Image Deblurring with Neural Networks Using Fourier Optics

July 17, 2025

In this task, you will create an image deblurring system by creating a dataset and training a neural network. The dataset will be generated by simulating the optical blur caused by a camera lens with chromatic aberrations and depth-dependent point spread functions (PSFs). You will calculate the PSF using principles from Fourier optics and convolve it with sharp images to produce blurred images. Afterward, by using ground truth and blurred images, you will train a neural network that performs deblurring.

1 System Setup

Consider a simple imaging system where a planar object is located at a certain depth z from the lens. The object emits monochromatic light of wavelength λ. Due to the optical system's properties, the image captured by the sensor is not a perfect magnified replica of the object but is blurred. This blur can be described mathematically using the concept of convolution.

The image captured by the sensor, denoted as $I^s_{z,\lambda}(x, y)$, can be approximated as:

$$I^s_{z,\lambda}(x, y) = I_{z,\lambda}(x, y) * h_{z,\lambda}(x, y), \tag{1}$$

where:

- $I_{z,\lambda}(x, y)$ is the ideal (sharp) image of the object at depth z and wavelength λ.
- $h_{z,\lambda}(x, y)$ is the point spread function (PSF) of the system at depth z and wavelength λ.
- $*$ denotes the convolution operation.

This equation represents the forward model of image formation, where the observed image is the convolution of the ideal image with the PSF.

Point Spread Function (PSF) Derivation

The PSF characterizes how a point source of light is spread out by the optical system. It depends on several factors, including the wavelength of light, the depth of the object, and lens aberrations like chromatic aberration.

Under paraxial approximation and perfectly incoherent illumination, the PSF can be derived using Fourier optics, which is given as following in relation with the so-called generalized pupil function $Q_{\lambda,z}(s,t)$:

$$h_{\lambda,z}(x,y) \propto \left| \mathcal{F}\left\{Q_{\lambda,z}(s,t)\right\} \Big|_{\left(\frac{x}{\lambda z_i}, \frac{y}{\lambda z_i}\right)} \right|^2, \tag{2}$$

where:

- $\mathcal{F}\{\cdot\}$ denotes the Fourier transform.
- (s,t) are coordinates in the lens (pupil) plane.
- (x,y) are coordinates in the image (sensor) plane.
- z_i is the image distance (distance from the lens to the image sensor).

The generalized pupil function $Q_{\lambda,z}(s,t)$ captures the effects of the lens aperture and phase aberrations due to defocus and chromatic aberration:

$$Q_{\lambda,z}(s,t) = A(s,t) \exp\left[j\frac{\pi}{\lambda}\Delta D[s^2+t^2]\right], \tag{3}$$

where:

- $A(s,t)$ is the aperture function, defining the shape and size of the lens aperture.
- j is the imaginary unit ($j^2 = -1$).
- ΔD is the defocus parameter.
- $s^2 + t^2$ represents the radial distance squared in the pupil plane.

For a circular aperture of radius R, the aperture function is defined as:

$$A(s,t) = \begin{cases} 1, & \text{if } s^2+t^2 \le R^2 \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

The defocus parameter ΔD quantifies the deviation from perfect focus due to object depth and lens imperfections:

$$\Delta D = \left(\frac{1}{z} + \frac{1}{z_i} - \frac{1}{f_\lambda}\right), \tag{5}$$

where:

- z is the object distance (distance from the object to the lens).
- z_i is the image distance.
- f_λ is the focal length of the lens at wavelength λ, accounting for chromatic aberration.

This parameter represents the mismatch between the optical path lengths, leading to defocus blur.

Longitudinal Chromatic Aberration

Longitudinal chromatic aberration is a type of lens aberration where the lens has a different focal length for different wavelengths of light. This occurs because the refractive index of the lens material varies with wavelength. To model chromatic aberration, we consider that the focal length f_λ depends on the wavelength λ:

$$f_\lambda = f_G * \frac{(n_G - 1)}{(n_\lambda - 1)}, \tag{6}$$

where:

- f_G is the focal length at green wavelength λ_G.
- n_λ is the wavelength-dependent refractive index of the material.

By incorporating f_λ into the defocus parameter, we account for the blurring effects caused by chromatic aberration.

2 Generating the Blurred Image Dataset

Implementation Steps

1. **Calculate the PSF for Each Color Channel:** For given wavelengths λ and corresponding focal length f_λ, and the object depth z,

    - Calculate the defocus parameter ΔD for each wavelength.
    - Compute the generalized pupil function $Q_{\lambda,z}(s,t)$.
    - Perform the Fourier transform to obtain $h_{\lambda,z}(x,y)$.

    For each image in the dataset, a single depth value, z, is picked randomly from the range $[z_{near}, z_{far}] = [1.84, 2.20]$ m. This depth value is assumed to be the object depth for PSF calculation, see Eq.5.

2. **Convolve Each Channel with its Respective PSF:**

    - For each channel, perform the convolution operation with the corresponding PSF. In practice, we normalize the PSF prior to the convolution, independently for each color channel, such that $\sum_{x,y} h_{\lambda,z}(x,y) = 1$.

3 Neural Network for Image Deblurring

With the blurred images generated, the next step is to train a neural network to perform deblurring. The network aims to learn an inverse mapping from the blurred images to the sharp images. You can utilize any neural network that is suitable for deblurring, e.g., ResNet, U-Net, etc.

Figure 1: Example of the ground truth (left) and blurred image (right)

Evaluation Metrics

To assess the performance of the deblurring network, consider the following metrics:

- **Peak Signal-to-Noise Ratio (PSNR):** Measures the reconstruction quality compared to the ground truth.
- **Structural Similarity Index (SSIM):** Evaluates perceived image quality based on luminance, contrast, and structure.

4 Parameters, Dataset and Helper functions

The necessary parameters are provided in *parameters.py*, and the convolution in Fourier domain can be found in *util.py*.

To train and evaluate networks, the dataset, which is a subset of imagenet, is provided within their corresponding folders, i.e. *train* and *test*. Please note that this dataset only contains ground truth images. You need to apply abovementioned computations to obtain blurred (sensor) images, see 2 for details. Here is the link to download dataset.

5 Deliverables

Please submit the source code for your implementation, along with a report detailing the steps you followed and including the experimental output, e.g., comparison of ground truth images and corresponding reconstructed network outputs. Indicate also average PSNR and SSIM values between sharp image and deblurred network output for the images in the test dataset (that are not to be used during training). Your code should be able to run as it is and you will see the required function signature in *evaluate.py*. Please note that the deblurring performance of your neural network will be also tested independently with another set of test images from a different dataset.

In a separate literature review report, briefly describe the image deblurring problem in the context of linear inverse problems (i.e., assume the blurring function or kernel is linear), and present existing approaches to it, including both deep learning based and traditional optimization based approaches (where there is no ground truth data is available). Please, keep the discussion brief and submit a report of not more than two-three pages.

6 References

For further reading and a deeper understanding of the concepts:

- Goodman, J. W. *Introduction to Fourier Optics.* Roberts and Company Publishers.
- Hecht, E. *Optics.* Addison-Wesley.
- Goodfellow, I. and Bengio, Y. and Courville, A. *Deep Learning.* MIT Press. `http://www.deeplearningbook.org`