

EECS 203: Discrete Mathematics
W24
Discussion 9 Notes

1 Definitions

- Product Rule:
- Sum Rule:
- Subtraction Rule (Inclusion-Exclusion):
- Division Rule:
- Permutation:
- Combination:
- Permutations with Replacement:
- Combinations and Permutations With and Without Repetition:
- Permutations with Indistinguishable/Categorized Objects:
- Binomial Theorem:
- Distinguishable:
- Indistinguishable:
- Experiment:
- Sample Space:
- Event:
- Probability of an Event (Equally Likely Outcomes):
- Probability of Events:
- Conditional Probability:
- Independence:
- Conditional Probability and Independence:

2 Exercises

1. Basic Permutations and Combinations ★

- How many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 100 different people who have entered a contest?
- How many poker hands of five cards can be dealt from a standard deck of 52 cards? Also, how many ways are there to select a set of 47 cards from a standard deck of 52 cards? The order of the hand of five cards and the order of the set of 47 cards do not matter.
- How many permutations of the letters ABCDEFGH contain the string ABC?
- How many bit strings of length n contain exactly r 1's?

2. Standing in Line ★

How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other? [Hint: First position the men and then consider possible positions for the women.]

3. Forming a Committee

Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have the same number of men and women?

4. Permutations with Objects of Different Types ★

How many different strings can be made by reordering the letters of the word SUCCESS?

5. Hanging Jerseys

Robert went out to the store and bought 10 Michigan basketball jerseys and 15 Michigan football jerseys to adorn his walls due to Michigan's recent successes. Each jersey has a different player's name so he could tell them apart. However, once he got back to his room he realized that he only had room to hang up 6 jerseys! If he doesn't care where each jersey is positioned on his walls, how many ways are there to select the jerseys that will be put up if:

- he would like to hang up more or equal number of football jerseys than basketball jerseys
 - he would like to hang up an equal number of football and basketball jerseys, but he can't hang up Surya's basketball jersey without also hanging up Ashu's football jersey?
- Note: he could hang up Ashu's jersey without hanging up Surya's jersey.**

Provide a brief justification for each part.

6. Probability Intro ★

What is the probability that when two dice are rolled, the sum of the numbers on the two dice is 8?

7. Poker Hands ★

- Find the probability that a hand of five cards in poker contains four cards of one rank.
- What is the probability that a poker hand contains a full house, that is, three of one rank and two of another rank?

8. Independent vs Mutually Exclusive Events

Two six-sided dice, Dice A and Dice B, are rolled. Using these dice, provide examples of:

- A pair of independent events.
- A pair of mutually exclusive events.

Recall that events E and F are independent if $P(E \cap F) = P(E) \cdot P(F)$, and they are mutually exclusive if $P(E \cap F) = 0$.

9. Conditional Probability ★

A bit string of length four is generated at random so that each of the 16 bit strings of length four is equally likely. What is the probability that it contains at least two consecutive 0s, given that its first bit is a 0?

10. Independent Events

Assume that each of the four ways a family can have two children is equally likely. Are the events E , that a family with two children has two boys, and F , that a family with two children has at least one boy, independent?