In this short letter we report our progress studying the AR(2) model

$$x_t = \beta_2 x_{t-1} + \beta_1 x_{t-2} + \epsilon_t \tag{1}$$

reproducing data from V1.

This model can be rewritten as

$$\begin{aligned} \dot{I} &= w_{ie} E, \\ \dot{E} &= w_{ee} E + w_{ei} I + \epsilon_t \end{aligned} \tag{2}$$

where $w_{ee} = -(1+\beta_1)$, $w_{ei} = \beta_2 + \beta_1 - 1$ and $w_{ie} = 1$.

Phasic dependency of inputs

First, we studied the model in (2) without noise. As one can see the origin is a fixed point. For the choice of parameters $\beta_1, \beta_2 = \{-0.9606, 1.8188\}$, its eigenvalues are given by

$$\mu_{\pm} = \lambda \pm i\omega = -0.0197 \pm 0.376i \tag{3}$$

so it is a stable focus very near a Hopf bifurcation.

To study the phasic dependency of perturbations over the AR(2) model, first, we have computed the isochrons for the focus in (3). Since the addition of perturbations to the dynamics in (1) is equivalent to perturb the E variable in (2), we have studied the effect of perturbations in the E direction. In Fig. 1, we show how depending on which phase the stimulus is applied, phase or amplitude modulations can be observed.

Frequency and Amplitude correlation

Next, we study the possible correlations between the amplitude of the oscillation and its frequency. To that aim we will work with (2) in the noisy case ($\epsilon_t \neq 0$). First, we explain how we define and compute the amplitude and frequency of our data. We run system (2) for large t. Then, we look for the maximal value $x(t_i)$ for any local interval of points above 0. The amplitude A and the period T of the oscillation are thus defined

$$\begin{aligned} A &= \frac{x(t_{i+1}) - x(t_i)}{2} - min(x(t_{i+1}), x(t_i)), \\ T &= t_{i+1} - t_i \end{aligned}$$

where $min(x(t_{i+1}), x(t_i))$ corresponds to the minimum value of $x(t)$ for $t \in [t_i, ..., t_{i+1}]$.

Our results in Fig. 2, show almost not dependency between A and T. As we argue next, the mismatch between our results and the observed correlation in the data between period and amplitude, might be due to the linearity of the model in (2). The model (2) we are working with can be written in polar coordinates as

$$\dot{r} = \lambda r \qquad \dot{\theta} = \omega \tag{4}$$

where λ and ω are given in (3). So, the angular frequency in (4) is constant. However, since the model is near a Hopf bifurcation, it can be also approached via its normal form

$$\dot{r} = r(\lambda - br^2), \qquad \dot{\theta} = \omega + ar^2. \tag{5}$$

Please notice the inclusion of non linear dynamics in $\dot{\theta}$ trough the parameter a. If a is negative, the far from the origin, the slower the speed, or in other words, the larger the amplitude the larger the period. However, for a precise determination of the value of a, the model will be required to have some sort of non-linearity.

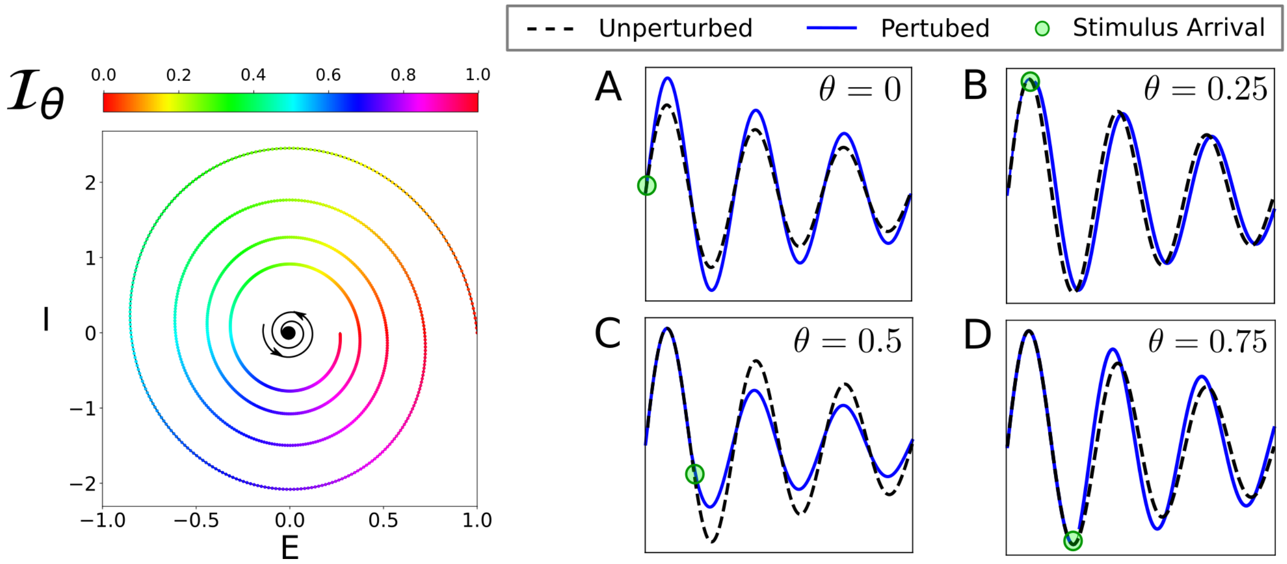

Figure 1: **Phase and amplitude modulations for the AR(2) model** Left panel shows the distribution of phases around the stable focus. If a positive signal in the E direction is applied at $\theta = 0$ ($\theta = 0.5$) we see a positive (negative) modulation in the amplitude but not in phase (Panels A and C). By contrast, phase modulation is seen when stimulating at $\theta = 0.25$ (Panel B). For $\theta = 0.75$ we see a change in both amplitude and phase (Panel D).

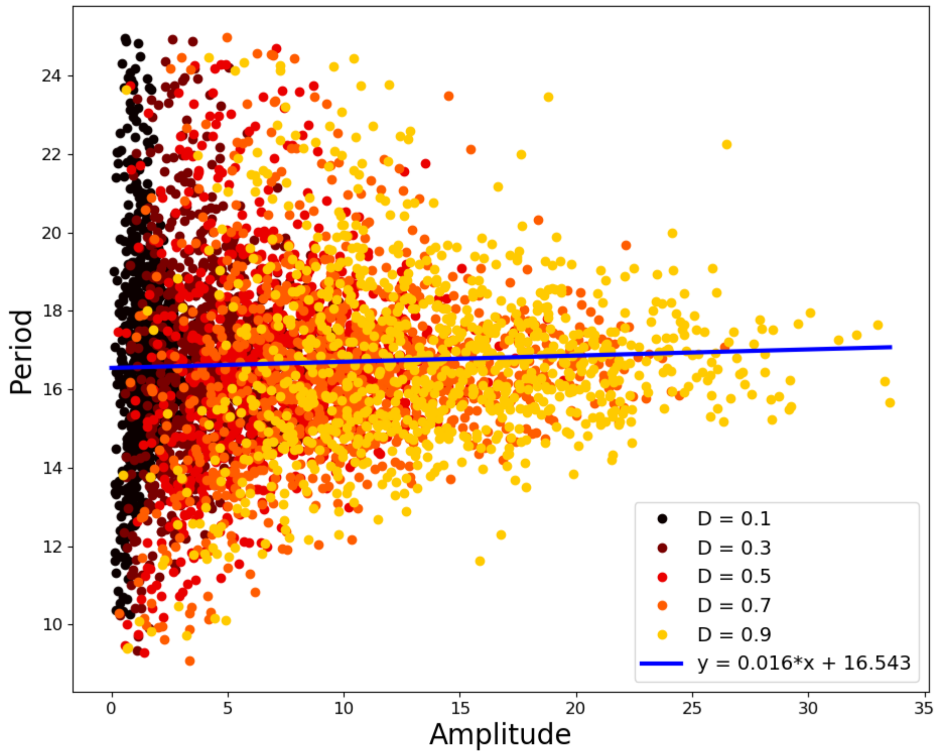

Figure 2: **Relationship between the amplitude and the period of the oscillation** For different noise amplitudes (denoted as D), we plot the period and amplitude of the different oscillations observed in the data generated by the integration of the AR(2) model. Results show almost no correlation between the two variables.