# Groupwork 7 Problems

## 1. Get to the Point [10 points]

Consider an arbitrary set $A$. We say a function $f : A \to A$ has a *fixed point* iff there exists $a \in A$ such that $f(a) = a$.

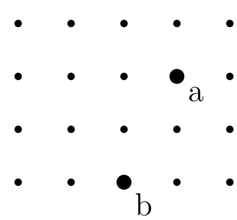
Consider the notation $f^{(n)}$ to mean $\underbrace{f \circ \cdots \circ f}_{n \text{ times}}$, where $n \in \mathbb{Z}^+$. Essentially, $n$ copies of $f$ are composed together.

Prove by **induction** that if $f$ is a function with a fixed point, then for all positive integers $n$, $f^{(n)}$ has a fixed point.

**Solution:**

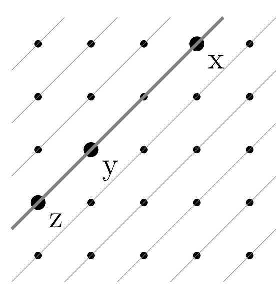
## 2. Going Off the Grid [8 points]

In a grid, we say that a point $a$ *dominates* a point $b$ iff $a$ lies strictly above and to the right of $b$. For example, in the picture below, $a$ dominates $b$.




Prove using the Pigeonhole Principle that if we choose $4n - 1$ points from an $n \times n$ grid ($n \geq 4$), there must be three chosen points $x, y, z$ such that $x$ dominates $y$ and $y$ dominates $z$. Make sure to state what your pigeons are and what your holes are, as well as how many of each you have.

**Hint:** If $x, y, z$ lie on the same increasing diagonal as shown in the picture below, then $x$ dominates $y$ and $y$ dominates $z$.



**Solution:**