Groupwork 6 Problems

1. Multiple Multiples [12 points]

Let $a, b \in \mathbb{Z}$. Show that $7a - 8b$ is a multiple of 5 if and only if $19a - 21b$ is a multiple of 5.

Solution:

2. Rapidly Rising [18 points]

For this problem, we will say a function $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ is "rapidly rising" if:

$$\forall x_1, x_2 \in \mathbb{Z}^+ \; [x_1 < x_2 \to 2f(x_1) < f(x_2)]$$

(a) Prove that $f(x) = 3^x$ is rapidly rising.

Hint: It may be easier to show $f(x_2) > 2f(x_1)$ than the other way around.

(b) Is a rapidly rising function always one-to-one? Is a one-to-one function from $\mathbb{Z}^+ \to \mathbb{Z}^+$ always rapidly rising? Is a one-to-one function (again from $\mathbb{Z}^+ \to \mathbb{Z}^+$) always strictly increasing? Briefly explain your answer; a formal proof is not necessary but is encouraged.

Note: $f : \mathbb{N} \to \mathbb{N}$ is strictly increasing if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$.

(c) Prove that, for any rapidly rising function f, it must **not** be onto.

Solution: