

EECS 203: Discrete Mathematics
Winter 2024
Discussion 8b Notes

1 Definitions

- Degree Sequence:
- Special Simple Undirected Graphs:
 - K_n Complete Graphs:
 - C_n Cycles:
 - W_n Wheels:
 - Q_n Hypercubes:
- Path:
- Simple Path:
- Spanning Tree:
- Bipartite Graph:
- Graph Coloring:
- Bipartite Theorem (3 Equivalent Statements):
- Euler Path:
- Euler Cycle/Circuit:
- Euler's Theorem:
- Product Rule:
- Sum Rule:
- Subtraction Rule (Inclusion-Exclusion):
- Division Rule:

2 Exercises

1. Bipartite Intro

Draw each of the following graphs. Which of these graphs are bipartite?

- a) C_6 , a cycle with 6 nodes
- b) W_4 , a wheel with a center node and 4 spoke nodes
- c) Q_3 , a graph representing a 3-dimensional cube, with nodes on each corner

2. Bipartite Conclusion

For which values of n are these graphs bipartite? Explain your answer.

- a) K_n
- b) C_n
- c) W_n
- d) Q_n

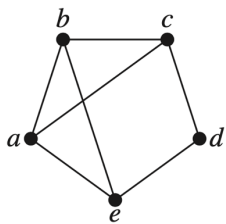
3. Bipartite Graphs

Does there exist a bipartite graph with degree sequence 3,3,3,3,3,3,3,3,3,5,6,9 (in other words, a graph with ten nodes of degree 3, one of degree 5, one of degree 6, and one of degree 9)? If not, explain why.

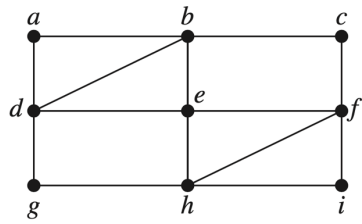
4. Euler Paths and Circuits

For each of the following graphs: Determine whether the graph has an Euler circuit. Construct such a circuit if one exists. If no Euler circuit exists, determine whether the graph has an Euler path. Construct such a path if one exists.

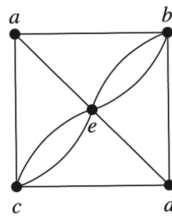
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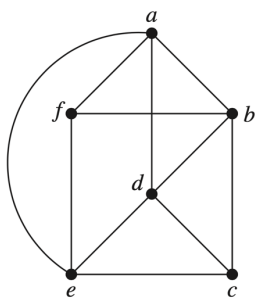
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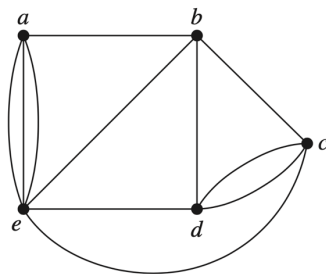
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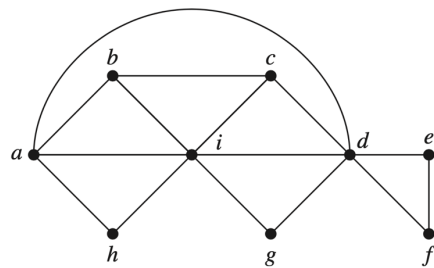
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5.



6.



5. Product Rule

- a. The chairs of an auditorium are to be labeled with an uppercase English letter followed by a positive integer not exceeding 100. What is the largest number of chairs that can be labeled differently?
- b. How many different license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits?
- c. How many functions are there from a set with m elements to a set with n elements?
- d. How many one-to-one functions are there from a set with m elements to one with n elements?

6. Sum Rule

- a. A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 19 possible projects, respectively. No project is on more than one list. How many possible projects are there to choose from?
- b. A wired equivalent privacy (WEP) key for a wireless fidelity (WiFi) network is a string of either 10, 26, or 58 hexadecimal digits. How many different WEP keys are there?

7. Inclusion Exclusion

- a. How many bit strings of length eight either start with a 1 bit or end with the two bits 00?
- b. A computer company receives 350 applications from college graduates for a job planning a line of new web servers. Suppose that 220 of these applicants majored in computer science, 147 majored in business, and 51 majored both in computer science and in business. How many of these applicants majored neither in computer science nor in business?

8. Division Rule

How many different ways are there to seat four people around a circular table, where two seatings are considered the same when each person has the same left neighbor and the same right neighbor?