PRACTICE Exam 2
EECS 203, Winter 2024

Name (ALL CAPS): ______________________________

Uniqname (ALL CAPS): ______________________________

8-Digit UMID: ______________________________

Instructions

- When you receive this packet, fill in your name, Uniqname, and UMID above.
- Once the exam begins, make sure you have problems 1-17 in this booklet.
- Write your UMID in the blank at the top of every other page.
- No one may leave within the last 10 minutes of the exam.
- After you complete the exam, sign the Honor Code below. If you finish when time is called, your proctor will give you time to sign the Honor Code.
- Do not detach the scratch paper at the end of the packet.
- Do not discuss the exam until solutions have been released!

Materials

- No electronics allowed, including calculators.
- You may use one 8.5″ by 11″ note sheet, front and back, created by you.
- You may not use any other sources of information.

Honor Code

This exam is administered under the College of Engineering Honor Code. Your signature endorses the pledge below. We will not grade your exam without your signature.

I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code. I further agree not to discuss any aspect of this examination in any way, shape, or form until the solutions have been published.

Signature: ______________________________

Part A: Single Answer Multiple Choice

Instructions

- There are 5 questions in this section.
- Shade **only one** circle corresponding to your answer choice.
- If you shade more than one circle, your answer will be marked as incorrect.

Example.	a b c d e

Make sure to SHADE A BUBBLE next to the question title, as shown above.

Problem 1. (4 points)	a b c d e

Consider a set $A \subseteq \{1, 2, 4, 5, 10, 20, 25, 50, 100\}$. What is the minimum cardinality of A that guarantees there exist two distinct elements a and b in A such that $ab = 100$?

(a) 5
(b) 6
(c) 7
(d) 8
(e) 9

Problem 2. (4 points)	a b c d e

Suppose $a \equiv 4 \pmod{13}$ and $b \equiv 7 \pmod{13}$. Compute $(a^5 + ab^2) \bmod 13$.

(a) 1
(b) 4
(c) 7
(d) 9
(e) 11

Problem 3. (4 points) ⓐ ⓑ ⓒ ⓓ ⓔ

All 25 of the EECS 203 IAs are participating in a donut-eating contest sponsored by the GSIs. Anyone who ate the most donuts won the contest. If 520 donuts were eaten during the contest, what is the minimum number of donuts a winner could have eaten?
Note: There could be multiple winners.

(a) 20

(b) 21

(c) 22

(d) 23

(e) 24

Problem 4. (4 points) ⓐ ⓑ ⓒ ⓓ ⓔ

How many initial conditions are needed for the following recurrence?

$$a_n = a_{n-1} + a_{n-4} + 2^n$$

(a) 1

(b) 2

(c) 3

(d) 4

(e) Not enough information

Problem 5. (4 points) ⓐ ⓑ ⓒ ⓓ ⓔ

Let f and g be functions from $\mathbb{R} \to \mathbb{R}$ with $f(x) = x^2 - 4$ and $g(x) = 3x - 2$. Find $(f \circ g)(x)$.

(a) $3x^2 - 14$

(b) $3x^2 - x$

(c) $x^2 + 3x - 6$

(d) $3x^3 - 2x^2 - 12x + 8$

(e) $9x^2 - 12x$

Part B: Multiple Answer Multiple Choice

Instructions

- There are 4 questions in this section.
- Shade whichever boxes you believe are correct. **This could be all answers, no answers, or anything in between.**
- If there are no correct answers, leave all the boxes blank.

Example	a b c d e

Make sure to SHADE 0 OR MORE BOXES next to the question title, as shown above.

Problem 6. (4 points)	a b c d e

Let P be a predicate over $\mathbb{N}$ that is not always true, and such that $\forall n, P(n+1) \to P(n)$. Which of the following **must** be true?

(a) If $P(0)$, then $P(203)$.

(b) If $P(203)$, then $P(0)$.

(c) There is some $c \in \mathbb{N}$ such that $P(c)$ is true but $P(c+1)$ is false.

(d) There is some $c \in \mathbb{N}$ such that $P(n)$ is true if and only if $n < c$.

(e) None of the above

Problem 7. (4 points)	a b c d e

- Let S be the set of University of Michigan students.
- Let M be the set of all college majors formally offered by the University of Michigan.
- Consider the function $f: S \to M$ that maps each student to their declared major.

Assume each student has declared exactly one major. Select each of the following that **must** be true.

(a) If $|S| > |M|$, then there exists a major declared by multiple students.

(b) If $|S| > |M|$, then each major has at least one student.

(c) If $|S| < |M|$, then no two students have declared the same major.

(d) If $|S| < |M|$, then some major has no student declared.

(e) f has an inverse, $f^{-1}\colon M \to S$.

Problem 8. (4 points)

a b c d e

Let $x \equiv 4 \pmod 5$ and $y \equiv 7 \pmod{10}$. Which of the following statements **must** be true?

(a) $xy \equiv 3 \pmod 5$

(b) $x - y \equiv 3 \pmod 5$

(c) $x + y \equiv 1 \pmod 5$

(d) $x + y \equiv 1 \pmod{10}$

(e) $xy \equiv 8 \pmod{10}$

Problem 9. (4 points)

a b c d e

Which of the following sets are **countably infinite**?

(a) $\mathbb{R} - \mathbb{Q}$

(b) $[0, 203] \cap \mathbb{Z}$

(c) $\mathbb{Q} \times \mathbb{Q}$

(d) $\{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid y = x^2\}$

(e) $\{(x, y) \in \mathbb{R} \times \mathbb{Z} \mid y = x^2\}$

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Part C: Short Answer

Instructions

- There are 5 questions in this section.
- Write your solution in the space provided.
- Show your work and include justification.

Problem 10: Onto Proof/Disproof (6 points)

Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 5x^3 - 10$. Prove or disprove that f is onto.

○ Prove ○ Disprove

Problem 11: Subset Proof (7 points)

Suppose you want to prove the following claim using strong induction:

Claim: $P(n) \;\; \forall n \geq 5$.

(a) Fill in the blanks to complete the inductive step below.
Note: Write all your answers in the Answer Packet. Nothing written below will be graded.

Inductive Step:
Let $k \geq$ ___.
Assume $P(j)$ is true for all _______ $\leq j \leq$ _______.
Since $P(k-4)$, ... [*specific deductions omitted*] ..., then $P(k)$.

(b) Given the inductive step in Part (a), for which values of n will $P(n)$ need to be proven using base cases?

Problem 12: Weak Induction Proof (7 points)

Prove by weak induction that for all integers $n \geq 1$, $\quad 1 + 3 + 5 + ... + (2n - 1) = n^2$.

Proof:

Problem 13: Cardinality Proof (7 points)

Let $A = [0, 1]$ and $B = [-2, -1] \cup [1, 2]$.

(a) Provide a function f that shows that $|A| \leq |B|$

(b) Provide a function g that shows that $|A| \geq |B|$

For each function you define, make sure to state its domain and codomain, and state which relevant property/properties your function has (e.g., one-to-one or onto).
You do **not** need to prove that your functions have the property/properties you indicate.

Proof:

(a) $|A| \leq |B|$

f: __________ $\rightarrow$ __________ $\qquad$ $f(x) =$

f is ______________________________ (State property/properties)

(b) $|A| \geq |B|$

g: __________ $\rightarrow$ __________ $\qquad$ $g(x) =$

g is ______________________________ (State property/properties)

Problem 14: Pigeonhole (7 points)

(a) Name the smallest integer n such that you can guarantee that if you select n distinct integers, there must be two distinct integers you selected, a and b, such that

$$a \equiv \pm b \pmod{10}.$$

You do not need to justify your answer or show work.

(b) Use the Pigeonhole Principle to prove that your value of n satisfies the claim.
Make sure to include what your pigeons and holes are, and how many of each you have.

(a) $n =$ ____________

(b)

Part D: Free Response

Instructions

- There are 3 questions in this section.
- Write your solution in the space provided.
- Write down your answer with care: answers that are unreadable (such as too faint or too messy) will not be graded. If you have multiple answers, you must indicate which one you want graded.
- **Show your work** and include justification.

Problem 15: Composition Proof (10 points)

Let $f : A \to B$ and $g : B \to C$. Prove:

If f is onto and g is **not** one-to-one, then $g \circ f$ is **not** one-to-one.

Hint: It could be helpful to write out what it means for g to be **not** one-to-one. Do the same for $g \circ f$.

Problem 16: Brad is Blessing (10 points)

Brad is U of M's favorite dog and he spreads smiles by doing 1 of his favorite 4 activities every day:

- Dog therapy, Celebrity photo op, Fetching tennis balls, or Resting

He has two restrictions:

- He will never Rest 2 days in a row.
- He always plays Fetch the day after a Celebrity photo op.

(a) Find a recurrence relation representing the number of ways Brad can spend n days.

Recurrence Relation: ____

Justify your answer by showing your work below

(b) Find the initial conditions for your recurrence relation in part (a). For full credit, you must provide the fewest initial conditions possible to satisfy your recurrence.
List your initial conditions below. Show your work.

Problem 17: Recurrence Inequality (10 points)

Consider the following recurrence with initial conditions $a_1 = 1$ and $a_2 = -1$:

$$a_n = -a_{n-1} + 6a_{n-2}$$

Use strong induction to prove: for all $n > 0, \quad |a_n| < 3^n$.

Note: You may use without proof that for all $x, y \in \mathbb{Z}$, $|x + y| \leq |x| + |y|$.

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This is a space for scratch work. DO NOT DETACH THIS PAPER FROM YOUR EXAM.

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