W24

EECS 203 Discussion 8b

Graphs (Paths & Cycles), Counting Intro

Admin Notes

- **Homework/Groupwork 8**
    - **Due Apr. 4th (This upcoming Thursday)**
- **Exam 2**
    - Grades should be released early next week

Colorability

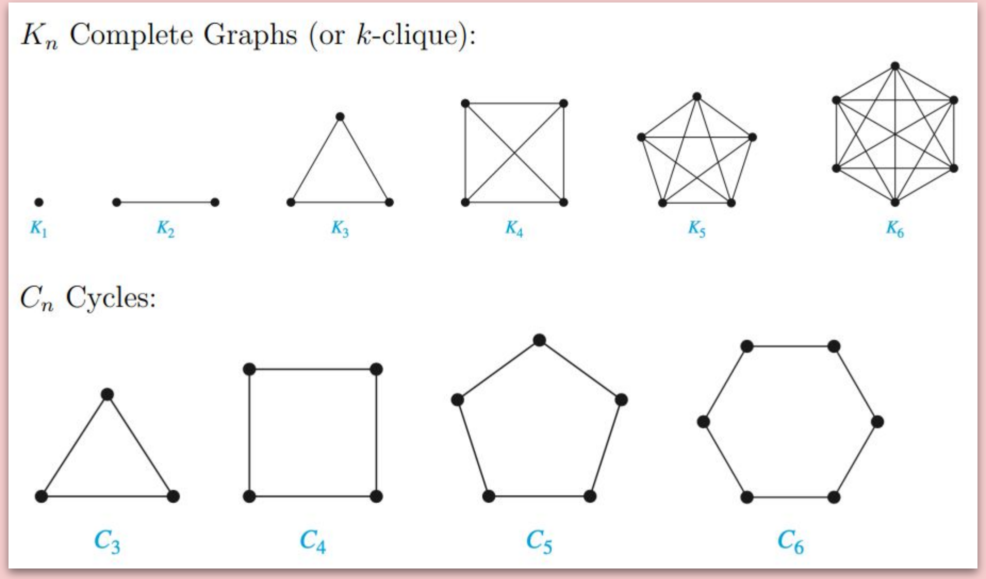
Bipartite Graphs/Colorability

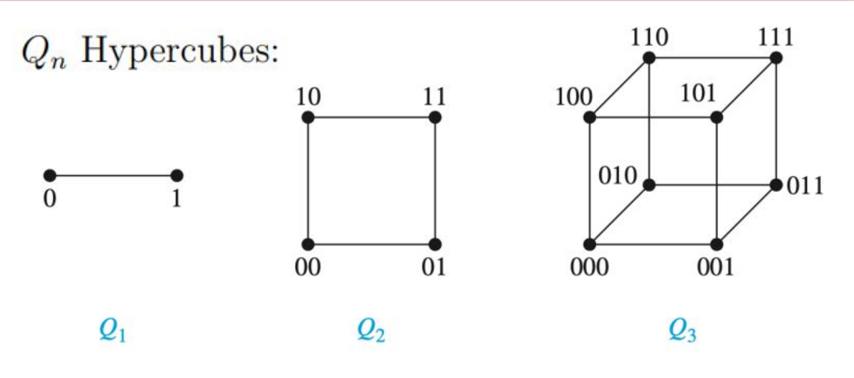
- **Bipartite Graph:** a simple graph G is called bipartite if its vertex set V can be partitioned into two disjoint sets V1 and V2 such that <u>every edge in the graph connects a vertex in V1 and a vertex in V2</u>. The pair **(V1, V2)** is called a **bipartition** of the vertex set V.

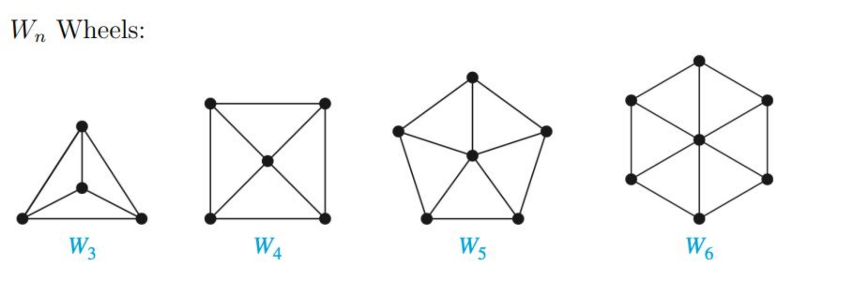
- **Bipartite Theorem (3 Equivalent Statements):** The following statements are equivalent:
    - G is **bipartite.**
    - G is **2-colorable.**
      (There is a function $f : V \Rightarrow \{red, blue\}$ such that $\mathbf{u, v \in E \Rightarrow f(u) \neq f(v)}$)
    - G **does not contain odd cycle** (C_{2k+1}) subgraphs.

Special Graphs

You only need to know **complete graphs** and **cycles**. (The others will be defined later.)







Problem 1

1. Bipartite Intro

Draw each of the following graphs. Which of these graphs are bipartite?

a) C_6, a cycle with 6 nodes

b) W_4, a wheel with a center node and 4 spoke nodes

c) Q_3, a graph representing a 3-dimensional cube, with nodes on each corner

Problem 2

2. Bipartite Conclusion

For which values of n are these graphs bipartite? Explain your answer.

a) K_n

b) C_n

c) W_n

d) Q_n

Problem 3

3. Bipartite Graphs

Does there exist a bipartite graph with degree sequence 3,3,3,3,3,3,3,3,3,3,5,6,9 (in other words, a graph with ten nodes of degree 3, one of degree 5, one of degree 6, and one of degree 9)? If not, explain why.

Graph Connectivity

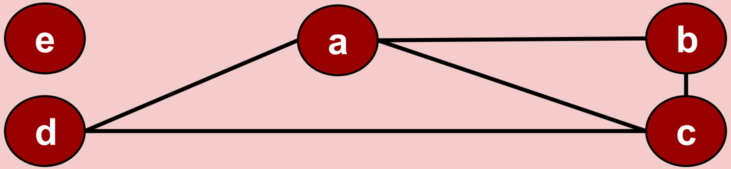
Graph Connectivity

- **Path:** a path $(u_0, u_1, \ldots u_k)$ is a sequence of vertices in which consecutive vertices in the sequence are adjacent in the graph (connected by an edge).
  - Note parentheses **()** because a path DOES indicate an order

- **Simple Path:** a path that <u>does not repeat any vertices</u>

Graph Connectivity

- **Connected Vertices:** Two vertices u and v are connected if there is a path from u to v: **(u, ..., v)**
  - Note that vertices *don't have to be adjacent* to be connected
    - **Ex from pic below: (d,b)** are connected but not adjacent
- **Connected Component:** A nonempty set of vertices in which every pair of vertices in the set is connected. **Example below: 2 connected components**



- **Connected Graph:** a graph G in which there is a path connecting any two vertices u, v $\in$ G. In other words, there is only one connected component in the graph. Example above is NOT a connected graph.

Special Types of Graph Paths

- **Euler Path:** A Euler (pronounced "oiler") path is a path that uses **every edge** of a graph exactly once. An Euler path can start and end at the same vertex OR at different vertices.

- **Euler Circuit:** An Euler path that **starts and ends at the same vertex**. Sometimes, this is also referred to as an **Euler cycle**, but note that an Euler circuit is not necessarily an actual cycle, since it can visit the same vertex multiple times, as long as it doesn't repeat an edge.

- **Euler's Theorem:** A connected graph (or multigraph) has an Euler cycle **if and only if** every vertex has even degree.

Special Types of Graph Paths

- **Hamiltonian Path:** A Hamiltonian path (or Hamilton path) is a path between two vertices of a graph that visits **every vertex** in the graph exactly once.

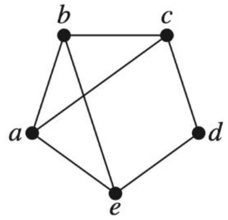
- **Hamiltonian Cycle:** If a Hamiltonian path exists whose endpoints are adjacent, then the resulting graph cycle (**starting and ending at same vertex**) is called a Hamiltonian cycle (or Hamilton cycle).

Problem 4

4. Euler Paths and Circuits

For each of the following graphs: Determine whether the graph has an Euler circuit. Construct such a circuit if one exists. If no Euler circuit exists, determine whether the graph has an Euler path. Construct such a path if one exists.

1.



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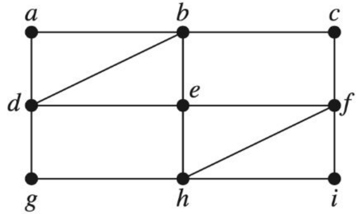

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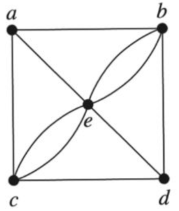

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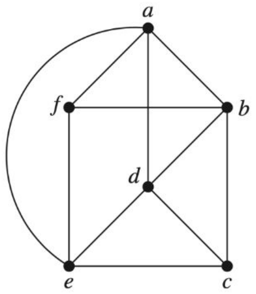

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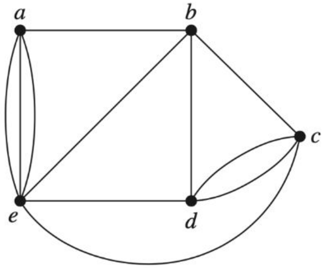

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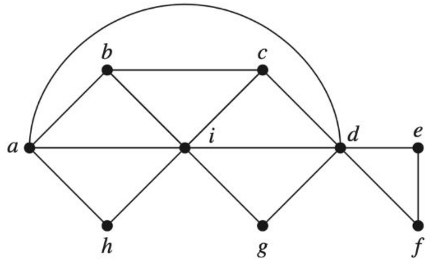

Intro to Counting

Counting Rules

- **Product Rule:** Suppose a procedure can be broken down into a sequence of *k* tasks, where you have to do each task to complete the procedure. If there are n_k ways to do the k-th task, then there are $\mathbf{n_1 \cdot n_2 \cdot n_3 \cdot \ldots \cdot n_k}$ ways to do the entire procedure.
- **Sum Rule:** Suppose there are *k* distinct, disjoint methods to complete a procedure such that k-th method can be done in n_k ways, then there are $\mathbf{n_1 + n_2 + n_3 + \ldots + n_k}$ ways to do exactly one of these tasks.
- **Division Rule**: If there are *N* ways to choose an object, and each object can be chosen in exactly *k* ways, there are ***N/k*** objects.

Counting Rules

- **Difference Rule**: Any process with *n* total choices, which has *k* extra choices that shouldn't have been counted, means ***n - k*** possible choices total

- **Inclusion Exclusion:** If a task can be done in either n_1 or n_2 ways, then the number of ways to do the task is $n_1 + n_2$ minus the number of ways to do the task that are common to the two different ways.

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$

Problem 5

5. Product Rule

a. The chairs of an auditorium are to be labeled with an uppercase English letter followed by a positive integer not exceeding 100. What is the largest number of chairs that can be labeled differently?

b. How many different license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits?

c. How many functions are there from a set with m elements to a set with n elements?

d. How many one-to-one functions are there from a set with m elements to one with n elements?

Problem 6

6. Sum Rule

a. A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 19 possible projects, respectively. No project is on more than one list. How many possible projects are there to choose from?

b. A wired equivalent privacy (WEP) key for a wireless fi- delity (WiFi) network is a string of either 10, 26, or 58 hexadecimal digits. How many different WEP keys are there?

Problem 7

7. Inclusion Exclusion

a. How many bit strings of length eight either start with a 1 bit or end with the two bits 00?

b. A computer company receives 350 applications from college graduates for a job planning a line of new web servers. Suppose that 220 of these applicants majored in computer science, 147 majored in business, and 51 majored both in computer science and in business. How many of these applicants majored neither in computer science nor in business?

Problem 8

8. Division Rule

How many different ways are there to seat four people around a circular table, where two seatings are considered the same when each person has the same left neighbor and the same right neighbor?