W24

EECS 203 Discussion 9

Counting, Probability Intro

Admin Notes

- **Homework/Groupwork 9**
  - **Due Apr. 11th (This upcoming Thursday)**
- **Exam 2**
  - Grades have been released!
  - Please submit regrade requests before the deadline!

Permutations and Combinations

RECALL…

- **Product Rule:** this (m ways) AND that (n ways)

  m·n ways

- **Sum Rule:** this (m ways) OR that (n ways)
  *no ways that do both

  m + n ways

- **Subtraction Rule:** this (m ways) OR that (n ways)
  *r ways that do both

  m + n – r ways

- **Division Rule:** N ways, k ways to choose each objects

  N/k ways

- **Difference Rule:** N ways total, k ways we don't want to choose

  N – k ways

Permutations and Combinations

- **Permutation:** **P(n,k)** is the number of ways to choose k things out of n things where the order of selection matters
  - n choices for first object, (n-1) choices for second object, . . .

$$P(n,k) = n(n-1)(n-2)\cdots(n-k+1) = n! \,/\, (n-k)!$$

- **Combination:** **C(n,k)** is the number of ways to choose k things out of n things (where the order of selection does not matter)
  - Permutation formula – divided by the number of orders those k things could've been chosen (division rule)

$$C(n,k) = n! \,/\, (n-k)!k!$$

Distinguishability &
Distributing Objects into Bins

Distinguishable vs Indistinguishable Objects

Objects can either be distinguishable or indistinguishable…

- **Distinguishable:** different from each other, “labeled”
    - different people
    - labelled boxes
    - different cards in a standard deck of cards
- **Indistinguishable:** considered identical, “unlabeled”
    - chocolate chip cookies
    - occurrences of S in SUCCESS
    - 0’s in a bit string
    - copies of the same book
    - unlabelled boxes

Distributing Objects Into Bins

Depending on if the objects and bins are distinguishable/indistinguishable changes the way we approach the problem.

- **Distinguishable Objects and Distinguishable Bins**
- **Indistinguishable Objects and Distinguishable Bins**
  - NOT COVERING THIS SEMESTER
- **Distinguishable Objects and Indistinguishable Bins**
- **Indistinguishable Objects and Indistinguishable Bins**

Problem

1. Basic Permutations and Combinations ⋆

a. How many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 100 different people who have entered a contest?

b. How many poker hands of five cards can be dealt from a standard deck of 52 cards? Also, how many ways are there to select a set of 47 cards from a standard deck of 52 cards? The order of the hand of five cards and the order of the set of 47 cards do not matter.

c. How many permutations of the letters ABCDEFGH contain the string ABC?

d. How many bit strings of length n contain exactly r 1's?

Problem

2. Standing in Line ⋆

How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other? [Hint: First position the men and then consider possible positions for the women.]

Problem

3. Forming a Committee

Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have the same number of men and women?

Problem

4. Permutations with Objects of Different Types ⋆

How many different strings can be made by reordering the letters of the word SUCCESS?

Problem

5. Hanging Jerseys

Robert went out to the store and bought 10 Michigan basketball jerseys and 15 Michigan football jerseys to adorn his walls due to Michigan's recent successes. Each jersey has a different player's name so he could tell them apart. However, once he got back to his room he realized that he only had room to hang up 6 jerseys! If he doesn't care where each jersey is positioned on his walls, how many ways are there to select the jerseys that will be put up if:

(a) he would like to hang up more or equal number of football jerseys than basketball jerseys

(b) he would like to hang up an equal number of football and basketball jerseys, but he can't hang up Surya's basketball jersey without also hanging up Ashu's football jersey? **Note: he could hang up Ashu's jersey without hanging up Surya's jersey.**

Provide a brief justification for each part.

Discrete Probability

Intro to Probability

- **Experiment:** Procedure that yields an outcome.
- **Sample Space**: Set of all possible outcomes in an experiment, usually denoted by ***S***
- **Event:** A subset of the sample space, usually denoted by ***E***
- **Probability of an Event (Equally Likely Outcomes):**
  The probability of an event $E \subseteq S$ given all elements in S are equally likely is

  $$P(E) = |E| / |S|$$

  - **Example:** P(rolling a 2 or 6) = 2/6 *fair 6-sided die
- **General Probability of Events:**
  The probability of an event is the sum of the probabilities of each outcome in the event. *NOTE: The probability of all outcomes in a sample space add up to 1.

  $$P(E) = \sum_{s \in E} p(s)$$

Problem

6. Probability Intro ⋆

What is the probability that when two dice are rolled, the sum of the numbers on the two dice is 8?

Problem

7. Poker Hands ⋆

a. Find the probability that a hand of five cards in poker contains four cards of one rank.

b. What is the probability that a poker hand contains a full house, that is, three of one rank and two of another rank?

Conditional Probability & Independence

Conditional Probability and Independence

- **Conditional Probability:** The probability of E_1 given E_2, denoted $\mathbf{P(E_1 \mid E_2)}$, is

$$P(E_1|E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)}$$

- **Independence:** Events E and F are independent if and only if:

  $$\mathbf{P(E \cap F) = P(E) * P(F)}$$

  *Note: This is not the same thing as two events being mutually exclusive

- **Conditional Probability and Independence:** If $\mathbf{P(E) = P(E \mid F)}$, then E and F are independent.

Problem

8. Independent vs Mutually Exclusive Events

Two six-sided dice, Dice A and Dice B, are rolled. Using these dice, provide examples of:

(a) A pair of independent events.

(b) A pair of mutually exclusive events.

Recall that events E and F are independent if $P(E \cap F) = P(E) \cdot P(F)$, and they are mutually exclusive if $P(E \cap F) = 0$.

Problem

9. Conditional Probability ⋆

A bit string of length four is generated at random so that each of the 16 bit strings of length four is equally likely. What is the probability that it contains at least two consecutive 0s, given that its first bit is a 0?

Problem

10. Independent Events

Assume that each of the four ways a family can have two children is equally likely. Are the events E, that a family with two children has two boys, and F, that a family with two children has at least one boy, independent?