Q. In the set of natural numbers, Prove that the relation R defined as.

$aRb \iff a = b^k \; \forall a, b, k \in N$

is a partial order relation.

Soln

Reflexive

Let $a \in N \; \forall a \in N$

$aRa \implies a = a^k \; ; k \in N$

$\implies a = a \quad (\text{if } k = 1)$

This is true

POORNIMA

Date	Unit No.	Lecture No.	Faculty	Subject Name	Subject Code	Main Topics:-

$\Rightarrow$ R is Reflexive.

(ii) Antisymmetric

Let $a, b \in R$

$aRb \Rightarrow a = b^{k_1}$

$bRa \Rightarrow b = a^{k_2}$ ~~[illegible]~~ $\forall k, \in N$

~~[illegible]~~

$\Rightarrow a = (a^{k_2})^{k_1}$

$= a = a^{k_1 k_2}$ ($\because k_1 k_2 = 1$

$\Rightarrow a = a$ is true.

$\Rightarrow a = b^{\cancel{k}}$

$\Rightarrow a = b$

that means R is Antisymmetric

(iii) Transitive

Let $a, b, c \in R$
aRb & bRc then aRc

$aRb = a = b^{k_1}$

$bRc = b = c^{k_2}$

$\Rightarrow a = (c^{k_2})^{k_1}$ ($k_1 k_2 = k \in N$)

$= a = c^k$

$\Rightarrow aRc$ $\Rightarrow$ R is Transitive.

Main Ideas, Questions & Summary:

Library / Website Ref.:-

⇒ its Reflexive, Antisymmetric & transitive therefore it is a Partial order Relation.

Q. Consider a set $A = \{a, b, c, d, e, f\}$ and Relation R define on set A Given.

$R = \{(a,a), (a,b), (b,a), (b,b), (c,c), (d,d), (d,e), (d,f), (e,d), (e,e), (e,f), (f,d), (f,e), (f,f)\}$

Write the Matrix Representation & prove it is an Equivalance Relation.

Soln

$$M_R = \begin{array}{c|cccccc} & a & b & c & d & e & f \\ \hline a & 1 & 1 & 0 & 0 & 0 & 0 \\ b & 1 & 1 & 0 & 0 & 0 & 0 \\ c & 0 & 0 & 1 & 0 & 0 & 0 \\ d & 0 & 0 & 0 & 1 & 1 & 1 \\ e & 0 & 0 & 0 & 1 & 1 & 1 \\ f & 0 & 0 & 0 & 1 & 1 & 1 \end{array}$$

⇒ The diagonal element of the Matrix 'M_R' are non-zero therefore the Relation is Reflexive

⇒ The Matrix 'M_R' and its Transpose 'M_R^T' are identical, therefore Relation is Symmetric.

$(M_R = M_R^T) \rightarrow$ Symmetric.

⇒ The Matrix '$M_R^2 = M_R$' therefore Relation is Transitive

⇒ Therefore Relation is Equivalence.