

Grading of Groupwork 6

Using the solutions and Grading Guidelines, grade your Groupwork 6 Problems:

- Use the table below to grade your past Groupwork submission and calculate scores.
- While grading, mark up your past submission. Include this with the table when you submit your grading.
- Write whether your submission achieved each rubric item. If it didn't achieve one, say why not.
- For extra credit, write positive comment(s) about your work.
- You don't have to redo problems correctly, but it is recommended!
- See "All About Groupwork" on Canvas for more detailed guidance, and what to do if you change groups.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)	Total:
Problem 1												/12
Problem 2												/18
Total:												/30

Groupwork 7 Problems

1. Get to the Point [10 points]

Consider an arbitrary set A . We say a function $f : A \rightarrow A$ has a *fixed point* iff there exists $a \in A$ such that $f(a) = a$.

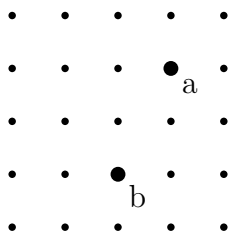
Consider the notation $f^{(n)}$ to mean $\underbrace{f \circ \cdots \circ f}_{n \text{ times}}$, where $n \in \mathbb{Z}^+$. Essentially, n copies of f are composed together.

Prove by **induction** that if f is a function with a fixed point, then for all positive integers n , $f^{(n)}$ has a fixed point.

Solution:

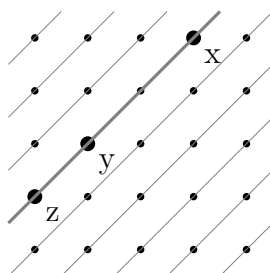
2. Going Off the Grid [8 points]

In a grid, we say that a point a *dominates* a point b iff a lies strictly above and to the right of b . For example, in the picture below, a dominates b .



Prove using the Pigeonhole Principle that if we choose $4n - 1$ points from an $n \times n$ grid ($n \geq 4$), there must be three chosen points x, y, z such that x dominates y and y dominates z . Make sure to state what your pigeons are and what your holes are, as well as how many of each you have.

Hint: If x, y, z lie on the same increasing diagonal as shown in the picture below, then x dominates y and y dominates z .



Solution: