EECS 203: Discrete Mathematics
Winter 2024
Discussion 5b Notes

1 Definitions

- **Strong Induction:**
- **Recurrence Relation:**

2 Mathematical Induction Exercises

1. Mathematical Induction - Sets Edition

Prove that a set with n elements has $n(n-1)/2$ subsets containing exactly two elements whenever n is an integer greater than or equal to 2.

3 Strong Induction Exercises

2. Faulty Induction

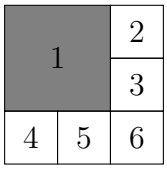
Find the flaw with the following "proof" that every postage of three cents or more can be formed using just three-cent and four-cent stamps.

Base Case: We can form postage of three cents with a single three-cent stamp and we can form postage of four cents using a single four-cent stamp.

Inductive Step: Assume that we can form postage of j cents for all non-negative integers j with $j \leq k$ using just three-cent and four-cent stamps. We can then form postage of $k + 1$ cents by replacing one three-cent stamp with a four-cent stamp or by replacing two four-cent stamps by three three-cent stamps.

3. Squares Strong Induction*

Prove that a square can be subdivided into any number of squares $n \geq 6$. Note that sub-squares don't need to be the same size. For example, here's how you would subdivide a square into 6 squares.



4. Jigsaw Puzzle Induction

A jigsaw puzzle is put together by successively joining pieces that fit together into blocks. A move is made each time a piece is added to a block, or when two blocks are joined. Use strong induction to prove that no matter how the moves are carried out, exactly $n-1$ moves are required to assemble a puzzle with n pieces.

5. Forming Discussion Groups 1⋆

Tom is trying to do a group activity in his next discussion session. He wants to form groups of size 5 or 6.

(a) Show Tom that if there are 23 students attending his discussion, he will be able to split the students into groups of 5 or 6.

(b) In fact, there is some cutoff $p \in \mathbb{N}$ where $\forall n \geq p$, n students can be split into groups of 5 or 6. Find the smallest possible value of p.

(c) Now prove to Tom that if at least p students attends his discussion, he can successfully split the students in to groups of 5 or 6.

4 Recurrence Relation Exercises

6. Forming Discussion Groups 2⋆

In the previous question, we proved that Tom can split a total of n students into groups of 5 or 6 when $n \geq 20$ using induction.

(a) Give a recurrence relation for the minimum number of groups, $G(n)$ that needs to be formed for a class of n students to be split into groups of 5 or 6.

(b) What are the initial conditions?

7. Lobster Recurrence

A model for the number of lobsters caught per year is based on the assumption that the number of lobsters caught in a year is the average of the number caught in the two previous years. Find a recurrence relation for $L(n)$, where $L(n)$ is the numbers of lobsters caught in year n, under the assumption for this model.

8. Stair Climbing

(a) Find a recurrence relation for the number of ways to climb n stairs if the person climbing the stairs can take one, two, or three stairs at a time.

(b) What are the initial conditions?