오즈비 (Odds Ratio) = $\frac{p}{1-p}$ (p : 성공확률)

로짓함수 (Logit Function)

$$\log\left(\frac{p}{1-p}\right) = z$$

$$\frac{p}{1-p} = e^z$$

$$p = e^z(1-p)$$

$$p = e^z - e^z p$$

$$p + e^z p = e^z$$

$$p(1+e^z) = e^z$$

$$p = \frac{e^z}{1+e^z}$$

손실함수

$$L = -(y\log(a) + (1-y)\log(1-a))$$

$y=1 \rightarrow$ $L = -(1\times\log(a) + (1-1)\log(1-a))$ 1일 때 좌항만 작동

대입 $= -\log(a)$

$y=0 \rightarrow$ $L = -(0\times\log(a) + (1-0)\log(1-a))$ 0일 때 우항만 작동

대입 $= -\log(1-a)$

$$\frac{\partial}{\partial x}\frac{1}{f(x)} = \frac{\partial}{\partial x}(f(x))^{-1}$$

$$= (f(x))^{-2}\frac{\partial}{\partial x}f(x)$$

$$= \frac{f'(x)}{f^2(x)}$$

$$\frac{\partial}{\partial x}\log x = \frac{1}{x} \quad \cdots (\log x = \ln x)$$

$$\log e = 1 , \qquad e^{\log x} = x$$

$$\frac{\partial L}{\partial a} = \frac{\partial}{\partial a}(-(y\log(a) + (1-y)\log(1-a)))$$

$$= -y\frac{\partial}{\partial a}\log(a) + (1-y)\frac{\partial}{\partial a}\log(1-a))$$

$$= -(y\times\frac{1}{a} + (1-y)\times(-1)\times\frac{1}{1-a})$$

$$= -(y\frac{1}{a} - (1-y)\frac{1}{1-a})$$

$$\frac{\partial}{\partial x}\log(1-x) = (1-x)'\times\frac{1}{1-x}$$

$$= -1\times\frac{1}{1-x}$$

$$= -\frac{1}{1-x}$$

$$a = \frac{1}{1+e^{-z}}$$

$$\begin{aligned}
\frac{\partial a}{\partial z} &= \frac{\partial}{\partial z}\left(\frac{1}{1+e^{-z}}\right) \\
&= \frac{\partial}{\partial z}(1+e^{-z})^{-1} \\
&= -(1+e^{-z})^{-2}\frac{\partial}{\partial z}(e^{-z}) \\
&= -(1+e^{-z})^{-2}(-e^{-z})
\end{aligned}$$

$$\begin{aligned}
&= \frac{e^{-z}}{(1+e^{-z})^2} \\
&= \frac{1}{1+e^{-z}}\frac{e^{-z}}{1+e^{-z}} \\
&= \frac{1}{1+e^{-z}}\left(1-\frac{1}{1+e^{-z}}\right) \\
&= a(1-a)
\end{aligned}$$

$$\frac{\partial L}{\partial w_i} = \frac{\partial L}{\partial a}\frac{\partial a}{\partial z}\frac{\partial z}{\partial w_i}$$

$$z = \sum_{i=1}^{n} w_i x_i + b$$

$$\frac{\partial z}{\partial w_i} = x_i$$

$$\begin{aligned}
&= -\left(y\frac{1}{a} - (1-y)\frac{1}{1-a}\right) \times a(1-a) \times x_i \\
&= -\left(y\frac{1}{a}\times a(1-a) - (1-y)\frac{1}{1-a}\times a(1-a)\right)\times x_i \\
&= -(y(1-a) - (1-y)a)x_i \\
&= -(y - ay - a + ay)x_i \\
&= -(y-a)x_i
\end{aligned}$$