Unit - II

Propositional logic and FSM

Statement or Proposition

An statement is a declarative sentence which is either, True or False, but not both, or In other words an statement is a declarative sentence which has a definite Truth value.

Ex (1) Blood is Red (T)

(2) The capital of Madhya Pradesh is Bhopal (T)

(3) $5+4=10$ (F)

(4) $3>9$ (F)

Following are not statements :-

(1) How are you? (Interrogative)

(2) please go from here? (Request)

(3) May God help you? (wish)

statement is denoted by p, q, r, and s

Ex p = Taj Mahal is in Agra.

Logical connectives or sentence connectives

Logical connectives or sentence connectives are the words or Symbols used to combine two sentences or statements to form a compound sentence or compound Statement.

Connectives word	Name of Connective	Symbol	Rank
Not	Denial or Negative	$\sim$ or $\neg$	1
and	Conjunction	$\wedge$	2
OR	Disjunction	$\vee$	3
If.. then	Conditional	$\Rightarrow$	4
Iff or If and only If	Biconditional	$\Leftrightarrow$	5

Ex (1) If p = It is 4 'O' clock

and q = the train is late.

Then state in words the following results.

(i) $p \vee q$ (ii) $p \wedge q$ (iii) $p \wedge (\sim q)$ (iv) $q \vee \sim p$ (v) $(\sim p) \wedge q$

(vi) $\sim p \wedge \sim q$ (vii) $\sim p \vee \sim q$ (viii) $\sim (p \wedge q)$ (ix) $\sim p \Rightarrow p$

Sol:- (i) $p \vee q$ = it is 4 (O) clock or the train is late.

(ii) $p \wedge q$ = It is 4 (O) clock and the train is late.

(iii) $p \wedge \sim q$ = it is 4 (O) clock and the train is not late.

(vi) $\sim p \wedge \sim q$ = It is not 4 'O' clock and the train is not late.

(v) $\sim p \wedge q$ = It is not 4 (O) clock and the train is late.

(iv) $q \vee \sim p$ = the train is late or it is not 4 'O' clock.

(vii) $\sim p \vee \sim q$ = It is not 4 (O) clock ~~and~~ or the train is not late.

(viii) $\sim (p \wedge q)$ = It is not true It is 4 (O) clock and the train is late.

(ix) $\sim p \Rightarrow q$ = If it is not 4 (O) clock then the train is late.

Ex (2) let p = It is ~~clod~~ cold, q = It is raining

Give a simple verbal sentence which describes each of the following statements:-

i) $\sim p$ (ii) $p \wedge q$ (iii) $p \vee q$ (iv) $q \Leftrightarrow p$ (v) $p \Rightarrow \sim q$ (vi)

Truth Table

i) Conjunction (∧) :- (AND)

let p and q be two statement, Then conjunction of p and q is denoted by $p \wedge q$ and Read as 'p and q' or 'p Meet q' and its Truth Table have $2^2 = 4$ No. of Truth values.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

ii) Disjunction (∨) OR :-

The disjunction of p and q is denoted by $p \vee q$ and ~~use~~ using the words 'OR',

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

iii) Negation (~) :- (NOT)

let P be any statement, then Negation of this P is denoted by ~P, and is read as 'not P'.

p	~p
T	F
F	T

(iv) Conditional ⇒ (If or then)

let P and q be two statement then condition of P and q is False if P is True and q is false, otherwise it is True. it is denoted by $P \Rightarrow q$ and Read as 'P Implies q'.

Truth Table

P	q	$P \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

(v) Bi-Conditional (⇔)(I/f)

let P and q be two statements; then P by conditional q is True If P and q both have same Truth values, other wise it is False. it is denoted by $P \Leftrightarrow q$ and Read as P I/f q or P If and only If q.

Truth Table

P	Q	$P \Leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

(vi) NOR or Joint Denial ($\downarrow$)

Let p and q be two statements, the statement of Type $P \downarrow Q$ Read as Neither P Nor Q is called joint denial or NOR statement. In other words NOR (or Joint denial) is the Negation of OR of two statement. Thus $P \downarrow Q = \sim(P \vee Q)$

It Means that $P \downarrow Q$ is True only when p and q both are false

P	Q	$P \downarrow Q$
T	T	F
T	F	F
F	T	F
F	F	T

(vii) NAND ($\uparrow$)

The connective NAND is denoted by the symbol $\uparrow$, and is defined as "Negation of AND of Two Statements"

If p and q are two statements then Their NAND is denoted by $P \uparrow Q$

$$P \uparrow Q = \sim(P \wedge Q)$$

$P \uparrow Q$ is false only in the case when p and q are True.

P	Q	$P \uparrow Q$
T	T	F
T	F	T
F	T	T
F	F	T

(viii) <u>XOR ($\oplus$)</u>

The XOR of two statement is True If only If one of the two statements is True. it is denoted by $P \oplus Q$

Truth Table

P	Q	$P \oplus Q$
T	T	F
T	F	T
F	T	T
F	F	F

<u>Kinds of Conditional</u>

If P and Q are any two statement, then some other conditional related to, conditional $P \Rightarrow Q$ are also used which are given in the following Table

	Conditional	Name of Kind
I.	$P \Rightarrow Q$	Direct Implication
II.	$Q \Rightarrow P$	Converse Implication
III.	$\sim P \Rightarrow \sim Q$	Inverse or opposite Implication
IV.	$\sim Q \Rightarrow \sim P$	Contrapositive Implication.

Tautology:-

If the Truth value of a Compound statement is True i.e the statement is always logically True is called a Tautology.

Contradiction:-

If the Truth value of a compound statement is false Then the statement is called Contradiction.

Logically Equivalence:-

Two compound statements P and Q are said to be Logically equivalence or Tautologically equivalent. If the Truth values of both statements are Identical. it is denoted by $P \cong Q$ or $P \Leftrightarrow Q$

Also $P \cong Q$ then $P \Leftrightarrow Q$ will be Tautology.

Falsy:-

Contingency:- A Contingency is a Proposition which is either True or False depending on the Truth values of its components or Propositions.

Q Construct a Truth Table for each compound statements.

(i) $p \wedge (\sim q \vee q)$ (ii) $\sim(p \vee q) \vee (\sim p \wedge \sim q)$

(iii) $(p \vee q) \Rightarrow p$ (iv) $(p \Rightarrow q) \wedge \sim q$

(v) $(p \vee q) \wedge \sim r \Rightarrow q$ (vi) $(p \Leftrightarrow q) \wedge (r \vee q)$

Sol:- (i) given p and q be Two statements Then its Truth value have $2^2 = 4$ No. of Truth values

Then To construct the Truth value of $p \wedge (\sim q \vee q)$

Truth Table

p	q	$\sim q$	$(\sim q \vee q)$	$p \wedge (\sim q \vee q)$
T	T	F	T	T
T	F	T	T	T
F	T	F	T	F
F	F	T	T	F

A.

(ii) $\sim(p \vee q) \vee (\sim p \wedge \sim q)$

Truth Table

p	q	$\sim p$	$\sim q$	$p \vee q$	A: $\sim(p \vee q)$	B: $\sim p \wedge \sim q$	$A \vee B$
T	T	F	F	T	F	F	F
T	F	F	T	T	F	F	F
F	T	T	F	T	F	F	F
F	F	T	T	F	T	T	T

A

Q1 Prove that $P \wedge q \Rightarrow P \vee q$ is a Tautology.

Sol:- Let P and q be two statements then its Truth value have $2^2 = 4$ No. of Truth values.

Truth Table

P	q	$P \wedge q$	$P \vee q$	$(P \wedge q) \Rightarrow (P \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

Hence all the values in the last Truth Table, are True. So that given compound statement is a Tautology.

Q.2. Show that the Proposition $P \wedge (q \wedge \sim P)$ is a Contradiction.

Sol:- Let P and q be two statements, Then the compound statement $P \wedge (q \wedge \sim P)$ is a contradiction, if all the Truth values in compound statement are false. To construct the Truth Table.

P	q	$\sim P$	$q \wedge \sim P$	$P \wedge (q \wedge \sim P)$
T	T	F	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	F	F

Hence all the entries in the last column are false so given statement is a contradiction.

Q. 3 Show that $p \Rightarrow q$ is logically equivalent to $\sim p \vee q$

OR

Prove that

$$p \to q \cong \sim p \vee q$$

Sol:- To construct the Truth Table for the same.

Truth Table.

p	q	$\sim p$	$p \to q$	$\sim p \vee q$	$p \to q \Leftrightarrow \sim p \vee q$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Hence all the entries in the last column are True so that given statement is a Tautology.

OR Hence all the entries in the column 4 and 5 are Identical so that both statements are Logically equivalent.

$$p \to q \cong \sim p \vee q$$

Q. 4 Prove that

$(p \Leftrightarrow q) \vee (q \Leftrightarrow r) \Rightarrow (p \Leftrightarrow r)$ is a Tautology.

Q. 5 Use Truth Table to Prove that De Morgan's law?

(i) $\sim(p \vee q) \cong \sim p \wedge \sim q$ (ii) $\sim(p \wedge q) \cong \sim p \vee \sim q$

Some Algebric laws

i) Idempotent laws:-

$p \vee p \cong p$

$p \wedge p \cong p$

ii) Involution law:-

$\sim(\sim p) \cong p$

(iii) Commutative laws.

$p \vee q \cong q \vee p$

$p \wedge q \cong q \wedge p$

(iv) Associative laws -

$p \vee (q \vee r) \cong (p \vee q) \vee r$

$p \wedge (q \wedge r) \cong (p \wedge q) \wedge r$

(v) Complement laws:-

$p \vee \sim p \cong T$

$p \wedge \sim p = F$

(vi) Distributive law

$p \wedge (q \vee r) \cong (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \cong (p \vee q) \wedge (p \vee r)$

Q. Simplify the following:-

i) $(P \vee \phi) \wedge \sim P$ (ii). $\sim(P \vee \phi) \vee (\sim P \wedge \phi)$

Sol:-

$\phi = F$

$\pi = T$

i) $(P \vee \phi) \wedge \sim P = P \wedge \sim P = F = \phi$

$\sim\phi = \pi$

ii) $\sim(P \vee \phi) \vee (\sim P \wedge \phi) = (\sim P \wedge \sim\phi) \vee (\sim P \wedge \phi)$

$= (\sim P \wedge \pi) \vee (\sim P \wedge \phi)$

$= \sim P \vee (\phi) = \sim P$ A

Q. Prove that

1) $\sim(\sim P \wedge \sim P) \equiv P$

ii) $\sim(\sim(\sim P \wedge \sim Q)) \equiv \sim(P \vee Q) \equiv \sim P \wedge \sim Q$

iii) $\sim(\sim(P \wedge Q)) \equiv (P \wedge Q)$

iv) $(\sim P \wedge \sim Q) \vee \sim Q \equiv \sim(P \wedge Q) \equiv \sim P \vee \sim Q$.

Sol:- i) L.H.S $= \sim(\sim P \wedge \sim P) = \sim(\sim(P \vee P)) = \sim(\sim P) = P =$ R.H.S

ii) L.H.S $= \sim(\sim(\sim P \wedge \sim Q)) = \sim(\sim(\sim(P \vee Q)) = \sim(P \vee Q) = \sim P \wedge \sim Q =$ R.H.S

iii) L.H.S $= \sim(\sim(P \wedge Q)) = (P \wedge Q)$

$\because \sim(\sim P) = P$ by complement law

iv) $(\sim P \wedge \sim Q) \vee \sim Q = \sim P \wedge (\sim Q \vee Q)$

$(\sim P \vee \sim Q) \wedge (\sim Q \vee Q) = (\sim P \vee \sim Q) \wedge \pi = \sim(P \wedge Q) = \sim P \vee \sim Q$

A

Argument :-

In logical Mathematics, we require the Process of Reasoning. Given a Certain set of Propositions (ie statements), we are required to derive other Propositions by logical reasoning.

The given set of Propositions is called Premises. (or Hypothesis) and the Proposition derived from this set is called Conclusion.

Argument :- Definition

An argument is ~~called~~ a Process which yield a conclusion (ie another Proposition) from a given set of Propositions, called Premises.

let Premises (ie given set of Prepositions) be $P_1, P_2 - P_n$ and let argument yield the conclusion. (ie another Proposition) q then Such an argument is denoted by

$$P_1, P_2, -- P_n \vdash q$$

Valid Argument :- (i)

An argument $P_1, P_2 -- P_n \vdash q$ is called valid if q is True whenever all its Premises, $P_1, P_2 -- P_n$ are True.

(ii) an argument is called valid if and only if the Premises Implies the conclusion.

Thus The argument $P_1, P_2, P_3 - P_n \vdash q$ is said to be Valid If and only if the statement

$$(P_1 \wedge P_2 \wedge P_3 - \wedge P_n) \rightarrow q$$ is a Tautology.

Fallacy Argument :- An argument which is not valid is said to be a fallacy or an invalid argument

Representation of an argument

an argument $P_1, P_2 - P_n \vdash q$ is written as

$$\left.\begin{matrix} P_1 \\ P_2 \\ \vdots \\ P_n \end{matrix}\right\} \text{Premises}$$
$$\overline{\quad q \quad} \text{ (conclusion)}$$

Q. Show that the following argument is valid.

$$\frac{\begin{array}{c} p \vee q \\ \sim p \end{array}}{q}$$

an argument is valid If and only If
$(p_1 \wedge p_2 \cdots \wedge p_n) \rightarrow q$ is a Tautology

- ie $[(p \vee q) \wedge \sim p] \rightarrow q$

Truth Table

p	q	$\sim p$	$(p \vee q)$	$\overset{A}{(p \vee q) \wedge \sim p}$	$A \rightarrow q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

Hence given argument is a valid argument

Law of syllogism (or Transitive Rule):-

We know that the statement
$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is a Tautology

$$\frac{\left.\begin{array}{c} p \rightarrow q \\ q \rightarrow r \end{array}\right\} \text{premises}}{p \rightarrow r \quad \text{conclusion.}}$$

is a valid argument, this is called law of Syllogism

Rule of Detachment:–

we know that the statement
$[p \wedge (p \to q)] \to q$ is a Tautology

and therefore the argument

$$\begin{array}{l} p \\ \underline{p \to q} \end{array} \Bigg\} \text{ Premises}$$
$$q \quad \text{Conclusion}$$

is a valid argument. this is called rule of detachment.

Q. Show that the argument $p, p \to q, q \to r \mid\!\!- r$ is valid.

Sol:– First Method:–

$$\begin{array}{l} p \\ \underline{p \to q} \end{array} \Bigg\} \text{ (Premises)}$$
q (Conclusion by Rule of Detachment
$\underline{q \to r}$ (Premises)
r Conclusion by Rule of detachment)

is a valid argument.

Q. Show that the argument
$p, p \to q \mid\!\!- q$ is valid.

Q. Test the validity of the following argument:–

If a man is bachelor, Then he is worried
If a man is worried, Then he dies young } Premises

Bachelors die young (conclusion)

Sol:–
$$\begin{array}{l} p \to q \\ \underline{q \to r} \end{array} \Bigg\} \text{ Premises}$$
Given argument is True, $p \to r$ (conclusion)

Q. Test the validity of the argument

If two sides of a Triangle are equal, then the opposite angles are equal

Two sides of a Triangle are not equal

∴ The opposite angles are not equal.

let p = two sides of a Triangle are equal,

q = opposite angle are equal.

argument,

$$\left.\begin{array}{c} p \to q \\ \sim p \end{array}\right\} \text{(Premises)}$$

$$\sim q \quad \text{(Conclusion)}$$

p	q	$\sim p$	$\sim q$	$p \to q$	A $((p\to q)\wedge \sim p)$	$A \to \sim q$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	T	F
F	F	T	T	T	T	T

Hence given argument is not a valid argument is it a fallacy

Propositional Function or open sentence or Predicate

In the Previous sections we have discussed the simple statements and the logical Techniques to Combine simple statements into Compound statements.
We cannot Apply Those Techniques to the arguments of the following forms:—

All human are mortal.
Newton is human.
Therefore, Newton is mortal.

The validity of such Type argument is depends upon the inner logical structure of simple statements it Contains.
The Premise of the above argument is a singular Proposition, it state That the individual Newton has the characteristic of being a human, we say 'Newton' the subject Term and 'human' the Predicate Terms.
The individual are not necessarily Persons, but may be any things, such as Planets, stars cities, animals etc, of which the characteristics may be Predicated. Similarly the characteristic can be designated by not only adjectives but also by nouns, Pronouns or verb.

In symbolic notation, we shall use small letters to denote the individuals such as the individual 'Newton' will be denoted by the small letter n (the first letter of the name) and is called individual constant.
The characteristic will be denoted by the capital letter such as 'Mortal' by M and human by H,
The Symbolic notation for Newton is mortal will be used as $M(n)$ and for 'Newton is human' as $H(n)$
let us use the symbolic representations $H(x)$ to denote the pattern common to all such singular Propositions,
The small letter x is called 'individual variable' and takes values from the set of individual constants.
when an individual constant is substituted for x, a singular Proposition is produced such as $H(n)$, $H(b)$, $H(l)$
The Singular Propositions $H(n)$, $H(b)$, $H(l)$ are either True or false i.e, They have their Truth values but there is no Truth value (T or F) of $H(x)$ since $H(x)$ is not a Proposition
The Expression of the type $H(x)$ are called Propositional functions or open Sentences,

Definition:-

An expression denoted by $P(x)$ is called a Propositional function or simply an open sentence or a predicate on A, if $P(a)$ is True or false for each $a \in A$ i.e $P(a)$ has a truth value for each $a \in A$.

In other words $P(x)$ is called a Propositional function or an open sentence if $P(x)$ becomes a statement whenever any element $a \in A$ is substituted for the value of x.

Ex let $P(x)$ be '$x+4<9$' then $P(x)$ is not a Propositional function of the set of Natural Numbers N

Clearly $P(x)$ ~~be $x+4<9$~~ is True for $x=1,2,3,4$ and false for $x=5,6,7$ ——

Hence $P(x)$ becomes a statement whenever any element $a \in N$ is substituted.

Ex. let $P(x)$ be $x+3>6$ then $P(x)$ is a Propositional function on the set of natural numbers N.

Ex let $P(x)$ be $x+3>6$ then $P(x)$ is not a Propositional function on the set of complex numbers C, because inequalities are not defined on C.

Truth set :-

Let $P(x)$ be a Propositional function and D be its domain. The set of elements $d \in D$ with the Property that $P(d)$ is True is called the Truth Set, $T(P)$ of $P(x)$

Symbolically : $T(P) = \{x : x \in D, P(x) \text{ is True}\}$

or In short $T(P) = \{x : P(x)\}$

Ex ①. If $x+3>6$ be a Propositional function defined on N, then find the Truth Set of $P(x)$

$P(x) = x+3>6 \quad , x \in N$

$T(P) = \{x : x \in N, P(x) \text{ is True}\}$

$= \{4, 5, 6, 7, 8 ---\}$

Ex ② If $P(x)$ be '$x+2<3$' and $D \equiv N$ then find the Truth Set.

Sol :- $T(P) = \{x : x \in D \equiv N, P(x) \text{ is True}\}$

$= \{\phi\}$ Ans

Ex ③ If $P(x)$ be '$x>5$' and $D \equiv N$ then

$T(P) = \{x : x>5, x \in N\}$

$= \{6, 7, 8 ---\}$

Ans

Quantifiers

In above Ex, we have seen that the Propositional function namely $P(x) = x+3>6$ defined on N become a True statement for $x = 4, 5, 6$ --- i.e $P(x)$ is a True statement for some $x \in N$, while in Ex Namely $P(x) = x>2$ defined on $D = \{3, 4, 5 ---\}$, $P(x)$ is a True statement for every $x \in D$,

Thus we observe that a Propositional function $P(x)$ becomes a True statement, by Imposing restriction for some 'x' or for every x'
These restriction namely 'for every' and 'for some' are called quantifiers.

There are two Types of quantifiers
1) Universal Quantifier The symbol $\forall$ which is read as 'for every' or for all, is called the universal quantifiers.
Let $P(x)$ be a Propositional function defined on the set D, If for every $x \in D$ $P(x)$ is true statement, then by the use of Universal quantifier ($\forall$) it is written as

$$(\forall x \in D)\ P(x) \text{ or } \forall x, P(x) \text{ or } \forall x\, P(x)$$

Here we observe that the Truth set of $P(x)$ is the entire set D — (1)

$$T(P) = \{x : x \in D, P(x)\} = D$$

Ex (1) the statement $(\forall n \in N)(n+2>1)$ is True since its Truth set

$$T(P) = \{n : n \in N, n+2>1\}$$
$$= \{1, 2, 3 ---\} = N \quad \forall n \in N$$

(2) the statement $(\forall n \in N)(n+1>5)$ is false since its Truth set

$$T(P) = \{5, 6, 7 ---\} \neq N$$

ii) Existential Quantifiers :-

The symbol $\exists$ which is read as 'There exist' or for some or for at least one' is called the existential quantifier.

Let $P(x)$ be a propositional function defined on the set D, If there exists an $x \in D$, such that $P(x)$ is a true statement or for some $x \in D$, $P(x)$ is a true statement) or for at least $x \in D$, $P(x)$ is a true statement, then by the use of existential quantifier ($\exists$) it is written as

$$(\exists x \in D)\, P(x) \text{ or } \exists x\, P(x)$$

Hence we observe that the truth set is not the empty set i.e

$$T(P) = \{ x : x \in D,\ P(x) \} \neq \phi$$

Therefore, we may stat that

i) If $T(P) \neq \phi$ then $\exists\ x\ P(x)$ is true

ii) If $T(P) = \phi$ then $\exists\ x\ P(x)$ is false.

Ex The statement $(\exists\ n \in N)\ (n+7<6)$ is false since

$$T(P) = \{ n : n+7<6 \} = \phi$$

Ex. The statement $(\exists\ x \in R)\ (x^2-1=0)$ R is the set of Real Numbers is true since

$$T(P) = \{ x : x^2-1=0 \} = \{1,-1\} \neq \phi$$

Negation of a Quantifier

Consider the following Proposition 'All Indians are honest' If M denotes the set of Indians, then the above in symbolic form it written as

$$(\forall\ x \in M)\ (x \text{ is honest})$$

The above statement will become false, If we say that 'there is an Indian who is not honest' or in symbolic form

$$(\exists\ x \in M)\ (x \text{ is not honest})$$

Therefore the negation of the statement 'All Indians are honest' will be 'There is an Indian who is not honest' or in symbolic form

$(\exists\, x \in M)$ (x is not honest)

Thus the negation of $\forall x P(x)$ is $\exists x [\sim P(x)]$ where $P(x)$ denotes x is honest.

De. Morgan's laws

$$\sim(\forall\, x \in D)\, P(x) \equiv (\exists\, x \in D) \sim P(x)$$
$$\sim(\exists\, x \in D)\, P(x) \equiv (\forall\, x \in D) \sim P(x)$$

Q. let $Q(x) \equiv x$ is a Rational number
$R(x) \equiv x$ is a Real Number
$E(x,y) \equiv$ '$x = y$'
$G(x,y) \equiv$ '$x > y$'

Translate the following sentences into symbols:-

(i) π is a Real number $= R(\pi)$
(ii) e is a real number $= R(e)$
(iii) $4/5$ is a Rational number $= Q(4/5)$
(iv) $\sqrt{3}$ is an Irrational number $= \sim Q(\sqrt{3})$
(v) Every rational number is a Real number $= \forall x [Q(x) \Rightarrow R(x)]$
(vi) Some Real Numbers are rational $= \exists x [R(x) \Rightarrow Q(x)]$
(vii) The square of every Real Number is not negative
$= \forall x [R(x) \Rightarrow \sim G(0, x^2)]$.

Note $G(0,x^2)$ Means $0 > x^2$, ie x^2 is Negative and therefore $\sim G(0,x^2)$ Means x^2 is not Negative.

Ex (2) Negate each of the following statements:—

i) $\forall x\,(|x| = x)$ (ii) $\forall x\,(x+1 > x)$

(iii) $\forall x\,(x \neq 1, x \neq 2)$ (iv) $\exists x\,(x^2 < 0)$ (v) $\forall x, (x \neq 0) \Rightarrow (x^2 > 0)$

(vi) $\exists x\,(|x| = 0)$ (vii) $\exists x\,(x^2 = x)$ (viii) All men are Mortal.

Sol:— i) $\forall x\,(|x| = x)$ it Negation $\exists x \sim (|x| = x)$

or $\exists x\,(|x| \neq x)$

ii) $\forall x\,(x+1 > x)$ it Negation $\exists x, \sim (x+1 > x)$

or $\exists x, (x+1 \leq x)$

(iii) $\forall x\,(x \neq 1, x \neq 2)$ its Negation $\exists x, \sim (x \neq 1, x \neq 2)$

or $\exists x, (x = 1, x = 2)$

$\exists x\,(x^2 + 3x + 2 = 0)$

(iv) $\exists x\,(x^2 < 0)$ its Negation

$\forall x\,(x^2 \geq 0)$

(v) $\forall x, (x \neq 0) \Rightarrow (x^2 > 0)$ it Negation $\exists x\,(x \neq 0 \wedge x^2 \geq 0)$

(vi) $\exists x\,(|x| = 0)$ its Negation $\forall x\,(|x| \neq 0)$

(vii) $\exists x\,(x^2 = x)$ its Negation $\forall x.\,(x^2 \neq x)$

(viii) $\forall x$ (Men are Mortal its Negation $\exists x, \sim (Px)$

or $\exists x$, { ~~All~~ Some men are not Mortal)

8

Q. let $D = \{1, 2, 3, 4, 5, 6 \cdots\}$

Negate the following statements.

i) $(\forall x \in D)(x+4 \geq 8)$ (ii) $(\forall x \in D)(x+2 < 9)$

(iii) $(\exists x \in D)(x+5 = 9)$

(iv) $(\exists x \in D)(x+1 > 6)$ (v) $\exists (x \in D)(x^2 = 6)$ (vi) $(\forall x \in D)(x^2 > 25)$

Q. Negate the statement (He is Poor and laborious)

Sol:- we know that $\sim(P \wedge q) \equiv \sim P \vee \sim q$

Hence Negation of given statement is

it is false that he is poor and laborious.

= He is not Poor or He is not labourious.

Q Negate the statement

i) $\exists x P(x) \vee \forall y Q(y)$

ii) $\forall x P(x) \wedge \exists y Q(y)$

Normal forms

Disjunctive Normal form (DNF) — Sum of Product: $(\ )\vee(\ )\vee(\ )$ — $(p\vee q)\wedge(p\vee q)$

Conjunctive Normal form (CNF) — Product of Sums: $(\ )\wedge(\ )\wedge(\ )$

Disjunctive Normal forms :-

A formula, (or statement) which contains a sum of elementary Products is called in disjunctive normal form.

Method for obtaining Disjunctive Normal form

we know that $P\leftrightarrow Q$ and $(P\wedge Q)\vee(\sim P\wedge\sim Q)$ are equivalent statement. (can be Proved by Truth Table)

Q. Express the following formula into disjunctive Normal form

$$\sim(p\vee q)\leftrightarrow(p\wedge q)$$

Sol:- let $P=\sim(p\vee q)$ and $Q=(p\wedge q)$

$\therefore\ \sim(p\vee q)\leftrightarrow(p\wedge q)$

$$\Rightarrow P\leftrightarrow Q\equiv(P\wedge Q)\vee(\sim P\wedge\sim Q)$$

$$=[\sim(p\vee q)\wedge(p\wedge q)]\vee[\sim(\sim(p\vee q))\wedge\sim(p\wedge q)]$$

$$=[\sim p\wedge\sim q\wedge p\wedge q]\vee[(p\vee q)\wedge(\sim p\vee\sim q)]$$

$$=[\sim p\wedge\sim q\wedge p\wedge q]\vee[(p\vee q)\wedge\sim p\vee(p\vee q)\wedge\sim q]$$

$$=[\sim p\wedge\sim q\wedge p\wedge q]\vee[(p\wedge\sim p)\vee(q\wedge\sim p)\vee(p\wedge\sim q)\vee(q\wedge\sim q)]$$

$$=(\sim p\wedge\sim q\wedge p\wedge q)\vee(p\wedge\sim p)\vee(q\wedge\sim p)\vee(p\wedge\sim q)\vee(q\wedge\sim q)$$

which is the Required DNF

Conjunctive Normal form (CNF)

A formula, which contains the product of elementary sums is called in Conjunctive Normal form (CNF)

Q. Express the following formula into CNF.

$$\sim(P \vee Q) \leftrightarrow (P \wedge Q)$$

Sol:- we have $P \leftrightarrow Q \equiv (P \to Q) \wedge (Q \to P)$

and $\{ P \to Q \equiv \sim P \vee Q \}$

$\sim(P \vee Q) \leftrightarrow (P \wedge Q)$

$\Rightarrow [\sim(P \vee Q) \to (P \wedge Q)] \wedge [(P \wedge Q) \to \sim(P \vee Q)]$

$\Rightarrow [\sim(\sim(P \vee Q)) \vee (P \wedge Q)] \wedge [\sim(P \wedge Q) \vee \sim(P \vee Q)]$

$\Rightarrow [(P \vee Q) \vee (P \wedge Q)] \wedge [(\sim P \vee \sim Q) \vee (\sim P \wedge \sim Q)]$

$[(P \vee Q) \vee P \wedge (P \vee Q) \vee Q] \wedge [(\sim P \vee \sim Q) \vee \sim P \wedge (\sim P \vee \sim Q) \vee \sim Q]$

$[(P \vee P \vee Q) \wedge (P \vee Q \vee Q)] \wedge [(\sim P \vee \sim Q \vee \sim P) \wedge (\sim P \vee \sim Q \vee \sim Q)]$

$\Rightarrow [(P \vee Q) \wedge (P \vee Q)] \wedge [(\sim P \vee \sim Q) \wedge (\sim P \vee \sim Q)]$

$(P \vee P \vee Q) \wedge (P \vee Q \vee Q) \wedge (\sim P \vee \sim Q \vee \sim P) \wedge (\sim P \vee \sim Q \vee \sim Q)$

which is a Required CNF A

Finite State Machine (FSM)

Finite State Automata:-

A finite State automaton (or simply an automaton) briefly written as FSA, denoted by M is a 5-Tuple (A, S, Y, S_0, F) where

i) A is a finite set (alphabet) of Inputs, ie A is input Alphabet.

ii) S is a finite set of Internal states.

iii) y is a subset of S, whose elements are said to be 'yes' or 'accepting' states.

(iv) S_0 is an Initial state in S.

(v) F from $S \times A$ into S is a Next-state function.

Note:- Automata is plural of automaton. Consider

Ex (I) consider an automaton $M = (A, S, Y, S_0, F)$ where

i) $A = \{a, b\}$ (ii) $S = \{S_0\ S_1\ S_2\}$ (iii) $Y = \{S_0, S_1\}$

(iv) S_0 is initial state.

(v) $F : S \times A \to S$ defined by

$$F(S_0, a) = S_0 \quad F = (S_1, a) = S_0,\ F(S_2, a) = S_2$$

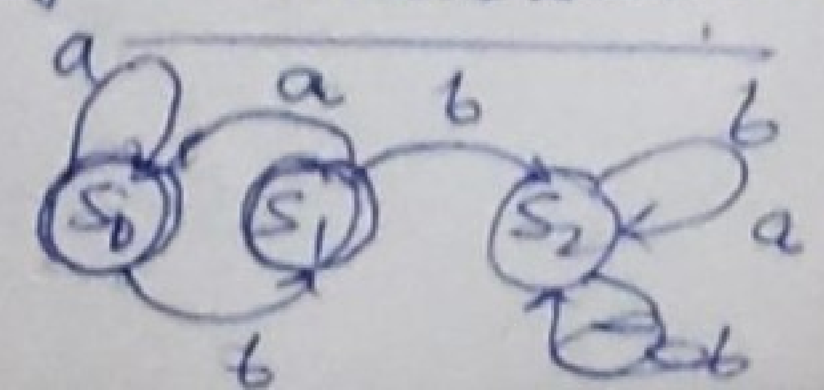
$$F(S_0, b) = S_1,\ F(S_1, b) = S_2\ \ F(S_2 b) = S_2$$

This Automaton M has two inputs symbols namely a, b and has Three Terminal states S_0, S_1, S_2 the 'yes' state are two namely S_0, S_1, the initial state is S_0,

The Next state function F can be represented by the following Table.

F	a	b
S_0	S_0	S_1
S_1	S_0	S_2
S_2	S_2	S_2

State diagram of an Automaton



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DETAILED LECTURE NOTES

PAGE NO. ....

Finite state Machine (FSM)

A Machine with a finite Number of states is said to be a finite state Machine, and a Machine which has an Infinite Number of states is called ~~as~~ an Infinite state Machine.

A finite state Machine is similar to a finite state automaton with the difference that, finite state Machine 'Prints' an output using an output alphabet. The output alphabet is distinct from input Alphabet.

Definition:- A finite state Machine M is defined as a 6 Tuple (S, A, O, f, g, s_0)

where i) $A = \{a_1\ a_2\ ---\}$ is a finite set of input Alphabet.

ii) $S = \{s_0\ s_1\ s_2\ ---\}$ is a finite set whose elements are called Internal states of the Machine.

iii) $s_0 \in S$ is an initial state of the Machine.

iv) $O = \{O_1\ O_2\ ---\}$ is a finite set of output alphabet.

v) f is a function from $S \times A$ into S called the Transition function or next-state function

vi) g is a function from $S \times A$ into O called the output.

Note!- The finite State Machine at any instant of time, is in some state, when Machine. recieves an input Symbol from the set, the Machine Moves to Next state according to the function f (Transition function), then the Machine, at each state produces an output symbol. according to the function g (output function)

The initial position of the Machine is The State S_0

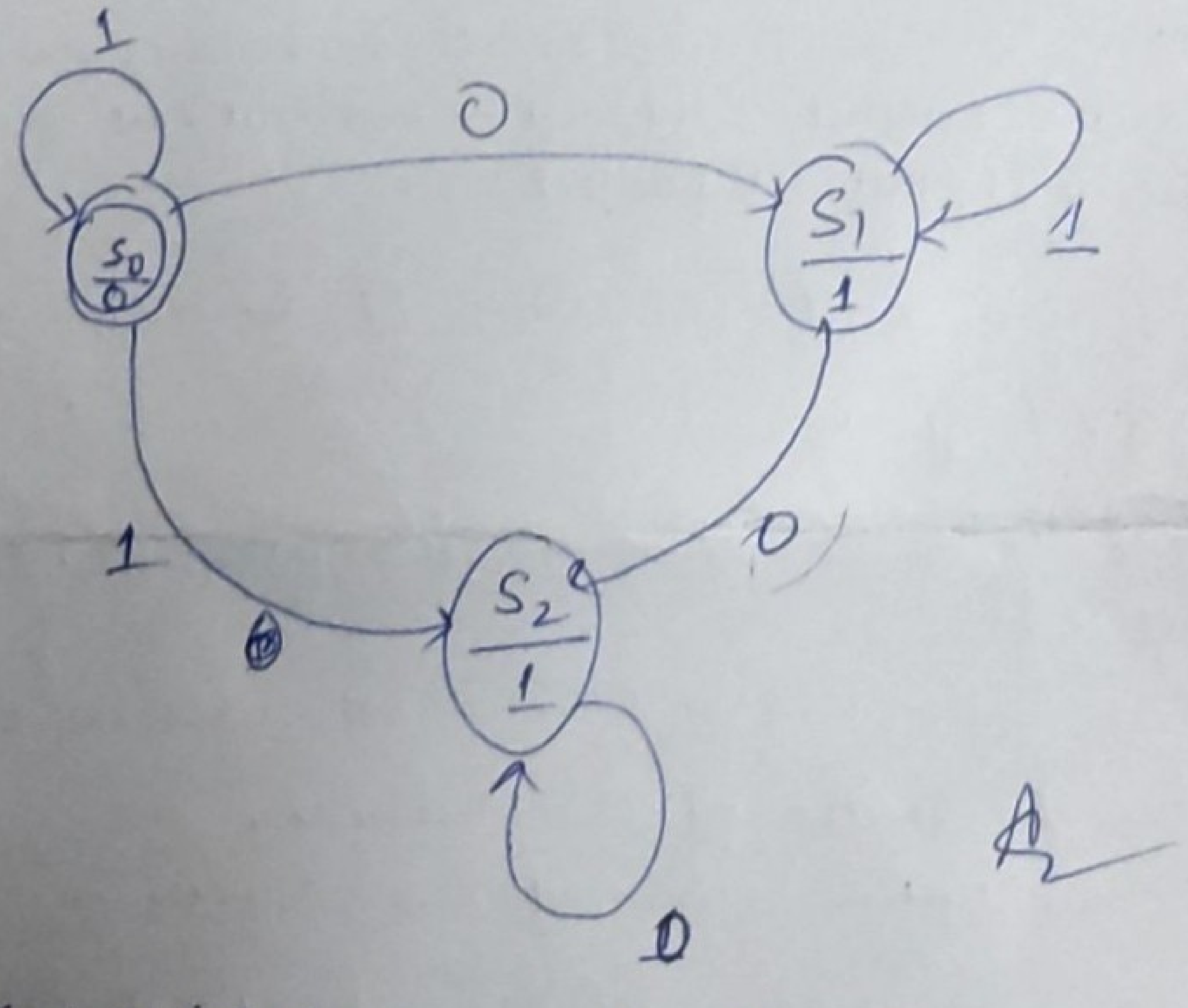
Example

let $A = \{0, 1\}$ $S = \{s_0\ s_1\ s_2\}$

$O = \{0, 1\}$ and the function f and g be given by the following table, known as state table:-

State	Input 0	Input 1	output
⇒ s_0	s_1	s_0	0
s_1	s_2	s_1	1
s_2	s_2	s_0	1

Draw the state graph of the FSM $M = \{S, A, O, f, g\ s_0\}$



Q. Find the output string by the Finite state Machine in above example(1) if the input string is (i) 011101 (ii) 01101

(i)

Input	State	Outputs
0	s_1	1
1	s_1	1
1	s_1	1
1	s_1	1
0	s_2	1
1	s_0	0

(ii)

Input	State	Output
0	s_1	1
1	s_1	1
1	s_1	1
0	s_2	1
1	s_1	