Grey Forest: The Sparks In The Dark Silence

Tianhao Wang, 1003907061

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"Look, the silly boy lit a fire."

1 Description

On a quiet sunset, an ant on the tombstone of Yang Dong was listening a historical whisper between Ye Wenjie and Luo Ji. The whisper about the cornerstones of cosmic sociology.
"First, there are two axioms in the universe. "

1. The survival and continuation is the first basic need for any civilization in the universe;
2. Civilization is always expanding, but the total amount of matter and energy is conserved in the universe.

Given the axiom 1 and axiom 2 in our universe, any civilization could be a potential competitor for one civilization at some time.
"Second, *technology innovation explosion* exists in our universe. "

- It is possible that the technology of developed civilizations realized in a long time could be innovated rapidly in the form of an explosion in a much shorter time.

Thus, no matter how developed you are, a civilization you have detected might easily surpass your own technological level in a few centuries and become a threat. That is, instead of at some time, any civilization can be a competitor for one civilization at any time.
"Third, the extreme distances between stars creates an insurmountable *chains of suspicion*. "

- A civilization can never be certain of other civilizations' true intention and technology level before communication.
- No matter whether intentional or not, any communication or non-extinction attack will accelerate the speed of technology development.

That is, the cost of communication (trigger technology innovation of competitors) is way higher than the cost of casting interstellar attack if accessible given far enough distance.

Therefore, it is in every civilization's best interest to preemptively strike and destroy any developing civilization before it can become a threat.

> "*Every civilization is a hunter in the dark forest.*"

Many years later, the Three-body civilization invaded the earth. A tiny little spaceship called "droplets" destroyed all the space fleets of human. Human society was in despair.

But Luo did not. Above the ice lake, he was thinking about the cosmological sociology that Ye told himself and accidentally fell into the ice lake. Within the dead blackness, he saw the truth of the universe.

> "*The Three-body civilization who views humans as bugs seems to have forgotten one fact: the bugs have never been truly defeated.*"

On a rainy night, digging a grave for himself with a high fever, Luo embarked on a plan for three years.

Through a program called "snow engineering", Luo connect 3614 H-bombs around the solar orbit to vital-sign monitoring system. Those bombs will be ignited at his death and create 3614 space dust cloud around the sun. If observed from a long range, the sun will create a signal by the visual light and other high-frequent waves under the cover of the dust. Each bomb in this system is placed perfectly to present three images: the relative location of the Three-body world, the solar system and the earth. "*I, as a wallfacer, Luo Ji, speak to the Three-body world!*", he said,

> "*... I know the sophons are around me, but you have never answered our calls. Your silence is the biggest scorn. We have been tolerated this for over two centuries. And now, you may keep silence if you will.*
> *You only have 30 seconds.*"

Luo threatened to suicide to establish a dark forest deterrence with the Three-body civilization. Afraid of being destroyed by some unknown highly-developed civilization, the Three-body world promised to compromise, immediately stopped the invasion and subsequently accepted many conditions of humanity. Humans ushered in a new peace.

Luo saved the earth and created a deterrence balance between the Tree-body world and the earth. Meanwhile, an ant has climbed over the tombstone, and slightly walking to the sunrise.

2 Formal Description

This paper models the strategic situation as a discounted repeated game at $\beta \in (0,1)$ with asymmetric information, in which there are only 2 players $\{P_1, P_2\}$ representing 2 civilizations with enough distance.

Notice that if one have detected a civilization, then one has also confirmed that the other civilization will eventually be able to detect you. Further, the more developed the other detected civilization is, the faster it would be.

Therefore, my model considers that each player knows the existence of each other. However, they do not know each others' type $\theta_{-i} \in \{H, L\}$, where H represents the civilization poss high enough technology to destroy a galaxy via interstellar weapon based on indefensible universe rule such as dual-vector foil, while L represents the civilization does not.

Further, denote A as the indefensible attack, C as communication and N as silence, then we can have the action set $A_H = \{A, C, N\}$ for the high type and $A_L = \{C, N\}$ for the low type [1].

Now assume the speed of the interstellar weapon is $1/2$ of light speed. Define 1 unit of period Δt as the value of the distance in light year. Then in each period t, players choose their actions simultaneously and these actions would be observed (or, detected) in the next period $t+1$, so that players could react based on what they observed. Note that here the P_{-i} is able to detect the trace of the attack to find the location of P_i.

Given the set up above, the game would keep on repeating but end at period $t+1$ if any of the players choose A in the period t, i.e., even though the action of P_{-i} in period t is A, P_i could still be alive and choose his final action $a_{i,t}$ in period $t+1$.

Also, to simplify the model, assume that the low type need several technology explosions to transfer into the high type. That is, assume the type remains static in the short run, for some large enough $T \in N$. And this paper would only analysis the situation in the short run [2].

Therefore, there would be only 3 sates in the short run, $\omega \in \Omega = \{LL, LH, HH\}$, in the sates set of the world. Following the assumptions above, the pay-off tables at the next period $t+1$ given the actions at period t are as follows.

[1] Note that action R, running away, is not considered in this model. In fact, since a civilization is a group of people instead of a signal person, one can easily conclude that no matter what action $\delta_{i;t}$ player P_i chooses at period t, it is always better off if it performs $\delta_{i;t}$ with a few subgroups of this civilization running away their home-planet. To simplified the model, only the civilizations stay in their home-planet are counted as players.

[2] There are 2 reasons for this assumption. First, *in the long run we all died*. Second, the short run is not "short" for humans. It is only "short" in terms of universe history.

LL	C	N
C	G, G	-1, 1
N	1, -1	0, 0

Table 1: Low type - Low type

LH	A	C	N
C	-M, K	2G, 0	-1, 1
N	-M, K	1, -1	0, 0

Table 2: Low type - High type

HH	A	C	N
A	-M,-M	K, -M	K, -M
C	-M, K	0, 0	-1, 1
N	-M, K	1, -1	0, 0

Table 3: High type - High type

Here, $1 < \min\{K, G\} < \max\{K, G\} << M$, where M represents the cost of being destroyed, K represents the the benefit of obtaining potential resources after destroying the counter civilization and G represents the potential benefit of technology development after successful communication.

Note that the unsuccessful communication, i.e., P_i chooses C but P_{-i} chooses N, will reveal few information about P_i to P_{-i}, which will give tiny advantages to P_{-i} and disadvantages to P_i. To estimate this effect, I assign 1 for the P_i and -1 for the P_i.

Also, notice that even the successful communication between the high type will give 0 pay-off to both players. This is due to an extra assumption that the technology level of the high type is so high that the communication between them may not be very beneficial, especially when compared with the relatively acuter conflict of interest for their resources need.

In summary, the low type has an intention to search for communication opportunity, and the high type has an incentive to attack other civilization in any given period t. However, given the extra period $t+1$ for their counter parties, the actions they choose in period t would act as a signal. Thus, low type may try to pretend as a high type civilization, and the high type need to think twice for potential retaliatory attack in the next period [1].

[1] In the short run, the low type would not be able to transfer into the high type. Thus, given the discounted factor $\beta \in (0,1)$ for time, the threat of those low type would not be too large. Thus I also assign 0 for N. However, one may still expend the model into the long run condition. Then the only thing need to do is to assign $-m$ for each pay-off cell where A is not chosen. Here $m = \alpha M$, for some discount factor $\alpha \in (0,1)$.

Note that in the new model considering the long run effect, the property in the summary will still hold. Therefore to simplify

3 Model Analysis

Denote $\delta_{-i;t} = \begin{cases} C^{\alpha}N^{1-\alpha}, & \text{if } \theta_{-i} = L \\ A^{\beta_1}C^{\beta_2}N^{1-\beta_1-\beta_2}, & \text{if } \theta_{-i} = H \end{cases}$ as the action of P_{-i} at period t for some $\alpha, \beta_1, \beta_2 \in \mathbb{R}$, s.t., $\alpha \in [0,1]$ and $\beta_1 + \beta_2 \in [0,1]$. Denote $p = Pr(\theta_{-i} = H)$ as the belief toward the world at period t for notation convenience.

Then to show the properties mentioned in the summary part above at each sub-game given period t, we can have the analysis as follows.

1. *For low type $\theta_i = L$:*

$$\begin{aligned} EU_L(C, \delta_{-i;t}; t) &= (1-p) \cdot EU_L(C, C^{\alpha}N^{1-\alpha}; t) + p \cdot EU_L(C, A^{\beta_1}C^{\beta_2}N^{1-\beta_1-\beta_2}; t) \\ &= (1-p) \cdot \alpha G - p \cdot \beta_1 M + 2p \cdot \beta_2 G \leq -p\beta_1 M = EU_L(N, \delta_{-i;t}; t) \end{aligned}$$

   Therefore, action $a_{i;t} = C$ weakly dominates N for the low type player P_i at sub-game given period t.

2. *For High type $\theta_i = H$:*

$$\begin{aligned} EU_H(A, \delta_{-i;t}; t) &= p \cdot K + (1-p) \cdot [-\beta_1 M + \beta_2 K + (1-\beta_1-\beta_2)K] \\ EU_H(C, \delta_{-i;t}; t) &= p \cdot 0 + (1-p) \cdot [-\beta_1 M - (1-\beta_1-\beta_2)] \\ EU_H(N, \delta_{-i;t}; t) &= p \cdot 0 + (1-p) \cdot [-\beta_1 M + \beta_2] \end{aligned}$$

   Therefore, action $a_{i;t} = A$ weakly dominates N, and action $a_{i;t} = N$ weakly dominates C, for the high type player P_i at sub-game given period t.

Therefore, given the properties above, it is reasonable to only consider the following strategies for the low type as

- Always keep silence, i.e., $a_L(h) = N$, $\forall\ h \in H$;
- Always try to communicate, i.e., $a_L(h) = C$, $\forall\ h \in H$;

parameters, I decided to assign them to be 0.

- Wait for communication signal, i.e., $a_L(h) = \begin{cases} N, & \text{if } h = \{NN, \cdots, NN\} \\ C, & \text{otherwise} \end{cases}$.

and the following strategies for the high type as

- Always keep silence, i.e., $a_L(h) = N$, $\forall\ h \in H$;
- Always attack, i.e., $a_H(h) = A$, $\forall\ h \in H$;
- Wait for attack signal, i.e., $a_H(h) = \begin{cases} N, & \text{if } h = \{NN, \cdots, NN\} \\ A, & \text{otherwise} \end{cases}$;

to simplify the analysis. And for most of the other strategies, they would either be hard to form an equilibrium, or fall into one of the categorical strategies listed above.

Since the actions in period t act as a signal for the next period $t+1$, I will discuss the the equilibria in the category of separating equilibrium and pooling equilibrium respectively.

3.1 Separating Equilibrium

Thm 3.1 *There exists one and only one separating equilibrium, which is*

- *Low type would always choose C for any period $t \in \mathbb{N}$, i.e., $a_L(h) = C$, $\forall\ h \in H$.*
- *High type would always choose A in the first period to end the game, i.e., $a_H(h) = A$, $\forall\ h \in H$.*
- *Both types have prior belief toward the world that $p = Pr(\theta_{-i} = H; \emptyset)$, s.t.,$p \neq 0$ and*

$$Pr(\theta_{-i} = H; h) = \begin{cases} 1, & \textit{if } \exists\ h \in H, s.t., a_{-i}(h) = A \\ 0, & \textit{if } \exists\ h \in H, s.t., a_{-i}(h) = C \\ p, & \textit{otherwise} \end{cases}.$$

.

pf:

(**Existence**) Let period t for each sub-game be given.

By the properties shown at the beginning of the analysis section, we know that for the best response of the low type for each sub-game at period t,

$$C \in BR_L(\delta_{i;t}), \quad \forall\ \delta_{i;t} \in A_{-i;t},$$

and for the best response of the high type for each sub-game at period t,

$$A \in BR_H(\delta_{i;t}), \quad \forall\ \delta_{i;t} \in A_{-i;t}.$$

Therefore, the strategies $a_L(h) = C$ and $a_H(h) = N$, $\forall\ h \in H$, are not only a Nash Equilibrium, but also a Sub-game Perfect Nash Equilibrium.

(**Uniqueness**) Assume there exists a separating equilibrium where the high type chooses N (or C) while the low type chooses C (or N) at some period $t \in N$. Without loss generality, we may only discuss the first case where $a_H = N$ while $a_L = C$, for some period $t \in \mathbb{N}$.

Assume that $a_H = N$ and $a_L = C$ for some period t. Then in next period $t+1$, P_i would be able to identify the type of P_{-i}. By the property we have shown at the beginning, the high type always has the incentive to destroy any civilization, especially the low type that cannot perform retaliatory attack in the next period.

Therefore, if P_i receive a signal of communication C, he will know that $\theta_{-i} = L$, thus choosing A in period $t+1$. By the assumption that $|G| << |M|$, P_L has an intention to choose N to pretend to be a high type civilization. Similarly argument for $a_H = C$ and $a_L = N$.

■

This SPNE is the equilibrium which is closest to the "Dark Forest Theory" conclusion, as described in the plot:

The universe is a dark forest. Every civilization is a hunter with a gun. It sneaks in the forest like a ghost, gently plucking the branches off the road, trying not to make any sound. Even the breathing need to be careful. He must be careful, because there are hunters like him everywhere in the forest. If he finds another hunter, there are only one thing he can do: shoot and destroy it. In this forest, the others are hell; In the forest, the others are eternal threats.

Here however, things are slightly different. Even though all the high type are attacking any potential civilization, the low type are trying to build communication with each other. The essential difference is due to the assumption that the technology level of low type are so low that they are not able to hide or attack. Since the high type will attack them anyway once detected, the low type civilizations will try their best to build communication with any potential low type civilization, and develop into a high type civilization as soon as possible.

In fact, we can also find similar strategies for the low type specifically from our human history. During the the World War II, any Western European country cannot compare with Germany, but the unit can; After The founding of New China, the Chinese government ran the South-South Cooperation program with those "low type" countries in South Africa to against the pressure from the U.S. and the Soviet Unit. In a word, it is reasonable to see the corporation within the low type.

Need to note that in this SPNE, even though the type of a civilization remain static in the short run, the type would improve in the long run. And once a low type gets upgrade, he will destroy the "friend" that helps him to get upgrade as soon as possible, based on the result of the SPNE.

3.2 Pooling Equilibrium

Recall the notation for δ_{-i} as the action of player P_{-i} and $p = Pr(\theta = H)$ defined at the beginning of this section.

Thm 3.2 *There exists a pooling equilibrium, such that*

- *L type always choose* $a_L(h) = N, \ \forall \ h \in H$.
- *H type choose* $a_H(h) = \begin{cases} N, & \text{if } h = \{NN, \cdots, NN\} \\ A, & \text{otherwise} \end{cases}$.
- *Both types have prior belief toward the world that* $p = Pr(\theta_{-i} = H; \emptyset)$*, s.t.,* $0 < p < \frac{G}{G+M}$ *and*

$$Pr(\theta_{-i} = H; h) = \begin{cases} 1, & \text{if } \exists\ h \in H, s.t., a_{-i}(h) = A \\ 0, & \text{if } \exists\ h \in H, s.t., a_{-i}(h) = C \\ p, & \text{otherwise} \end{cases}.$$

.

pf:

Assume that it is a Nash equilibrium.

1. For the low type, the best alternative strategies is "always communication". Thus, the pay-off from deviation is

$$EU_L(C, \delta_{-i}; h) = p \cdot (-\beta M) + (1-p) \cdot \sum_{t=1}^{T} 0 = -p\beta M < 0 = EU_L(N, \delta_{-i}; h)$$

   Therefore, low type would not deviate unless $p = 0$.

2. For the high type, the best alternative strategies is "always attack". Thus, the pay-off from deviation is

$$EU_H(A, \delta_{-i}; h) = -p \cdot \beta M + (1-p) \cdot K$$

   Therefore, high type would not deviate unless $p \leq \frac{K}{K+\beta M}$.

Thus, since both parties have no intention to deviate by unilaterally changing their strategies, it is a Nash equilibrium. ■

In this Nash equilibrium, even though the high type still have incentive to perform attack, they do not dare to do so. This essential property results from the assumption that the players still get the chance to attack in the next period $t+1$ even if the counter parties perform an attack in the previous period t. Therefore, the high type is able to threat the counter party and stay peace.

This equilibrium also gives an explanation about why we are not able to find any extra-civilization. If possible, all the civilizations would prefer to keep silence. Any action apart from that may lead to destruction but no benefit.

Also, need to note that since $K << M$, $\frac{K}{K+M} \to 0$. Thus, the range for the prior belief is still wide enough to imply this equilibrium to most of the universe.

Thm 3.3 *There exists a pooling equilibrium, such that*

- *L type choose* $a_L(h) = \begin{cases} N, & \text{if } h = \{NN, \cdots, NN\} \\ C, & \text{otherwise} \end{cases}$.

- *H type choose* $a_H(h) = \begin{cases} N, & \text{if } h = \{NN, \cdots, NN\} \\ A, & \text{otherwise} \end{cases}$.
- *Both types have prior belief toward the world that* $p = Pr(\theta_{-i} = H; \emptyset)$*, s.t.,* $\frac{K}{K+\beta M} < p < \frac{G}{G+M}$ *and*

$$Pr(\theta_{-i} = H; h) = \begin{cases} 1, & \text{if } \exists\, h \in H, s.t., a_{-i}(h) = A \\ 0, & \text{if } \exists\, h \in H, s.t., a_{-i}(h) = C \\ p, & \text{otherwise} \end{cases}.$$

.

pf:

Assume that it is a Nash equilibrium.

1. For the low type, the pay-off from deviation is

$$EU_L(C, \delta_{-i}; h) = -(1-p) \cdot \beta M + p \cdot \sum_{t=0}^{T} \beta^t G < 0; \quad (G << M)$$

$$EU_L(N, \delta_{-i}; h) = 0.$$

   Therefore, low type would not deviate unless $p \geq \frac{M}{G+M}$.

2. For the high type, the pay-off from deviation is still the same as the proof above. Again, high type would not deviate unless $p \leq \frac{K}{K+\beta M}$.

Thus, since both parties have no intention to deviate by unilaterally changing their strategies, it is a Nash Equilibrium. ■

Notice that even though theorem 3.3 is different from the theorem 3.2, the resulting history in equilibrium would be the same, which is $\{NN, NN, \cdots, NN\}$. That is, both types would still keep silence.

However, need to note that the essential difference between theorem 3.2 and theorem 3.3 is that, as long as one naive low type civilization do not act "rationally" and decided to communicate, and as long as the counter party is also a low type luckily, then communication between civilization would happen.

Also, since $K << M$ and $G << M$, $\frac{K}{K+M} \to 0$ and $G << M$, $\frac{M}{G+M} \to 1$. Thus again, the range for the prior belief would still be wide enough to imply this equilibrium to most of the universe.

4 Conclusion

Even though I analyzed three possible equilibria in this paper, none of them are exactly the same as "Dark Forest Theory" described in the novel. The critical reasons are as follows.

- My model allows player P_i to react according to the actions players P_{-i} acted in the last period. That is, any attack or communication will reveal the information of P_{-i} to P_i, while the "Dark Forest Theory" does not.

One may argue that it would be extremely hard to locate one point out of such a huge universe 3D map by tracking the trajectory of an attack. But in fact, even the Three-body world can locate the earth based on only twice radio waves. Note that the Tree-body civilization cannot perform the galaxy-level devastating attack, thus counted as a low type player in this model. Therefore, if even some of the low type players are able to do it, I believe that this assumption is actually more realistic.

- I assumed that players already know the existence of each other while the the novel does not.

However, notice that if none of the civilization know the existence of other civilizations, then the "Dark Forest Theory" cannot hold. After all, the high type civilizations need to select a target to attack. In fact, the "Dark Forest Theory" implicitly assumed that at least one player P_i knows the existence, or at least the potential existence of one another player P_{-i}. Then if they have detected a civilization, they has also confirmed that the other civilization either has already detected them or will eventually be able to detect them.

Therefore, at the perspective of P_i, it would be reasonable to assume that P_{-i} has also known the existence of themselves

Also, notice that the action, keeping silence N, is in both action sets A_L and A_H. Thus, even there does not exist a civilization in a planet, one can still view it as a player with observed action N for each period. Therefore, a potential existence is enough for a civilization to view it as a player, no matter whether it exists or not. At least, it can be viewed as a low type in practice.

Related to the existence, there is another difference from this model to the explanation of the plot, which is given by a paragraph of analogy.

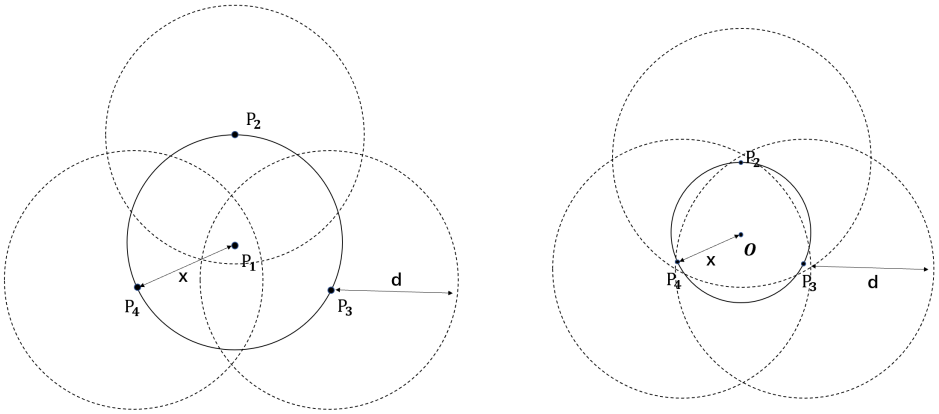
"... *Each civilization is a solder in the forest, where the friendly troops and enemies are mixed,*

thus not able to clarify whether any other solders are friend or enemy. Once a solder finds another, he could choose to verify, shoot or stay hided..."

——Luo Ji

Noticed that here the solder is able to hide in grass or the back of a tree or a stone instead of simply keeping silence. When players choose silence, it feels that it would be harder for other players to detected them. But in the universe, there is nothing for a civilization to hide in. We are in fact not in a dark forest, but in a dark plain. A huge dark plain. The only hope for not yet detected is that the other civilizations are instantly not looking in my direction. There is no place for us to hide.

Meanwhile, even though only two players is analyzed in this model, n players can be considered in this model.



Here, x represents the distance between players P_i's and d represents the farthest detection distance for players P_i.

In case (a), P_1 knows the existence of P_2, P_3 and P_4, but P_2, P_3 and P_4 only know the existence of P_1. Therefore, we can still view the game as three 2-players games between P_1 and P_2, P_3 and P_4 respectively. The same equilibria can be obtained follow the analysis above.

In case (b), P_2, P_3 and P_4 know the existence of each other. Analysis would become a little complicated here. However, those equilibria can still be obtained, following the similar argument but with more players considered in.

Besides, note that the assumption that there are only two players in each game is actually close to the real wold condition because the universe is too large. It would be rare to have $n > 2$ players detected

each other at the same time, at least at the earth perspective [1] . Therefore, this model may still fit in practice.

In summary, this paper revealed three possible equilibria.

In the separating equilibrium 3.1, the high type are always attacking and the low type are always communicating. If this is the equilibrium our universe is in, the reason we still exists is that we are living far away from the center of the universe, which is in the blood and fire. Try to not give any communication out of our detection distance but actively finding other potential civilizations to communicate would be our best response.

While in the pooling equilibria 3.2 and 3.3, the high type are keeping silence and wait the attack signal while the low type are pretending as the high type by keeping silence as well, with some of them probably also waiting for the communication signal. If this is the equilibria our universe is in, we might be in a dangerous deep sea with lots of sea monsters staring at us. Even though around us there are still lots of little shaking shrimps like us, the best things we may do is also keep silence and wait for the day we grow into a great monster.

"**Where are they?**"

The classical Fermi paradox is still there, with lots of models and theories trying to explain it. Although we still have no way to get the answer for now, our thought may still be able to go though the vast stardust and gradually approach the essence of the universe.

Based on Liu Cixin's novel "Three-body Problem II: Dark Forest", this paper used a simple model to describe a possible gray forest, where there are still full of hostility and hatred. However, the difference is that there might be some silly boys lighting sparks in the dark forest, and waiting for a communication signal in the endless darkness. Those sparks will bring a few rays of light to the dark forest, and provide some sort of warm solace to lonely travelers in the universe. As is described in the plot,

"*In this world, beauty is side with crisis, but light lives with truth.*"

[1] The solar system is far away from the center of the universe.