EECS 203: Discrete Mathematics
Winter 2024
Discussion 2 Notes

1 Definitions

- **Quantifiers:**
  - **Universal quantifier:**
  - **Existential quantifier:**
- **Nested Quantifiers:**
- **Argument:**
- **Valid Argument/Proof:**
- **Even:**
- **Odd:**
- **Integer:**
- **Rational Numbers:**
- **Divisibility:**
- **Prime Numbers:**
- **Composite Numbers:**
- **Proof:**
- **Direct Proof:**
- **Disproof:**
- **Without loss of generality (WLOG):**

1. Quantifiers and Negations $\star$

Find the negation of each of these propositions. Simplify so that your answers do not include the negation symbol.

a) $\exists x[-4 < x \leq 1]$

b) $\forall z \exists x \exists y[x^3 + y^3 = z^3]$

2. Quantified Statement Counterexamples

Find a counterexample, if possible, to these quantified statements, where the domain for all variables is integers.

a) $\forall x \exists y(x = 1/y)$

b) $\forall x \exists y(y^2 - x < 100)$

c) $\forall x \forall y(x^2 \neq y^3)$

3. Quantifier Translations $\star$

Let $P(x)$ be "x is perfect"; let $F(x)$ be "x is your friend"; and let the domain of quantifiers be all people. Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

a) No one is perfect.

b) Not everyone is perfect.

c) All your friends are perfect.

d) At least one of your friends is perfect

e) Everyone is your friend and is perfect.

f) Not everybody is your friend or someone is not perfect.

4. Odd Proof

Prove or disprove: The product of two odd numbers is odd.

5. Even Proof

Prove (using a direct proof) that if $m + n$ and $n + p$ are even integers, where m, n, and p are integers, then $m + p$ is even.

6. Negation Station

For each of the following statements, write the statement's negation. Then, determine which is true: the original statement or the negated statement? (You do not need a rigorous proof.)

Reminder: Two numbers, x and y, are **multiplicative inverses** if $xy = 1$.

a. For all real numbers x and y, if $x + y = 0$, then one of them is negative and the other is positive.

b. For all nonzero rational numbers x and y, if they are multiplicative inverses, then $x \neq y$.

c. Each non-zero rational number has a rational multiplicative inverse.

d. Each non-zero integer has an integer multiplicative inverse.

7. Quantifier Proofs

Building on the last question, prove or disprove each of the following statements.
(If you find it helpful to translate the statements to logical connectives and symbols first, you can, but it's not required that you; you can just work with the English statements directly.)

a. For all nonzero rational numbers x and y, if they are multiplicative inverses, then $x \neq y$.

b. Each non-zero rational number has a multiplicative inverse that is also a rational number.

8. Divides Proof

Prove that if n is odd, then $4|(n^2 - 1)$.