EECS 203 Discussion 1b

Introduction to Logic

Important Forms

- Two beginning-of-semester surveys on Canvas
  - **FCI BoT** Survey and **Better Belonging in Computer Science (BBCS) Entry** Survey
  - **Due:** Friday, Feb. 2nd @11:59pm
- Exam Date Confirmation Survey
  - **Due:** Friday, Feb. 2nd @11:59pm
  - Please fill this out, even if you don’t have an exam conflict!
- They are each worth a few points, so make sure to fill them out!

Upcoming Homework

- Assignment 0 was due Jan. 18th
- Homework/Groupwork 1 will be due **Jan. 25th**
  - **Don’t forget to match pages!**
  - Please note as soon as you press submit you’ve successfully submitted by the deadline. **You can still match pages** with no rush without adding to your submission time.
- Groupwork
  - Groupwork can be done alone, but the problems tend to be more difficult, and the goal is for you to puzzle them out with others!
  - Your discussion section is a great place to find a group!
  - There is also a pinned Piazza thread for searching for homework groups.

Propositions

Problem 1

1. Negations ⋆

Negate the following statements. Any "not"s in your answer should directly precede a simple proposition, not an entire and/or statement.

a. You should study.

b. I do not like pizza.

c. I'm going to get a chai or a mocha today.

d. I'm a teacher and a student.

e. I don't like green and I don't like purple.

f. If it's raining, I'm using my umbrella.

g. $x > 2$

h. $1 + 1 = 2$

Important Truth Tables

p	q	$p \rightarrow q$	$p \leftrightarrow q$	$p \land q$	$p \lor q$
T	T	T	T	T	T
T	F	F	F	F	T
F	T	T	F	F	T
F	F	T	T	F	F

Problem 2

2. Truth Tables

Fill in the following truth table.
***Reminder:** $\wedge$ denotes "and", $\vee$ denotes "or", and $\rightarrow$ denotes "implies"/"if...then".

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$p \vee r$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \vee r)$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

Problem 3

3. Finding Truth Values of Compound Propositions ⋆

For each compound proposition, find its truth value when $p = T$, $q = F$, $r = F$, $s = F$, $t = T$, $u = F$, and $v = F$

a) $(q \to \neg p) \vee (\neg p \to \neg q)$

b) $(p \vee \neg t) \wedge (p \vee \neg s)$

c) $(p \to r) \vee (\neg s \to \neg t) \vee (\neg u \to v)$

d) $(p \wedge r \wedge s) \vee (q \wedge t) \vee (r \wedge \neg t)$

Note: p → q is only **false** in the case of **T → F**.

Problem 4

4. English to Logic Translation I

Let p, q, and r be the propositions defined as follows.

- p: Grizzly bears have been seen in the area.
- q: Hiking is safe on the trail.
- r: Berries are ripe along the trail.

Write these propositions in logic using p, q, r, logical connectives (including negations), and parentheses.
***Reminder:** $\wedge$ denotes "and", $\vee$ denotes "or", $\leftrightarrow$ denotes"if and only if", and $\neg$ denotes "not".

a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.

b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.

c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe

Problem 5

5. Logic to English Translation

Consider the following propositions:

- g: you can graduate
- m: you owe money to the university
- r: you have completed the requirements of your major
- b: you have an overdue library book

Translate the following statement to English: $g \rightarrow (r \wedge \neg m \wedge \neg b)$

Logical Equivalences

Logical Equivalence

Two compound propositions are **logically equivalent** if they have the same truth value for any input truth values.

Example:
p and ¬¬p

p	¬p	¬¬p
T	F	T
F	T	F

p and ¬¬p have the same truth tables, and thus are logically equivalent!

Examples of Logical Equivalences

De Morgan's Laws (and/or):

$$\neg(p \land q) \equiv \neg p \lor \neg q$$

$$\neg(p \lor q) \equiv \neg p \land \neg q$$

Identity Laws:

$$p \land T \equiv p$$

$$p \lor F \equiv p$$

Implication Breakout:

$$p \to q \equiv \neg p \lor q$$

Examples of Logical Equivalences

Commutative Laws:

$$p \wedge q \equiv q \wedge p$$

$$p \vee q \equiv q \vee p$$

Associative Laws:

$$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$$

$$p \vee (q \vee r) \equiv (p \vee q) \vee r$$

Distributive Laws:

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

The Contrapositive

Take an "if, then" statement **$p \rightarrow q$**. We define the **contrapositive** of this statement as **$\neg q \rightarrow \neg p$**. An implies statement and its contrapositive are logically equivalent.

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

Example:

- If **I am teaching**, then *my materials are prepared*.
- Contrapositive: If *my materials are NOT prepared*, then **I am NOT teaching**.

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg q \rightarrow \neg p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Logical Equivalence Tables (Rosen 1.3)

TABLE 8 Logical Equivalences Involving Biconditional Statements.

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

TABLE 7 Logical Equivalences Involving Conditional Statements.

$p \to q \equiv \neg p \vee q$
$p \to q \equiv \neg q \to \neg p$
$p \vee q \equiv \neg p \to q$
$p \wedge q \equiv \neg(p \to \neg q)$
$\neg(p \to q) \equiv p \wedge \neg q$
$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

Tautologies & Contradictions

Tautology, Contradiction, Satisfiability

- **Tautology:** A compound proposition that is **always true** regardless of its input values
  - **Example:** $\mathbf{p \lor \neg p}$
- **Contradiction:** A compound proposition that is **always false** regardless of its input values
  - **Example:** $\mathbf{p \land \neg p}$
- **Satisfiable:** A compound proposition is satisfiable if it **can be true** (there is at least one set of inputs that makes the proposition true)
  - **Example:** $\mathbf{p \land q}$

Problem 6

6. Tautologies

a) Determine whether $[\neg p \wedge (p \rightarrow q)] \rightarrow \neg q$ is a tautology.

b) Show that this conditional statement is a tautology by using truth tables.

$$[p \wedge (p \rightarrow q)] \rightarrow q$$

Note: a **tautology** is a **compound** proposition that is **always true** regardless of its input values

Problem 7

7. Promising Premises

For the following sets of premises and conclusions, determine whether each conclusion is valid, given the provided premise(s). A conclusion is valid if and only if it *must* be true given the premise(s). Show your work by explaining your thought process, or using a truth table, or using logical equivalences. For invalid conclusions, providing a counterexample is also sufficient to explain why it's invalid.

A note on notation: the statements above the line are the premises, and the statement below the line is the conclusion. The symbol $\therefore$ means "therefore". For example, in Part (a) there are two premises: Premise 1 is $p \vee q$ and Premise 2 is $\neg p$. You need to determine whether, together, those premises guarantee that the listed conclusion, q, is true.

a) $$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore\ q \end{array}$$

b) $$\begin{array}{l} r \to q \\ r \\ \hline \therefore\ p \vee q \end{array}$$

c) $$\begin{array}{c} (p \to q)\ \wedge\ (q \to r) \\ \hline \therefore\ r \to p \end{array}$$

d) $$\begin{array}{l} p \wedge q \\ q \to r \\ \hline \therefore\ r \end{array}$$

Problem 8

8. Logic Puzzle – Stolen Jewels

Robin Hood and his fellows Little John and Marian snuck in to a jewelry store; one of them stole a sapphire, one stole a diamond, and one stole an emerald. They were caught and put on trial, during which they made the following statements:
Robin: "John stole the sapphire."
Marian: "No, John stole the diamond."
John: "Both of them are lying. I didn't steal either."

It turns out that the one who stole the emerald lied, and the one who stole the sapphire told the truth. Who stole which gemstone?