EECS 203: Discrete Mathematics
Winter 2024
Homework 1

Due **Thursday, January 25th**, 10:00 pm

No late homework accepted past midnight.

Number of Problems: $7 + 1$ Total Points: $100 + 20$

- **Match your pages!** Your submission time is when you upload the file, so the time you take to match pages doesn't count against you.
- Submit this assignment (and any regrade requests later) on Gradescope.
- Justify your answers and show your work (unless a question says otherwise).
- By submitting this homework, you agree that you are in compliance with the Engineering Honor Code and the Course Policies for 203, and that you are submitting your own work.
- Check the syllabus for full details.

Individual Portion

1. Collaboration and Support [3 points]

(a) Give the names and uniqnames of 2 of your EECS 203 classmates (these could be members of your homework group or other classmates).

(b) When you have questions about the course content, where can you ask them? Where are *you* most likely to ask questions?

(c) Name one self-care action you plan to do this semester to maintain your overall well-being.

Solution:

a) Youssef Cherri (ycherri) and Jonathan Kertawidjaja (jonkert)
b) We go to the same discussion section, so I would ask them there. I would most likely ask questions where they are hard.
c) I take naps when I can't focus anymore.

2. Rock the Vote [12 points]

Let p and q be the following propositions:

- p: The election has been decided.
- q: The votes have been counted.

Express each of these propositions as an English statement:

(a) $\neg p \to \neg q$

(b) $\neg q \lor (\neg p \land q)$

Solution:

a) If the election has not been decided, then the votes have not been counted.
b) The votes have not been counted, or the election has not been decided and the votes have been counted.

3. Negation Station [16 points]

For each of the following propositions, give their negation in natural English. Your answer should not contain the original proposition. That is, you shouldn't negate it as "It is not the case that ..." or something similar.

Note: You do not need to show work besides your translation, but you may earn partial credit if you do.

(a) a is greater than 6 or at most 2.

(b) b is a perfect square, odd, and not divisible by 7.

(c) c is odd whenever it is prime and greater than 3.

(d) If d is divisible by 2, then it is even.

Solution:

a) a is less than 6 and at least 2.
b) b is not a perfect square, is even, and is divisible by 7.
c) c is even whenever it is not prime and is smaller than 3.
d) d is divisible by 2 and is odd. This is using the Implication breakout rule with negating an implies: $\neg(p \to q) = p \wedge \neg q$.

4. Lying and Politics [16 points]

Imagine a world with two kinds of people: knights and knaves, where knights always tell the truth and knaves always lie. There are three people A, B, and C, and one of them is the city mayor.

- A says "I am not the city mayor."
- B says "The city mayor is a knave."
- C says "All three of us are knaves."

Is the city mayor a knight or a knave? As part of your solution, determine everything you can about whether A, B, and C are knights or knaves.

Solution:

The mayor has to be a knave, either A or C.
C's proposition is the first one to eliminate here. It can only be False, making C a knave.

This is because if C is knight telling the truth then A will be the mayor but B's claim contradicts with that.
That leaves A's and B's to be either True or False. Suppose both A's and B's are True, then A is not the mayor, the mayor is a knave, which is C.
If A's is True and B's is not, then they contradict each other.
If A's is False and B's is True, then A is the mayor.
Thus, the two possible options are A or C being the mayor while both being knaves.

5. Is Equivalence Equivalent to Equality? [15 points]

Show that $(b \to a) \land (c \to a)$ is logically equivalent to $\neg(b \lor c) \lor a$. If you use a truth table, be sure to state why the table tells you what you claim. If you use logical equivalences, be sure to cite each law you use.

Solution:

$(b \to a) \land (c \to a)$	
$\equiv (\neg b \lor a) \land (\neg c \lor a)$	Implication Breakout Rule
$\equiv (\neg b \land \neg c) \lor a$	Distributive Laws
$\equiv \neg(b \lor c) \lor a$	DeMorgan's Laws
Thus, $(b \to a) \land (c \to a) \equiv \neg(b \lor c) \lor a$.	

6. Deduce, Reuse, Recycle [20 points]

Given that the following statements are **true**:

$$(p \land r) \to q \qquad \neg q \qquad r \lor s \qquad q \lor r$$

For each of the propositions, p, q, r, and s, state its truth value and explain. If it cannot be found, briefly explain why.

Solution:

p is False, q is False, r is True, and s cannot be determined.
$\neg q$ is True, thus q is False.
$q \lor r$ is True, meaning that one of q and r is True, since q is already False, r has to be True.
$r \lor s$ is True, meaning that at least one of r and s is True, but s cannot be determined

because it can be either True or False.
As q is False, $(p \wedge r)$ has to be False for $(p \wedge r) \rightarrow q$ is True. Since r is true, p has to be False.
Thus, p is False, q is False, r is True, and s cannot be determined.

7. Preposterous Propositions [18 points]

Consider the following truth table, where s, t, and w are unknown propositions.

p	q	r	s	t	w
T	T	T	F	T	F
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	F	T	T
F	T	F	F	F	F
F	F	T	F	T	T
F	F	F	F	T	F

Use the above truth table to answer the following questions. For each unknown proposition, s, t, and w:

- Find an equivalent compound proposition using p, q, and/or r.
- You may use **only** $\wedge$, $\vee$, $\neg$, and parentheses in each of your answers.
- You may use p, q, and r **at most once** in each of your answers.

Solution:

$s = (p \wedge q) \wedge \neg r$
t = $\neg q \vee r$
w = $\neg(p \wedge q) \wedge r$.
For s:
As it is mostly False, it can be concluded that it should be an $\wedge$
Since the only time it is True is when p and q are True and r is False, we can say that s is True when p and q are True and r is False.
For t:
As it is mostly True, it can be concluded that it should be an $\vee$
it looks very similar to r, except that it is True when q is False and r is True, so we can

say that t is True when q is False or r is True.
For w:
it also looks like r, except that it is False when p and q are True and r is True, so we can say that w is False when (p and q) are false and r is True.

Groupwork 1 Problems

1. Caught Red-Handed! [20 points]

Four friends have been identified as suspects for a recent hack. They have made the follow statements to the authorities:

- Redd says that “Blu did it”
- Violet says that “I did not do it”
- Blu says that “Grey did it”
- Grey says that “Blu lied when they said that I did it”

(a) If the authorities know exactly one of the four suspects is telling the truth, who did it?

(b) If the authorities know exactly one of the four suspects is lying, who did it?

Solution:

a) Violet is the hacker. Blu and Grey’s propositions contradict each other, thus one of them is lying for sure. Since the authorities know exactly one of them is lying, it has to be either Blu or Grey. Thus Redd and Violet are lying. Blu, therefore, did not do it. Violet is the one that did it. Regardless of whether Blu or Grey is lying.
b) Blu is the hacker. Same idea as a), but this time Redd and Violet are telling the truth. Therefore, Blu has to be the hacker.