

proposition: predicate w/o quantifier is not a proposition (i.e. $x = 0$ w/o for all/there exists x is not a proposition) **rows** = 2^n , n = num of predicates

negate: **always** negate quantifiers $\neg \forall x$ to $\exists x$ when writing out translation

quantifier/logic translation: **always** use: $\forall x(P(x) \rightarrow Q(x))$ and $\exists x(P(x) \wedge Q(x))$

quantifier scoping: $\forall x \exists y$ for all x there is at least one y , doesn't have to be the same y
 $\exists y \forall x$ there is only one y for all x , is the same y for all x

Distributing quantifier:

Nested quantifier: demorgan is done one layer after another: $\neg \forall x \forall y \forall z P(x) \equiv \exists x \neg \forall y \forall z P(x) \equiv \exists x \exists y \exists z \neg P(x)$

Only \wedge and \vee distribute

Useful demorgans/equivalence laws:

De Morgan's Law for quantifiers:

Negation	Equivalent Statement	When Is Negation True?	When False?
$\neg \exists x P(x)$	$\forall x \neg P(x)$	For every x , $P(x)$ is false.	There is an x for which $P(x)$ is true.
$\neg \forall x P(x)$	$\exists x \neg P(x)$	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x .

De Morgan's Law: $\neg(p \wedge q) \equiv \neg p \vee \neg q$, $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Definition of \rightarrow : $p \leq q \equiv T$, $p > q \equiv F$

Idempotent Laws: $p \vee p \equiv p$, $p \wedge p \equiv p$

Distributive Law: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$, $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Absorption Laws: $p \vee (p \wedge q) \equiv p$, $p \wedge (p \vee q) \equiv p$, this is because \vee only needs at most one to be true

Impl breakout/contrapos: $p \rightarrow q \equiv \neg p \vee q \equiv \neg q \rightarrow \neg p$ (**always breakout first to simplify stuff**)

Nega impl: $\neg(p \rightarrow q) \equiv p \wedge \neg q$

TABLE 7 Logical Equivalences Involving Conditional Statements.

$p \rightarrow q \equiv \neg p \vee q$
 $p \rightarrow q \equiv \neg q \rightarrow \neg p$
 $p \vee q \equiv \neg p \rightarrow q$
 $p \wedge q \equiv \neg(p \rightarrow \neg q)$
 $\neg(p \rightarrow q) \equiv p \wedge \neg q$
 $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
 $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$
 $(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$
 $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

TABLE 8 Logical Equivalences Involving Biconditional Statements.

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
 $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
 $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
 $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Tautology/must be true $A(x) \wedge (\neg A(x) \vee A(x)) \equiv A(x)$

Contradiction/must be false $p \wedge \neg p$

Satisfiable/can have one case to be true

counterexamples think:

$\forall x \exists y (y^2 = x) [x, y \in \mathbb{R}]$, false, x cannot be < 0 (should also check for zero)

$\forall y \exists x (y = 3x) [x, y \in \mathbb{Z}]$, false, $y = 1$, x can be a fraction.

$\exists x \forall y (y = 3x) [x, y \in \mathbb{Z}]$, false, $x = 4$, $y = 1, 2, 3, 203$

Proof 101:

always prove/disprove for all instead of there exists, then **let vars be arb. type of num**
to disprove $P(x)$ = prove $\neg P(x)$ (is true)

Proof:

by contradiction: seeking contradiction, assume the (negation), there is a contradiction, thus original is true

by cases: blah, then (WLOG) all cases exhausted

by contrapos: assume contrapo

truth table finding: if mostly false, then its \wedge + add \neg , ; if mostly true, then its \vee + add \neg ; if its half false & true, then its \leftrightarrow

Sets:

Cardinality: num of elements in a set. Just be careful with which one gets subtracted from

Power set: every single possible combos of sets including $\{\emptyset\}$

disjoint sets

minus: $A - B = \text{in } A \text{ but not in } B$

Sets proof 101:

Subset method: show that each side of the identity is a subset of the other side.

Proper subset: $A \subset B \wedge A \neq B$, A is a subset of B , and A and B are not the same set.

Inclusion-exclusion principle: Only applies to cardinality

$|A \cup B \cup C| (\text{this is total}) = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

More counterexamples for rational / irrational $x = \sqrt{2}, y = \sqrt{2}, x \times y = 2$, which is irrational times irrational making a rational

Counterexamples for positive / negative, $x = 0$

Prime numbers > 2 are odd. Or prime numbers only divide themselves and 1.

Tautology can always be satisfiable.
Satisfiable cannot always be tautology.

Madeleine's OH equations: $x \in \overline{(B \cap A)} \equiv x \in (\overline{B} \cup \overline{A}) \equiv x \notin (B \cap A) \equiv x \notin B \vee x \notin A$

TABLE 1 Set Identities.	
Identity	Name
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cap C) = (A \cup B) \cap C$ $A \cap (B \cup C) = (A \cap B) \cup C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws