

EECS 203: Discrete Mathematics
Winter 2024
Homework 7

Due **Thursday, Mar. 21**, 10:00 pm

No late homework accepted past midnight.

Number of Problems: $8 + 2$

Total Points: $100 + 18$

- **Match your pages!** Your submission time is when you upload the file, so the time you take to match pages doesn't count against you.
- Submit this assignment (and any regrade requests later) on Gradescope.
- Justify your answers and show your work (unless a question says otherwise).
- By submitting this homework, you agree that you are in compliance with the Engineering Honor Code and the Course Policies for 203, and that you are submitting your own work.
- Check the syllabus for full details.

Individual Portion

1. Growing your Growth Mindset [5 points]

- (a) Watch the linked video about developing a growth mindset. This is a different video than the one you saw in lecture.
- (b) Rewrite the last two fixed mindset statements as growth mindset statements.
- (c) Write down one of your recurring fixed mindset thoughts, then write a thought you can replace it with that reflects a growth mindset.

Video: Developing a Growth Mindset (tinyurl.com/eecs203growthMindset)

What to submit: Your three pairs of fixed and growth mindset statements (the two from the table, and one that you came up with on your own).

Fixed Mindset Statement	Growth Mindset Statement
When I have to ask for help or get called on in lecture, I get anxious and feel like people will think I'm not smart.	The question I have is likely the same question someone else in lecture may have. It's important for me to ask so I can better understand what I am learning.
I'm jealous of other people's success.	I am inspired and encouraged by other people's success. They show me what is possible.
I didn't score as high on the exam as I expected. I'm not going to do well in this class and should drop it.	I learned from my mistakes on exam 1, and exam 2 will be a new opportunity for me to practice what I've learned.
This class is hard for me, so I am not fit for this major.	[FILL IN YOUR OWN]
Either I'm good at Discrete Math, or I'm not.	[FILL IN YOUR OWN]
[FILL IN YOUR OWN]	[FILL IN YOUR OWN]

Solution:

2. Sketchy Compositions [15 points]

Consider $f: X \rightarrow Y$ and $g: Y \rightarrow X$. **Prove or disprove** each of the following statements.

- (a) If $f \circ g$ is one-to-one, then g must be one-to-one.
- (b) If $g \circ f$ is one-to-one, then g must be one-to-one.

Solution:

3. Flippy Function Fun! [15 points]

A function $f: A \rightarrow A$ is said to be *flippy* if for all $a \in A$, $f(f(a)) = a$. **Prove or disprove** each of the following statements

- (a) If $f: A \rightarrow A$ is flippy, then f is bijective. (Either prove f is both onto and one-to-one using their respective definitions, or provide a counterexample.)
- (b) If $f: A \rightarrow A$ and $g: A \rightarrow A$ are flippy, then $f \circ g$ must be flippy.

Solution:

4. A Hairy Situation [12 points]

Assume that nobody on Earth has more than 1,000,000 hairs on their head. Assume that the population of New York City in 2024 is 8,468,000 people. As of 2024, what is the maximum number of people in New York City that we can guarantee all have the same number of hairs on their heads?

Your explanation should use the Pigeonhole Principle. Make sure to state what the pigeons and holes are, as well as how many of each you have.

Solution:

5. A Paity Situation [14 points]

Suppose that 52 integers are chosen among the set of natural numbers less than 100. In other words, suppose that 52 integers are chosen from $\{0, 1, 2, 3, \dots, 99\}$. **Prove or disprove** that there must exist at least one pair of integers among those chosen whose difference is equal to 7.

Your proof or disproof should use the Pigeonhole Principle. Make sure to state what the pigeons and holes are, as well as how many of each you have.

Solution:

6. Super Sets [15 points]

Let A be the set of prime numbers less than 203. The universe of discourse is \mathbb{R} . State whether each of the following sets are empty, finite but nonempty, countably infinite, or uncountable. Briefly justify your answers.

(a) $\mathbb{Z} \times \mathbb{Z}$

(b) $(\mathbb{Z} \times \mathbb{Z}) - (\mathbb{Q} \times \mathbb{Q})$

(c) $\mathbb{R} - \mathbb{Q}$

(d) $\mathbb{Q} - \mathbb{R}$

(e) $A \cap \mathbb{Q}$

(f) $\overline{A} \cap \overline{\mathbb{Q}}$

Solution:

7. Cardinal Construction [12 points]

For each part, give *uncountable* sets A and B such that $A - B$ is

(a) uncountable.

(b) countably infinite.

(c) finite but nonempty.

(d) empty.

Solution:

8. Interesting Intervals [12 points]

Prove that $|[0, 3]| = |(2, 5) \cup (6, 7)|$. If you construct functions in your solution with certain properties, you may assert that they have those properties without proof.

Solution: