EECS 203: Discrete Mathematics
Winter 2024
Discussion 3 Notes

1 Proof Styles

1.1 Proofs by Contraposition

Proof by Contraposition: Prove a conditional (in the form "if p, then q") by proving that proving that if q is false, then p must also be false. This is done by assuming the negation of q and concluding the negation of p

1.1.1 Proof by Contraposition $\star$

Prove that if $n^2 + 2$ is even, then n is even using a proof by contrapositive.

1.1.2 Proof by Contraposition II

Prove by contrapositive that if $a^2 + a + 2 \geq b^2 + b + 2$, then $a \geq b$, where a and b are positive integers.
You may use without proving:

1. $c < d$ and $e < f \to (c + e) < (d + f)$
2. $c < d$ and $e < f \to ce < df$, where c, d, e, f are positive integers

1.2 Proofs by Contradiction

Prove p is true by assuming $\neg p$, and arriving at a contradiction, i.e. a conclusion that we know is false.
When using a proof by contradiction to prove "if p is true then q is true", we assume that p is true and that q is false, and derive a contradiction. This shows us that if p is true, then q is true.

$$\neg(p \to q) \equiv (p \wedge \neg q) \to F \to \neg(p \wedge \neg q) \equiv (p \to q)$$

A simpler way to view this: Assume p is true and show that

$$(\neg q \to \mathrm{F}) \to q$$

1.2.1 Contraposition vs Contradiction

Show that for an integer n: if $n^3 + 5$ is odd, then n is even, using

a) a proof by contraposition.

b) a proof by contradiction.

Note: The algebra in either case is the same. You don't need to rewrite the algebra for part (b), just reformat your proof from (a) into a proof by contradiction.

1.3 Choosing Proof Style

A number is considered **rational** if and only if it can be written as the ratio of two integers: $\frac{p}{q}$ where $q \neq 0$.

1.3.1 Proof Practice

Prove or disprove that the sum of a rational number and an irrational number must be irrational.

1.3.2 Odd Proof III

Prove that for all integers a and b, if a divides b and $a + b$ is odd, then a is odd.

1.3.3 Proofs

(a) Prove or disprove: For all nonzero rational numbers x and y, x^y is rational

(b) Prove or disprove: For all even integers x and all positive integers y, x^y is even.

(c) Prove or disprove: For all real numbers x and y, if x^y is irrational, then x or y is not a positive integer