

EECS 203: Discrete Mathematics
Winter 2024
Homework 3

Due **Thursday, Feb. 8**, 10:00 pm

No late homework accepted past midnight.

Number of Problems: $7 + 1$

Total Points: $100 + 30$

- **Match your pages!** Your submission time is when you upload the file, so the time you take to match pages doesn't count against you.
- Submit this assignment (and any regrade requests later) on Gradescope.
- Justify your answers and show your work (unless a question says otherwise).
- By submitting this homework, you agree that you are in compliance with the Engineering Honor Code and the Course Policies for 203, and that you are submitting your own work.
- Check the syllabus for full details.

1. On the Contrary [12 points]

Let n be an integer. Prove that if $4 \mid (n^2 - 1)$, then n is odd using

- (a) a proof by contraposition, and
- (b) a proof by contradiction.

Then,

- (c) compare your answers to parts (a) and (b). What is different? What is the same?

Solution:

Let p be $4 \mid (n^2 - 1)$, q be n is odd.

The original proposition can therefore be expressed as $\forall n(p \rightarrow q)$.

a) Proof:

To prove by contraposition,

the contrapositive of the original is $\forall n(\neg q \rightarrow \neg p)$.

contraposition

Assume n is an even integer

$n = 2k$, k is a random integer

definition of even

$n^2 - 1 \equiv 4k^2 - 1$

substitution

It does not divide 4

definition of divide.

Thus, the original proposition is true by contraposition.

b) Proof:

To prove by contradiction,

the negation of the original is $\exists n(p \wedge \neg q)$.

Implication Breakout Rule

Assume n is an even integer, $n^2 - 1$ divides 4

$n = 2k$, k is a random integer

definition of even

$n^2 - 1 \equiv 4k^2 - 1$

substitution

It does not divide 4

definition of divide.

Thus, the original proposition is true by contradiction.

2. An Even-Numbered Question about Even Numbers [16 points]

Prove or disprove the following statements:

- (a) For all integers x , if x is even, then x^2 is even.
- (b) For all integers x , if x^2 is even, then x is even.
- (c) For all integers x , if x is even, then $2x$ is even.

(d) For all integers x , if $2x$ is even, then x is even.

Solution:

3. Even Stevens [16 points]

Prove or disprove the following statement: “There is a finite amount of even numbers.”

Solution:

4. Pay it Forward (Or Don't, It's Up To You) [12 points]

Consider a centipede game, where there are two players: Ka-chun and Zyaire. The game starts by Ka-chun's decision of take or wait.

- If Ka-chun takes, Ka-chun earns \$1 while Zyaire earns nothing, and the game ends.
- If Ka-chun waits, then Zyaire can choose between take or wait. If Zyaire takes, Zyaire earns \$2 while Ka-chun earns nothing and the game ends. If Zyaire waits it becomes Ka-chun's turn to choose again.
- If they keep waiting the reward grows by \$1 each round, until Zyaire's choice of taking \$20 or waiting, when the game will end no matter what.

Both of Ka-chun and Zyaire want to maximize their rewards, and behave as perfect logicians.

- (a) Suppose Ka-chun and Zyaire made it to round 20. What happens in round 20?
- (b) Using your answer to (a), what would happen if they made it to round 19?
- (c) Building off of parts (a) and (b), argue that Ka-chun should take \$1 in the very first round.

Solution:

5. Proofs to the Max [12 points]

Prove that for all real numbers a , b , and c , if $\max\{a^2(b - c), -a\}$ is non-negative, then $a \leq 0$ or $b \geq c$.

Note: You can use the following facts in your proof:

- If x and y are positive, then $x \cdot y$ is positive.
- If x is positive and y is negative, then $x \cdot y$ is negative.
- If x and y are negative, then $x \cdot y$ is positive.

Solution:

6. Let's All Be Rational [16 points]

Show that these statements about a real number x are equivalent to each other:

- (i) x is rational
- (ii) $\frac{x}{2}$ is rational
- (iii) $3x - 1$ is rational.

Hint: One way to prove statements (i), (ii) and (iii) are equivalent is by proving (i) \rightarrow (ii), (ii) \rightarrow (iii), and (iii) \rightarrow (i).

Solution:

7. Irrational Proof [16 points]

Prove or disprove that the product of a nonzero rational number and an irrational number is irrational.

Solution:

Grading of Groupwork 2

Using the solutions and Grading Guidelines, grade your Groupwork 2 Problems:

- Use the table below to grade your past groupwork submission and calculate scores.
- While grading, mark up your past submission. Include this with the table when you submit your grading.
- Write whether your submission achieved each rubric item. If it didn't achieve one, say why not.
- For extra credit, write positive comment(s) about your work.
- You don't have to redo problems correctly, but it is recommended!
- See "All About Groupwork" on Canvas for more detailed guidance, and what to do if you change groups.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)	Total:
Problem 1												/20
Problem 2												/20
Total:												/40

Groupwork 3 Problems

1. Are These Equivalent? [30 points]

Let $P(x)$ and $Q(x)$ be arbitrary predicates.

- (a) Prove or disprove that for any domain of x , $\forall x(P(x) \leftrightarrow Q(x))$ must be logically equivalent to $\forall xP(x) \leftrightarrow \forall xQ(x)$.
- (b) Prove or disprove that for any domain of x , $\exists x(P(x) \leftrightarrow Q(x))$ must be logically equivalent to $\exists xP(x) \leftrightarrow \exists xQ(x)$.
- (c) Let $\Diamond x$ mean that “there exists **at most one** x .” Prove or disprove that for any domain of x , $\Diamond x(P(x) \leftrightarrow Q(x))$ must be logically equivalent to $\Diamond xP(x) \leftrightarrow \Diamond xQ(x)$.

Solution:
