Groupwork 7 Problems

Grading of Groupwork 7

Using the solutions and Grading Guidelines, grade your Groupwork 7 Problems:

- Use the table below to grade your past groupwork submission and calculate scores.
- While grading, mark up your past submission. Include this with the table when you submit your grading.
- Write whether your submission achieved each rubric item. If it didn't achieve one, say why not.
- For extra credit, write positive comment(s) about your work.
- You don't have to redo problems correctly, but it is recommended!
- See "All About Groupwork" on Canvas for more detailed guidance, and what to do if you change groups.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)	Total:
Problem 1	2	1	2	1	2	2						10/10
Problem 2	1	1	0	0	0	0						3/8
Total:												13/18

Comments

Pigeon hole can be hard to find. However, I figured out the first problem on my own, kuddos to me.

1. Get to the Point [10 points]

Consider an arbitrary set A. We say a function $f : A \to A$ has a *fixed point* iff there exists $a \in A$ such that $f(a) = a$.

Consider the notation $f^{(n)}$ to mean $\underbrace{f \circ \cdots \circ f}_{n \text{ times}}$, where $n \in \mathbb{Z}^+$. Essentially, n copies of f are composed together.

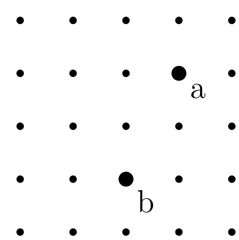
Prove by **induction** that if f is a function with a fixed point, then for all positive integers n, $f^{(n)}$ has a fixed point.

Solution:

Let P(n) be the statement that $f^{(n)}$ has a fixed point.
Inductive step: Assume P(k) is true for some $k \in \mathbb{Z}^+$.
Want to show: $P(k) \implies P(k+1)$ for all $k \in \mathbb{Z}^+$.
i.e. $\exists a \in A$ such that $f^{(k)}(a) = a \implies \exists b \in A$ such that $f^{(k+1)}(b) = b$.
$f^{(k+1)}(b) = f(f^{(k)}(b))$.
By inductive hypothesis, $f^{(k)}(b) = b$.
Then $f(f^{(k)}(b)) = f(b) = b$.
So b is a fixed point of $f^{(k+1)}$.
Thus, $P(k) \implies P(k+1)$ for all $k \in \mathbb{Z}^+$.
Base case: $n = 1$. Then $f^{(1)} = f$ has a fixed point by assumption.

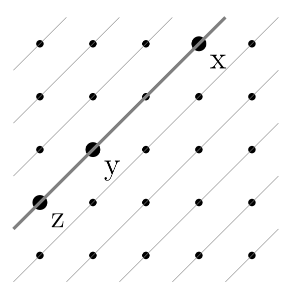
2. Going Off the Grid [8 points]

In a grid, we say that a point a *dominates* a point b iff a lies strictly above and to the right of b. For example, in the picture below, a dominates b.



Prove using the Pigeonhole Principle that if we choose $4n - 1$ points from an $n \times n$ grid ($n \geq 4$), there must be three chosen points x, y, z such that x dominates y and y dominates z. Make sure to state what your pigeons are and what your holes are, as well as how many of each you have.

Hint: If x, y, z lie on the same increasing diagonal as shown in the picture below, then x dominates y and y dominates z.



Solution:

Pigeons: the $4n-1$ points chosen from the grid.
Holes: the $n-1$ increasing diagonals of the grid.
the diagonal is 2n-1
Each point lies on exactly one increasing diagonal.
By the Pigeonhole Principle, at least one increasing diagonal contains at least $\lceil \frac{4n-1}{n-1} \rceil = 4$ points.
By the hint, there must be three chosen points x, y, z such that x dominates y and y dominates z. Thus, if we choose $4n-1$ points from an $n \times n$ grid ($n \geq 4$), there must be three chosen points x, y, z such that x dominates y and y dominates z.