PRACTICE Exam 1
EECS 203, Winter 2024

Name (ALL CAPS): ______________________________

Uniqname (ALL CAPS): ______________________________

8-Digit UMID: ______________________________

Instructions

- When you receive this packet, fill in your name, Uniqname, and UMID above.
- Once the exam begins, make sure you have problems 1-18 in this booklet.
- Write your UMID in the blank at the top of every other page.
- No one may leave within the last 10 minutes of the exam.
- After you complete the exam, sign the Honor Code below. If you finish when time is called, your proctor will give you time to sign the Honor Code.
- Do not detach the scratch paper at the end of the packet.
- Do not discuss the exam until solutions have been released!

Materials

- No electronics allowed, including calculators.
- You may use one 8.5″ by 11″ note sheet, front and back, created by you.
- You may not use any other sources of information.

Honor Code

This exam is administered under the College of Engineering Honor Code. Your signature endorses the pledge below. We will not grade your exam without your signature.

I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code. I further agree not to discuss any aspect of this examination in any way, shape, or form until the solutions have been published.

Signature: ______________________________

Part A: Single Answer Multiple Choice

Instructions

- There is 2 question in this section (this is fewer than normal).
- Shade **only one** circle corresponding to your answer choice.
- If you shade more than one circle, your answer will be marked as incorrect.

Example. ⓐ ⓑ ● ⓓ ⓔ

Make sure to SHADE A BUBBLE next to the question title, as shown above.

Problem 1. (4 points) ⓐ ⓑ ⓒ ⓓ ⓔ

Consider an arbitrary function $f :\to$. Which of the following is the correct **assumption** to begin a one-to-one proof?

(a) For arbitrary real number b assume there exists a real number a such that $f(a) = b$

(b) For arbitrary real numbers a, b assume $f(a) = f(b)$

(c) For arbitrary real numbers a, b assume $(f(a) = f(b)) \to (a = b)$

(d) For arbitrary real numbers a, b assume that $f(a) = b$

(e) For arbitrary real number a assume there exists a real number b such that $f(a) = b$

Problem 2. (4 points) ⓐ ⓑ ⓒ ⓓ ⓔ

Consider the recurrence relation

$$f(n) = 2f(n-1) + 4f(n-2) + f(n-5).$$

What is the **minimum** number of initial conditions required for this recurrence?

(a) 2

(b) 3

(c) 4

(d) 5

(e) Not enough information

Part B: Multiple Answer Multiple Choice

Instructions

- There are 10 questions in this section.
- Shade whichever boxes you believe are correct. **This could be all answers, no answers, or anything in between.**
- If there are no correct answers, leave all the boxes blank.

Example	a b c d e

Make sure to SHADE 0 OR MORE BOXES next to the question title, as shown above.

Problem 3. (4 points)	a b c d e

For each Inductive Step, determine whether it's possible to complete an inductive proof that $P(n)$ is true for all $n \geq n_0$ using the specified number of base cases.

(a) Inductive Step: $P(k) \to P(k+1)$. Number of Base Case(s): 1

(b) Inductive Step: $P(k-2) \to P(k)$. Number of Base Case(s): 1

(c) Inductive Step: $P(k-4) \to P(k)$. Number of Base Case(s): 4

(d) Inductive Step: $P(k-1) \to P(k+1)$. Number of Base Case(s): 2

(e) Inductive Step: $P(k-1) \to P(k+3)$. Number of Base Case(s): 4

Problem 4. (4 points)	a b c d e

Let $n \equiv 2 \pmod 6$. Which of the following statements are **guaranteed** to be true?

(a) $n \equiv 0 \pmod 2$

(b) $n \equiv 5 \pmod 2$

(c) $n \equiv 8 \pmod{12}$

(d) $n \equiv 2 \pmod{12}$

(e) $n \equiv 2 \pmod 3$

Problem 5. (4 points)

a b c d e

Which of these sets are uncountable?

(a) $[10, 17)$
(b) $[5, 6] - (5, 6)$
(c) $\mathbb{R} - \mathbb{Q}$
(d) $\{0, 1\} \times \mathbb{R}$
(e) $\mathbb{Q} \times \mathbb{Q}$

Problem 6. (4 points)

a b c d e

$$f(x) = \sqrt{x + 203}$$

Reminder: $\sqrt{a}$ returns the positive square root of a.

For which of the following domain-codomain pairs is $f(x)$ a function?

(a) $\mathbb{R}^+ \to \mathbb{R}^-$
(b) $\{4\} \to \{\sqrt{207}\}$
(c) $\mathbb{Q} \to \mathbb{R}^+$
(d) $\mathbb{Q}^+ \to \mathbb{R}$
(e) $\mathbb{R} \to \mathbb{R}$

Problem 7. (4 points)

a b c d e

If $x \equiv 3 \pmod 5$ and $y \equiv 1 \pmod 6$, compute $6x + 5y \bmod 15$.

(a) 1
(b) 5
(c) 8
(d) 12
(e) Not enough information

Problem 8. (4 points)

a b c d e

Which of the following definitions of $f(x)$ satisfy both of the following:

- are valid functions $f : [2,4] \to (8,16)$, and
- prove that $|[2,4]| \leq |(8,16)|$

(a) $f(x) = 7$
(b) $f(x) = \sqrt{x} + 8$
(c) $f(x) = 4x$
(d) $f(x) = \begin{cases} 10, & x \in \{2,4\} \\ 4x, & 2 < x < 4 \end{cases}$
(e) $f(x) = -2x + 18$

Problem 9. (4 points)

a b c d e

What is the least number of distinct integers you would need to draw from the set $\{1, 2, \ldots, 20\}$ that would guarantee you have numbers x and y such that $y - x = 10$.

(a) 10
(b) 20
(c) 5
(d) 11
(e) 2

Problem 10. (4 points)

a b c d e

Which of the following functions are **bijective**?

(a) $f\colon \mathbb{Z} \to \mathbb{Z},\ f(x) = x^3$
(b) $g\colon \mathbb{Z}^+ \to \mathbb{Z},\ g(x) = x^3$
(c) $p\colon \mathbb{R} \to \mathbb{R}^+,\ p(x) = |x|$
(d) $q\colon \mathbb{R}^+ \to \mathbb{R}^+,\ q(x) = |x|$
(e) $r\colon \mathbb{R} \to \mathbb{R}^+,\ r(x) = 2^x$

Problem 11. (4 points)

a b c d e

You're making friendship bracelets for Tako, your octopus friend. What is the minimum number of friendship bracelets you have to make to guarantee that Tako will have 5 or more bracelets on at least one tentacle? (Tako 8 tentacles.)

(a) 5
(b) 6
(c) 32
(d) 33
(e) 41

Problem 12. (4 points)

a b c d e

Consider two arbitrary functions $g : A \to B$ and $f : B \to C$. Which of the following must be true for $f \circ g$ to be a bijection?

(a) $|A| = |B|$

(b) $|A| = |C|$

(c) f is a bijection

(d) g is one-to-one

(e) g is onto

Part C: Short Answer

Instructions

- There are 1 questions in this section
- Write your solution in the space provided below the question
- Don't simplify your answers
- Show your work and include justification

Problem 13. (6 points)

Let $x \equiv 5 \pmod{6}$ and $y \equiv 7 \pmod{9}$. Compute the following values. If a value cannot be computed, write "N/A" as your answer.

(a) $4x + y \bmod 3$.

(b) $4x + y \bmod 18$.

Part D: Free Response

Instructions

- There are 5 questions in this section
- Write your solution in the space provided
- Write down your answer with care: **answers that are unreadable (such as too faint or too messy) will not be graded**
- If you have multiple answers, you must indicate which one you want graded. Otherwise, we will grade your least favorable answer.
- Show your work and include justification

Problem 14. (10 points)

Consider the function $f : \mathbb{R}^+ \to \mathbb{R}^+$, $f(x) = \left|\dfrac{2}{x}\right|$

(a) Prove or disprove f is one-to-one.

(b) Prove or disprove f is onto.

Problem 15. (9 points)

Prove using weak induction that for all positive integers n:

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + ... + n \cdot (n+1) = \frac{n(n+1)(n+2)}{3}$$

Problem 16. (8 points)

A function $f\colon \mathbb{R} \to \mathbb{R}$ is *strictly increasing* when, for all $x, y \in \mathbb{R}$,

$$f(x) < f(y) \text{ if and only if } x < y.$$

1. Using the definition of one-to-one, prove that if f is strictly increasing, then f is one-to-one.
2. Using the definition of strictly increasing, prove that if f and g are strictly increasing, then $f \circ g$ is strictly increasing.

Problem 17. (7 points)

Michelle the triathlete will do one of four things on any given day: Swim, Run, Bike, or Nap. To maximize training efficiency and prevent injuries, Michelle follows the following rules when constructing a training plan.

- If Michelle Runs on a given day, she must have Napped on the previous day.
- If Michelle Swims on a given day, she must have Run or Biked on the previous day.

Note that the above rules mean that she cannot Swim or Run on the first day.

(a) Find a recurrence relation for $f(n)$, the number of ways Michelle can train in n days.

(b) Find the initial conditions for your recurrence relation in part (a). For full credit, you must provide the fewest initial conditions possible to satisfy your recurrence.

Problem 18. (7 points)

The Martian monetary system uses colored beads instead of coins. In particular,

- A red bead is worth 3 Martian credits
- A green bead is worth 7 Martian credits
- A blue bead is worth 8 Martian credits.

Let $P(n)$ be the predicate "There is some combination of red, green and blue beads that is worth exactly n Martian credits." Prove that $\forall n \geq 6,\ P(n)$.

This is a space for scratch work. DO NOT DETACH THIS PAPER FROM YOUR EXAM.

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