Assignment 1

Tuesday, February 14, 2023 5:34 PM

1)

a) $\underbrace{A}_{n\times n}\underbrace{B}_{n\times n} x \Rightarrow \underset{n\times n}{C}\ \underset{n\times 1}{X} \Rightarrow n^3+n^2$

C

$n*n*n = n^3$ $\qquad n*n*1 = n^2$

$A(\underset{n\times n}{B}\ \underset{n\times 1}{x}) \Rightarrow \underset{n\times n}{A}\ \underset{n\times 1}{D} \Rightarrow n^2+n^2$

D

$n*n*1=n^2$ $\qquad n*n*1 = n^2$

b) $p = (I - (\underset{n\times 1}{x}\underset{1\times n}{x^T})/(\underset{1\times n}{x^T}\underset{n\times 1}{x}))\underset{n\times 1}{b} \Rightarrow \underset{n\times n}{Q}\ \underset{n\times 1}{b} \Rightarrow 2n^2+2n$

Q $\qquad +n^2$

$$\Downarrow$$

$$I - \frac{xx^T}{x^Tx}$$

$$\Downarrow \; +n$$

$$\cancel{I} - \underset{n\times 1}{x}\underset{1\times n}{x^T}(\underset{1\times n}{x^T}\underset{n\times 1}{x})^{-1}$$

$+n^2 \qquad +n$

c) $p = (I - (xx^T)/(x^Tx))b \Rightarrow \cancel{I}b - \frac{x(x^Tb)}{x^Tx} \Rightarrow b - x(x^Tb)(x^Tx)^{-1}$

n Scalar

Scalar $\|x\|^2$ n

$n+n+n = 3n$

d) $p = b - \dfrac{x(x^Tb)}{\|x\|^2} \Rightarrow p^T = \left(b - \dfrac{x(x^Tb)}{\|x\|^2}\right) \Rightarrow p^T = b^T - \dfrac{b^T x x^T}{\|x\|^2} \Rightarrow$

$\Rightarrow p^T x = b^T x - \dfrac{b^T x x^T x}{\|x\|^2} \Rightarrow p^T x = b^T x - \dfrac{b^T x(x^T x)}{\|x\|^2}$

$\Rightarrow p^T x = b^T x - \dfrac{b^T x \cancel{\|x\|^2}}{\cancel{\|x\|^2}} \Rightarrow p^T x = b^T x - b^T x = 0$

e) $\dfrac{5.7}{2} \approx \dfrac{2n^2+2n}{n} \Rightarrow 2n+2$ which is similar to what was expected

2

a) For a rank-1 matrix A the column space, $C(A)$, and the row space $C(A^T)$ have dimensions of 1. The nullspace of A, $N(A)$, has dimensions of $n-1$ while the of all vectors orthog to U

the row space $C(A^T)$ have ... the
nullspace of A, $N(A)$, has dimensions of $n-1$ while the
null space of A^T is $m-1$ consisting of all vectors orthog to u

b) For any column vectors $u, v \in \mathbb{R}^3$, the matrix
uv^T is rank 1, except when $v=0$ or $u=0$
in which case uv^T has a rank of 0

3

a) $C(AB)$ is contained in the column space of A

b) We could use the results of the multiplications by multiplying each of
the resulting vectors by the transpose of each x. This is possible because
rank B = rank B^TB. We can then assemble all of the resulting scalars into
a 10×10 matrix with the same rank as A, assuming none of the random vectors are $\perp$.

c) We can do something similar in this case, but instead of just vector multiplication
we can do matrix multiplication to get a resulting 1000×10 matrix. We can then
multiply by X^T since that still preserves rank and results in a 10×10 matrix C.

4

a) $H_4 x \Rightarrow 3$ ops / row for 4 rows $= 12$ ops

b) $H_4 x \Rightarrow \begin{pmatrix} H_2 & H_2 \\ H_2 & -H_2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \Rightarrow \begin{pmatrix} H_2 x_1 + H_2 x_2 \\ H_2 x_1 - H_2 x_2 \end{pmatrix}$

$\underset{2\times2}{H_2} \underset{2\times1}{x_1} \Rightarrow 4$ ops

$\underset{2\times2}{H_2} \underset{2\times1}{x_2} \Rightarrow 4$ ops

$4+4 = 8$

c) $H_8 x$

$\Downarrow$

7 ops for 8 rows $= 56$

$\begin{pmatrix} H_4 & H_4 \\ H_4 & -H_4 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \Rightarrow \begin{pmatrix} H_4 y_1 + H_4 y_2 \\ H_4 y_1 - H_4 y_2 \end{pmatrix}$

$\Downarrow$

$H_4 y_1 \sim 8$ ops

$H_4 y_2 \sim 8$ ops

$H_4 y_1 + H_4 y_2 \sim 4$ ops

$H_4 y_1 - H_4 y_2 \sim 4$ ops

$8+8+4+4 = 24$

5

a) If A commutes with P, then the other "?"
entries must be rotations of the first row.

b) see notebook