EECS 203: Discrete Mathematics
Winter 2024
Discussion 10 Notes

1 Definitions

- **Bayes' Theorem:**
- **Random Variable:**
- **Expected Value:**
- **Linearity of Expectation:**
- **Indicator Random Variable:**
- **Geometric Distribution:**
- **Expected Value of a Geometric Distribution:**
- **Bernoulli Trials and the Binomial Distribution:**

2 Exercises

1. Conditional Probability

A bit string of length four is generated at random so that each of the 16 bit strings of length four is equally likely. What is the probability that it contains at least two consecutive 0s, given that its first bit is a 0? (We assume that 0 bits and 1 bits are equally likely.)

2. Bayes' Theorem ⋆

An electronics company is planning to introduce a new camera phone. The company commissions a marketing report for each new product that predicts either the success or the failure of the product. Of new products introduced by the company, 60% have been successes. Furthermore, 70% of their successful products were predicted to be successes, while 40% of failed products were predicted to be successes. Find the probability that this new camera phone will be successful if its success has been predicted.

3. Conditional Probability ⋆

Jakub has created an app that classifies images as either being a Hot Dog or Not a Hot Dog, and he needs your help for some analysis.

Suppose that 4% of the images in a data set are images of hot dogs. Furthermore, suppose that when Jakub's app classifies an image, 97% of the hot dog images are classified correctly (as hot dogs), and 2% of the images that are not hot dogs are classified incorrectly (as hot dogs). What is the probability that:

a) an image classified as a hot dog is really a hot dog?

b) an image classified as a hot dog is **not** a hot dog?

c) an image classified as "not a hot dog" is a hot dog?

d) an image classified as "not a hot dog" is **not** a hot dog?

4. Random Variables and Expected Value

Suppose that a coin is flipped three times. Let X be a random variable representing the total number of heads across the three flips.

(a) What are the possible values of X and which outcome(s) are associated with each value of X?

(b) Find $E(X)$?

5. Bernoulli Trials and the Binomial Distribution (I) ⋆

A biased program is generating a 10-bit string. For any single bit, that the probability that the program generates a 0 is 0.9, and each of the 10 bits are generated independently.

(a) What is the probability that the bitstring will contain exactly 8 0's?

(b) What is the expected number of of 1's in bitstring?

6. Bernoulli Trials and the Binomial Distribution (II)

A group of six people play the game of "odd person out" to determine who will buy refreshments. Each person flips a fair coin. If there is a person whose outcome is not the same as that of any other member of the group, this person has to buy the refreshments.

(a) What is the probability that there is an odd person out after the coins are flipped once?

(b) Suppose the group continues to play the game until there is an odd person out. In each iteration of the game, every person flips their coin and then the group checks for an odd person out. What is the expected number of iterations until there is finally an odd person out?

7. Expected Value ⋆

Kim's Note: This question is in the homework too! Eep!

Kim's Note: Please:
1. Delete it from the Discussion Bank (or comment out with a HUGE note not to use it b/c it's in the HW bank)
2. Replace in this discussion with the next question, which is brand new
3. Add the next question to the Discussion Bank

Rohit recently became super passionate about rolling dice. He decides to roll a single (fair 6-sided) die 100 times. What is the expected number of times he rolls a 5 followed by a 6?

8. Expected Value ⋆

Kim's Note: NEW QUESTION - PLEASE PROOFREAD!
Then add to Discussion Bank once finalized

Rebecca is creating a new 15-digit passcode using only the digits 1, 2, and 3. She chooses each digit uniformly at random from that set of 3 digits, {1, 2, 3}. What is the expected number of times that the sequence 3113 appears in her passcode? For example, it appears 4 times in the passcode

$$\overline{3113}\underline{11}3\underline{11}3\underline{1132}3\underline{113}.$$