PolygoN

1 Objective

Given n points on a $2-$dimensional plane, we need to build all possible simple closed polygons whose vertices are all n points given.

1.1 Definitions

Definition 1. *A Vector of Points* VP *is an ordered array of unique points in the form of* (x, y) *where* x *represents the* x *dimension and* y *represents the* y *dimension. The order is such that for any given pair of consecutive points* $(x_n, y_m), (x_{n+1}, y_m)\, x_n < x_{n+1}$ *and for any given pair of consecutive points* $(x_n, y_m), (x_n, y_{m+1}) y_m < y_{m+1}$.

Example 2. $\text{VP} = \{(0,0), (1,3), (4,2), (4,3), (8,1), (9,7)\}$

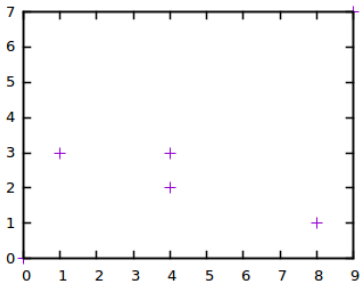

Figure 1.

A point can be $0-$connected, $1-$connected or $2-$connected depending on the number of sides to which it is connected to. We can also say $0-$connected means «free», $1-$connected means «open» and $2-$connected means «closed». Therefore we can have free, open or closed points.

A initial segment between any two i, j points from VP is said to be a line connecting those two points that does not intersect with any sides of the polygon. At the beginning, every pair of points i, j are connected by an initial segment. As the construction of the polygon progresses, some of these initial segments will be intersected by sides of the polygon and therefore erased.

The Initial Segments Matrix ISM is a $n \times n$ matrix where n is the number of points given and for a given $\text{ISM}_{i,j}\, i = \text{VP}_i \wedge j = \text{VP}_j$ we find a 0 if no initial segment still exists between those two i, j points and 1 if the initial segment still exists.

Distance between two points is defined as follows. When there is a side of a polygon that connects two points, their distance is zero and they are said to be connected. When there is an initial segment not intersecting any existing side of the polygon, their distance is 1 and they are said to be linked. When two points are linked via a number of initial segments not intersecting and existing side of the polygon, their distance is the minimum number of initial segments that will connect the two points.

The Distance Matrix is a matrix that represents for a given coordinate x, y $(x,\ y \subset P)$ the distance $d(x, y)$ for the two points x, y.

2 Theorems

Theorem 3. *When a free point has distance 1 only to 1-connected or 2-connected points, that point is considered isolated and the polygon cannot be.*

3 Approach

A $2-$connected point immediately erases all initial segments connected to it.

A new proposed side for a polygon can only be connected to a $1-$connected point if it is the only $1-$connected point remaining (ie: the first point).

3.1 Rules to discard impossible polygon

1. Check if there are no $0-$connected points left. If the two remaining $1-$connected points can be connected, we store this as a valid polygon.
2. Check the status of the latest point connected to the polygon. This point has to have at least one initial segment with a $0-$connected point.
3. Check the status of the initial point. If no initial segments
4. If 2) is passed, check the status of all $0-$connected points to ensure they all have existing initial segments to at least one $1-$connected point and one $0-$connected point.
5. If 3) is passed the polygon can still be.

If a point remains connected by initial segments to only 1-connected points, that point is considered isolated and the polygon cannot be.

Rules:

1. **check the status of the latest point connected to the polygon (at least 1 initial segment with a 0-connected point)**
2. **If 1) is OK, check the status of all 0-connected points (there is a 1-connected-free path using initial segments between the 0-connected point and some other 0/1-connected point. The exception to this rule is that there is only one 0-connected point left and there are initial segments to two 1-connected points**
3. **If 2) is OK, check the status of all 1-connected points**
4. **if 3) is OK, the polygon can still be**