EECS 203: Discrete Mathematics
Winter 2024
Discussion 4 Notes

1 Definitions

- **Types of Proofs:**

  - **Direct Proof:**
  - **Proof by Contraposition:**
  - **Proof by Contradiction:**
  - **Proof by Cases:**

- **Set:**

- **Universe:**

- **Set Operations:**

  - **Union** $S \cup T$
  - **Intersection** $S \cap T$
  - **Complement** $\overline{S}$
  - **Minus** $S - T$

- **Inclusion−Exclusion Principle:**

- **Subset:**

- **Proper Subset:**

- **Disjoint:**

- **Power Set:**

- **Cardinality:**

- **Cartesian Product:**

- **Empty Set:**

2 Exercises

1. Proof by Cases/Contradiction ⋆

Prove that there is no rational solution to the equation $x^3 + x + 1 = 0$. **Hint:** Use the fact that 0 is an even number.

You can use the following lemmas without proving:

- Odd × Even = Even

- Odd × Odd = Odd

- Even × Even = Even

- Odd + Even = Odd

- Odd + Odd = Even

- Even + Even = Even

2. Prime Proof ⋆

Show that for any prime number p, $p^2 + 11$ is composite (not prime). Recall that a prime p is defined to be a natural number ≥ 2 such that p and 1 are the only factors that divide p.

3. Proving the Triangle Inequality

Prove the triangle inequality, which states that if x and y are real numbers, then $|x| + |y| \geq |x + y|$ (where $|x|$ represents the absolute value of x, which equals x if $x \geq 0$ and equals $-x$ if $x < 0$).

4. Set Exploration ⋆

a) What is $|\emptyset|$?

b) Let $A = \{1, 2, 3\}$, $B = \{\emptyset\}$, $C = \{\emptyset, \{\emptyset\}\}$, $D = \{4, 5\}$, and $E = \{\emptyset, 5\}$.

    i. Is $\emptyset \in A$?

    ii. Is $\emptyset \subseteq A$?

    iii. Is $\emptyset \in B$?

    iv. Is $\emptyset \subseteq B$?

    v. Is $\emptyset \in C$?

    vi. Is $\emptyset \subseteq C$?

    vii. What is $A \cap D$?

    viii. What is $B \cap C$?

    ix. What is $B \cap E$?

    x. What is $|B|$, $|C|$, $|E|$?

c) Let A and C be the sets defined above.

    i. What is $P(A)$?

    ii. What is $P(C)$?

    iii. Find a formula for the size of the power set of S, $|P(S)|$, in terms of $|S|$.

    iv. What is $C \times A$?

    v. What is A^2? ($A^2 = A \times A$)

    vi. Find a formula for the size of the Cartesian product of A and B, $|A \times B|$ in terms of $|A|$ and $|B|$.

5. Double Subset Equality ⋆

Prove the set equivalence: $A - (B \cap C) = (A - B) \cup (A - C)$

6. Subset Proofs

Let A, B, and C be sets. Prove that

a) $(A \cap B \cap C) \subseteq (A \cap B)$

b) $(A - B) - C \subseteq A - C$

7. Power Sets

Can you conclude that $A = B$ if A and B are two sets with the same power set?

8. More Power Sets ⋆

Determine whether each of these sets is the power set of a set, where a and b are distinct elements.

a) $\emptyset$

b) $\{\emptyset, \{a\}\}$

c) $\{\emptyset, \{a\}, \{\emptyset, a\}\}$

d) $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

9. Power Set of a Cartesian Product

Prove or disprove that if A and B are sets, then $\mathcal{P}(A \times B) = \mathcal{P}(A) \times \mathcal{P}(B)$.