EECS 203: Discrete Mathematics
Winter 2024
Discussion 7 Notes

1 Definitions

- **Pigeonhole Principle:**
- **Generalized Pigeonhole Principle:**
- **Function** $f : A \to B$:
- **Domain:**
- **Codomain:**
- **Range:**
- **Onto:**
- **One-to-One:**
- **Bijection:**
- **Function Inverse** f^{-1}:
- **Function Composition** $f \circ g$:
- **Adding and Multiplying Functions:**
- **Countably Infinite**:
- **Uncountably Infinite**:
- **Schroder-Bernstein Theorem:**

2 Exercises

1. Pigeonhole Principle $\star$

How many distinct numbers must be selected from the set $\{1, 3, 5, 7, 9, 11, 13, 15\}$ to guarantee that at least one pair of these numbers add up to 16?

2. More Pigeonhole Principle

(a) Undergraduate students at a college belong to one of four groups depending on the year in which they are expected to graduate. Each student must choose one of 21 different majors. How many students are needed to assure that there are two students expected to graduate in the same year who have the same major?

(b) What is the minimum number of students, each of whom comes from one of the 50 states, who must be enrolled in a university to guarantee that there are at least 100 who come from the same state?

3. Even More Pigeonhole Principle $\star$

Sophia has a bowl of 15 red, 15 blue, and 15 orange pieces of candy. Without looking, Sophia grabs a handful of pieces.

(a) What is the smallest number of pieces of candy Sophia has to grab to make sure she has at least 4 of the same color?

(b) What is the smallest number of pieces of candy Sophia has to grab to make sure she has 3 orange candies?

4. Pigeonhole Principle Is All You Need

A computer network consists of six computers. Each computer is directly connected to at least one of the other computers. Show that there are at least two computers in the network that are directly connected to the same number of other computers.

5. Different Infinities $\star$

Determine whether each of these sets is finite, countably infinite, or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.

(a) The set of all integers greater than 10.

(b) The set of all integers with absolute value less than 1,000,000.

(c) The set of all real numbers between 0 and 2.

(d) The set $A \times \mathbb{Z}$, where $A = \{2, 3\}$.

6. Different Infinities with Sets

Give an example of two uncountable sets A and B such that $A \cap B$ is:

a) Finite.

b) Countably infinite.

c) Uncountably infinite.

7. Cardinality Proof $\star$

Show that $|(0,1)| \geq |\mathbb{Z}^+|$.

8. Schroder-Bernstein Theorem $\star$

Show that $(0, 1)$ and $[0, 1]$ have the same cardinality.

9. Countability

(a) Find a countably infinite subset A of $(0, 1)$.

(b) Find a bijection between A and $A \cup \{0, 1\}$.

(c) Find an explicit one-to-one and onto mapping from the open interval $(0, 1)$ to the closed interval $[0, 1]$.