W24

EECS 203 Discussion 1b

Introduction to Logic

Important Forms

- Two beginning-of-semester surveys on Canvas
  - **FCI BoT** Survey and **Better Belonging in Computer Science (BBCS) Entry** Survey
  - **Due:** Friday, Feb. 2nd @11:59pm
- Exam Date Confirmation Survey
  - **Due:** Friday, Feb. 2nd @11:59pm
  - Please fill this out, even if you don’t have an exam conflict!
- They are each worth a few points, so make sure to fill them out!

Upcoming Homework

- Assignment 0 was due Jan. 18th
- Homework/Groupwork 1 will be due **Jan. 25th**
    - **Don’t forget to match pages!**
    - Please note as soon as you press submit you’ve successfully submitted by the deadline. **You can still match pages** with no rush without adding to your submission time.
- Groupwork
    - Groupwork can be done alone, but the problems tend to be more difficult, and the goal is for you to puzzle them out with others!
    - Your discussion section is a great place to find a group!
    - There is also a pinned Piazza thread for searching for homework groups.

Propositions

Problem 1

1. Negations ⋆

Negate the following statements. Any "not"s in your answer should directly precede a simple proposition, not an entire and/or statement.

a. You should study.

b. I do not like pizza.

c. I'm going to get a chai or a mocha today.

d. I'm a teacher and a student.

e. I don't like green and I don't like purple.

f. If it's raining, I'm using my umbrella.

g. $x > 2$

h. $1 + 1 = 2$

Solution

1. Negations ⋆

Negate the following statements. Any "not"s in your answer should directly precede a simple proposition, not an entire and/or statement.

a. You should study.

b. I do not like pizza.

c. I'm going to get a chai or a mocha today.

d. I'm a teacher and a student.

e. I don't like green and I don't like purple.

f. If it's raining, I'm using my umbrella.

g. $x > 2$

h. $1 + 1 = 2$

Solution:

a. You should not study.

b. I do like pizza.

c. I'm not going to get a chai and I'm not going to get a mocha today.

d. I'm not a teacher or I'm not a student.

e. I like green or purple.

f. It's raining and I'm not using my umbrella.
(The original statement is equivalent to it's not raining OR I'm using my umbrella.)

g. $x \leq 2$

h. $1 + 1 \neq 2$

Important Truth Tables

p	q	$p \rightarrow q$	$p \leftrightarrow q$	$p \wedge q$	$p \vee q$
T	T	T	T	T	T
T	F	F	F	F	T
F	T	T	F	F	T
F	F	T	T	F	F

Problem 2

2. Truth Tables

Fill in the following truth table.
***Reminder:** $\wedge$ denotes "and", $\vee$ denotes "or", and $\rightarrow$ denotes "implies"/"if…then".

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$p \vee r$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \vee r)$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

Solution

Solution:

p	q	r	$p \to q$	$q \to r$	$(p \to q) \land (q \to r)$	$p \lor r$	$[(p \to q) \land (q \to r)] \to (p \lor r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	F

Problem 3

3. Finding Truth Values of Compound Propositions ⋆

For each compound proposition, find its truth value when $p = T$, $q = F$, $r = F$, $s = F$, $t = T$, $u = F$, and $v = F$

a) $(q \to \neg p) \vee (\neg p \to \neg q)$

b) $(p \vee \neg t) \wedge (p \vee \neg s)$

c) $(p \to r) \vee (\neg s \to \neg t) \vee (\neg u \to v)$

d) $(p \wedge r \wedge s) \vee (q \wedge t) \vee (r \wedge \neg t)$

Note: p → q is only **false** in the case of **T → F**.

Solution

3. Finding Truth Values of Compound Propositions ⋆

For each compound proposition, find its truth value when $p = T$, $q = F$, $r = F$, $s = F$, $t = T$, $u = F$, and $v = F$

a) $(q \to \neg p) \lor (\neg p \to \neg q)$

b) $(p \lor \neg t) \land (p \lor \neg s)$

c) $(p \to r) \lor (\neg s \to \neg t) \lor (\neg u \to v)$

d) $(p \land r \land s) \lor (q \land t) \lor (r \land \neg t)$

Solution:

a) $(q \to \neg p) \lor (\neg p \to \neg q)$
$\equiv (F \to \neg T) \lor (\neg T \to \neg F)$
$\equiv (F \to F) \lor (F \to T)$
$\equiv T \lor T$
$\equiv T$

b) $(p \lor \neg t) \land (p \lor \neg s)$
$\equiv (T \lor \neg T) \land (T \lor \neg F)$
$\equiv (T \lor F) \land (T \lor T)$
$\equiv T \land T$
$\equiv T$

c) $(p \to r) \lor (\neg s \to \neg t) \lor (\neg u \to v)$
$\equiv (T \to F) \lor (\neg F \to \neg T) \lor (\neg F \to F)$
$\equiv (T \to F) \lor (T \to F) \lor (T \to F)$
$\equiv F \lor F \lor F$
$\equiv F$

d) $(p \land r \land s) \lor (q \land t) \lor (r \land \neg t)$
$\equiv (T \land F \land F) \lor (F \land T) \lor (F \land \neg T)$
$\equiv (T \land F \land F) \lor (F \land T) \lor (F \land F)$
$\equiv F \lor F \lor F$
$\equiv F$

Problem 4

4. English to Logic Translation I

Let p, q, and r be the propositions defined as follows.

- p: Grizzly bears have been seen in the area.
- q: Hiking is safe on the trail.
- r: Berries are ripe along the trail.

Write these propositions in logic using p, q, r, logical connectives (including negations), and parentheses.
***Reminder:** $\wedge$ denotes "and", $\vee$ denotes "or", $\leftrightarrow$ denotes"if and only if", and $\neg$ denotes "not".

a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.

b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.

c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe

Solution

Solution:

a) $r \land \neg p$

b) $\neg p \land q \land r$

c) $r \to (q \leftrightarrow \neg p)$

d) $\neg q \land \neg p \land r$

4. English to Logic Translation I

Let p, q, and r be the propositions defined as follows.

- p: Grizzly bears have been seen in the area.
- q: Hiking is safe on the trail.
- r: Berries are ripe along the trail.

Write these propositions in logic using p, q, r, logical connectives (including negations), and parentheses.
***Reminder:** $\land$ denotes "and", $\lor$ denotes "or", $\leftrightarrow$ denotes"if and only if", and $\neg$ denotes "not".

a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.

b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.

c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe

Problem 5

5. Logic to English Translation

Consider the following propositions:

- g: you can graduate
- m: you owe money to the university
- r: you have completed the requirements of your major
- b: you have an overdue library book

Translate the following statement to English: $g \rightarrow (r \wedge \neg m \wedge \neg b)$

Solution

5. Logic to English Translation

Consider the following propositions:

- g: you can graduate
- m: you owe money to the university
- r: you have completed the requirements of your major
- b: you have an overdue library book

Translate the following statement to English: $g \to (r \land \neg m \land \neg b)$

Solution:
You can graduate only if you have completed the requirements of your major and you do not owe money to the university and you do not have an overdue library book. Equivalently, if you can graduate, then you have completed the requirements of your major and you do not owe money to the university and you do not have an overdue library book.

Logical Equivalences

Logical Equivalence

Two compound propositions are **logically equivalent** if they have the same truth value for any input truth values.

Example:
p and ¬¬p

p	¬p	¬¬p
T	F	T
F	T	F

p and ¬¬p have the same truth tables, and thus are logically equivalent!

Examples of Logical Equivalences

De Morgan's Laws (and/or):

$$\neg(p \land q) \equiv \neg p \lor \neg q$$

$$\neg(p \lor q) \equiv \neg p \land \neg q$$

Identity Laws:

$$p \land T \equiv p$$

$$p \lor F \equiv p$$

Implication Breakout:

$$p \rightarrow q \equiv \neg p \lor q$$

Examples of Logical Equivalences

Commutative Laws:

$$p \land q \equiv q \land p$$

$$p \lor q \equiv q \lor p$$

Associative Laws:

$$p \land (q \land r) \equiv (p \land q) \land r$$

$$p \lor (q \lor r) \equiv (p \lor q) \lor r$$

Distributive Laws:

$$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$$

$$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$$

The Contrapositive

Take an "if, then" statement $p \to q$. We define the **contrapositive** of this statement as $\neg q \to \neg p$. An implies statement and its contrapositive are logically equivalent.

$$p \to q \equiv \neg q \to \neg p$$

Example:

- If **I am teaching**, then *my materials are prepared*.
- Contrapositive: If *my materials are NOT prepared*, then **I am NOT teaching**.

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg q \to \neg p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Logical Equivalence Tables (Rosen 1.3)

TABLE 8 **Logical Equivalences Involving Biconditional Statements.**

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

TABLE 7 **Logical Equivalences Involving Conditional Statements.**

$p \rightarrow q \equiv \neg p \vee q$
$p \rightarrow q \equiv \neg q \rightarrow \neg p$
$p \vee q \equiv \neg p \rightarrow q$
$p \wedge q \equiv \neg(p \rightarrow \neg q)$
$\neg(p \rightarrow q) \equiv p \wedge \neg q$
$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$
$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$
$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

TABLE 6 **Logical Equivalences.**

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

Tautologies & Contradictions

Tautology, Contradiction, Satisfiability

- **Tautology:** A compound proposition that is **always true** regardless of its input values
  - **Example: $p \lor \neg p$**
- **Contradiction:** A compound proposition that is **always false** regardless of its input values
  - **Example: $p \land \neg p$**
- **Satisfiable:** A compound proposition is satisfiable if it **can be true** (there is at least one set of inputs that makes the proposition true)
  - **Example: $p \land q$**

Problem 6

6. Tautologies

a) Determine whether $[\neg p \wedge (p \to q)] \to \neg q$ is a tautology.

b) Show that this conditional statement is a tautology by using truth tables.

$$[p \wedge (p \to q)] \to q$$

Note: a **tautology** is a **compound** proposition that is **always true** regardless of its input values

Solution

6. Tautologies

a) Determine whether $[\neg p \wedge (p \to q)] \to \neg q$ is a tautology.

b) Show that this conditional statement is a tautology by using truth tables.

$$[p \wedge (p \to q)] \to q$$

Solution:

a) This is not a tautology. We can show this by finding values for p and q which make the proposition false. In order to make the proposition false, we can set $p = F$ and $q = T$.

$$\begin{aligned}
&[\neg p \wedge (p \to q)] \to \neg q \\
&\equiv [\neg F \wedge (F \to T)] \to \neg T \\
&\equiv [T \wedge T] \to F \\
&\equiv T \to F \\
&\equiv F
\end{aligned}$$

Solution

6. Tautologies

a) Determine whether $[\neg p \wedge (p \to q)] \to \neg q$ is a tautology.

b) Show that this conditional statement is a tautology by using truth tables.

$$[p \wedge (p \to q)] \to q$$

b) As seen in the truth table, all combinations of boolean assignments result in the statements being true. Therefore, it is a tautology.

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Of note, we can also show this statement is equivalent to true (a tautology) by our logical equivalences.

$[p \wedge (p \to q)] \to q$

$\equiv [p \wedge (\neg p \vee q)] \to q$ Implication Breakout

$\equiv [(p \wedge \neg p) \vee (p \wedge q)] \to q$ Distributive Law

$\equiv [F \vee (p \wedge q)] \to q$ ($p \wedge \neg p \equiv F$, contradiction)

$\equiv [(p \wedge q) \to q]$ Identity Law

$\equiv \neg(p \wedge q) \vee q$ Implication Breakout

$\equiv (\neg p \vee \neg q) \vee q$ De Morgan's

$\equiv \neg p \vee (\neg q \vee q)$ Associative

$\equiv \neg p \vee T$ ($\neg q \vee q \equiv T$, tautology)

$\equiv T$ Domination Law

Note: we **will not** ask you to do lengthy line-by-line logical equivalence calculations like this one on homework or exams.

You only need to know the **names** of the logical equivalence laws explicitly mentioned in lecture.

In general, **truth tables always work** for logical equivalence proofs.

Problem 7

7. Promising Premises

For the following sets of premises and conclusions, determine whether each conclusion is valid, given the provided premise(s). A conclusion is valid if and only if it *must* be true given the premise(s). Show your work by explaining your thought process, or using a truth table, or using logical equivalences. For invalid conclusions, providing a counterexample is also sufficient to explain why it's invalid.

A note on notation: the statements above the line are the premises, and the statement below the line is the conclusion. The symbol $\therefore$ means "therefore". For example, in Part (a) there are two premises: Premise 1 is $p \lor q$ and Premise 2 is $\neg p$. You need to determine whether, together, those premises guarantee that the listed conclusion, q, is true.

a) $$\begin{array}{l} p \lor q \\ \neg p \\ \hline \therefore\ q \end{array}$$

b) $$\begin{array}{l} r \to q \\ r \\ \hline \therefore\ p \lor q \end{array}$$

c) $$\begin{array}{c} (p \to q)\ \land\ (q \to r) \\ \hline \therefore\ r \to p \end{array}$$

d) $$\begin{array}{l} p \land q \\ q \to r \\ \hline \therefore\ r \end{array}$$

Solution

a) $\dfrac{\begin{array}{l} p \vee q \\ \neg p \end{array}}{\therefore\ q}$

b) $\dfrac{\begin{array}{l} r \to q \\ r \end{array}}{\therefore\ p \vee q}$

c) $\dfrac{(p \to q)\ \wedge\ (q \to r)}{\therefore\ r \to p}$

d) $\dfrac{\begin{array}{l} p \wedge q \\ q \to r \end{array}}{\therefore\ r}$

Solution:

a) This conclusion is valid.

From the second premise, we know that $\neg p$ is true, i.e., that p is false. So the first premise, $p \vee q$, becomes $F \vee q$. The only way for this proposition to be true is for q to be true. In logic, when $p \equiv F$, we have $p \vee q \equiv F \vee q \equiv q$.

Alternatively, we can look at a truth table to evaluate the situation where both premises are true:

p	q	$p \vee q$	$\neg p$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	T

Row 3 is the only row where the premises, $p \vee q$ and $\neg p$, are both true, and row 3 also shows q as true.

Solution

a) $\dfrac{\begin{array}{l} p \lor q \\ \neg p \end{array}}{\therefore\ q}$

b) $\dfrac{\begin{array}{l} r \to q \\ r \end{array}}{\therefore\ p \lor q}$

c) $\dfrac{(p \to q)\ \land\ (q \to r)}{\therefore\ r \to p}$

d) $\dfrac{\begin{array}{l} p \land q \\ q \to r \end{array}}{\therefore\ r}$

b) This conclusion is valid

Putting together the two premises, r and $r \to q$, we can conclude q. And if q is true, then ($anything \lor q$) is also true. Therefore, $p \lor q$ is true. Another way to think about this is, if q is true, then $p \lor q$ will always be true, regardless of the value of p.

Alternatively sing a truth table:

r	q	p	$(r \to q)$	$(p \lor q)$
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	F

We see that in rows 1 and 2, where r and $r \to q$ are both true, $p \lor q$ is also true. r and $r \to q$ implies $p \lor q$.

Solution

a) $\dfrac{\begin{array}{l} p \vee q \\ \neg p \end{array}}{\therefore\ q}$

b) $\dfrac{\begin{array}{l} r \to q \\ r \end{array}}{\therefore\ p \vee q}$

c) $\dfrac{(p \to q)\ \wedge\ (q \to r)}{\therefore\ r \to p}$

d) $\dfrac{\begin{array}{l} p \wedge q \\ q \to r \end{array}}{\therefore\ r}$

c) This conclusion is not valid. Using a truth table:

p	q	r	$(p \to q)$	$(q \to r)$	$((p \to q) \wedge (q \to r))$	$(r \to p)$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	T	T	F
F	T	F	T	F	F	T
F	F	T	T	T	T	F
F	F	F	T	T	T	T

We see that $((p \to q) \wedge (q \to r))$ does not imply $(r \to p)$. In rows 5 and 7, we see the premises are true, while the conclusion is false, making this conclusion invalid. Thus, $((p \to q) \wedge (q \to r))$ DOES NOT imply $(r \to p)$. (To visualize this you could write out this final implies statement in another column in the truth table.)

Counterexample Method:
Let p be that it's Tuesday, q be that I eat pizza, and r be that I get a stomach ache. Then the premises tell us that if it's Tuesday, then I eat pizza and if I eat pizza, then I have a stomach ache. That doesn't imply that if I have a stomach ache, then it's Tuesday. I can still get a stomach ache on any other day.

Solution

a) $\dfrac{\begin{array}{l} p \vee q \\ \neg p \end{array}}{\therefore\ q}$

b) $\dfrac{\begin{array}{l} r \to q \\ r \end{array}}{\therefore\ p \vee q}$

c) $\dfrac{(p \to q)\ \wedge\ (q \to r)}{\therefore\ r \to p}$

d) $\dfrac{\begin{array}{l} p \wedge q \\ q \to r \end{array}}{\therefore\ r}$

d) This conclusion is valid.

The first premise is $p \wedge q$, which tells us that both p and q are true. Then the second premise, $q \to r$, tells us that whenever q is true, then r must also be true. Putting those together, we can deduce that r must be true.

Alternately, we can show the validity of the conclusion r using a truth table:

p	q	r	$(p \wedge q)$	$(q \to r)$	$(p \wedge q) \wedge (q \to r)$	$[(p \wedge q) \wedge (q \to r)] \to r$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	F	T	F	T
F	T	F	F	F	F	T
F	F	T	F	T	F	T
F	F	F	F	T	F	T

As the last column of the truth table shows, together the two premises, $p \wedge q$ and $q \to r$ [always] imply the conclusion, r.

Problem 8

8. Logic Puzzle – Stolen Jewels

Robin Hood and his fellows Little John and Marian snuck in to a jewelry store; one of them stole a sapphire, one stole a diamond, and one stole an emerald. They were caught and put on trial, during which they made the following statements:
Robin: "John stole the sapphire."
Marian: "No, John stole the diamond."
John: "Both of them are lying. I didn't steal either."

It turns out that the one who stole the emerald lied, and the one who stole the sapphire told the truth. Who stole which gemstone?

Solution

8. Logic Puzzle – Stolen Jewels

Robin Hood and his fellows Little John and Marian snuck in to a jewelry store; one of them stole a sapphire, one stole a diamond, and one stole an emerald. They were caught and put on trial, during which they made the following statements:
Robin: "John stole the sapphire."
Marian: "No, John stole the diamond."
John: "Both of them are lying. I didn't steal either."

It turns out that the one who stole the emerald lied, and the one who stole the sapphire told the truth. Who stole which gemstone?

Solution: If John stole the sapphire, then he must have told the truth, which implies Robin lied, a contradiction. If John stole the emerald, then his claim is actually correct, contradicting that the one who had stolen the emerald lied. So John stole the diamond, Robin lied, and Marian told the truth. Hence, John stole the diamond, Robin stole the emerald, and Marian stole the sapphire.