EECS 203 Exam 1 Review

Day 1

Today’s Review Topics

- Propositional Logic
- Predicates and Quantifiers

Propositional Logic

Cheat Sheet Suggestions

TABLE 1 The Truth Table for the Negation of a Proposition.

p	$\neg p$
T	F
F	T

TABLE 2 The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

TABLE 3 The Truth Table for the Disjunction of Two Propositions.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

TABLE 2 De Morgan's Laws.

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

TABLE 4 The Truth Table for the Exclusive Or of Two Propositions.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

TABLE 5 The Truth Table for the Conditional Statement $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Cheat Sheet Suggestions

"if p, then q"
"if p, q"
"p is sufficient for q"
"q if p"
"q when p"
"a necessary condition for p is q"
"q unless $\neg p$"
"p implies q"
"p only if q"
"a sufficient condition for q is p"
"q whenever p"
"q is necessary for p"
"q follows from p"

Compound Proposition	Expression in English
¬p	"It is not the case that p"
p∧q	"Both p and q"
p∨q	"p or q (or both)"
p⊕q	"p or q (but not both)"
p→q	"if p then q" "p implies q"
p↔q	"p if and only if q"

Quick Recap

- Proposition - declarative statement that is either true or false
- $p \rightarrow q$
  - Logically equivalent to $\neg p \lor q$
  - Converse: $q \rightarrow p$
  - Contrapositive: $\neg q \rightarrow \neg p$
  - Inverse: $\neg p \rightarrow \neg q$
  - The original implication and the contrapositive have the same truth value, while the converse and inverse have the same truth values.
- Tautology - compound proposition that is always true
- Contradiction - compound proposition that is always false
- Satisfiable/Consistent - some assignment of truth values that make the compound proposition true
- How many propositions does a truth table with 256 rows have?

Truth Tables

If we have 2 propositions, how many rows will there be in the truth table?

If we have 5 propositions, how many rows will be in the truth table?

If we have n propositions, how many rows will be in the truth table?

Solution

2^2, 2^5, 2^n

Which of the following expressions is a contradiction?

(a) $(p \wedge q) \leftrightarrow (p \wedge r)$

(b) $(p \wedge q) \wedge T \wedge (\neg q \vee \neg p)$

(c) $(r \rightarrow q) \rightarrow (p \wedge \neg p)$

(d) $F \vee ((\neg\neg p \rightarrow q) \leftrightarrow \neg r)$

(e) $(q \wedge \neg q) \leftrightarrow (r \wedge \neg r)$

Solution

b, because we can perform DeMorgan’s on $\neg q \vee \neg p \equiv \neg(q \wedge p)$ which is a contradiction with $p \wedge q$. Also for any truth values we use for p and q, the compound proposition is false

Given:

- c : school is canceled
- s : it snows two feet
- t : the temperature is -40 degrees

Which of the following is a propositional logic translation of the sentence:

"School will be canceled whenever the temperature is -40 degrees or it snowed two feet."

(a) $(s \wedge t) \rightarrow c$

(b) $(s \vee t) \rightarrow c$

(c) $\neg c \leftrightarrow \neg(s \vee t)$

(d) $c \rightarrow (s \wedge t)$

(e) $c \rightarrow (s \vee t)$

"if p, then q"
"if p, q"
"p is sufficient for q"
"q if p"
"q when p"
"a necessary condition for p is q"
"q unless $\neg p$"

"p implies q"
"p only if q"
"a sufficient condition for q is p"
"q whenever p"
"q is necessary for p"
"q follows from p"

Solution: (b)
"p whenever q"means that if q happens, then p must also happen; however, p can happen even if q does not happen. So "p whenever q"$\equiv q \to p$.

Suppose we have the following premises:

(i) If you are in Ann Arbor and it is not winter, then it is not snowing $[(a \wedge \neg w) \rightarrow \neg s]$

(ii) If you are not in Ann Arbor, then you are on vacation $[\neg a \rightarrow v]$

(iii) It is snowing $[s]$

(iv) If you are not enrolled in school then it is not the case that either you are on vacation or it is winter $[\neg e \rightarrow \neg(v \vee w)]$

Which is **NOT** a valid conclusion?

(A) You are on vacation or it is winter $[v \vee w]$

(B) You are not in Ann Arbor and it is winter $[\neg a \wedge w]$

(C) You are not in Ann Arbor or it is winter $[\neg a \vee w]$

(D) You are enrolled in school $[e]$

Solution

Solution: B. Combining premises (i) and (iii), we have that $\neg(a \wedge \neg w)$, i.e., $\neg a \vee w$, is correct. Combining $\neg a \vee w$ and premise (ii), we have $v \vee w$ is correct. Thus, e is correct based on $v \vee w$ and premise (iv).

Show that $(p \wedge q) \rightarrow r$ is **not** logically equivalent to $(p \rightarrow r) \wedge (q \rightarrow r)$.

TABLE 2 **The Truth Table for the Conjunction of Two Propositions.**

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

TABLE 5 **The Truth Table for the Conditional Statement $p \rightarrow q$.**

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Solution

Let $p = T$, $q = F$, and $r = F$. Then

$$\begin{aligned}(p \wedge q) \to r &\equiv (T \wedge F) \to F \\ &\equiv F \to F \\ &\equiv T\end{aligned}$$

and

$$\begin{aligned}(p \to r) \wedge (q \to r) &\equiv (T \to F) \wedge (F \to F) \\ &\equiv F \to T \\ &\equiv F\end{aligned}$$

The two compound propositions give different truth values for these inputs, therefore they are **not** logically equivalent.

Alternate Solution

Truth table method:

p	q	r	$(p \wedge q)$	$(p \to r)$	$(q \to r)$	$(p \wedge q) \to r$	$(p \to r) \wedge (q \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	T	T	T
T	F	F	F	F	T	T	F
F	T	T	F	T	T	T	T
F	T	F	F	T	F	T	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

The last two columns differ on the 4th line (and also the 6th line), therefore the two compound propositions are **not** logically equivalent.

Show that $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$ is a tautology. You can use truth tables or logical equivalences.

TABLE 2 **The Truth Table for the Conjunction of Two Propositions.**

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

TABLE 3 **The Truth Table for the Disjunction of Two Propositions.**

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

TABLE 5 **The Truth Table for the Conditional Statement $p \to q$.**

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Truth Table Solution

$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$(p \vee q) \wedge (p \to r) \wedge (q \to r)$	$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	T

5 Minute Break

https://paveldogreat.github.io/WebGL-Fluid-Simulation/

Predicates and Quantifiers

Cheat Sheet Suggestions

TABLE 1 Quantifiers.

Statement	*When True?*	*When False?*
$\forall x P(x)$	$P(x)$ is true for every x.	There is an x for which $P(x)$ is false.
$\exists x P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every x.

TABLE 2 De Morgan's Laws for Quantifiers.

Negation	*Equivalent Statement*	*When Is Negation True?*	*When False?*
$\neg\exists x P(x)$	$\forall x \neg P(x)$	For every x, $P(x)$ is false.	There is an x for which $P(x)$ is true.
$\neg\forall x P(x)$	$\exists x \neg P(x)$	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x.

Cheat Sheet Suggestions

TABLE 1 Quantifications of Two Variables.

Statement	*When True?*	*When False?*
$\forall x \forall y P(x, y)$ $\forall y \forall x P(x, y)$	$P(x, y)$ is true for every pair x, y.	There is a pair x, y for which $P(x, y)$ is false.
$\forall x \exists y P(x, y)$	For every x there is a y for which $P(x, y)$ is true.	There is an x such that $P(x, y)$ is false for every y.
$\exists x \forall y P(x, y)$	There is an x for which $P(x, y)$ is true for every y.	For every x there is a y for which $P(x, y)$ is false.
$\exists x \exists y P(x, y)$ $\exists y \exists x P(x, y)$	There is a pair x, y for which $P(x, y)$ is true.	$P(x, y)$ is false for every pair x, y.

It's true that:

– $\forall x\,[P(x) \wedge Q(x)] \equiv [\forall x\, P(x)] \wedge [\forall x\, Q(x)]$

But it's not true that:

– $\forall x\,[P(x) \vee Q(x)] \equiv [\forall x\, P(x)] \vee [\forall x\, Q(x)]$

Likewise, it's true that:

– $\exists x\,[P(x) \vee Q(x)] \equiv [\exists x\, P(x)] \vee [\exists x\, Q(x)]$

But it's not true that:

– $\exists x\,[P(x) \wedge Q(x)] \equiv [\exists x\, P(x)] \wedge [\exists x\, Q(x)]$

Problem 6. (4 points)

Let $S(x, y)$ be the statement that "person x is shorter than person y". If Atreya is taller than Nouman but shorter than twins Eric and Paul (who are the same height), which of the following is true?

(a) $S(\text{Atreya, Nouman})$

(b) $S(\text{Eric, Eric})$

(c) $S(\text{Eric, Paul})$

(d) $S(\text{Nouman, Eric})$

(e) $S(\text{Paul, Nouman})$

Solution

d, we know from the statement that Nouman is shorter than Atreya who is shorter than the twins Eric and Paul

Small note on translations

When we translate a sentence such as “Someone in this class is going to ace the exam” to proposition logic, we use $\exists x(C(x) \land A(x))$, where C(x) is x is in this class and A(x) is x is going to ace the exam. We do not want to use the $\rightarrow$ here, because for a person that isn’t a student, the implication would be true, which is not what we want.

When we translate a sentence such as “Everyone in this class is going to ace the exam” to proposition logic, we use $\forall x(C(x) \rightarrow A(x))$, where C(x) is x is in this class and A(x) is x is going to ace the exam. We do not want to use the $\land$ here, because the translation would give us false for those not in the class, even though those people do not matter.

Let $H(x,t)$ be the statement that "person x is happy at time t". Translate the following sentence:

"All the time someone is happy, but no one is happy all the time."

a) $\forall t \exists x H(x,t) \land \neg \exists x \forall t H(x,t)$

b) $\forall t \exists x H(x,t) \to \neg \exists x \forall t H(x,t)$

c) $\exists x \forall t H(x,t) \land \neg \forall t \exists x H(x,t)$

d) $\exists x \forall t H(x,t) \to \neg \forall t \exists x H(x,t)$

Solution

A, these are two separate statements connected by the “but”, which is equivalent to an “and” statement. The quantifiers then fall into place.

Let $L(x,y)$, $C(x,y)$, and $R(x,y)$ be the statements "x eats lunch with y", "x has a class with y", and "x is roommates with y" respectively. The domain for x and y is students at the University of Michigan.

Translate the following expressions of quantifiers, logical connectives, and predicates into English in the clearest way possible.

(a) $\forall x \forall y((C(x,y) \wedge R(x,y)) \rightarrow L(x,y))$

(b) $\exists x \forall y(((x \neq y) \wedge C(x,y)) \rightarrow \neg L(x,y))$

(c) $\forall x \exists y((x \neq y) \wedge (C(x,y) \vee R(x,y)) \wedge \neg L(x,y))$

Solution:

(a) All students who have a class together and are roommates will eat lunch together.

(b) There exists at least one student who does not eat lunch with any other students in any of their classes. (Alternately, there exists a student who, if they are in a class with another student, they do not eat lunch with that other student)

(c) Every student has another roommate or classmate with whom they don't eat lunch. (Alternately, For all students, there exists a different student with whom they either share a class or a room, but with whom they do not eat lunch.

13. Rewrite each of the following statements so that the negation appears before the predicates

(a) $\neg\forall x(\exists y\forall z P(x,y,z) \wedge \exists z\forall y P(x,y,z))$

(b) $\neg(\exists x\exists y\neg P(x,y) \wedge \forall x\forall y Q(x,y))$

Solution:

(a) $\exists x(\forall y\exists z\neg P(x,y,z) \vee \forall z\exists y\neg P(x,y,z))$

(b) $\forall x\forall y P(x,y) \vee \exists x\exists y\neg Q(x,y)$

Choose the true statements from the following if the domain of discourse is $\mathbb{R}$.

(a) $\forall x \forall y \exists z (x^2 + y^2 = z^2)$

(b) $\forall x [(x > 4) \to |x - 4| \geq 1]$

(c) $\forall x \exists y P(x, y) \to \exists y \forall x P(x, y)$ for all predicates $P(x, y)$

(d) $\exists y \forall x P(x, y) \to \forall x \exists y P(x, y)$ for all predicates $P(x, y)$

Solution: (a), (d)
(a) is true because for every x and y, $x^2 + y^2 \geq 0$ and we can use $z = \sqrt{x^2 + y^2}$.
(b) is not true. 4.5 as a counterexample.
(c) is also not true.
(d) is true. Note that while $\forall x \exists y P(x, y)$ and $\exists y \forall x P(x, y)$ are not logically equivalent, the latter does imply the former. This is because if we have a single y value that works for every x, we can use that same value for each x in turn. However, if each x has a y value that works, it might be a different y for each one.

Good luck studying!