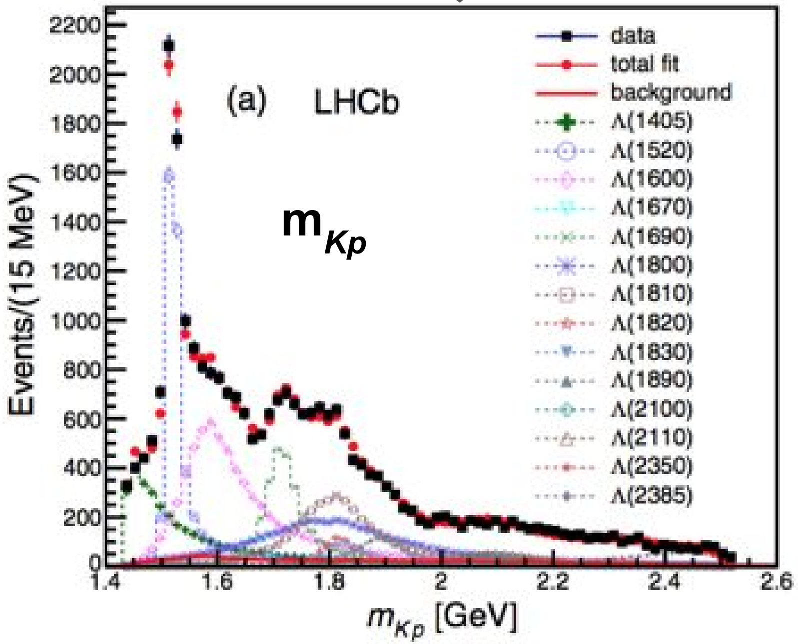


# $\Lambda^* \to$ Kp activity

| State | $J^P$ | $M_0$ (MeV) | $\Gamma_0$ (MeV) |
|---|---|---|---|
| $\Lambda(1405)$ | $1/2^-$ | $1405.1^{+1.3}_{-1.0}$ | $50.5 \pm 2.0$ |
| $\Lambda(1520)$ | $3/2^-$ | $1519.5 \pm 1.0$ | $15.6 \pm 1.0$ |
| $\Lambda(1600)$ | $1/2^+$ | 1600 | 150 |
| $\Lambda(1670)$ | $1/2^-$ | 1670 | 35 |
| $\Lambda(1690)$ | $3/2^-$ | 1690 | 60 |
| $\Lambda(1800)$ | $1/2^-$ | 1800 | 300 |
| $\Lambda(1810)$ | $1/2^+$ | 1810 | 150 |
| $\Lambda(1820)$ | $5/2^+$ | 1820 | 80 |
| $\Lambda(1830)$ | $5/2^-$ | 1830 | 95 |
| $\Lambda(1890)$ | $3/2^+$ | 1890 | 100 |
| $\Lambda(2100)$ | $7/2^-$ | 2100 | 200 |
| $\Lambda(2110)$ | $5/2^+$ | 2110 | 200 |
| $\Lambda(2350)$ | $9/2^+$ | 2350 | 150 |
| $\Lambda(2585)$ | ? | $\approx 2585$ | 200 |

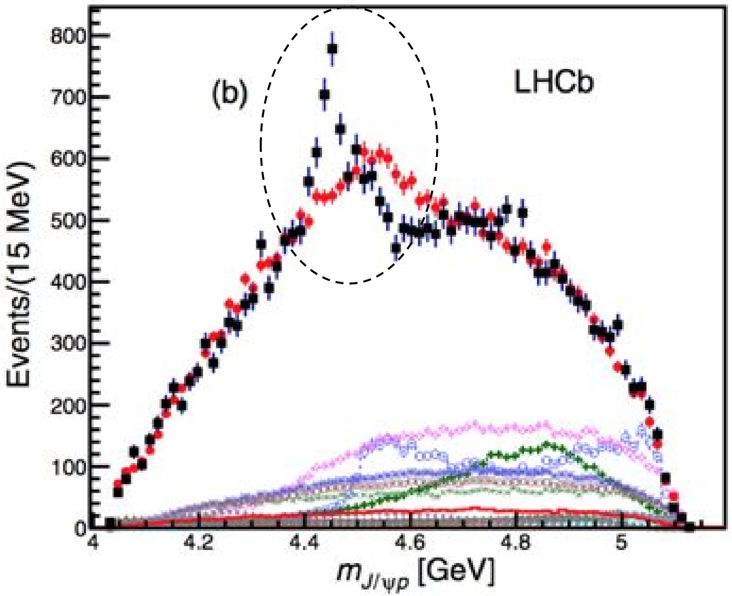
LHCb assumes $5/2^-$



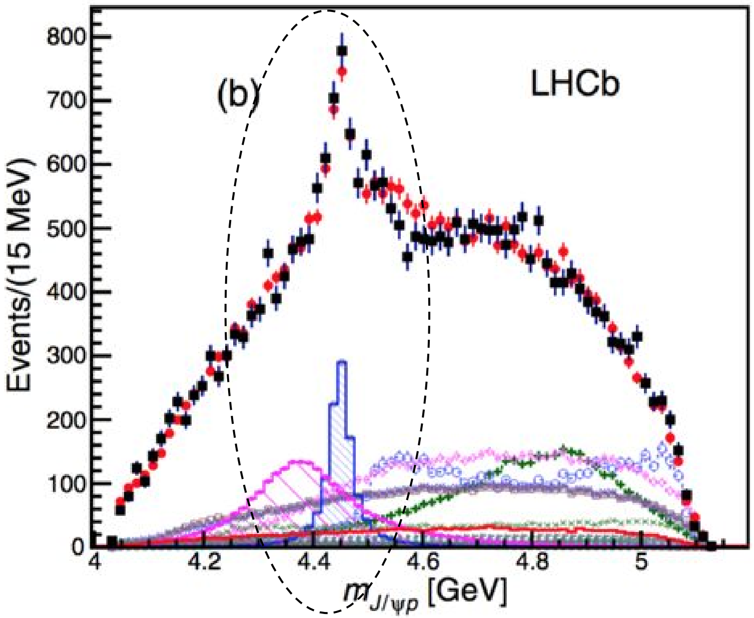
# Λ* → Kp activity was not enough ...

- Λ* → Kp reflections could not describe the **J/ψ p** distribution. Unless ...




**Adding two resonances**

**$P_c(4450) \rightarrow J/\psi\ p$**
**$P_c(4380) \rightarrow J/\psi\ p$**

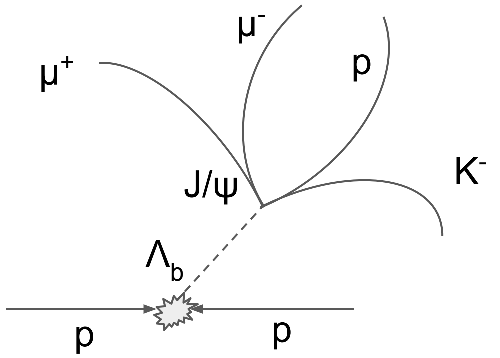


# Goal

**Reproduce the LHCb analysis using CMS 2012 data**:

1. 6-D analysis of $\Lambda_b \rightarrow J/\psi\, Kp$ decays. → Ongoing work. **This talk**.
    a. $\Lambda_b \rightarrow J/\psi\, Kp$ **reconstruction** and selection optimization.
    b. $\Lambda_b \rightarrow J/\psi\, Kp$ amplitudes fit: **signal model**, background description (reflections, modeling), efficiencies (MC), etc.
2. Addition of 1-2 extra resonant components. → We are not there yet.
    a. Confirm or reject LHCb pentaquarks.

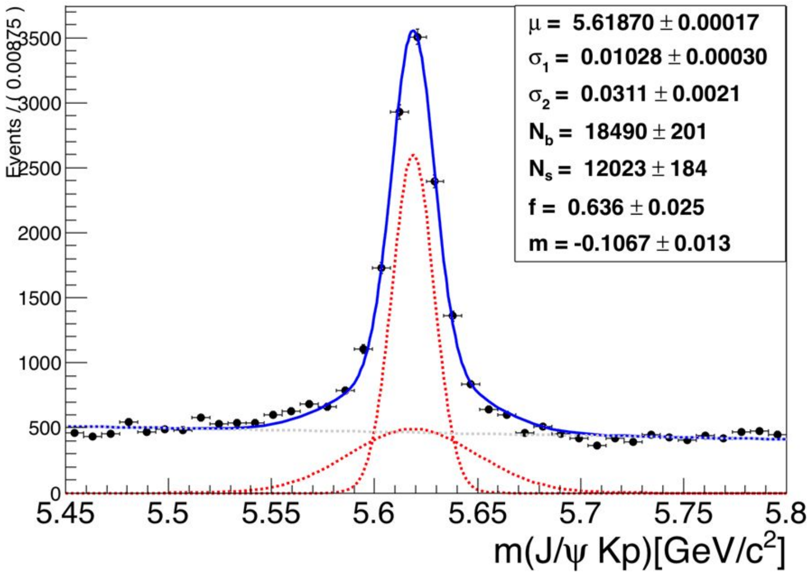
# Decay reconstruction

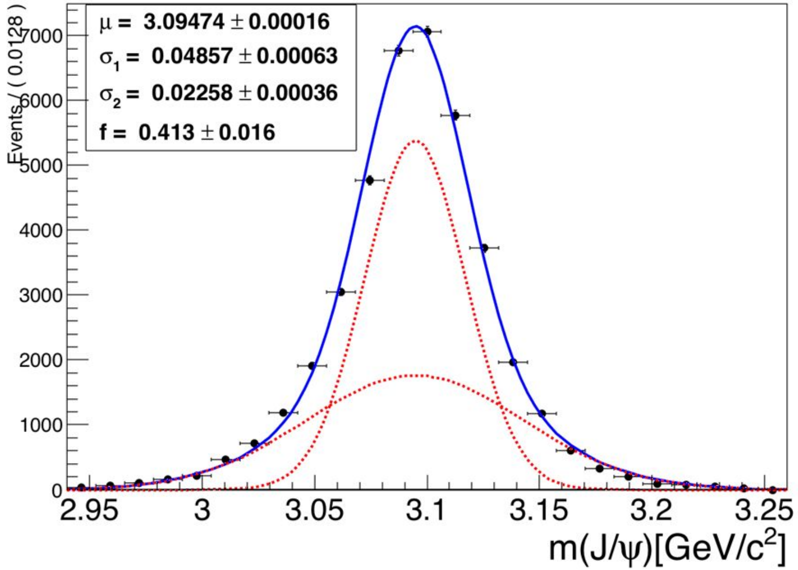



- 2012 “Muonia” sample (so far).
- Displaced J/ψ triggers.
  - Muons must match trigger.
- Soft-muon selection.
- J/ψ mass window: (2.95,3.25)
- Two muons and two high purity tracks, making a vertex:
  - Assumed to be proton and a kaon. Two possible assumptions: $Kp = t_1 t_2$ or $t_2 t_1$. Testing both.
  - pT > 0.7 GeV.
- Kinematic vertex fit of 4 tracks:
  - Dimuon mass constrained to the J/ψ W.A. mass to improve the $\Lambda_b$ mass resolution and correct muon momenta.
  - Multiple candidates kept (so far).

# MC simulation

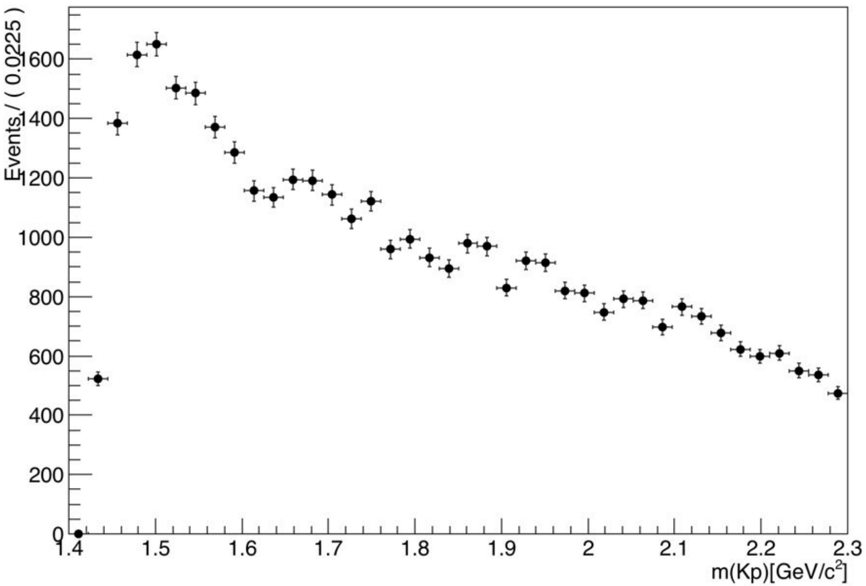
- $\Lambda_b \to J/\psi(\to\mu^+\mu^-)\,Kp$ private MC generated using phase space models in order to test reconstruction code, optimize selection, understand contaminations, efficiencies, etc.

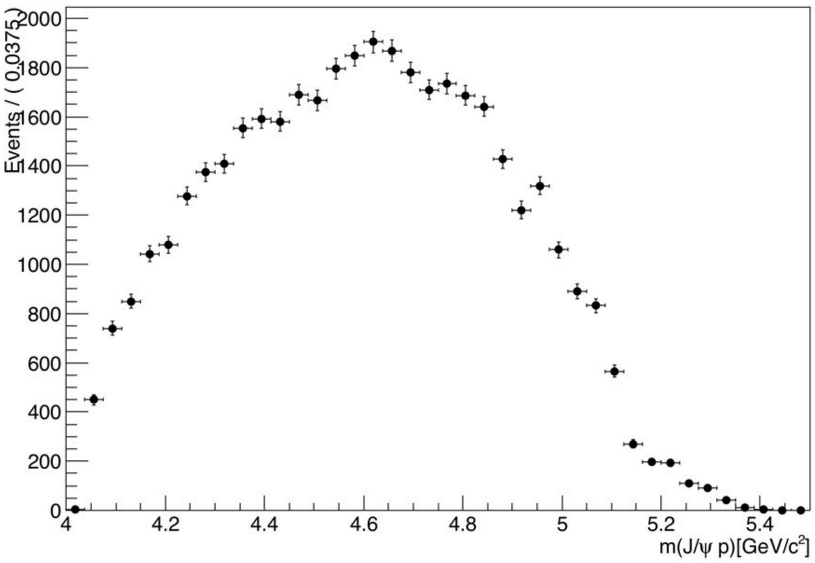



- Working on MC-matching.
- Official MC was requested: 50 M, no generation filters.

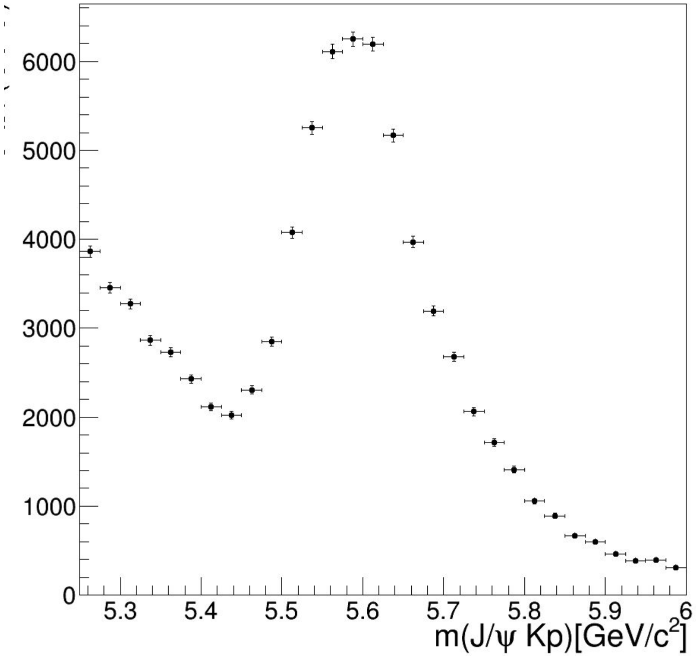
# MC simulation



# Data after some cuts

- Cuts
- pt(J/ψ) > 15
- $c\tau/\sigma_{c\tau}$ >5
- pt(Kp) >7
- $\chi^2_{vtx}$(J/ψKp), $\chi^2_{vtx}$(J/ψK) > 5
- $\chi^2_{vtx}$(J/ψp), $\chi^2_{vtx}$(Kp) > 5



# Decay angles (Helicity formalism)

The choice of the $\hat{z}_0^{\{\Lambda_b^0\}}$ direction for the $\Lambda_b^0$ spin quantization is arbitrary.

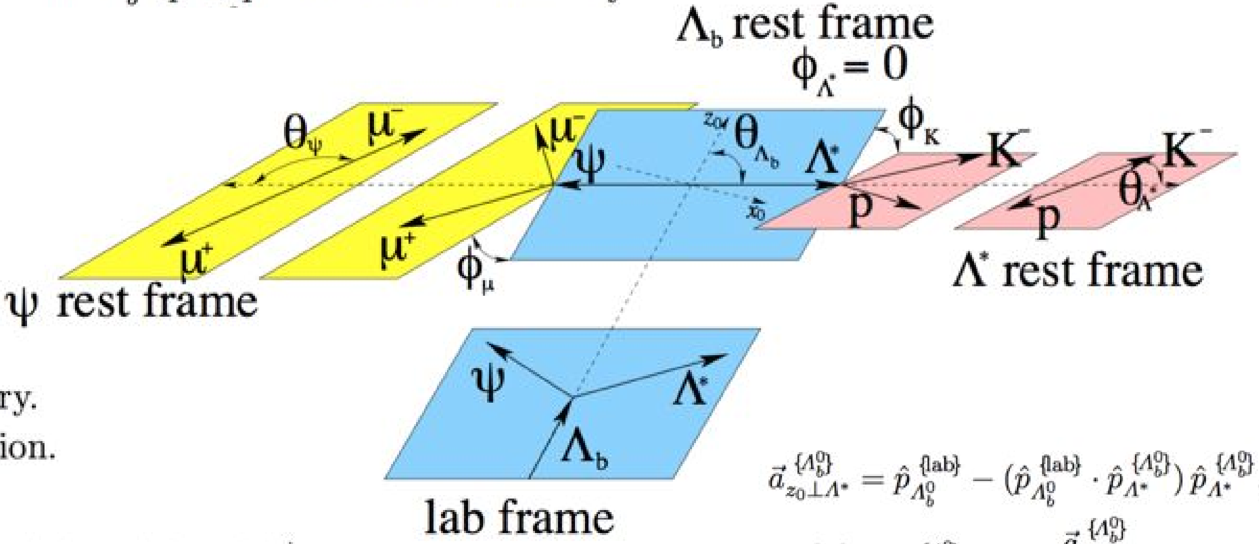
$$\hat{z}_0^{\{\Lambda_b^0\}} = \hat{p}_{\Lambda_b^0}^{\{\text{lab}\}}$$

$$\cos\theta_{\Lambda_b^0} = \hat{p}_{\Lambda_b^0}^{\{\text{lab}\}} \cdot \hat{p}_{\Lambda^*}^{\{\Lambda_b^0\}}$$

$$\cos\theta_{\Lambda^*} = -\hat{p}_{\psi}^{\{\Lambda^*\}} \cdot \hat{p}_{K}^{\{\Lambda^*\}}$$

The choice of $\hat{x}_0^{\{\Lambda_b^0\}}$ direction in the $\Lambda_b^0$ rest frame is also arbitrary.

$\Rightarrow$ $\phi_{\Lambda^*}$ angle zero by definition.




Etc.:

$$\phi_K = \operatorname{atan2}\left(-(\hat{p}_{\psi}^{\{\Lambda^*\}} \times \hat{x}_0^{\{\Lambda^*\}}) \cdot \hat{p}_K^{\{\Lambda^*\}},\ \hat{x}_0^{\{\Lambda^*\}} \cdot \hat{p}_K^{\{\Lambda^*\}}\right)$$

$$\cos\theta_\psi = -\,\hat{p}_{\Lambda^*}^{\{\psi\}} \cdot \hat{p}_{\mu}^{\{\psi\}}, \qquad \phi_\mu = \operatorname{atan2}\left(-(\hat{p}_{\Lambda^*}^{\{\psi\}} \times \hat{x}_0^{\{\psi\}}) \cdot \hat{p}_\mu^{\{\psi\}},\ \hat{x}_0^{\{\psi\}} \cdot \hat{p}_\mu^{\{\psi\}}\right)$$

$$\vec{a}_{z_0\perp\Lambda^*}^{\{\Lambda_b^0\}} = \hat{p}_{\Lambda_b^0}^{\{\text{lab}\}} - (\hat{p}_{\Lambda_b^0}^{\{\text{lab}\}} \cdot \hat{p}_{\Lambda^*}^{\{\Lambda_b^0\}})\, \hat{p}_{\Lambda^*}^{\{\Lambda_b^0\}},$$

$$\hat{x}_0^{\{\Lambda^*\}} = \hat{x}_3^{\{\Lambda_b^0\}} = -\frac{\vec{a}_{z_0\perp\Lambda^*}^{\{\Lambda_b^0\}}}{|\vec{a}_{z_0\perp\Lambda^*}^{\{\Lambda_b^0\}}|}.$$

$$\hat{x}_0^{\{\psi\}} = \hat{x}_0^{\{\Lambda^*\}} = \hat{x}_3^{\{\Lambda_b^0\}}$$

# Probability Density Function

$$\boxed{\mathcal{P}(m_{Kp}, \Omega|\overrightarrow{\omega}) = (1-\beta)\,\mathcal{P}_{\text{sig}}(m_{Kp}, \Omega|\overrightarrow{\omega}) + \beta\,\mathcal{P}_{\text{bkg}}(m_{Kp}, \Omega).}$$

Bkg. model from:
- $\Lambda_b$ Sidebands
- B → J/ψ X MC

$$\mathcal{P}_{\text{sig}}(m_{Kp}, \Omega|\overrightarrow{\omega}) = \frac{1}{I(\overrightarrow{\omega})}\,|\mathcal{M}(m_{Kp}, \Omega|\overrightarrow{\omega})|^2\,\Phi(m_{Kp})\epsilon(m_{Kp}, \Omega).$$

Efficiency from reconstructed PHSP MC

Normalization integral

phase space function equal to $p\,q$

$p$ is the $\Lambda^*$ momentum in the $\Lambda_b^0$ rest frame ($p = |\vec{p}_{\Lambda^*}^{\,\{\Lambda_b^0\}}|$).

$q$ is the $K^-$ momentum in the $\Lambda^*$ rest frame ($q = |\vec{p}_K^{\,\{\Lambda^*\}}|$)

$$|\mathcal{M}^{\Lambda^*}|^2 = \frac{1+\mathrm{P}^{\Lambda_b^0}}{2}\sum_{\lambda_p}\sum_{\Delta\lambda_\mu}\left|\mathcal{M}_{(\lambda_{\Lambda_b^0}=+1/2),\lambda_p,\Delta\lambda_\mu}\right|^2 + \frac{1-\mathrm{P}^{\Lambda_b^0}}{2}\sum_{\lambda_p}\sum_{\Delta\lambda_\mu}\left|\mathcal{M}_{(\lambda_{\Lambda_b^0}=-1/2),\lambda_p,\Delta\lambda_\mu}\right|^2$$

$$\mathcal{M}^{\Lambda^*}_{\lambda_{\Lambda_b^0},\lambda_p,\Delta\lambda_\mu} = \sum_n R_{\Lambda_n^*}(m_{Kp})\,\mathcal{H}^{\Lambda_n^*\to Kp}_{\lambda_p}\sum_{\lambda_\psi} e^{i\lambda_\psi\phi_\mu}\,d^1_{\lambda_\psi,\Delta\lambda_\mu}(\theta_\psi)$$
$$\times\sum_{\lambda_{\Lambda^*}}\mathcal{H}^{\Lambda_b^0\to\Lambda_n^*\psi}_{\lambda_{\Lambda^*},\lambda_\psi}\,e^{i\lambda_{\Lambda^*}\phi_K}\,d^{\frac{1}{2}}_{\lambda_{\Lambda_b^0},\lambda_{\Lambda^*}-\lambda_\psi}(\theta_{\Lambda_b^0})\,d^{J_{\Lambda_n^*}}_{\lambda_{\Lambda^*},\lambda_p}(\theta_{\Lambda^*})$$

# Matrix Element for $\Lambda_b \rightarrow J/\psi\ \Lambda^*$

$\Lambda_b$ helicity amplitudes

$\Lambda_b$ L-S amplitudes

Clebsch-Gordan Coefficients

$$\mathcal{H}^{A\to BC}_{\lambda_B,\lambda_C} = \sum_L \sum_S \sqrt{\frac{2L+1}{2J_A+1}} B_{L,S} \left( \begin{array}{cc|c} J_B & J_C & S \\ \lambda_B & -\lambda_C & \lambda_B - \lambda_C \end{array} \right) \times \left( \begin{array}{cc|c} L & S & J_A \\ 0 & \lambda_B - \lambda_C & \lambda_B - \lambda_C \end{array} \right)$$

Spin ½ Wigner's D-Matrix

$\Lambda^*$ helicity amplitudes:

$$\mathcal{H}^{\Lambda_n^*\to Kp}_{+\frac{1}{2}} = (1,0)$$

$$\mathcal{H}^{\Lambda_n^*\to Kp}_{-\frac{1}{2}} = (P_{\Lambda_n^*}(-1)^{J_{\Lambda_n^*}-\frac{3}{2}},0)$$

$$\boxed{\begin{aligned} \mathcal{M}^{\Lambda^*}_{\lambda_{\Lambda_b^0},\lambda_p,\Delta\lambda_\mu} \equiv \sum_n \sum_{\lambda_{\Lambda^*}} \sum_{\lambda_\psi} & \mathcal{H}^{\Lambda_b^0\to\Lambda_n^*\psi}_{\lambda_{\Lambda^*},\lambda_\psi} D^{1/2}_{\lambda_{\Lambda_b^0},\lambda_\Lambda^*-\lambda_\psi}(0,\theta_{\Lambda_b^0},0)^* \\ & \times \mathcal{H}^{\Lambda_n^*\to Kp}_{\lambda_p,0} D^{J_{\Lambda_n^*}}_{\lambda_{\Lambda^*},\lambda_p}(\phi_K,\theta_{\Lambda^*},0)^* R_{\Lambda_n^*}(m_{Kp}) \\ & \times D^1_{\lambda_\psi,\Delta\lambda_\mu}(\phi_\mu,\theta_\psi,0)^*, \end{aligned}}$$

**14 resonances:**
***n*** = 1, 2, …, 14
⇒ **148 amplitudes**,
2 fixed by the normalization.

Spin 1 Wigner's D-Matrix

Spin $J_{\Lambda^*}$ Wigner's D-Matrix

$$R_X(m) = B'_{L^X_{\Lambda_b^0}}(p,p_0,d)\left(\frac{p}{M_{\Lambda_b^0}}\right)^{L^X_{\Lambda_b^0}} \times \mathrm{BW}(m|M_{0X},\Gamma_{0X}) B'_{L_X}(q,q_0,d)\left(\frac{q}{M_{0X}}\right)^{L_X}.$$

# Mass dependent terms

$$\boxed{R_{\Lambda_n^*}(m_{Kp}) = \underbrace{B'_{L^{\Lambda_n^*}_{\Lambda_b^0}}(p, p_0, d)\left(\frac{p}{M_{\Lambda_b^0}}\right)^{L^{\Lambda_n^*}_{\Lambda_b^0}}}_{\text{Barrier factor for } \Lambda_b \to J/\psi\, \Lambda^*} \mathrm{BW}(m_{Kp}|M_0^{\Lambda_n^*}, \Gamma_0^{\Lambda_n^*})\, \underbrace{B'_{L_{\Lambda_n^*}}(q, q_0, d)\left(\frac{q}{M_0^{\Lambda_n^*}}\right)^{L_{\Lambda_n^*}}}_{\text{Barrier factor for } \Lambda^* \to Kp}}$$

$$\mathrm{BW}(m|M_0, \Gamma_0) = \frac{1}{M_0^2 - m^2 - iM_0\Gamma(m)}$$

Blatt-Weisskopf functions:

$$B'_0(p, p_0, d) = 1\,,$$

$$B'_1(p, p_0, d) = \sqrt{\frac{1 + (p_0\, d)^2}{1 + (p\ \ d)^2}}\,,$$

$$B'_2(p, p_0, d) = \sqrt{\frac{9 + 3(p_0\, d)^2 + (p_0\, d)^4}{9 + 3(p\ \ d)^2 + (p\ \ d)^4}}\,,$$

etc.

$$\Gamma(m) = \Gamma_0 \left(\frac{q}{q_0}\right)^{2L_{\Lambda^*}+1} \frac{M_0}{m} B'_{L_{\Lambda^*}}(q, q_0, d)^2$$

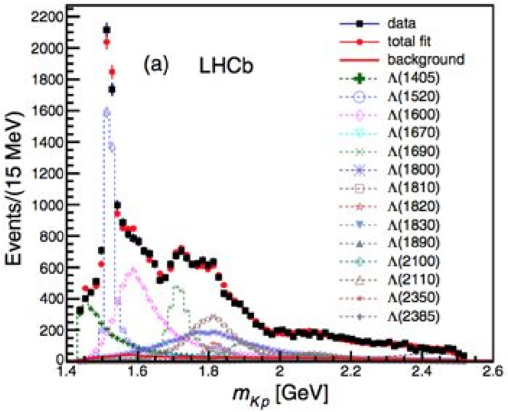
$p_0$ and $q_0$ denote values of $p$ and $q$ at $m = M_{0X}$

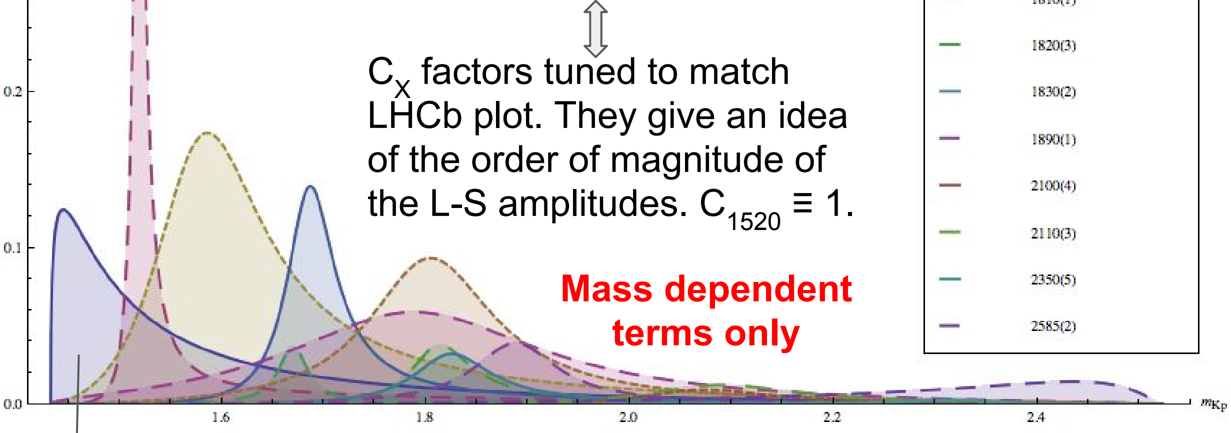
$d = 3.0\ \mathrm{GeV}^{-1}$ = size of the decaying particle

# Signal model

- Signal model built using **Mathematica**.
  - Programming optimized for further analytical integration.
  - No approximations.
- Used built-in functions (WignerD, etc.). All were cross-checked with the literature.

$|C_X R_X(m_{Kp})|^2 \, \Phi(m_{Kp})$




$C_X$ factors tuned to match LHCb plot. They give an idea of the order of magnitude of the L-S amplitudes. $C_{1520} \equiv 1$.

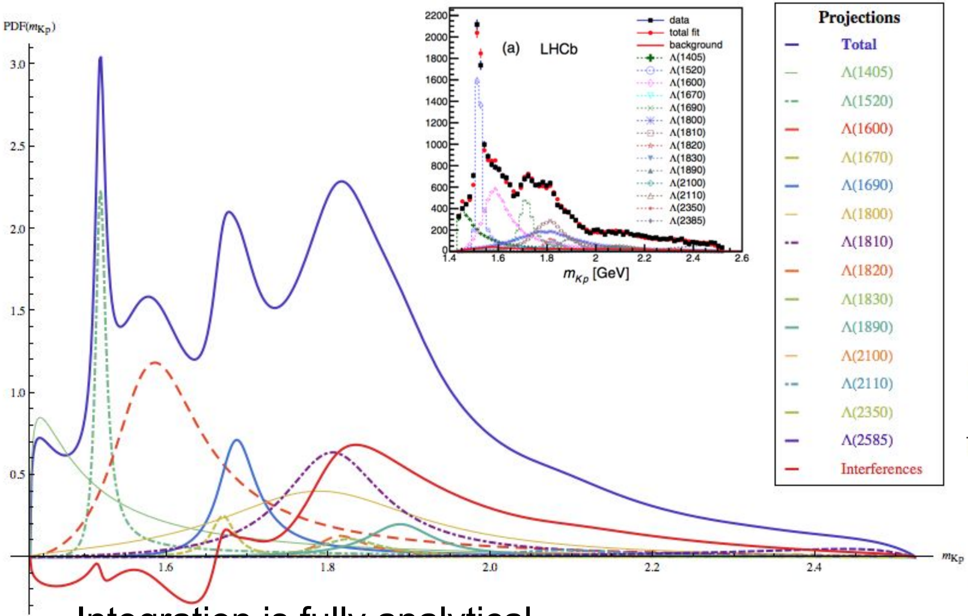
**Mass dependent terms only**

Λ*(1405) ⇨ two-component Flatté-like parametrization

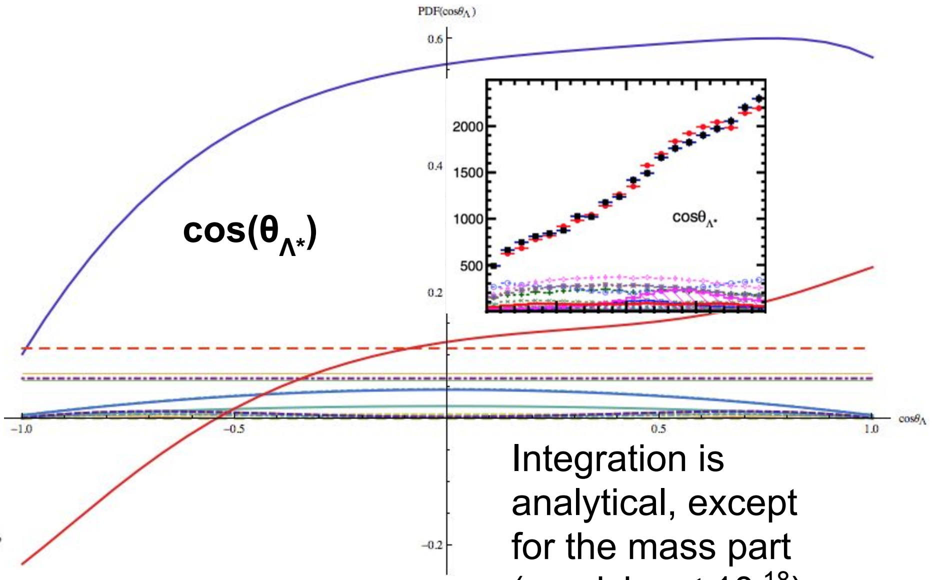
$$\Gamma(m) = \Gamma(m)_{K^-p} + \Gamma(m)_{\Sigma\pi}$$

# 5D integration of complete signal model

- **Example:** $B^X_{L,S}$ amplitudes set to $\mathrm{Re}(B^X_{L,S}) = C_X$ and $\mathrm{Im}(B^X_{L,S}) = 0$.




Integration is fully analytical.

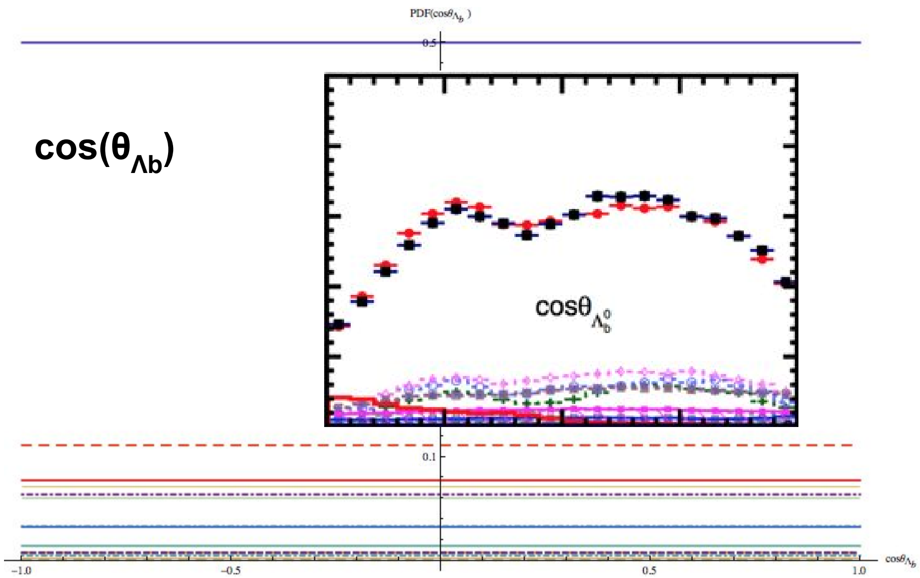



Integration is analytical, except for the mass part (precision $\lesssim 10^{-18}$).

**Interferences are important!**

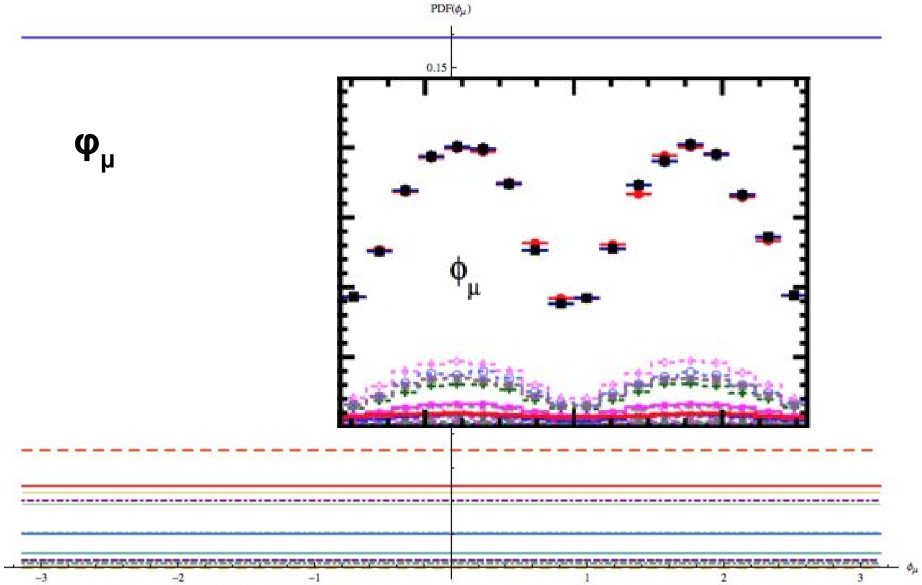
# 5D integration of complete signal model (II)

**$\cos(\theta_{\Lambda b})$**




Flat distribution expected for **$\cos(\theta_{\Lambda b})$** due to $P_{\Lambda b} = 0$

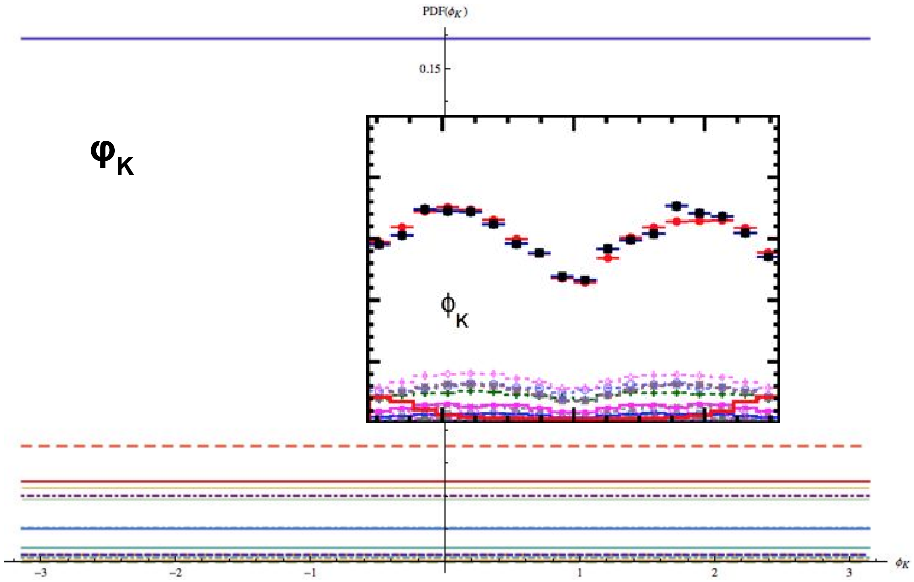
**$\varphi_\mu$**

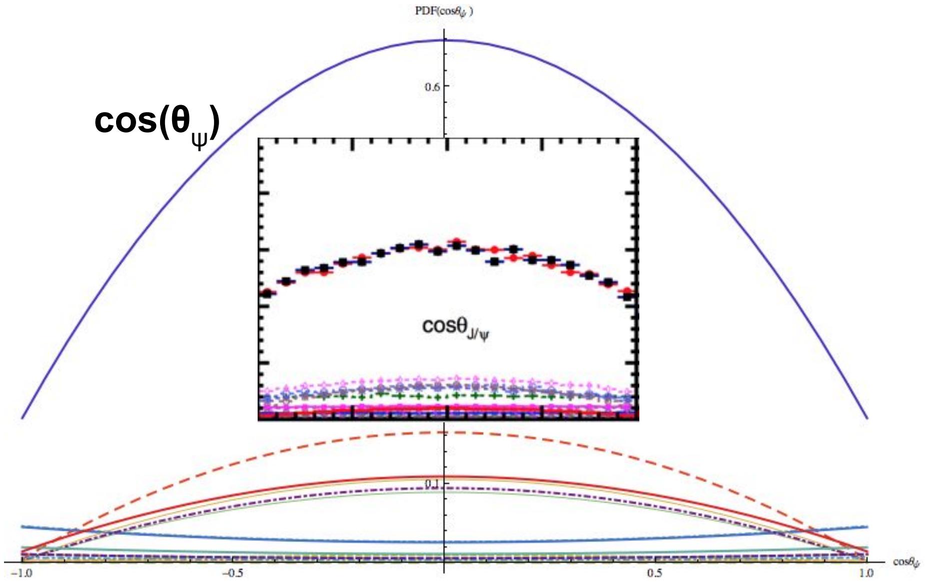



Flat distributions expected for all azimuthal angles.

**Efficiencies are important!**

# 5D integration of complete signal model (III)

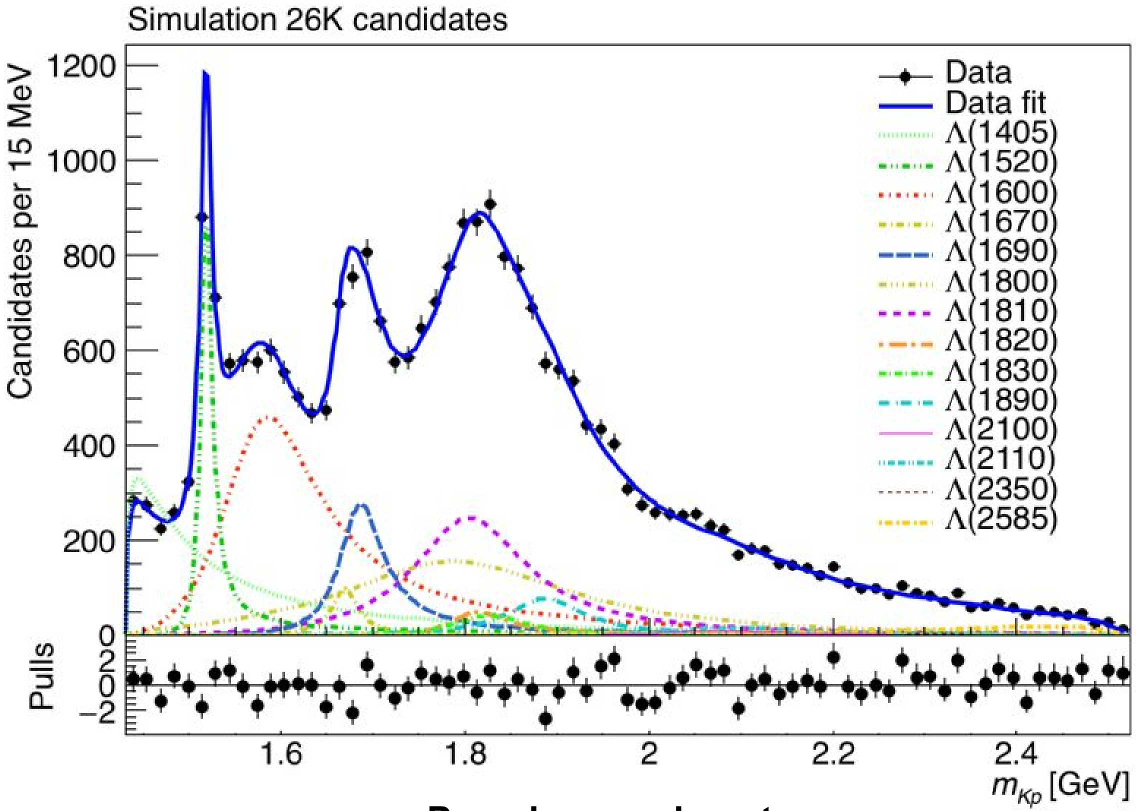



Flat distributions expected
for all azimuthal angles

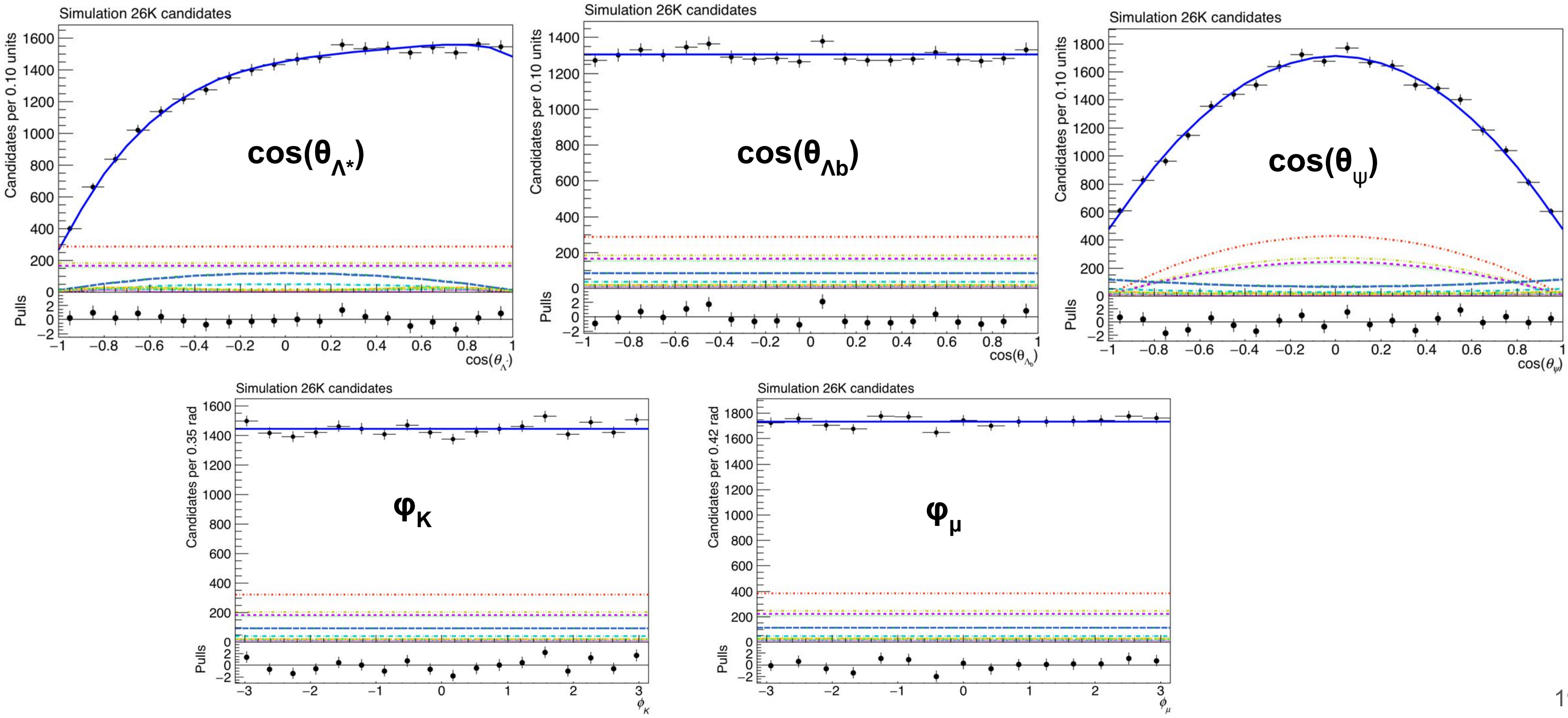
# Fitting code

- Signal model was ported to RooFit.
  - Programming optimized for fast evaluation and negligible precision loss.
- RooFit generates pseudo-experiments.
- RooFit performs 5D integration numerically (or can use “advertised” integrals).
- Fitting tests ongoing.




**Pseudo-experiments**

# Angular projections using RooFit



# Summary

- Decay reconstruction is done.
- Reflections and other contaminations are being studied in data and MC.
    - Selection will be optimized using data (sidebands) and MC (signal and backgrounds) using multivariate techniques such as BDTs.
- 6D signal code is all set.
    - We are “learning” to fit at this level, with only a few resonances.
    - Then we will introduce efficiencies.
    - Then background.
    - ...
- A lot of work to be done yet.
- Group:
    - Analysis: Iraq Rabadan, Rogelio Reyes.
    - Advise: Ivan Heredia, Heriberto Castilla.
    - Collaboration with $\Lambda_b \rightarrow \mu^+\mu^- Kp$ team: Cecilia Duran, Jhovanny Mejia, Eduard de la Cruz.

# Spares

# Resonance fractions

The fraction for a given resonance is the ratio of the phase space integrals of $|\mathcal{M}|^2_X$ calculated for the resonance amplitude X taken alone and for the total $|\mathcal{M}|^2$ summing over all contributions.

The sum of the fractions is not necessarily unity due to the potential presence of interference between two resonances.

Interference between different spin-J states vanishes.

| | LHCb Fit | Our example |
|---|---|---|
| Particle | Fit fraction (%) cFit | **Fraction (%)** |
| $P_c(4380)^+$ | $8.42 \pm 0.68$ | 0 |
| $P_c(4450)^+$ | $4.09 \pm 0.48$ | 0 |
| $\Lambda(1405)$ | $14.64 \pm 0.72$ | 12.05 |
| $\Lambda(1520)$ | $18.93 \pm 0.52$ | 6.64 |
| $\Lambda(1600)$ | $23.50 \pm 1.48$ | 22.25 |
| $\Lambda(1670)$ | $1.47 \pm 0.49$ | 1.32 |
| $\Lambda(1690)$ | $8.66 \pm 0.90$ | 6.48 |
| $\Lambda(1800)$ | $18.21 \pm 2.27$ | 14.24 |
| $\Lambda(1810)$ | $17.88 \pm 2.11$ | 12.76 |
| $\Lambda(1820)$ | $2.32 \pm 0.69$ | 1.51 |
| $\Lambda(1830)$ | $1.76 \pm 0.58$ | 1.46 |
| $\Lambda(1890)$ | $3.96 \pm 0.43$ | 2.82 |
| $\Lambda(2100)$ | $1.65 \pm 0.29$ | 0.45 |
| $\Lambda(2110)$ | $1.62 \pm 0.32$ | 0.93 |
| Λ(2350) | ? | 0.08 |
| Λ(2585) | ? | 1.52 |
| TOTAL | **127.11** | **84.51** |
| Interferences | **-27.11 ?** | **15.49** |
| TOTAL + Interf. | 100 ? | 100 |