

EECS 203: Discrete Mathematics  
Winter 2024  
Homework 3

Due **Thursday, Feb. 8**, 10:00 pm

No late homework accepted past midnight.

Number of Problems:  $7 + 1$

Total Points:  $100 + 30$

- **Match your pages!** Your submission time is when you upload the file, so the time you take to match pages doesn't count against you.
- Submit this assignment (and any regrade requests later) on Gradescope.
- Justify your answers and show your work (unless a question says otherwise).
- By submitting this homework, you agree that you are in compliance with the Engineering Honor Code and the Course Policies for 203, and that you are submitting your own work.
- Check the syllabus for full details.

## 1. On the Contrary [12 points]

Let  $n$  be an integer. Prove that if  $4 \mid (n^2 - 1)$ , then  $n$  is odd using

- (a) a proof by contraposition, and
- (b) a proof by contradiction.

Then,

- (c) compare your answers to parts (a) and (b). What is different? What is the same?

### Solution:

Let  $p$  be  $4 \mid (n^2 - 1)$ ,  $q$  be  $n$  is odd.

The original proposition can therefore be expressed as  $\forall n p \rightarrow q$ .

a) Proof:

To prove by contraposition, the contrapositive of the original is  $\forall n \neg q \rightarrow \neg p$  contraposition

Assume  $n$  is an even integer

$n = 2k$ ,  $k$  is a random integer definition of even

$$n^2 - 1$$

$$\equiv 4k^2 - 1$$

substitution

It does not divide 4

definition of divide

Thus, the original proposition is true because the contrapositive is true.

## 2. An Even-Numbered Question about Even Numbers [16 points]

**Prove or disprove** the following statements:

- (a) For all integers  $x$ , if  $x$  is even, then  $x^2$  is even.
- (b) For all integers  $x$ , if  $x^2$  is even, then  $x$  is even.
- (c) For all integers  $x$ , if  $x$  is even, then  $2x$  is even.
- (d) For all integers  $x$ , if  $2x$  is even, then  $x$  is even.

### Solution:

### 3. Even Stevens [16 points]

**Prove or disprove** the following statement: “There is a finite amount of even numbers.”

<b>Solution:</b>
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### 4. Pay it Forward (Or Don't, It's Up To You) [12 points]

Consider a centipede game, where there are two players: Ka-chun and Zyaire. The game starts by Ka-chun's decision of take or wait.

- If Ka-chun takes, Ka-chun earns \$1 while Zyaire earns nothing, and the game ends.
- If Ka-chun waits, then Zyaire can choose between take or wait. If Zyaire takes, Zyaire earns \$2 while Ka-chun earns nothing and the game ends. If Zyaire waits it becomes Ka-chun's turn to choose again.
- If they keep waiting the reward grows by \$1 each round, until Zyaire's choice of taking \$20 or waiting, when the game will end no matter what.

Both of Ka-chun and Zyaire want to maximize their rewards, and behave as perfect logicians.

- (a) Suppose Ka-chun and Zyaire made it to round 20. What happens in round 20?
- (b) Using your answer to (a), what would happen if they made it to round 19?
- (c) Building off of parts (a) and (b), argue that Ka-chun should take \$1 in the very first round.

<b>Solution:</b>
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### 5. Proofs to the Max [12 points]

Prove that for all real numbers  $a$ ,  $b$ , and  $c$ , if  $\max\{a^2(b - c), -a\}$  is non-negative, then  $a \leq 0$  or  $b \geq c$ .

**Note:** You can use the following facts in your proof:

- If  $x$  and  $y$  are positive, then  $x \cdot y$  is positive.
- If  $x$  is positive and  $y$  is negative, then  $x \cdot y$  is negative.
- If  $x$  and  $y$  are negative, then  $x \cdot y$  is positive.

**Solution:**

## 6. Let's All Be Rational [16 points]

Show that these statements about a real number  $x$  are equivalent to each other:

- (i)  $x$  is rational
- (ii)  $\frac{x}{2}$  is rational
- (iii)  $3x - 1$  is rational.

**Hint:** One way to prove statements (i), (ii) and (iii) are equivalent is by proving (i)  $\rightarrow$  (ii), (ii)  $\rightarrow$  (iii), and (iii)  $\rightarrow$  (i).

**Solution:**

## 7. Irrational Proof [16 points]

Prove or disprove that the product of a nonzero rational number and an irrational number is irrational.

**Solution:**

## Grading of Groupwork 2

Using the solutions and Grading Guidelines, grade your Groupwork 2 Problems:

- Use the table below to grade your past groupwork submission and calculate scores.
- While grading, mark up your past submission. Include this with the table when you submit your grading.
- Write whether your submission achieved each rubric item. If it didn't achieve one, say why not.
- For extra credit, write positive comment(s) about your work.
- You don't have to redo problems correctly, but it is recommended!
- See "All About Groupwork" on Canvas for more detailed guidance, and what to do if you change groups.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)	Total:
Problem 1												/20
Problem 2												/20
Total:												/40

## Groupwork 3 Problems

### 1. Are These Equivalent? [30 points]

Let  $P(x)$  and  $Q(x)$  be arbitrary predicates.

- (a) Prove or disprove that for any domain of  $x$ ,  $\forall x(P(x) \leftrightarrow Q(x))$  must be logically equivalent to  $\forall xP(x) \leftrightarrow \forall xQ(x)$ .
- (b) Prove or disprove that for any domain of  $x$ ,  $\exists x(P(x) \leftrightarrow Q(x))$  must be logically equivalent to  $\exists xP(x) \leftrightarrow \exists xQ(x)$ .
- (c) Let  $\Diamond x$  mean that “there exists **at most one**  $x$ .” Prove or disprove that for any domain of  $x$ ,  $\Diamond x(P(x) \leftrightarrow Q(x))$  must be logically equivalent to  $\Diamond xP(x) \leftrightarrow \Diamond xQ(x)$ .

<b>Solution:</b>
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