Phill-DS-1213, 1220, 0103, 0110

Tree 的名詞與概念

Node 節點 → data field, 資料欄位

Link, edge, branch 分支 → 連接兩個節點的線

degree 分支度

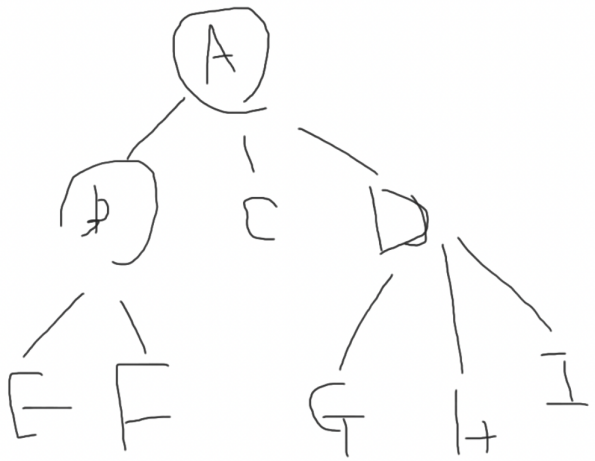
- 某一個node : 多少個 subtrees → 跟多少個 nodes 相連 → $T_1, T_2, \ldots, T_N$ = N
- 一棵樹 degree → all nodes 最大 degree
- leaf node → degree =0
- non-leaf node → degree <> 0

hierarchical relationship

- child (children) → children of a node → roots of subtrees → X’s children
- parent → children’s parent → owns your subtree
- sibling → sibling nodes → parent node 相同
- ancestors → from root ~ X → path 包含的所有的 nodes → ancestors
- level : root level =1, children of root level=2, X level =N → X’s children level =N+1
- level of tree → height, depth → all nodes , maximum level

height=3

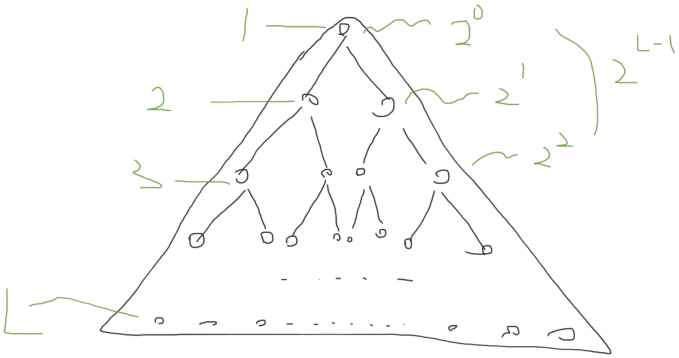
root = A
node #= 9
A's degree=3
leaf nodes = EFGHI
GHI parent =D
H siblings = GI
I ancestors = AD



x = y+z;

Nodes and edges 的計算

- 每一個 node 都一定有一個 edge → root 例外
- 任何一棵樹 → Node # N 跟 edge # B 有何關係 → N = B + 1
- 二元樹 binary tree
  - level-L 節點個數最多是多少? 2^{L-1}



  - depth K binary tree , nodes # = $2^K - 1$, K≥1

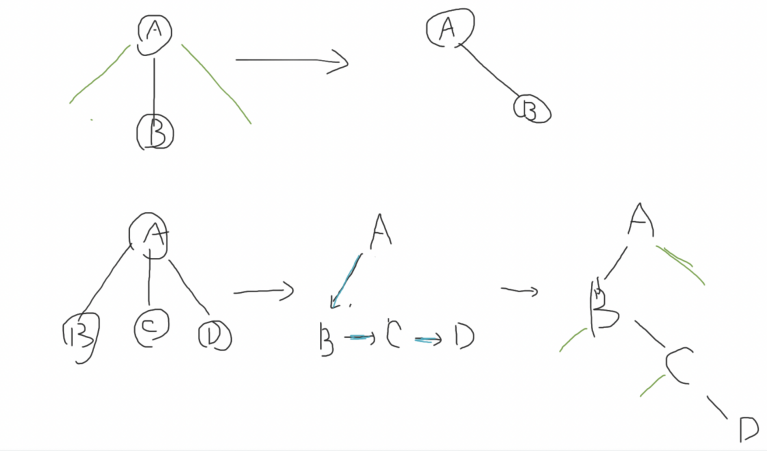
二元樹：skewed binary (傾斜樹), complete binary tree (perfect binary)

- skew binary tree → binary tree, left subtree or right subtree 是空集合
- full binary tree

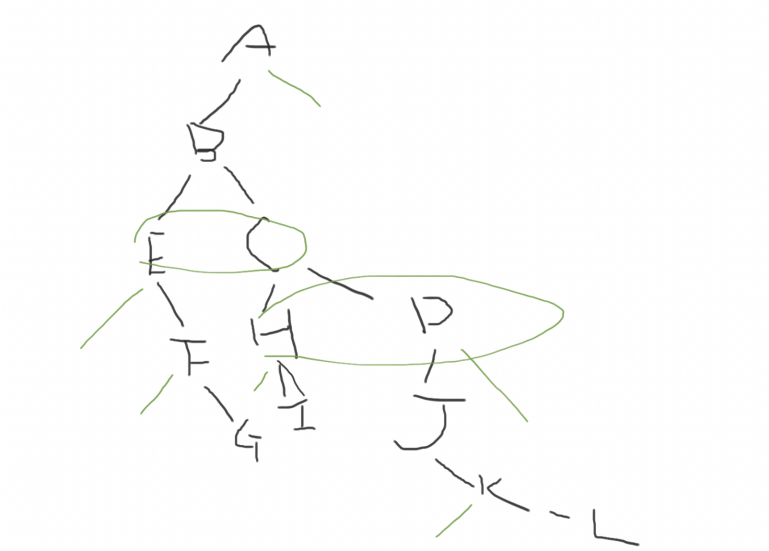
- 節點最多的 binary tree
- 高度 K → $2^K - 1$ nodes

各種 tree → binary tree

- 所有的 tree 都可轉成 unique binary tree
- 轉換原則

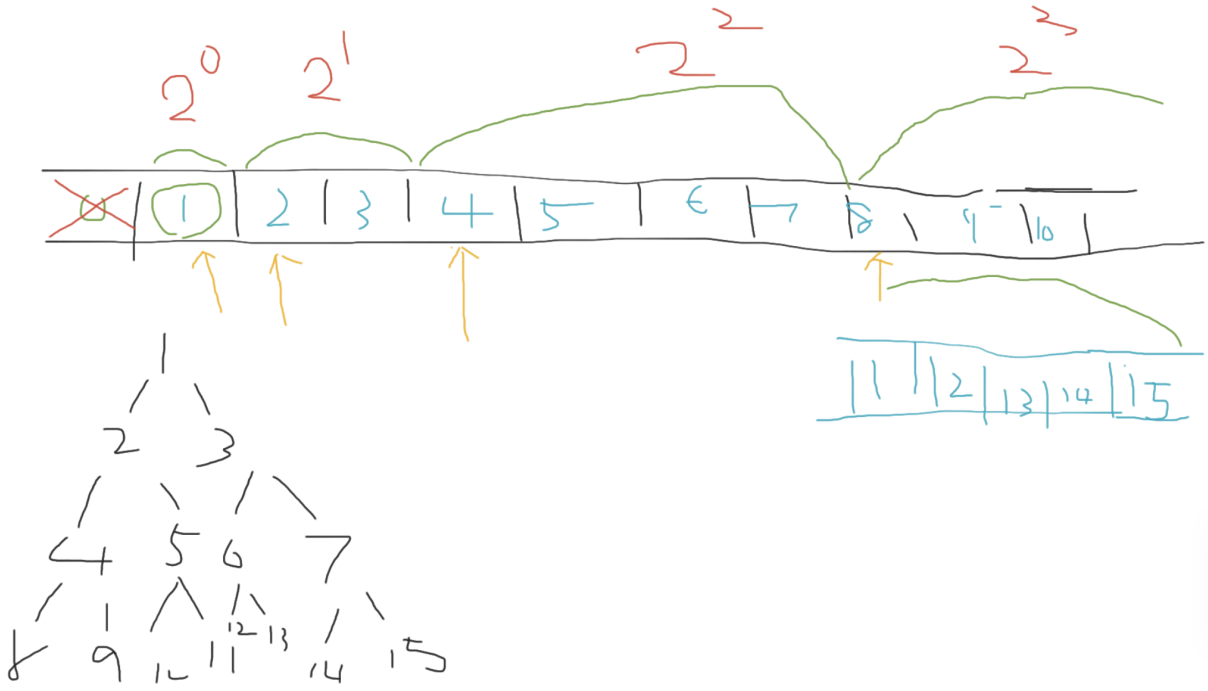


- 鎖定 root
- 連結 最 left 的 child node
- 連結 sibling → siblings → end
- 舊的 link 拿掉
- 橫向的 link , 旋轉45度



怎麼實作二元樹 (陣列)

陣列的實作

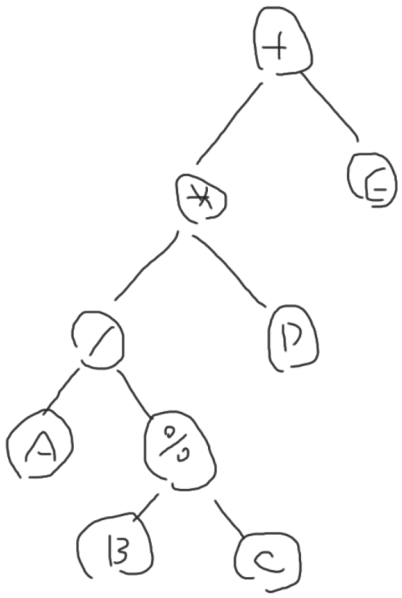


- complete binary tree → efficient
- skewed binary tree → 浪費空間

Binary tree Traverse

- inorder traverse 中序 → 原則: 先找左子樹 自己 再找右子樹

(A/(B%C))*D+E



- preorder 前序 → 原則: 先找自己 再找左子樹 再找右子樹
- postorder 後序 → 先找左子樹 再找右子樹 自己

實作

```
#include <iostream>
using namespace std;

struct TreeNode{
  int value;
  int leftChildIndex;
  int rightChildIndex;
};
```

```
class BinaryTree{
  private:
    TreeNode* nodes; // root ->index
    int capacity;
    int nextIndex; //即將要插入的 index

  public:
    BinaryTree(int capacity) : capacity(capacity), nextIndex(1){
      nodes = new TreeNode[capacity+1];
    }

    void insert(int value){
      nodes[nextIndex].value = value;
      nodes[nextIndex].leftChildIndex = -1; //初始值
      nodes[nextIndex].rightChildIndex = -1;

            if(nextIndex>1){ //root? 非root 就要處理向上的問題
              int parentIndex = nextIndex /2 ; //商數即parent

              if(nextIndex%2 ==0){
                nodes[parentIndex].leftChildIndex = nextIndex;
              }
              else{
                nodes[parentIndex].rightChildIndex = nextIndex;
              }
            }
            nextIndex++;
    }

    void preorderPrint(int index){
      if(index>0 && index < nextIndex){ //index 值不會錯 -> core
        cout << nodes[index].value << " ";
        preorderPrint(nodes[index].leftChildIndex);
        preorderPrint(nodes[index].rightChildIndex);
      }
    }
```

```
};

int main()
{
    BinaryTree tree(10); //給 capacity

    tree.insert(1);
    tree.insert(2);
    tree.insert(3);
    tree.insert(4);
    tree.insert(5);
    tree.insert(6);
    tree.insert(7);
    tree.insert(8);
    tree.insert(9);
    tree.insert(10);

    cout << "traverse:"<< endl;
    tree.preorderPrint(1); //從 root 開始 traverse
    cout << "end" << endl;

    return 0;
}
```

Linked List version

```
#include <iostream>
using namespace std;

struct TreeNode{
  int data;
  TreeNode* llink; //左子樹 link
  TreeNode* rlink;

  TreeNode(int value): data(value), llink(NULL), rlink(NULL){}
```

```
};

class BinaryTree{
  private:
    TreeNode* root;

  public:
    BinaryTree() : root(NULL){}

    void insert(int data){
      if(root ==NULL){ //樹是空的
        root = new TreeNode(data);
        return;
      }

      TreeNode* temp = root;
      while(true){
        if(data < temp->data){ //插入的值比魁儡變數值小 -> 向左走
          if(temp->llink==NULL){ //左是空
            temp->llink = new TreeNode(data); //直接assign成左noc
            return;
          }
          temp = temp->llink; //推移魁儡變數
        }
        else{
          if(temp->rlink == NULL){
            temp->rlink = new TreeNode(data);
            return;
          }
          temp = temp->rlink;
        }
      }
    }

    TreeNode *getRoot(){ //get root 實體 -> getter
      return root;
```

```
    }

    ~BinaryTree(){
      destroyTree(root);
    }

  private:
    void destroyTree(TreeNode *node){
      if(node != NULL){ //不要 free 掉空tree
        destroyTree(node->llink);
        destroyTree(node->rlink);
        delete node;
      }
    }
};

int main()
{
    BinaryTree tree;

    tree.insert(5);
    tree.insert(3);
    tree.insert(7);
    tree.insert(2);
    tree.insert(4);
    tree.insert(6);
    tree.insert(8);
    return 0;
}
```

練習

- 用上面的 class
- 計算 binary tree 有多少個 nodes? →
  - private → countNodes()

- public → totalNodes()