EECS 203: Discrete Mathematics
Winter 2024
Discussion 5a Notes

1 Definitions

- **Mathematical Induction**:
- **Induction Steps**:
  - **Base Case**:
  - **Inductive Hypothesis**:
  - **Inductive Step**:

2 Exercises

1. Bandar's Blunder ⋆

Bandar writes a proof for the following statement:

$$n! > n^2 \text{ for all } n \geq 4.$$

His proof is incorrect, and it's your task to help him identify his mistake!

Proof:

Inductive step:
Let k be arbitrary. Assume $P(k) : k! > k^2$. We need to show $P(k+1) : (k+1)! > (k+1)^2$

$$\begin{aligned}
(k+1)! &= (k+1) \cdot k! \\
&> (k+1) \cdot k^2 && \text{(By the Inductive Hypothesis)} \\
&= (k+1)(k \cdot k) \\
&\geq (k+1)(2 \cdot k) && \text{(Because } k \geq 2\text{)} \\
&= (k+1)(k+k) \\
&\geq (k+1)(k+1) && \text{(Because } k \geq 1\text{)} \\
&= (k+1)^2
\end{aligned}$$

This proves $(k+1)! > (k+1)^2$.

Base Case:
Prove $P(0) : 0! > 0^2$, $0! = 1 > 0^2 = 0$

Thus by mathematical induction, $n! > n^2$ for all $n \geq 0$.

What is wrong with Bandar's proof?

2. Sum Mathematical Induction

Using induction, prove that for all integers $n \geq 1$:

$$\sum_{r=1}^{n} (r+1) \cdot 2^{r-1} = n \cdot 2^n$$

3. REVIEW: Satisfiability $\star$

Determine whether each of these compound propositions is satisfiable.

(a) $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$

(b) $(p \to q) \wedge (p \to \neg q) \wedge (\neg p \to q) \wedge (\neg p \to \neg q)$

4. REVIEW: Nested Quantifier Translations

Let P(x, y) be the statement "Student x has taken class y," where the domain for x consists of all students in your class and for y consists of all computer science courses at your school. Express each of these quantifications in English.

a) $\exists x \exists y P(x, y)$

b) $\exists x \forall y P(x, y)$

c) $\forall x \exists y P(x, y)$

d) $\exists y \forall x P(x, y)$

e) $\forall y \exists x P(x, y)$

f) $\forall x \forall y P(x, y)$

5. REVIEW: Direct Proof

Use a direct proof to show that the product of any two odd numbers must be odd.

6. REVIEW: Proof by Contradiction $\star$

Prove that for all integers n, if $n^2 + 2$ is even, then n is even using a proof by contradiction.

7. REVIEW: Proof by Contrapositive $\star$

Prove that for all integers x and y, if xy^2 is even, then x is even or y is even.

8. REVIEW: Proof by Cases/Disproofs $\star$

a) Prove or Disprove that for all integers n, $n^2 + n$ is even

b) Prove or Disprove that for all integers a and b, $\frac{a}{b}$ is a rational number.

9. REVIEW: Sets $\star$

Let our domain U be the set of the 26 lowercase letters in the English alphabet. Let $A = \{i, a, n\}, B = \{s, h, u, b\}, C = \{i, s, a, b, e, l\}$. Compute the following, where complements are taken within U. Write your answers in list notation.

Hint: For parts (b) and (c), simplifying the expressions using set identities may make the sets quicker to compute.

(a) $(A \cup B) - C$

(b) $\overline{\overline{B \cup C} \cup A}$

(c) $(A \times B) \cap (A \times C)$