EECS 203: Discrete Mathematics
Winter 2024
Homework 6

Due **Thursday, Mar. 14th**, 10:00 pm

No late homework accepted past midnight.

Number of Problems: $7 + 2$ Total Points: $100 + 30$

- **Match your pages!** Your submission time is when you upload the file, so the time you take to match pages doesn't count against you.
- Submit this assignment (and any regrade requests later) on Gradescope.
- Justify your answers and show your work (unless a question says otherwise).
- By submitting this homework, you agree that you are in compliance with the Engineering Honor Code and the Course Policies for 203, and that you are submitting your own work.
- Check the syllabus for full details.

Individual Portion

1. Mod Warm-up [12 points]

Find the integer a such that

(a) $a \equiv 58 \pmod{18}$ and $0 \leq a \leq 17$;

(b) $a \equiv -142 \pmod{7}$ and $0 \leq a \leq 6$;

(c) $a \equiv 17 \pmod{29}$ and $-14 \leq a \leq 14$;

(d) $a \equiv -11 \pmod{21}$ and $110 \leq a \leq 130$.

Show your intermediate steps or briefly explain your process to justify your work.

Solution:

2. Multiple Modular Madness [14 points]

For each of the questions below, answer "always," "sometimes," or "never," then explain your answer. Your explanation should justify why you chose the answer you did, but does not have to be a rigorous proof.

Hint: Recall that if $a \equiv b \pmod{m}$ then there exists an integer k such that $a = b + mk$.

(a) Suppose $a \equiv 2 \pmod{21}$. When is $a \equiv 2 \pmod{7}$?

(b) Suppose $b \equiv 2 \pmod{7}$. When is $b \equiv 2 \pmod{21}$?

(c) Suppose $c \equiv 5 \pmod{8}$. When is $c \equiv 4 \pmod{16}$?

(d) Suppose $d \equiv 3 \pmod{21}$. When is $d \equiv 0 \pmod{6}$?

Solution:

3. How Low Can You Go? [12 points]

Suppose $a \equiv 3 \pmod{10}$ and $b \equiv 8 \pmod{10}$. In each part, find c such that $0 \leq c \leq 9$ and

(a) $c \equiv 14a^2 - b^3 \pmod{10}$

(b) $c \equiv b^{15} - 99 \pmod{10}$

(c) $c \equiv a^{97} \pmod{10}$

Show your work! You should be doing the arithmetic/making substitutions **without using a calculator**. Your work must **not** include numbers above 100.

Solution:

4. Be There or Be Square [16 points]

Prove that if n is an odd integer, then $n^2 \equiv 1 \pmod{8}$.

Note: You **cannot** use the fact that all integers are equivalent to one of 0-7 (mod 8) without proof.

Solution:

5. Functions and Fakers [16 points]

Determine if each of the examples below are functions or not.

- If it is not a function, explain why not.
- If it is a function, state whether or not it is bijective, and briefly justify your answer.

All domains and codomains are given as intended.

(a) $f\colon \mathbb{R}^\times \to \mathbb{R}^\times$ such that $f(x) = x^{-1}$.

Note: The set $\mathbb{R}^\times$ is the set $\mathbb{R} - \{0\}$. Additionally, recall that $x^{-1} = \frac{1}{x}$.

(b) $g\colon \mathbb{R} \to \mathbb{R}$ such that $g(x) = y$ iff $y \leq x$.

(c) h: **U-M Courses** $\to \{\text{EECS}, \text{MATH}\}$ which maps each class to its department.

(d) k: **U-M Courses** $\to \mathbb{N}$ which maps each class to its course number

For example, $h(\text{EECS } 203) = \text{EECS}$ and $k(\text{EECS } 203) = 203$.

Note: For the purpose of parts (c) and (d), two courses are considered "equal" if and only if they have the same department and course number. In particular, cross-listed courses are treated as distinct elements of **U-M Courses**.

Solution:

6. Fantastic Functions [18 points]

For each of the functions below, determine whether it is (i) one-to-one, (ii) onto. Prove your answers.

(a) $f\colon \mathbb{R} \to \mathbb{R} - \mathbb{R}^-,\ f(x) = e^{2x+1}$.

(b) $g\colon \mathbb{R} - \left\{-\frac{2}{5}\right\} \to \mathbb{R} - \left\{\frac{3}{5}\right\},\ g(x) = \frac{3x-1}{5x+2}$.

(c) $h\colon \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z},\ h(m,n) = |m| - |n|$

Solution:

7. Comp$\circ$sition(Functions) [12 points]

For each of the following pairs of functions f and g, find $f \circ g$ and $g \circ f$. Make sure to include the domain and codomain of each composed function you give. If either can't be computed, explain why.

(a) $f\colon \mathbb{N} \to \mathbb{Z}^+,\ f(x) = x + 1$

$g\colon \mathbb{Z}^+ \to \mathbb{N},\ g(x) = x^2 - 1$

(b) $f\colon \mathbb{Z} \to \mathbb{R},\ f(x) = \left(\frac{3}{2}x + 3\right)^3$

$g\colon \mathbb{R} \to \mathbb{R}_{\geq 0},\ g(x) = |x|$

Note: $\mathbb{R}_{\geq 0}$ is the set of real numbers greater than or equal to 0.

Solution: