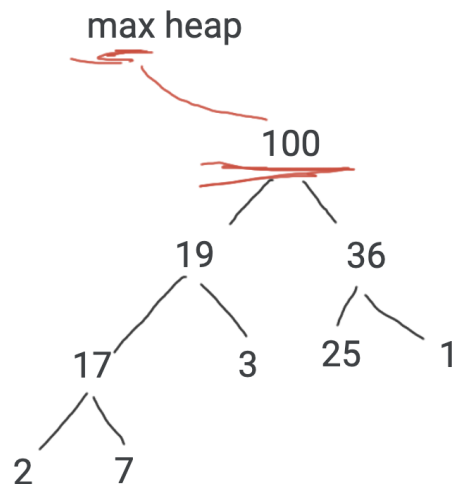


# Phill-DS-0220 0306

## Heap

- tree, binary tree 不是 binary search tree → 上下有大小, 左右不分大小
- 四種 heap → max heap, min heap, min-max heap, deap
- 應用
  - heap sort → efficient  $n \log n$
  - priority queue 的陽春版

## Max heap

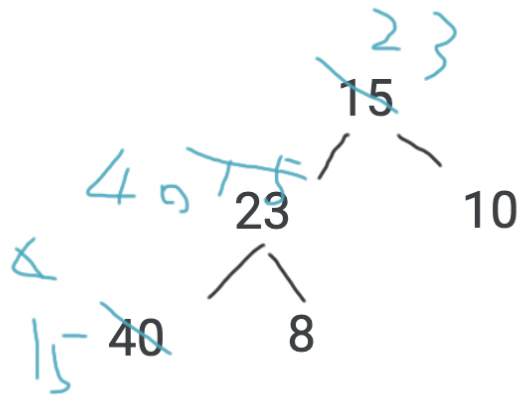


- 大小
- 排列方式 保證 最大值永遠在根節點
  - priority queue
  - $O(n \log n)$
  - sorting

# 建立 heap

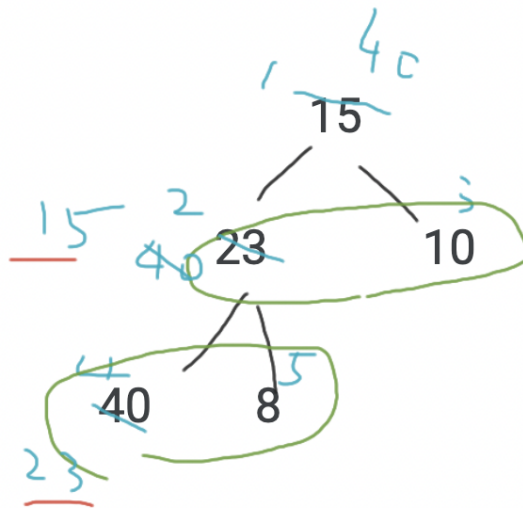
由上到下(調整)

- 鐵三角比較：上下比兩次 or 左右比再上比
- 逐層比較由上往下
- 比完再向上比一次 以免造成層與層之間大小不對的狀況



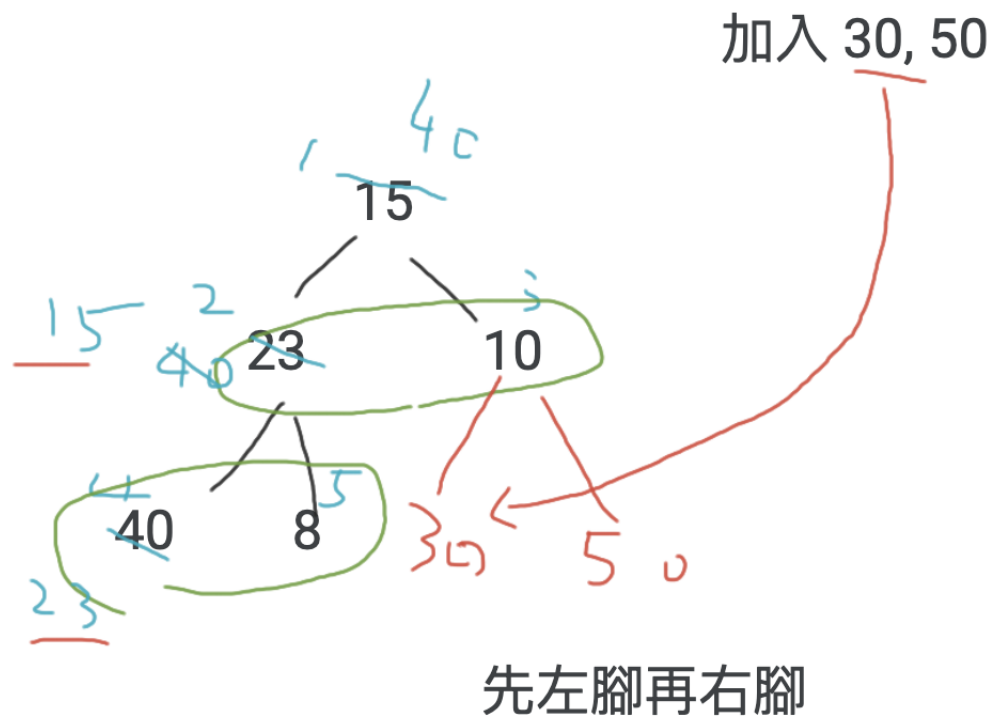
由下往上

- 對節點做編號 以大號碼開始處理
- 鐵三角同層比較 由下往上進行
- 比完以後還要向下比一次



## 加入元素

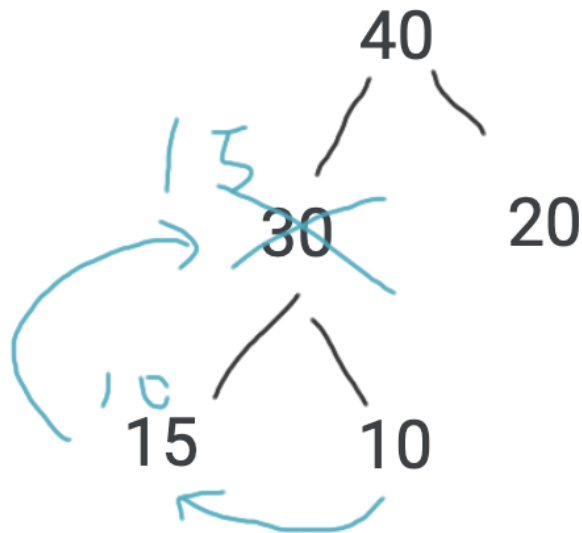
- 要加入的元素加入最下方
- 藉由調整的機制還原成 heap 的原則
- note: 實作上 希望先加左 再加右



## 刪除元素

- 刪除節點
- 先提左腳
- 轉右腳到左腳(維持先左後右的原則)

30



實作

```
#include <iostream>
using namespace std;

class MaxHeap{
private:
    int* heapArray; //指標開頭將產生實體
    int capacity;
    int currentSize;

    void shiftUp(int index){
        int temp = heapArray[index]; //把現在的值放進魁儡變數
        while(index>0){ //因為到root 要停
            int parentIndex = (index-1) /2;
            if(heapArray[parentIndex] < temp){ //要換
```

```

        heapArray[index] = heapArray[parentIndex]; //換
        index = parentIndex;
    }
    else{
        break;
    }
}
heapArray[index] = temp; //我的最高點
}

```

```

void shiftDown(int index){
    int temp = heapArray[index];
    int childIndex = 2*index +1;
    while(childIndex < currentSize){
        if(temp < heapArray[childIndex]){
            heapArray[index] = heapArray[childIndex];
            index = childIndex;
            childIndex = 2*index +1;
        }
        else{
            break;
        }
    }
    heapArray[index] = temp; //我的最低點
}

```

```

public:
    MaxHeap(int cap): capacity(cap), currentSize(0){
        heapArray = new int[capacity];
    }

```

```

~MaxHeap(){
    delete[] heapArray; //c++11 連續空間歸還語法
}

```

```

void insert(int val){

```

```

        if(currentSize == capacity){
            cout << "滿了" << endl;
            return;
        }
        heapArray[currentSize] = val; //插入最後一個 先左再右的原則
        shiftUp(currentSize); //向上調整
        currentSize++; //多了一個
    }

    int removeMax(){
        int maxItem = heapArray[0];
        heapArray[0] = heapArray[currentSize-1]; //把最下面的提上來
        currentSize--;
        shiftDown(0);
        return maxItem;
    }

    bool isEmpty(){
        return currentSize==0;
    }
};

```

## Heap 的應用

- 使用上面的 class
- Heap Sort (由大到小)

```
int numbers[] = {8,5,2,9,10,6,3};
```

```

#include <iostream>
using namespace std;

class MaxHeap{
private:

```

```

int* heapArray; //指標開頭將產生實體
int capacity;
int currentSize;

void shiftUp(int index){
    int temp = heapArray[index]; //把現在的值放進魁儡變數
    while(index>0){ //因為到root 要停
        int parentIndex = (index-1) /2;
        if(heapArray[parentIndex] < temp){ //要換
            heapArray[index] = heapArray[parentIndex]; //換
            index = parentIndex;
        }
        else{
            break;
        }
    }
    heapArray[index] = temp; //我的最高點
}

void shiftDown(int index){
    int temp = heapArray[index];
    int childIndex = 2*index +1;
    while(childIndex < currentSize){
        if(temp < heapArray[childIndex]){
            heapArray[index] = heapArray[childIndex];
            index = childIndex;
            childIndex = 2*index +1;
        }
        else{
            break;
        }
    }
    heapArray[index] = temp; //我的最低點
}

public:

```



```

MaxHeap(int cap): capacity(cap), currentSize(0){
    heapArray = new int[capacity];
}

~MaxHeap(){
    delete[] heapArray; //c++11 連續空間歸還語法
}

void insert(int val){
    if(currentSize == capacity){
        cout << "滿了" << endl;
        return;
    }
    heapArray[currentSize] = val; //插入最後一個 先左再右的原則
    shiftUp(currentSize); //向上調整
    currentSize++; //多了一個
}

int removeMax(){
    int maxItem = heapArray[0];
    heapArray[0] = heapArray[currentSize-1]; //把最下面的提上來
    currentSize--;
    shiftDown(0);
    return maxItem;
}

bool isEmpty(){
    return currentSize==0;
}

};

int main() {
    int numbers[] = {8,5,2,9,10,6,3};
    int length = sizeof(numbers)/sizeof(numbers[0]);

    MaxHeap heap(length);

```

```

for(int i=0; i<length;i++)
    heap.insert(numbers[i]); //每次插入後會自動調整 heapify

int j=0;
while(!heap.isEmpty()){ //提出到空就停止
    numbers[j++]=heap.removeMax();
}

for(int z=0; z<length; z++)
    cout << numbers[z]<< " ";

return 0;
}

```

## 練習

### Priority Queue

- item 會有一個優先值 20, 5, 15, 40, 10
- class PriorityQueue → 使用 class MaxHeap
  - enqueue : 照順序 enqueue
  - dequeue : 依照優先權最高的取得

```

#include <iostream>
using namespace std;

class MaxHeap{
private:
    int* heapArray;
    int capacity;
    int currentSize;

    void shiftUp(int index){
        int temp = heapArray[index];

```

```

while(index > 0){
    int parentIndex = (index-1) / 2; /*** -- 父節點的正确索引
    if(heapArray[parentIndex] < temp){
        heapArray[index] = heapArray[parentIndex];
        index = parentIndex;
    }
    else{
        break;
    }
}
heapArray[index] = temp;
}

void shiftDown(int index){
    int temp = heapArray[index];
    while(index < currentSize){
        int leftChildIndex = 2 * index + 1;
        int rightChildIndex = 2 * index + 2;
        int largest = index;

        if(leftChildIndex < currentSize && heapArray[leftChildIndex] > temp){
            largest = leftChildIndex;
        }

        if(rightChildIndex < currentSize && heapArray[rightChildIndex] > temp){
            largest = rightChildIndex;
        }

        if(largest != index){
            swap(heapArray[index], heapArray[largest]); /*** -- 1
            index = largest;
        }
        else{
            break;
        }
    }
}

```

```

    }

public:
    MaxHeap(int cap): capacity(cap), currentSize(0){
        heapArray = new int[capacity];
    }

    ~MaxHeap(){
        delete[] heapArray;
    }

    void insert(int val){
        if(currentSize == capacity){
            cout << "Heap is full" << endl; /*** -- heap已滿的提示信,
            return;
        }
        heapArray[currentSize] = val;
        shiftUp(currentSize);
        currentSize++;
    }

    int removeMax(){
        if(isEmpty()){ /*** -- 檢查是否為空，以避免在空堆上進行操作
            return -1; // Error code
        }
        int maxItem = heapArray[0];
        heapArray[0] = heapArray[currentSize-1];
        currentSize--;
        shiftDown(0);
        return maxItem;
    }

    bool isEmpty(){
        return currentSize == 0;
    }
};

```

```

class PriorityQueue{
private:
    MaxHeap maxHeap;

public:
    PriorityQueue(int capacity): maxHeap(capacity){}

    void enqueue(int value){
        /*** -- 刪除了不必要的空條件判斷，因為MaxHeap已經有滿的檢查
        maxHeap.insert(value);
        cout << "Priority "<< value << " has been enqueued." << endl;
    }

    int dequeue(){
        int value = maxHeap.removeMax(); //優先權最高
        if(value == -1){
            cout << "Queue is empty." << endl;
        }
        return value;
    }

    bool isEmpty(){
        return maxHeap.isEmpty();
    }
};

int main() {
    PriorityQueue q(10);

    q.enqueue(20);
    q.enqueue(5);
    q.enqueue(15);
    q.enqueue(40);
    q.enqueue(10);

```

```
while(!q.isEmpty()){
    int dequeuedValue = q.dequeue();
    if(dequeuedValue != -1){ /*** -- 檢查dequeue返回值，避免針
        cout << "Dequeued: " << dequeuedValue << endl;
    }
}
return 0;
}
```