Report 2021.03.01

***Abstract*—**

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I. Introduction

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A. Network Model

System consists of $M+1$ base stations (BSs), including one Macrocell Base Station (MBS) and M Smallcell Base Stations (SMSs), denoted by the set $\mathcal{M} = \{0, 1, 2, ..., M\}$, where base station 0 indicates MBS, and N user equipments (UEs) $\mathcal{N} = \{1, 2, ..., N\}$. All SBSs are associated with the MBS by wirelinks. Each UE i has a task $I_i = \{\beta_i; \alpha_i; \tau_i\}$, where

- β_i [cycles] specifies the amount of CPU computation to complete the task;
- α_i [bits] is the input data size including information of system settings, program codes, and other parameters being transmitted in case of remote computation;
- τ_i [seconds] is upper limit on latency for completing the task.

Each task I_i is atomic and can not be devided into subtasks.

Consider all base stations have the same number of subchannels S. The set of subchannels of a base station: $\mathcal{S} = \{1, 2, ..., S\}$. Each UE in the coverage area of the same BS m ($m \in \mathcal{M}$) can do its task locally or offload its task to the BS m using NOMA, over subchannel j ($j \in \mathcal{S}$), one of the subchannels from the set $\mathcal{S}$.

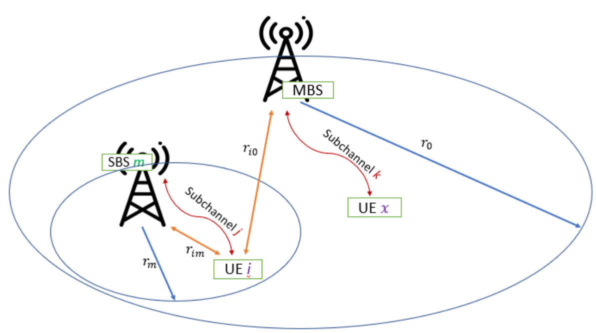

Fig. 1. Network Model.

Devices are associated with the MBS or SBSs.

- We consider devices first communicate with the nearest SBS to perform computation offloading.
- Denote r_m is the coverage radius of base station m ($m \in \mathcal{M}$), r_{im} is the distance between UE i ($i \in \mathcal{N}$) and base station m.
- If $r_{im} < r_m$, device i can communicate with SBS m, for $m \neq 0$.
- If device i does not lie within the coverage area of any SBS, it will communicate with the MBS (with condition of $r_{i0} < r_0$).

B. Local and Remote Computations

For local computation, the time (in seconds) denoted by T_i^l that UE i ($i \in \mathcal{N}$) takes to complete its task is

$$T_i^l = \frac{\beta_i}{f_i^l}, \tag{1}$$

where f_i^l is the computing capability(in cycles/second) of UE i. The energy E_i^l (in Joules) consumed for this task is

$$E_i^l = \kappa_i \beta_i \left(f_i^l\right)^2, \tag{2}$$

where κ_i is the energy coefficient depending on UE i's chip architecture.

For remote computation, there are 3 types of costs in terms of time and energy that UE i suffers when offloading the task to the MEC server: i) the time and energy for transmitting the task to server; ii) the time for executing the task at the server; and iii) the time for transmitting the output back from server.

Define a binary subchannel association (SA) variable x_{ijm}, where $i \in \mathcal{N}$, $j \in \mathcal{S}$, $m \in \mathcal{M}$:

- $x_{ijm} = 0$ indicates that UE i computes its task locally;
- $x_{ijm} = 1$ indicates that UE i offload its task to BS m via subchannel j.

Since each UE is assumed to use at most one subchannel for offloading, the constraint below should be satisfied:

$$\sum_{j \in \mathcal{S}} x_{ijm} \leq 1, \forall i \in \mathcal{N}, \forall m \in \mathcal{M}. \tag{3}$$

Also, since each UE is assume to associate with at most one base station, we have:

$$\sum_{m \in \mathcal{M}} \sum_{j \in \mathcal{S}} x_{ijm} \leq 1, \forall i \in \mathcal{N}. \tag{4}$$

Offloading decision of UE i can be determined by $\sum_{m \in \mathcal{M}} \sum_{j \in \mathcal{S}} x_{ijm}$:

- $\sum_{m \in \mathcal{M}} \sum_{j \in \mathcal{S}} x_{ijm} = 1$ means that UE i decides to offload the task.
- $\sum_{m \in \mathcal{M}} \sum_{j \in \mathcal{S}} x_{ijm} = 0$ means that UE i decides to execute the task locally.

We consider the system with NOMA as the multiple access scheme in the uplink to each BS. All BSs use the same frequency resource, and the operational frequency band B is divided into S equal subbands of size $W = B/S$ [Hertz].

Denote $|h_{ijm}|^2$ is the channel power gain between UE i ($i \in \mathcal{N}$) and SBS m ($m \in \mathcal{M}$) via subchannel j ($j \in \mathcal{S}$).

Denote P_{tol}^m is a predefined threshold required for SIC decoder at the BS m. Denote A_j is the set of UEs offoading to the same subchannel j. The following condition should be satisfied:

$$\frac{p_{ij}\left|h_{ijm}\right|^2 (r_{im})^{-\alpha}}{\sum\limits_{\substack{k\in\mathcal{A}_j\\|h_{kj}|^2\le|h_{ij}|^2}} p_{xj}\left|h_{xjm}\right|^2 (r_{xm})^{-\alpha}} \ge P_{tol}, \forall i \in \mathcal{A}_j. \quad (5)$$

The data rate of UE i through subchannel j to BS m can be given as:

$$R_{ijm} = W log_2\left[1 + \frac{p_{ij}\left|h_{ijm}\right|^2 (r_{im})^{-\alpha}}{n_0 + \sum\limits_{\substack{k\in\mathcal{A}_j\\|h_{kj}|^2\le|h_{ij}|^2}} p_{xj}\left|h_{xjm}\right|^2 (r_{xm})^{-\alpha}}\right] \quad (6)$$

where p_{ij} is the transmit power of UE i with the chosen subchannel j, n_0 is the noise power, α is the path loss exponent.

Denote p_i is the transmit power of UE i, p_i^0 is the maximum power budget, we have

$$p_i = \sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{S}} x_{ijm} p_{ij} \le p_i^0. \quad (7)$$

The transmission time taken by UE i for offloading is

$$T_i^{off} = \sum_{j\in\mathcal{S}} \frac{x_{ijm}\alpha_i}{R_{ijm}}. \quad (8)$$

The energy cost that UE i suffers from the offloading is

$$E_i^{\text{off}} = \frac{p_i}{\xi_i} T_i^{\text{off}} = \frac{p_i}{\xi_i}\alpha_i \sum_{j\in\mathcal{S}} \frac{x_{ij}}{R_{ij}}, \quad (9)$$

where ξ_i is power amplifier efficiency.

At the server, the time for executing the task is

$$T_i^{\text{exe}} = \frac{\beta_i}{f_{im}}, \quad (10)$$

where f_{im} [cycles/second] is the server m's computation resource allocated to execute the task from UE i. It implies that when $f_{im} = 0$, UE i executes locally. Moreover,

$$\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{S}} x_{ijm} f_{im} \le f_m^0, \quad (11)$$

where f_m^0 is the total computing resource of the server of BS m.

Total time and energy that UE i consumes for the offloading:

$$T_i^r = T_i^{\text{off}} + T_i^{\text{exe}} = \alpha_i \sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{S}} \frac{x_{ijm}}{R_{ijm}} + \frac{\beta_i}{f_{im}}, \quad (12)$$

and

$$E_i^r = E_i^{\text{off}} = \frac{p_i}{\xi_i}\alpha_i \sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{S}} \frac{x_{ijm}}{R_{ijm}}. \quad (13)$$

The total time and energy taken by UE i for its task I_i:

$$T_i = (1 - \sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{S}} x_{ijm}) T_i^l + \sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{S}} x_{ijm} T_i^r, \quad (14)$$

$$E_i = (1 - \sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{S}} x_{ijm}) E_i^l + \sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{S}} x_{ijm} E_i^r. \quad (15)$$

The upper limit on latency for completing the task I_i is τ_i, so we have:

$$T_i \le \tau_i, \forall i \in \mathcal{N}. \quad (16)$$

III. Problem Formulation

Our problem is finding the subchannel associations, computation resource allocations and transmission powers to mimimize the total energy of our system. This problem can be formulated as:

$$\begin{aligned} \min_{\boldsymbol{X},\boldsymbol{P},\boldsymbol{F}} E &= \min_{\boldsymbol{X},\boldsymbol{P},\boldsymbol{F}} \sum_{i\in\mathcal{N}} E_i \\ &= \min_{\boldsymbol{X},\boldsymbol{P},\boldsymbol{F}} \sum_{i\in\mathcal{N}} \left[\left(1 - \sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{S}} x_{ijm}\right) E_i^l \right. \\ &\left. + \left(\sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{S}} x_{ijm}\right) E_i^r\right] \end{aligned} \quad (17)$$

$$\text{s.t. C1: } x_{ijm} \in \{0,1\}, \forall i \in \mathcal{N}, \forall j \in \mathcal{S},\ \forall m \in \mathcal{M}; \quad (18)$$

$$\text{C2: } \sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{S}} x_{ijm} \le 1,\ \forall i \in \mathcal{N}; \quad (19)$$

$$\text{C3: } p_{ij} \ge 0,\ \forall i \in \mathcal{N},\ \forall j \in \mathcal{S}; \quad (20)$$

$$\text{C4: } \sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{S}} x_{ijm} p_{ij} \le p_i^0,\ \forall i \in \mathcal{N}; \quad (21)$$

$$\text{C5: } \frac{p_{ij}\left|h_{ijm}\right|^2 (r_{im})^{-\alpha}}{\sum\limits_{\substack{k\in\mathcal{A}_j\\|h_{kj}|^2\le|h_{ij}|^2}} p_{xj}\left|h_{xjm}\right|^2 (r_{xm})^{-\alpha}} \ge P_{tol},\ \forall i \in \mathcal{A}_j; \quad (22)$$

$$\text{C6: } f_{im} \ge 0,\ \forall i \in \mathcal{N},\ \forall m \in \mathcal{M}; \quad (23)$$

$$\text{C7: } \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{S}} x_{ijm} f_{im} \le f_m^0,\ \forall m \in \mathcal{M}; \quad (24)$$

$$\text{C8: } T_i \le \tau_i,\ \forall i \in \mathcal{N},\ T_i \text{ is defined by (14).} \quad (25)$$

$$(26)$$

Denote $\Gamma(f(x)) = \begin{cases} 0, & \text{if } f(x) \le 0, \\ 1, & \text{if } f(x) > 0 \end{cases}$;

and

$$h_2(\boldsymbol{P_{u}}_i) = \sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{S}} x_{ijm} - 1,\ i \in \mathcal{N}, \tag{27}$$

$$h_4(\boldsymbol{P_{p}}_i) = \sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{S}} x_{ijm}p_{ij} - p_i^0,\ i \in \mathcal{N}, \tag{28}$$

$$\begin{aligned} h_5(\boldsymbol{P_{s}}_i) = P_{tol} \times & \sum_{\substack{k\in\mathcal{A}_j \\ |h_{kj}|^2 \le |h_{ij}|^2}} p_{xj}\left|h_{xjm}\right|^2 (r_{xm})^{-\alpha} \\ & - p_{ij}\left|h_{ijm}\right|^2 (r_{im})^{-\alpha},\ i \in \mathcal{N}, \end{aligned} \tag{29}$$

$$h_7(\boldsymbol{P_{f}}_m) = \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{S}} x_{ijm}f_{im} - f_m^0,\ m \in \mathcal{M}, \tag{30}$$

$$\begin{aligned} & h_8(\boldsymbol{P_{t}}_i) = T_i - \tau_i \\ & = (1 - \sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{S}} x_{ijm})T_i^l + \sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{S}} x_{ijm}T_i^r - \tau_i, \\ & \qquad (i \in \mathcal{N}). \end{aligned} \tag{31}$$

IV. Plan for next week

- Complete problem formation
- Realign equation (6) and (17) for a better look.