EECS 203: Discrete Mathematics
Winter 2024
Discussion X Notes

1 Definitions

- **Graph,** $G = (V, E)$**:**
- **Simple Graph:**
- **Directed Graph:**
- **Multigraph:**
- **Loops:**
- **Adjacent Vertices:**
- **Degree,** $deg(v)$**:**
- **In-Degree,** $deg^-(v)$**:**
- **Out-Degree,** $deg^+(v)$**:**
- **Neighborhood,** $N(v)$**:**
- **In-Neighborhood,** $N^-(v)$**:**
- **Out-Neighborhood,** $N^+(v)$**:**
- **The Handshake Theorem:**
- **Handshake Theorem Equivalent for Directed Graphs:**
- **Degree Sequence:**
- **Special Simple Undirected Graphs:**
  - K_n Complete Graphs:
  - C_n Cycles:
  - W_n Wheels:
  - Q_n Hypercubes:

- **Acyclic:**
- **Tree:**
- **Tree Theorems (2):**
- **Spanning Tree:**

2 Exercises

1. Graphs Intro

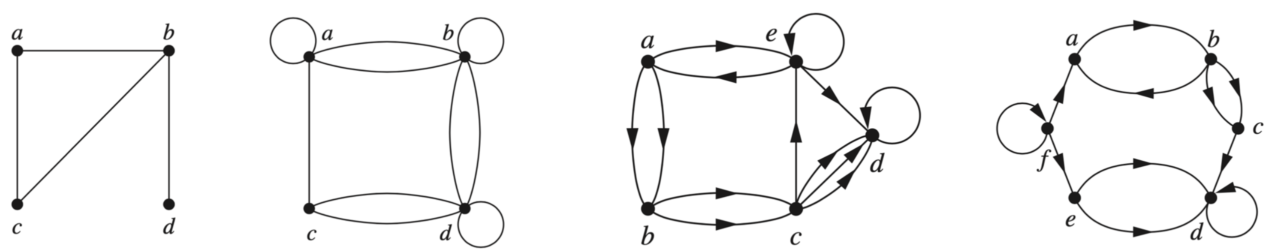
For the following graphs:

a) Identify whether the graph has directed or undirected edges, whether it has multiple edges, and whether it has one or more loops.

b) For each undirected graph, identify whether or not it is simple. If it is not simple, find a set of edges to remove to make it simple.

c) Find $deg(b)$ or if the graph is directed, find $deg^-(b)$ and $deg^+(b)$.

d) Write out its degree sequence. For this part, treat the directed graphs as if they were undirected.



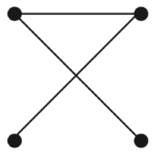
2. Edges and Vertices

Suppose a graph has 21 edges, and 3 vertices of degree 4. All other vertices have degree 2. How many vertices are in the graph?

3. Trees

Which of the following graphs are trees? If it is not a tree, are you able to construct a spanning tree of the graph?

a)



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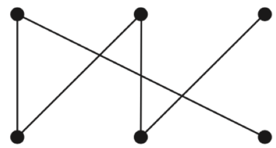

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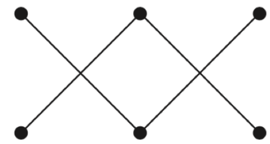

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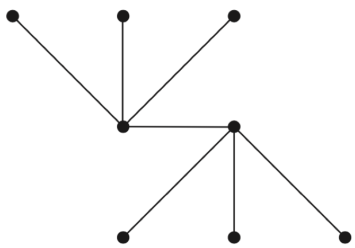

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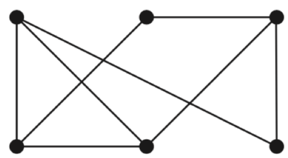

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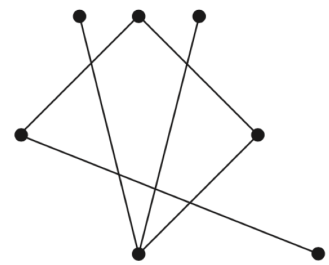

4. Isomorphic Graphs

Determine whether each given pair of graphs is isomorphic. Exhibit an isomorphism or provide an argument that none exists.

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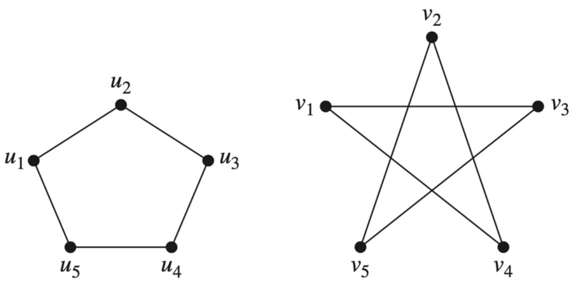

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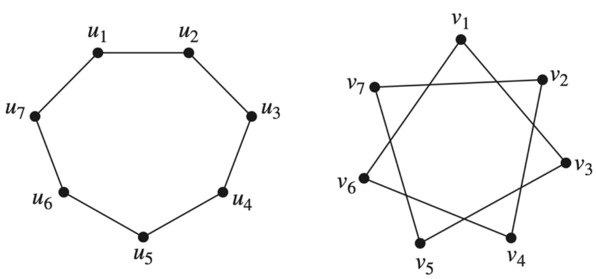

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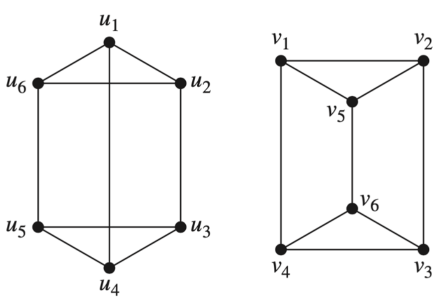

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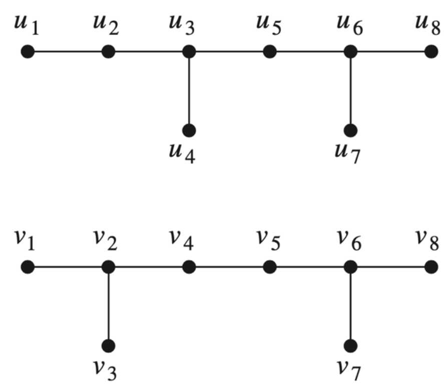

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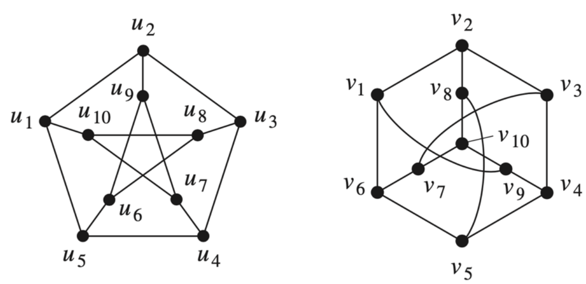

3 Exam 2 Review

5. Inductive Conclusions

Suppose that $P(n)$ is an unknown predicate. Determine for which positive integers n the statement $P(n)$ must be true, and justify your answer, if

a) $P(1)$ is true, and for all positive integers n, if $P(n)$ is true, then P(n + 2) is true.

b) $P(1)$ and $P(2)$ are true, and for all positive integers n, if $P(n)$ and $P(n+1)$ are true, then $P(n+2)$ is true.

c) $P(1)$ is true, and for all positive integers n, if $P(n)$ is true, then $P(2n)$ is true.

d) $P(1)$ is true, and for all positive integers n, if $P(n)$ is true, then $P(n+1)$ is true.

6. Strong Induction

Prove that every positive integer is either a power of 2, or can be written as the sum of distinct powers of 2.

7. Mod

Let $a \equiv 38 \pmod{15}$, $b \equiv 2 \pmod{15}$, and $c \equiv 3 \pmod{5}$. Compute the following if possible:

1. $d \equiv a^{24} \pmod{15}$
2. $e \equiv a^3 b^7 + b^{13} \pmod{15}$
3. $g \equiv a + c \pmod{15}$
4. $h \equiv a + c \pmod{5}$

8. Composition and Onto

If f and $f \circ g$ are onto, does it follow that g is onto? Justify your answer.