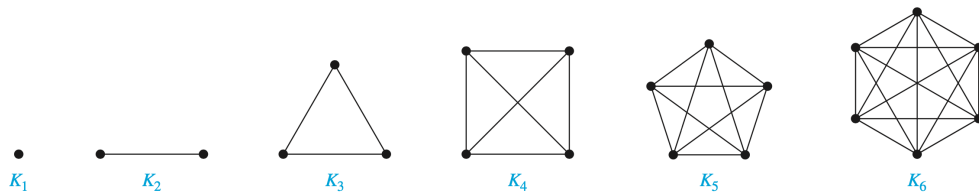# EECS 203: Discrete Mathematics
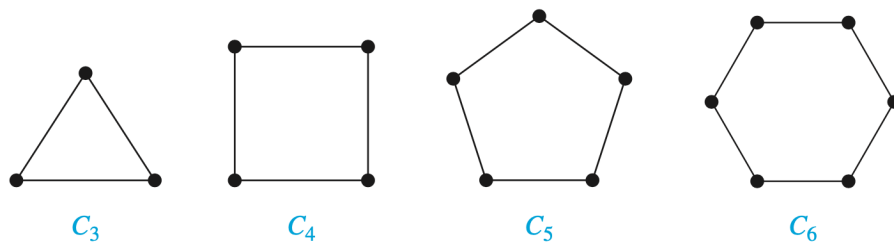# Winter 2024
Discussion 8b Notes

1 Graphs

1.1 Definitions

- **Degree Sequence:** The degree sequence of a graph is the sequence of the degrees of the vertices of the graph in nonincreasing order.
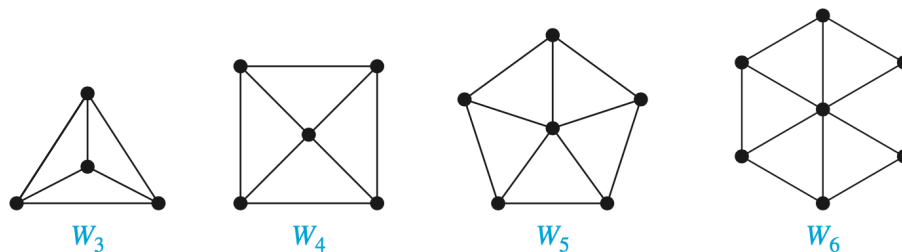
- **Special Simple Undirected Graphs:**
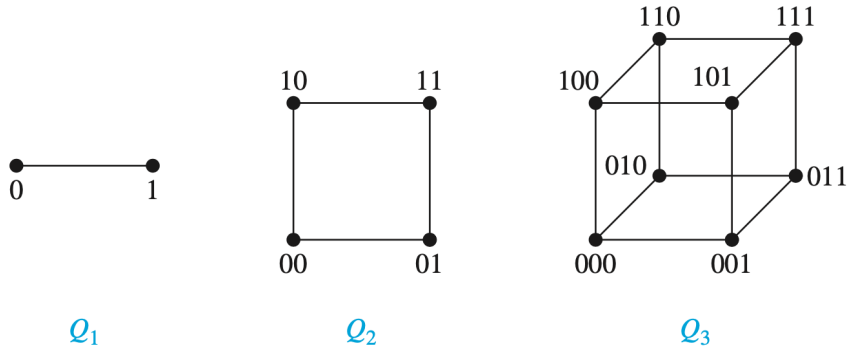
  - K_n Complete Graphs:

    K_1 K_2 K_3 K_4 K_5 K_6

  - C_n Cycles:

    C_3 C_4 C_5 C_6

  - W_n Wheels:

    W_3 W_4 W_5 W_6

– Q_n Hypercubes:

Q_1 Q_2 Q_3

- **Path:** A path $(u_0, u_1, ..., u_k)$ is a sequence of vertices in which consecutive vertices in the sequence are adjacent in the graph (connected by an edge).

- **Simple Path:** A simple path is a path that does not repeat any vertices.

- **Spanning Tree:** Let G be a simple graph. A spanning tree of G is a subgraph of G that is a tree containing every vertex of G.

- **Bipartite** if its vertex set V can be partitioned into two disjoint sets V_1 and V_2 such that every edge in the graph connects a vertex in V_1 and a vertex in V_2. The pair (V_1, V_2) is called a bipartition of the vertex set V.

- **Bipartite Theorem (3 Equivalent Statements):** The following statements are equivalent:

   – G is bipartite.

   – G is 2-colorable. (There is a function $f : V \implies \{red, blue\}$ such that $u, v \in E \implies f(u) \neq f(v)$.)

   – G does not contain odd cycle (C_{2k+1}) subgraphs.

2

1.2 Exercises

1.2.1 Bipartite Intro

Draw each of the following graphs. Which of these graphs are bipartite?

a) C_6, a cycle with 6 nodes

b) W_4, a wheel with a center node and 4 spoke nodes

c) Q_3, a graph representing a 3-dimensional cube, with nodes on each corner

1.2.2 Bipartite Conclusion

For which values of n are these graphs bipartite? Explain your answer.

a) K_n

b) C_n

c) W_n

d) Q_n

### 1.2.3   Bipartite Graphs

Does there exist a bipartite graph with degree sequence 3,3,3,3,3,3,3,3,3,3,5,6,9 (in other words, a graph with ten nodes of degree 3, one of degree 5, one of degree 6, and one of degree 9)? If not, explain why.
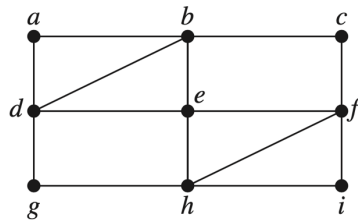
1.2.4   Euler Paths and Circuits

For each of the following graphs: Determine whether the graph has an Euler circuit. Construct such a circuit if one exists. If no Euler circuit exists, determine whether the graph has an Euler path. Construct such a path if one exists.
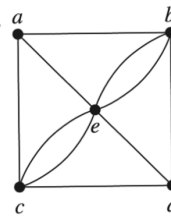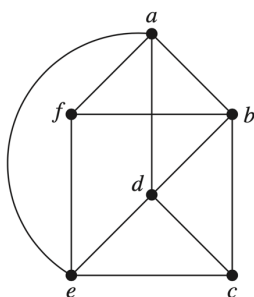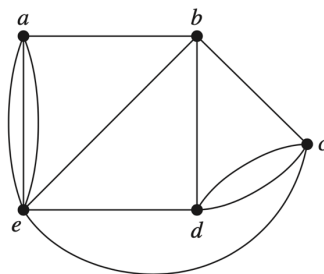
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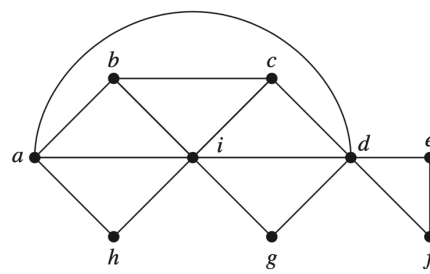

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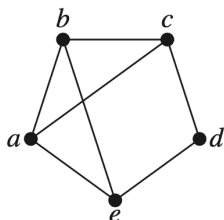
2 Intro to Counting

2.1 Definitions

- **Product Rule:** Suppose a procedure can be broken down into a sequence of k tasks, where you have to do each task to complete the procedure. If there are n_k ways to do the kth task, then there are

$$n_1 \cdot n_2 \cdot n_3 \cdot ... \cdot n_k$$

  ways to do the entire procedure.

- **Sum Rule:** Suppose that there are k distinct, disjoint methods to complete a procedure such that k^{th} method can be done in n_k ways, then there are

$$n_1 + n_2 + ... + n_k$$

  ways of doing exactly one of these tasks.

- **Subtraction Rule (Inclusion-Exclusion):** If a task can be done in either n_1 ways or n_2 ways, then the number of ways to do the task is $n_1 + n_2$ minus the number of ways to do the task that are common to the two different ways.

- **Division Rule:** If there are N ways to choose an object, and each object can be chosen in exactly k ways, there are N/k objects.

2.2 Exercises

2.2.1 Product Rule

a. The chairs of an auditorium are to be labeled with an uppercase English letter followed by a positive integer not exceeding 100. What is the largest number of chairs that can be labeled differently?

b. How many different license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits?

c. How many functions are there from a set with m elements to a set with n elements?

d. How many one-to-one functions are there from a set with m elements to one with n elements?

2.2.2 Sum Rule

a. A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 19 possible projects, respectively. No project is on more than one list. How many possible projects are there to choose from?

b. A wired equivalent privacy (WEP) key for a wireless fi- delity (WiFi) network is a string of either 10, 26, or 58 hexadecimal digits. How many different WEP keys are there?

2.2.3 Inclusion Exclusion

a. How many bit strings of length eight either start with a 1 bit or end with the two bits 00?

b. A computer company receives 350 applications from college graduates for a job planning a line of new web servers. Suppose that 220 of these applicants majored in computer science, 147 majored in business, and 51 majored both in computer science and in business. How many of these applicants majored neither in computer science nor in business?

2.2.4   Division Rule

a. How many different ways are there to seat four people around a circular table, where two seatings are considered the same when each person has the same left neighbor and the same right neighbor?