W24

EECS 203 Discussion 3

Proof by Contrapositive & Contradiction

Important Forms

- Two beginning-of-semester surveys on Canvas
    - **FCI BoT** Survey and **Better Belonging in Computer Science (BBCS) Entry** Survey
    - **Due:** Friday, Feb. 2nd @11:59pm
- Exam Date Confirmation Survey
    - **Due:** Friday, Feb. 2nd @11:59pm
    - Please fill this out, even if you don’t have an exam conflict!
- They are each worth a few points, so make sure to fill them out!

Upcoming Homework

- Homework/Groupwork 3 will be due **Feb. 8th**
  - **Don't forget to match pages!**
  - Please note as soon as you press submit you've successfully submitted by the deadline. **You can still match pages** with no rush without adding to your submission time.
- Groupwork
  - Groupwork can be done alone, but the problems tend to be more difficult, and the goal is for you to puzzle them out with others!
  - Your discussion section is a great place to find a group!
  - There is also a pinned Piazza thread for searching for homework groups.o

Proof Techniques

Proof Methods

- **Direct Proof:** Proves $p \rightarrow q$ by showing
  $p \rightarrow \text{stuff} \rightarrow q$
- **Proof by Contraposition:** Proves $p \rightarrow q$ by showing
  $\neg q \rightarrow \text{stuff} \rightarrow \neg p$
- **Proof by Contradiction:** Proves p by showing
  $\neg p \rightarrow F$
- **Proof by Cases:** next week

Some Methods of Proving $p \rightarrow q$

- **Direct Proof:**
  Proves $p \rightarrow q$ by showing **$p \rightarrow$ stuff $\rightarrow q$**
- **Proof by Contraposition:**
  Proves $p \rightarrow q$ by showing **$\neg q \rightarrow$ stuff $\rightarrow \neg p$**
  (Knowing $\neg q \rightarrow \neg p$ enables concluding $p \rightarrow q$ because $\neg q \rightarrow \neg p \equiv p \rightarrow q$)
- **Proof by Contradiction:**
  Proves p by showing **$\neg p \rightarrow F$**
  To prove $p \rightarrow q$, assume the negation: **$\neg(p \rightarrow q) \equiv \neg(\neg p \lor q) \equiv p \land \neg q$**
  Derive a contradiction (F) from this assumption by arriving at a mathematically incorrect statement (ex: $0 = 2$) or two statements that contradict each other ($x = y$ and $x \neq y$). Therefore, $p \rightarrow q$.

Problem 1

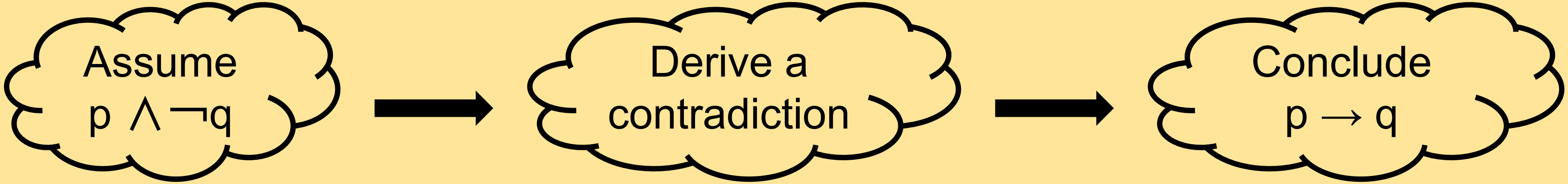
1. Proof by Contraposition $\star$

Prove that if $n^2 + 2$ is even, then n is even using a proof by contrapositive.

Proof by Contradiction

Proof by Contradiction

- When trying to prove p implies q, assume p is true and q is false. Derive a **contradiction**, (something that is always false, **ex:** $0 = 2$, **ex:** $x = y$ and $x \neq y$). Therefore, $p \rightarrow q$.
  - We assume the negation of what we want to prove
  - We arrive at something false
  - Therefore the negation of the thing we assumed must be true (ie the thing we wanted to prove)



$\neg(p \rightarrow q) \equiv (p \land \neg q)$ ⟶ **F** ⟶ $\neg(p \land \neg q) \equiv \neg p \lor q \equiv p \rightarrow q$

Problem 2

2. Contraposition vs Contradiction $\star$

Show that for an integer n: if $n^3 + 5$ is odd, then n is even, using

a) a proof by contraposition.

b) a proof by contradiction.

Note: The algebra in either case is the same. You don't need to rewrite the algebra for part (b), just reformat your proof from (a) into a proof by contradiction.

Problem 3

3. Proof Practice

Prove or disprove that the sum of a rational number and an irrational number must be irrational.

Problem 4

4. Odd Proof III

Prove that for all integers a and b, if a divides b and $a + b$ is odd, then a is odd.

Problem 5

5. Proofs ⋆

1. Prove or disprove: For all nonzero rational numbers x and y, x^y is rational
2. Prove or disprove: For all even integers x and all positive integers y, x^y is even.
3. Prove or disprove: For all real numbers x and y, if x^y is irrational, then x is not a positive integer or y is not a positive integer