EECS 203: Discrete Mathematics
Winter 2024
Discussion 1b Notes

1 Definitions

- **Logical Equivalence**:
- **DeMorgan's Laws**:
- **Contrapositive**:
- **Implication Breakout**:
- **Identity Laws:**
- **Distributive Laws:**
- **Commutative Laws:**
- **Associative Laws:**
- **Tautology**:
- **Contradiction**:
- **Satisfiable:**

2 Exercises

1. Negations ⋆

Negate the following statements. Any "not"s in your answer should directly precede a simple proposition, not an entire and/or statement.

a. You will study.

b. I do not like pizza.

c. I'm going to get a chai or a mocha today.

d. I'm a teacher and a student.

e. I don't like green and I don't like purple.

f. If it's raining, I'm using my umbrella.

g. $x > 2$

h. $1 + 1 = 2$

2. Truth Tables

Fill in the following truth table.
***Reminder:** $\wedge$ denotes "and", $\vee$ denotes "or", and $\rightarrow$ denotes "implies"/"if...then".

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$p \vee r$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \vee r)$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

3. Finding Truth Values of Compound Propositions $\star$

For each compound proposition, find its truth value when $p = T$, $q = F$, $r = F$, $s = F$, $t = T$, $u = F$, and $v = F$

a) $(q \rightarrow \neg p) \vee (\neg p \rightarrow \neg q)$

b) $(p \vee \neg t) \wedge (p \vee \neg s)$

c) $(p \rightarrow r) \vee (\neg s \rightarrow \neg t) \vee (\neg u \rightarrow v)$

d) $(p \wedge r \wedge s) \vee (q \wedge t) \vee (r \wedge \neg t)$

4. English to Logic Translation I

Let p, q, and r be the propositions defined as follows.

- p: Grizzly bears have been seen in the area.
- q: Hiking is safe on the trail.

- r: Berries are ripe along the trail.

Write these propositions in logic using p, q, r, logical connectives (including negations), and parentheses.
***Reminder:** $\wedge$ denotes "and", $\vee$ denotes "or", $\leftrightarrow$ denotes"if and only if", and $\neg$ denotes "not".

a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.

b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.

c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe

5. Logic to English Translation

Consider the following propositions:

- g: you can graduate
- m: you owe money to the university
- r: you have completed the requirements of your major
- b: you have an overdue library book

Translate the following statement to English: $g \rightarrow (r \wedge \neg m \wedge \neg b)$

6. Tautologies

a) Determine whether $[\neg p \wedge (p \rightarrow q)] \rightarrow \neg q$ is a tautology.

b) Show that this conditional statement is a tautology by using any method you like.

$$[p \wedge (p \rightarrow q)] \rightarrow q$$

7. Promising Premises

For the following sets of premises and conclusions, determine whether each conclusion is valid, given the provided premise(s). A conclusion is valid when it *must* be true given the premise(s). Show your work by explaining your thought process, or using a truth table, or using logical equivalences. For invalid conclusions, providing a counterexample is also sufficient to explain why it's invalid.

A note on notation: the statements above the line are the premises, and the statement below the line is the conclusion. The symbol $\therefore$ means "therefore". For example, in Part (a) there are two premises: Premise 1 is $p \vee q$ and Premise 2 is $\neg p$. You need to determine whether, together, those premises guarantee that the listed conclusion, q, is true.

a) $$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore\ q \end{array}$$

b) $$\begin{array}{l} r \rightarrow q \\ r \\ \hline \therefore\ p \vee q \end{array}$$

c) $$\begin{array}{l} (p \rightarrow q)\ \wedge\ (q \rightarrow r) \\ \hline \therefore\ r \rightarrow p \end{array}$$

d) $$\begin{array}{l} p \wedge q \\ q \rightarrow r \\ \hline \therefore\ r \end{array}$$

8. Logic Puzzle – Stolen Jewels

Robin Hood and his fellows Little John and Marian snuck in to a jewelry store; one of them stole a sapphire, one stole a diamond, and one stole an emerald. They were caught and put on trial, during which they made the following statements:
Robin: "John stole the sapphire."
Marian: "No, John stole the diamond."
John: "Both of them are lying. I didn't steal either."

It turns out that the one who stole the emerald lied, and the one who stole the sapphire told the truth. Who stole which gemstone?