# EECS 203: Discrete Mathematics
# Winter 2024
# Discussion 3 Notes

# 1 Proof Styles

## 1.1 Proofs by Contraposition

**Proof by Contraposition:** Prove a conditional (in the form "if $p$, then $q$") by proving that proving that if $q$ is false, then $p$ must also be false. This is done by assuming the negation of $q$ and concluding the negation of $p$

### 1.1.1 Proof by Contraposition $\star$

Prove that if $n^2 + 2$ is even, then $n$ is even using a proof by contrapositive.

### 1.1.2 Proof by Contraposition II

Prove by contrapositive that if $a^2 + a + 2 \geq b^2 + b + 2$, then $a \geq b$, where $a$ and $b$ are positive integers.
You may use without proving:

1. $c < d$ and $e < f \rightarrow (c + e) < (d + f)$
2. $c < d$ and $e < f \rightarrow ce < df$, where $c, d, e, f$ are positive integers

## 1.2 Proofs by Contradiction

Prove $p$ is true by assuming $\neg p$, and arriving at a contradiction, i.e. a conclusion that we know is false.
When using a proof by contradiction to prove "if $p$ is true then $q$ is true", we assume that $p$ is true and that $q$ is false, and derive a contradiction. This shows us that if $p$ is true, then $q$ is true.

$$\neg(p \rightarrow q) \equiv (p \wedge \neg q) \rightarrow F \rightarrow \neg(p \wedge \neg q) \equiv (p \rightarrow q)$$

A simpler way to view this: Assume $p$ is true and show that

$$(\neg q \rightarrow \mathrm{F}) \rightarrow q$$

### 1.2.1 Contraposition vs Contradiction

Show that for an integer $n$: if $n^3 + 5$ is odd, then $n$ is even, using

a) a proof by contraposition.

b) a proof by contradiction.

**Note:** The algebra in either case is the same. You don't need to rewrite the algebra for part (b), just reformat your proof from (a) into a proof by contradiction.

## 1.3 Choosing Proof Style

A number is considered **rational** if and only if it can be written as the ratio of two integers: $\frac{p}{q}$ where $q \neq 0$.

### 1.3.1 Proof Practice

Prove or disprove that the sum of a rational number and an irrational number must be irrational.

### 1.3.2 Odd Proof III

Prove that for all integers $a$ and $b$, if $a$ divides $b$ and $a + b$ is odd, then $a$ is odd.

### 1.3.3 Proofs

(a) Prove or disprove: For all nonzero rational numbers $x$ and $y$, $x^y$ is rational

(b) Prove or disprove: For all even integers $x$ and all positive integers $y$, $x^y$ is even.

(c) Prove or disprove: For all real numbers $x$ and $y$, if $x^y$ is irrational, then $x$ or $y$ is not a positive integer