

# Algebra II Planning Notes

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## General Notes

1. OER Bookdown Temple [Complete - June 22, 2019]
2. CCSS Alignment [Complete - June 24, 2019]
  - (a) Determine Essential, Supporting, Advanced, and Advanced Supporting Standards [Complete - June 23, 2019]
  - (b) Group Standards by Topic and or Unit [Complete - June 24, 2019]
3. Learning Targets [First Pass Complete - June 24, 2019, Need to Prune]
4. Scope & Sequence
5. Learning Target Quizzes
6. Guided Practice Exercises
7. Challenge Problems

## CCSS Alignment

### Essential Standards

1. CC.9-12.A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
2. CC.9-12.A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
3. CC.9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems.
4. CC.9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
5. CC.9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
6. CC.9-12.A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
7. CC.9-12.A.REI.4a Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x-p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form.

8. CC.9-12.A.REI.4b Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .
9. CC.9-12.A.REI.7 . Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
10. CC.9-12.A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
11. CC.9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
12. CC.9-12.A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.
13. CC.9-12.A.SSE.3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.\*
14. CC.9-12.A.SSE.3c Use the properties of exponents to transform expressions for exponential functions.
15. CC.9-12.F.BF.2 Build a function that models a relationship between two quantities. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
16. CC.9-12.F.BF.3 Build new functions from existing functions. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
17. CC.9-12.F.BF.4 Find inverse functions.
18. CC.9-12.F.BF.4a Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse.
19. CC.9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
20. CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
21. CC.9-12.F.IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
22. CC.9-12.F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
23. CC.9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.
24. CC.9-12.F.LE.1a Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
25. CC.9-12.F.LE.1b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

26. CC.9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
27. CC.9-12.F.LE.4 For exponential models, express as a logarithm the solution to  $ab^{ct} = d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.
28. CC.9-12.F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.
29. CC.9-12.F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
30. CC.9-12.F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
31. CC.9-12.F.TF.8 Prove the Pythagorean identity  $(\sin A)^2 + (\cos A)^2 = 1$  and use it to find  $\sin A$ ,  $\cos A$ , or  $\tan A$ , given  $\sin A$ ,  $\cos A$ , or  $\tan A$ , and the quadrant of the angle.
32. CC.9-12.G.GPE.4 Use coordinates to prove simple geometric theorems algebraically.
33. CC.9-12.G.SRT.8 . Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
34. CC.9-12.N.CN.2 Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
35. CC.9-12.N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.
36. CC.9-12.N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
37. CC.9-12.N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.
38. CC.9-12.S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
39. CC.9-12.S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
40. CC.9-12.S.ID.6a Fit a function to the data; use functions fitted to data to solve problems in the context of the data.

## Supporting Standards

1. CC.9-12.A.APR.2 Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x-a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x-a)$  is a factor of  $p(x)$ .
2. CC.9-12.A.APR.4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity  $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$  can be used to generate Pythagorean triples.
3. CC.9-12.A.APR.6 Rewrite rational expressions. Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
4. CC.9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

5. CC.9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
6. CC.9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
7. CC.9-12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.
8. CC.9-12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.\*
9. CC.9-12.A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity.
10. CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it.
11. CC.9-12.A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
12. CC.9-12.F.BF.1 Write a function that describes a relationship between two quantities.
13. CC.9-12.F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.
14. CC.9-12.F.BF.1b Combine standard function types using arithmetic operations.
15. CC.9-12.F.IF.3 . Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
16. CC.9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
17. CC.9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
18. CC.9-12.F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
19. CC.9-12.F.IF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
20. CC.9-12.F.IF.8b Use the properties of exponents to interpret expressions for exponential functions.
21. CC.9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
22. CC.9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
23. CC.9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
24. CC.9-12.F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
25. CC.9-12.G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
26. CC.9-12.G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

27. CC.9-12.G.GPE.2 Derive the equation of a parabola given a focus and directrix.
28. CC.9-12.G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.
29. CC.9-12.G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
30. CC.9-12.G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.
31. CC.9-12.N.CN.1 Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.
32. CC.9-12.N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
33. CC.9-12.S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
34. CC.9-12.S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.
35. CC.9-12.S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
36. CC.9-12.S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
37. CC.9-12.S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
38. CC.9-12.S.IC.6 Evaluate reports based on data.
39. CC.9-12.S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
40. CC.9-12.S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.
41. CC.9-12.S.ID.9 Distinguish between correlation and causation.

## Advanced Standards

1. CC.9-12.F.BF.1c (+) Compose functions.
2. CC.9-12.F.BF.4b (+) Verify by composition that one function is the inverse of another.
3. CC.9-12.F.BF.4c (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
4. CC.9-12.F.BF.4d (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. CC.9-12.F.IF.7d (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

## Advanced Supporting Standards

1. CC.9-12.A.APR.5 (+) Know and apply that the Binomial Theorem gives the expansion of  $(x + y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are any numbers, with coefficients determined for example by Pascal's Triangle.
2. CC.9-12.A.APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
3. CC.9-12.A.REI.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable.
4. CC.9-12.A.REI.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension  $3 \times 3$  or greater).
5. CC.9-12.F.BF.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
6. CC.9-12.F.TF.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3, \pi/4$  and  $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for  $\pi - x, \pi + x$ , and  $2\pi - x$  in terms of their values for  $x$ , where  $x$  is any real number.
7. CC.9-12.F.TF.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
8. CC.9-12.F.TF.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
9. CC.9-12.F.TF.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.\*
10. CC.9-12.F.TF.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
11. CC.9-12.G.GPE.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
12. CC.9-12.G.SRT.9 (+) Derive the formula  $A = (1/2)ab\sin(C)$  for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
13. CC.9-12.G.SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.
14. CC.9-12.G.SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
15. CC.9-12.N.CN.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
16. CC.9-12.N.CN.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
17. CC.9-12.N.CN.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.
18. CC.9-12.N.CN.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
19. CC.9-12.N.CN.8 (+) Extend polynomial identities to the complex numbers.

20. CC.9-12.N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
21. CC.9-12.N.VM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g.,  $v$ ,  $|v|$ ,  $\|v\|$ ,  $\vec{v}$ ).
22. CC.9-12.N.VM.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
23. CC.9-12.N.VM.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.
24. CC.9-12.N.VM.4 (+) Add and subtract vectors.
25. CC.9-12.N.VM.4a (+) Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
26. CC.9-12.N.VM.4b (+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
27. CC.9-12.N.VM.4c (+) Understand vector subtraction  $v-w$  as  $v + (-w)$ , where  $(-w)$  is the additive inverse of  $w$ , with the same magnitude as  $w$  and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
28. CC.9-12.N.VM.5 (+) Multiply a vector by a scalar.
29. CC.9-12.N.VM.5a (+) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as  $c(\vec{v}_x, \vec{v}_y) = (c\vec{v}_x, c\vec{v}_y)$ .
30. CC.9-12.N.VM.5b (+) Compute the magnitude of a scalar multiple  $c\vec{v}$  using  $\|c\vec{v}\| = |c|\vec{v}$ . Compute the direction of  $c\vec{v}$  knowing that when  $|c|\vec{v} \neq 0$ , the direction of  $c\vec{v}$  is either along  $\vec{v}$  (for  $c > 0$ ) or against  $\vec{v}$  (for  $c < 0$ ).
31. CC.9-12.N.VM.6 (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
32. CC.9-12.N.VM.7 (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
33. CC.9-12.N.VM.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.
34. CC.9-12.N.VM.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
35. CC.9-12.N.VM.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
36. CC.9-12.N.VM.11 (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
37. CC.9-12.N.VM.12 (+) Work with  $2 \times 2$  matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.
38. CC.9-12.S.CP.8 (+) Apply the general Multiplication Rule in a uniform probability model,  $P(A \text{ and } B) = [P(A)] \times [P(B|A)] = [P(B)] \times [P(A|B)]$ , and interpret the answer in terms of the model.

39. CC.9-12.S.CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.
40. CC.9-12.S.MD.1 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
41. CC.9-12.S.MD.2 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
42. CC.9-12.S.MD.3 (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.
43. CC.9-12.S.MD.4 (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.
44. CC.9-12.S.MD.5 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
45. CC.9-12.S.MD.5a (+) Find the expected payoff for a game of chance.
46. CC.9-12.S.MD.5b (+) Evaluate and compare strategies on the basis of expected values.
47. CC.9-12.S.MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
48. CC.9-12.S.MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).



# Standards Grouped by Topic

## General

### 1. Essential

- CC.9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems.
- CC.9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- CC.9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
- CC.9-12.N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

### 2. Supporting

- CC.9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- CC.9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- CC.9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- CC.9-12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.
- CC.9-12.A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity.
- CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

### 3. Advanced

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### 4. Advanced Supporting

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## Standards - Functions

### 1. Essential

- CC.9-12.A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- CC.9-12.F.BF.2 Build a function that models a relationship between two quantities. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- CC.9-12.F.BF.3 Build new functions from existing functions. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- CC.9-12.F.BF.4 Find inverse functions.
- CC.9-12.F.BF.4a Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse.
- CC.9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

### 2. Supporting

- CC.9-12.F.BF.1 Write a function that describes a relationship between two quantities
- CC.9-12.F.BF.1b Combine standard function types using arithmetic operations.
- CC.9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- CC.9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- CC.9-12.F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- CC.9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- CC.9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

### 3. Advanced

- CC.9-12.F.BF.1c (+) Compose functions.
- CC.9-12.F.BF.4b (+) Verify by composition that one function is the inverse of another.
- CC.9-12.F.BF.4c (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
- CC.9-12.F.BF.4d (+) Produce an invertible function from a non-invertible function by restricting the domain.

### 4. Advanced Supporting

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## Standards - Quadratics

### 1. Essential

- CC.9-12.A.REI.4a Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x-p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form.
- CC.9-12.A.REI.4b Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .
- CC.9-12.A.REI.7 . Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
- CC.9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- CC.9-12.A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.
- CC.9-12.A.SSE.3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- CC.9-12.N.CN.2 Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
- CC.9-12.N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.

### 2. Supporting

- CC.9-12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.\*
- CC.9-12.F.IF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- CC.9-12.N.CN.1 Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.

### 3. Advanced

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### 4. Advanced Supporting

- CC.9-12.N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
- CC.9-12.N.CN.8 (+) Extend polynomial identities to the complex numbers.

## Standards - Polynomials

### 1. Essential

- CC.9-12.A.APR.1 Add, subtract, and multiply polynomials.
- CC.9-12.A.APR.3 Identify zeros of polynomials when suitable factorizations are available.
- CC.9-12.A.APR.3 Use the zeros of polynomials to construct a rough graph of the function defined by the polynomial.
- CC.9-12.F.IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

### 2. Supporting

- CC.9-12.A.APR.2 Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x-a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x-a)$  is a factor of  $p(x)$ .
- CC.9-12.A.APR.4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity  $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$  can be used to generate Pythagorean triples.
- CC.9-12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.\*

### 3. Advanced

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### 4. Advanced Supporting

- CC.9-12.N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
- CC.9-12.N.CN.8 (+) Extend polynomial identities to the complex numbers.
- CC.9-12.A.APR.5 (+) Know and apply that the Binomial Theorem gives the expansion of  $(x+y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are any numbers, with coefficients determined for example by Pascal's Triangle.

## Standards - Exponential

### 1. Essential

- CC.9-12.A.SSE.3c Use the properties of exponents to transform expressions for exponential functions.
- CC.9-12.F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior.
- CC.9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.
- CC.9-12.F.LE.1a Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
- CC.9-12.F.LE.1b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- CC.9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- CC.9-12.F.LE.4 For exponential models, express as a logarithm the solution to  $ab^{ct} = d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.
- CC.9-12.F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

### 2. Supporting

- CC.9-12.F.IF.8b Use the properties of exponents to interpret expressions for exponential functions.
- CC.9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

### 3. Advanced

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### 4. Advanced Supporting

- CC.9-12.F.BF.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

## Standards - Rational

### 1. Essential

- CC.9-12.A.REI.2 Solve simple rational equations in one variable, and give examples showing how extraneous solutions may arise.

### 2. Supporting

- CC.9-12.A.APR.6 Rewrite rational expressions. Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.

### 3. Advanced

- CC.9-12.F.IF.7d (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

### 4. Advanced Supporting

- CC.9-12.A.APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

## Standards - Radical

### 1. Essential

- CC.9-12.A.REI.2 Solve radical equations in one variable, and give examples showing how extraneous solutions may arise.
- CC.9-12.N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.
- CC.9-12.N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

### 2. Supporting

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### 3. Advanced

- CC.9-12.F.BF.4d (+) Produce an invertible function from a non-invertible function by restricting the domain.

### 4. Advanced Supporting

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## Standards - Trigonometry

### 1. Essential

- CC.9-12.F.IF.7e Graph trigonometric functions, showing period, midline, and amplitude.
- CC.9-12.F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- CC.9-12.F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- CC.9-12.F.TF.8 Prove the Pythagorean identity  $(\sin A)^2 + (\cos A)^2 = 1$  and use it to find  $\sin A$ ,  $\cos A$ , or  $\tan A$ , given  $\sin A$ ,  $\cos A$ , or  $\tan A$ , and the quadrant of the angle.
- CC.9-12.G.GPE.4 Use coordinates to prove simple geometric theorems algebraically.
- CC.9-12.G.SRT.8 . Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

### 2. Supporting

- CC.9-12.F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- CC.9-12.G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
- CC.9-12.G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- CC.9-12.G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

### 3. Advanced

- CC.9-12.F.BF.4d (+) Produce an invertible function from a non-invertible function by restricting the domain.

### 4. Advanced Supporting

- CC.9-12.F.TF.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$  and  $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for  $\pi - x$ ,  $\pi + x$ , and  $2\pi - x$  in terms of their values for  $x$ , where  $x$  is any real number.
- CC.9-12.F.TF.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
- CC.9-12.F.TF.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
- CC.9-12.F.TF.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.\*
- CC.9-12.F.TF.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
- CC.9-12.G.SRT.9 (+) Derive the formula  $A = (1/2)ab \sin(C)$  for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
- CC.9-12.G.SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.
- CC.9-12.G.SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).



- CC.9-12.N.CN.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
- CC.9-12.N.CN.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
- CC.9-12.N.CN.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.
- CC.9-12.N.CN.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

## Standards - Vectors and Matrices

### 1. Essential

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### 2. Supporting

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### 3. Advanced

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### 4. Advanced Supporting

- CC.9-12.A.REI.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable.
- CC.9-12.A.REI.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension  $3 \times 3$  or greater).
- CC.9-12.N.VM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g.,  $v$ ,  $|v|$ ,  $\|v\|$ ,  $\vec{v}$ ).
- CC.9-12.N.VM.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- CC.9-12.N.VM.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.
- CC.9-12.N.VM.4 (+) Add and subtract vectors.
- CC.9-12.N.VM.4a (+) Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
- CC.9-12.N.VM.4b (+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
- CC.9-12.N.VM.4c (+) Understand vector subtraction  $v-w$  as  $v+(-w)$ , where  $(-w)$  is the additive inverse of  $w$ , with the same magnitude as  $w$  and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
- CC.9-12.N.VM.5 (+) Multiply a vector by a scalar.
- CC.9-12.N.VM.5a (+) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as  $c(\vec{v}_x, \vec{v}_y) = (c\vec{v}_x, c\vec{v}_y)$ .
- CC.9-12.N.VM.5b (+) Compute the magnitude of a scalar multiple  $c\vec{v}$  using  $\|c\vec{v}\| = |c|\vec{v}$ . Compute the direction of  $c\vec{v}$  knowing that when  $|c|\vec{v} \neq 0$ , the direction of  $c\vec{v}$  is either along  $\vec{v}$  (for  $c > 0$ ) or against  $\vec{v}$  (for  $c < 0$ ).
- CC.9-12.N.VM.6 (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
- CC.9-12.N.VM.7 (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
- CC.9-12.N.VM.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.
- CC.9-12.N.VM.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

- CC.9-12.N.VM.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
- CC.9-12.N.VM.11 (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
- CC.9-12.N.VM.12 (+) Work with  $2 \times 2$  matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

## Standards - Probability and Statistics

### 1. Essential

- CC.9-12.S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
- CC.9-12.S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
- CC.9-12.S.ID.6a Fit a function to the data; use functions fitted to data to solve problems in the context of the data.

### 2. Supporting

- CC.9-12.S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
- CC.9-12.S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.
- CC.9-12.S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
- CC.9-12.S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
- CC.9-12.S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
- CC.9-12.S.IC.6 Evaluate reports based on data.
- CC.9-12.S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
- CC.9-12.S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.
- CC.9-12.S.ID.9 Distinguish between correlation and causation.
- CC.9-12.A.APR.5 (+) Know and apply that the Binomial Theorem gives the expansion of  $(x+y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are numbers, with coefficients determined for example by Pascal's Triangle.

### 3. Advanced

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### 4. Advanced Supporting

- CC.9-12.S.CP.8 (+) Apply the general Multiplication Rule in a uniform probability model,  $P(A \text{ and } B) = [P(A)] \times [P(B|A)] = [P(B)] \times [P(A|B)]$ , and interpret the answer in terms of the model.
- CC.9-12.S.CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.
- CC.9-12.S.MD.1 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
- CC.9-12.S.MD.2 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
- CC.9-12.S.MD.3 (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.

- CC.9-12.S.MD.4 (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.
- CC.9-12.S.MD.5 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
- CC.9-12.S.MD.5a (+) Find the expected payoff for a game of chance.
- CC.9-12.S.MD.5b (+) Evaluate and compare strategies on the basis of expected values.
- CC.9-12.S.MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
- CC.9-12.S.MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

## Standards - Sequences and Series

### 1. Essential

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### 2. Supporting

- CC.9-12.A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
- CC.9-12.F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.
- CC.9-12.F.IF.3 . Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- CC.9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

### 3. Advanced

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### 4. Advanced Supporting

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## Standards - Conics

### 1. Essential

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### 2. Supporting

- CC.9-12.G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
- CC.9-12.G.GPE.2 Derive the equation of a parabola given a focus and directrix.

### 3. Advanced

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### 4. Advanced Supporting

- CC.9-12.G.GPE.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

## Learning Targets

2. D Level Learning Targets
3. C Level Learning Targets
4. B Level Learning Targets
5. A Level Learning Targets

## 1 Targets - Functions

1. Use technology to find the solutions to of the equation  $f(x) = g(x)$ . Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. [2] [CC.9-12.A.REI.11]
2. Sketch the graphs of functions using transformations of their parent function. [3] [CC.9-12.F.BF.3]
3. Be able to determine from a graph whether a function is odd, even, or neither. [3] [CC.9-12.F.BF.3]
4. Find the inverse of a function. [3] [CC.9-12.F.BF.4]
5. Combine functions using arithmetic operations. [3] [CC.9-12.F.BF.1b]
6. Determine the domain of a function from its graph. [3] [CC.9-12.F.IF.5]
7. Write functions that model relationships between two quantities. [3] [CC.9-12.F.BF.1; CC.9-12.F.BF.2]
8. Interpret key features of graph of functions. [3] [CC.9-12.F.IF.4]
9. Be able to algebraically determine whether a function is odd, even, or neither. [4] [CC.9-12.F.BF.3]
10. Determine the domain of a function algebraically. [4] [CC.9-12.F.IF.5]
11. Graph the inverse of a function given the graph of a function. [4] [CC.9-12.F.BF.4c]
12. Perform function composition. [5] [CC.9-12.F.BF.1c]
13. Verify two functions are inverses by composition. [5] [CC.9-12.F.BF.4b]



## 2 Targets - Quadratics

1. Interpret parts of an expression, such as terms, factors, and coefficients. [2] [CC.9-12.A.SSE.1a ]
2. Determine the minimum or maximum value of a quadratic function written in vertex form. [2] [CC.9-12.A.SSE.3b]
3. Solve quadratic equations by taking square roots. [2] [CC.9-12.A.REI.4b]
4. Solve quadratic equations by using the quadratic formula. [2] [CC.9-12.A.REI.4b]
5. Solve a simple system consisting of a linear equation and a quadratic equation in two variables graphically (with technology). [2] [CC.9-12.A.REI.7 ]
6. Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real. [2] [CC.9-12.N.CN.1 ]
7. Factor quadratic expressions. [3] [CC.9-12.A.SSE.3a]
8. Solve quadratic equations by factoring. [3] [CC.9-12.A.REI.4b]
9. Solve a simple system consisting of a linear equation and a quadratic equation in two variables graphically (without technology). [3] [CC.9-12.A.REI.7 ]
10. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. [4] [CC.9-12.F.IF.8a]
11. Add and subtract complex numbers. [3] [CC.9-12.N.CN.2 ]
12. Complete the square to transform any quadratic function into vertex form. [4] [CC.9-12.A.REI.4a] [CC.9-12.A.SSE.3b]
13. Solve quadratic equations by completing the square. [4] [CC.9-12.A.REI.4b]
14. Multiply complex numbers. [4] [CC.9-12.N.CN.2 ]
15. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically. [4] [CC.9-12.A.REI.7 ]
16. Derive the quadratic formula by completing the square on the general form of a quadratic equation. [5] [CC.9-12.A.REI.4a]

### 3 Targets - Polynomials

1. Add and subtract polynomials. [2] [CC.9-12.A.APR.1]
2. Determine the degree, leading coefficient, constant term, and maximum number of turning points for a polynomial function. [2] [CC.9-12.A.SSE.1a]
3. Use the zeros of polynomials to construct a rough graph of the function defined by the polynomial. [2] [CC.9-12.A.APR.3]
4. Multiply polynomials. [3] [CC.9-12.A.APR.1]
5. Divide polynomials by linear factors using synthetic division or polynomial long division. [3] [CC.9-12.A.APR.2]
6. Identify zeros of polynomials when suitable factorizations are available. [3] [CC.9-12.A.APR.3]
7. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.[4] [CC.9-12.F.IF.7c]
8. Use the remainder theorem to assist in factoring polynomials. [4] [CC.9-12.A.APR.2]
9. Know and apply that the Binomial Theorem gives the expansion of  $(x + y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are any numbers [4] [CC.9-12.A.APR.5 ]

## 4 Targets - Exponential

1. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. [2] [CC.9-12.F.LE.1b]
2. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. [2] [CC.9-12.F.LE.1c]
3. Use the properties of exponents to transform expressions for exponential functions. [2] [CC.9-12.A.SSE.3c]
4. Distinguish between situations that can be modeled with linear functions and with exponential functions. [3] [CC.9-12.F.LE.1 ]
5. Interpret the parameters in a linear or exponential function in terms of a context. [3] [CC.9-12.F.LE.5]
6. Use the properties of exponents to interpret expressions for exponential functions. [3] [CC.9-12.F.IF.8b]
7. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. [3] [CC.9-12.F.LE.1a]
8. Graph exponential functions showing intercepts and end behavior. [3] [CC.9-12.F.IF.7e]
9. Graph logarithmic functions showing intercepts and end behavior. [3] [CC.9-12.F.IF.7e]
10. For exponential models, express as a logarithm the solution to  $ab^{ct} = d$  where  $a, c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology. [4] [CC.9-12.F.LE.4]
11. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). [4] [CC.9-12.F.LE.2]
12. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.[5] [CC.9-12.F.BF.5]

## Targets - Rational

1. Using the provided procedure, graph rational functions without holes or oblique asymptotes that are written in factored form. [3] [CC.9-12.F.IF.7d]
2. Add, subtract, multiply, and divide rational expressions. [4] [CC.9-12.A.APR.7]
3. Solve simple rational equations in one variable, and give examples showing how extraneous solutions may arise. [4] [CC.9-12.A.REI.2]
4. Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ . [4] [CC.9-12.A.APR.6 ]
5. Graph rational functions, identifying zeros and asymptotes (horizontal and vertical) when suitable factorizations are available, and showing end behavior. [4] [CC.9-12.F.IF.7d]
6. Graph rational functions, identifying zeros, holes, and asymptotes (oblique and vertical) when suitable factorizations are available, and showing end behavior. [5] [CC.9-12.F.IF.7d]

## Targets - Radical

1. Convert expressions written as radicals to rational exponents. [2] [CC.9-12.N.RN.1]
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. [2]  
[CC.9-12.N.RN.2]
3. Solve radical equations in one variable[3] [CC.9-12.A.REI.2 ]

## Targets - Trigonometry

1. Add and subtract vectors graphically. [2] [CC.9-12.N.VM.4 ]
2. Add and subtract vectors algebraically. [2] [CC.9-12.N.VM.4 ]
3. Perform scalar multiplication graphically. [2] [CC.9-12.N.VM.5a]
4. Perform scalar multiplication algebraically. [2] [CC.9-12.N.VM.5]
5. Find the components of a vector given its initial and terminal points. [2] [CC.9-12.N.VM.2]
6. Know the definitions of trigonometric ratios for acute angles. [2] [CC.9-12.G.SRT.6]
7. Explain and use the relationship between the sine and cosine of complementary angles. [2] [CC.9-12.G.SRT.7]
8. Convert from degrees to radians and from radians to degrees. [2] [CC.9-12.F.TF.1]
9. Derive the equation of a circle of given center and radius using the Pythagorean Theorem. [3] [CC.9-12.G.GPE.1]
10. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. [3] [CC.9-12.G.SRT.8]
11. Know and be able to use the unit circle definition of the trigonometric functions. [3] [CC.9-12.F.TF.2]
12. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. [3] [CC.9-12.F.TF.7 ]
13. Graph trigonometric functions, showing period, midline, and amplitude. [3] [CC.9-12.F.IF.7e]
14. Use the addition and subtraction formulas for sine, cosine, and tangent to solve problems.[3] [CC.9-12.F.TF.9]
15. Determine the area of a sector of a circle. [3] [CC.9-12.G.C.5]
16. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. [3] [CC.9-12.N.VM.4b]
17. Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3, \pi/4$  and  $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for  $\pi - x, \pi + x$ , and  $2\pi - x$  in terms of their values for  $x$ , where  $x$  is any real number. [4] [CC.9-12.F.TF.3 ]
18. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. [4] [CC.9-12.F.TF.5]
19. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). [4] [CC.9-12.G.SRT.11]
20. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. [4] [CC.9-12.N.CN.3]
21. Solve problems involving velocity and other quantities that can be represented by vectors. [4] [CC.9-12.N.VM.3]
22. Simplify trigonometric expressions given the fundamental trigonometric identities. [4] [CC.9-12.F.TF.9]
23. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. [5] [CC.9-12.N.CN.4 ]

24. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. [5] [CC.9-12.N.CN.5]
25. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. [5] [CC.9-12.N.CN.6]
26. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.[5] [CC.9-12.F.TF.9]
27. Prove the Pythagorean identities and use them to find  $\sin A$ ,  $\cos A$ , or  $\tan A$ , given  $\sin A$ ,  $\cos A$ , or  $\tan A$ , and the quadrant of the angle. [5] [CC.9-12.F.TF.8]
28. Know and use Heron's formula and the formula  $A = (1/2)ab\sin(C)$  to find the area of a triangle. [5] [CC.9-12.G.SRT.9]
29. Prove trigonometric identities using the fundamental trigonometric definitions and identities. [5] [CC.9-12.F.TF.9]

## Targets - Probability and Statistics

1. CC.9-12.S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
2. CC.9-12.S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
3. CC.9-12.S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
4. CC.9-12.S.CP.8 (+) Apply the general Multiplication Rule in a uniform probability model,  $P(A \text{ and } B) = [P(A)] \times [P(B|A)] = [P(B)] \times [P(A|B)]$ , and interpret the answer in terms of the model.
5. CC.9-12.S.CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.
6. CC.9-12.S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
7. CC.9-12.S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
8. CC.9-12.S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
9. CC.9-12.S.ID.6a Fit a function to the data; use functions fitted to data to solve problems in the context of the data.
10. CC.9-12.S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
11. CC.9-12.S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.
12. CC.9-12.S.IC.6 Evaluate reports based on data.
13. CC.9-12.S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.
14. CC.9-12.S.ID.9 Distinguish between correlation and causation.
15. CC.9-12.S.MD.1 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
16. CC.9-12.S.MD.2 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
17. CC.9-12.S.MD.3 (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.
18. CC.9-12.S.MD.4 (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.
19. CC.9-12.S.MD.5 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
20. CC.9-12.S.MD.5a (+) Find the expected payoff for a game of chance.
21. CC.9-12.S.MD.5b (+) Evaluate and compare strategies on the basis of expected values.



22. CC.9-12.S.MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
23. CC.9-12.S.MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

## Targets - Sequences and Series

1. CC.9-12.A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
2. CC.9-12.F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.
3. CC.9-12.F.IF.3 . Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
4. CC.9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

## Targets - Conics

1. CC.9-12.G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
2. CC.9-12.G.GPE.2 Derive the equation of a parabola given a focus and directrix.
3. CC.9-12.G.GPE.3 Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.