EECS 203 Exam 1 Review

Day 2

Today’s Review Topics

- Proof Methods
  - Direct Proof
  - Proof by Contrapositive
  - Proof by Contradiction
  - Proof by Cases
- Sets

Proof Methods

Proofs Overview

- Direct Proof - Prove p → q by showing that **if p is true, then q must also be true.**
- Proof by Contraposition - Prove p → q by showing that **if not q, then not p.**
  - Assume not q and arrive at not p
- Proof by Contradiction
  - Prove p by **assuming ¬p and arriving at a contradiction, therefore proving p is true (can think of this as ¬-intro from natural deduction)**
  - Prove p → q by **assuming p and ¬q and arriving at a contradiction, therefore ¬(p and ¬q) is true which is equivalent to saying p → q is true**

Overview Cont.

- Proof by Cases
  - Prove that a predicate is true by **separating into all possible cases and showing that the predicate is true in each individual case.**
  - Proof by cases is similar to the idea of $\vee$ - elimination.

NOTE: Proof by Induction will not be covered in Exam 1

Proof Methods Table

$p \to q$	Assumptions	Want to Reach
Direct Proof	p	q
Proof By Contrapositive	$\neg q$	$\neg p$
Proof By Contradiction	$p \land \neg q$	F

Proving + Disproving Quantified Statements

	Prove	**Disprove**
$\forall x P(x)$	Show that **arbitrary** x **satisfies** P(x)	Find a **counterexample** x which does not satisfy P(x)
$\exists x P(x)$	Find an **example** x which satisfies P(x)	Show that an **arbitrary** x **does not satisfy** P(x)

NOTE: The above does not show proof by example. Proof by example is **never** valid.

WLOG

Without Loss of Generality (WLOG) – used when the same argument can be made for multiple cases

Example: Show that if x and y are integers and both $x \cdot y$ *and* $x+y$ are even, then both x and y are even.

Proof: Use a proof by contraposition. Suppose x and y are not both even. Then, one or both are odd. Without loss of generality, assume that x is odd. Then $x = 2m + 1$ for some integer k.

Case 1: y is even. Then $y = 2n$ for some integer n, so $x + y = (2m + 1) + 2n = 2(m + n) + 1$ is odd.

Case 2: y is odd. Then $y = 2n + 1$ for some integer n, so $x \cdot y = (2m + 1)(2n + 1) = 2(2m \cdot n + m + n) + 1$ is odd.

Prove that if n is an odd integer, then n^2 is odd.

Prove that if $a \cdot b < 0$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$, then $(a / b) < 0$.

Prove that if $n = ab$, where a and b are positive integers, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$.

Prove that if $3n + 2$ is odd, then n is odd.

Prove or Disprove: For all rational numbers x and y, x^y is also rational.

Prove or Disprove: There exists an integer n such that

$4n^2 + 8n + 16$ is prime

Which of the following describe the proof method(s) used to show the following statement? Mark all that apply.

Statement: If x is rational and y is irrational, then $x + y$ is irrational.
Proof: Assume that x is rational, y is irrational, and $x + y$ is rational. Notice that $y = (x + y) - x$. Since both $x + y$ and x are rational, and the difference of two rational numbers is also rational, this means that y is rational. But we assumed y was irrational. So it must be the case that whenever x is rational and y is irrational, $x + y$ is irrational.

(a) Proof by contrapositive

(b) Proof by cases

(c) Proof by contradiction

(d) Direct Proof

(e) Exhaustive proof Proving all cases possible

Identify the mistakes in the following proof, multiple answers

We prove that $0 = 2$ as follows.
S1. We have $4x^2 = 4x^2$.
S2. Rewriting the left and right hand sides, we get $(-2x)^2 = (2x)^2$.
S3. Taking the square root, we get $-2x = 2x$.
S4. Adding $x^2 + 1$ on both sides gives $-2x + x^2 + 1 = 2x + x^2 + 1$.
S5. By algebra, this can be written as $(x - 1)^2 = (x + 1)^2$.
S6. Taking the square root, we get $x - 1 = x + 1$.
S7. Subtracting $x - 1$ on both sides, we get $x - 1 - (x - 1) = x + 1 - (x - 1)$, i.e., $0 = 2$.

5 Minute Break

https://paveldogreat.github.io/WebGL-Fluid-Simulation/

Sets and Set Proofs

Overview/Definitions

Set: An unordered collection of distinct objects

Subset ($\subseteq$): A set *A* is considered to be a **subset** of *B* if every element in *A* is also in *B* (Note that, with this definition, *A* is a subset of itself)

Proper Subset ($\subsetneq$): A set *A* is considered to be a **proper subset** of *B* if *A* is a subset of *B*, and *B* contains at least one element not in *A*.

Power set (P(S)): A set containing all of the subsets of S as **elements** in the set.

Inclusion-Exclusion Principle: $|A \cup B| = |A| + |B| - |A \cap B|$

Sets Question 1

Which of the following are valid subsets of the set S where S = {1, {2}, ∅}? Select all that apply.

A. ∅
B. {∅}
C. 1
D. {1}
E. {2}

More definitions and Sets Question 2

Cardinality: The number of elements in a set, denoted |A|

Note that power sets of sets with n elements are of cardinality 2^n

Cartesian Product: A x B is the set of all pairs of elements from A and B, i.e. (a,b) where a ∈ A and b ∈ B. Note that |A x B| = |A| * |B|

What is the cardinality of {E,E,C,S} X {2,0,3}?

Sets Question 3

Prove that if $C \subseteq \text{comp}(A - B)$, then $A \cap C \subseteq B$. Note that comp() is the complement of the set.

Good luck studying!