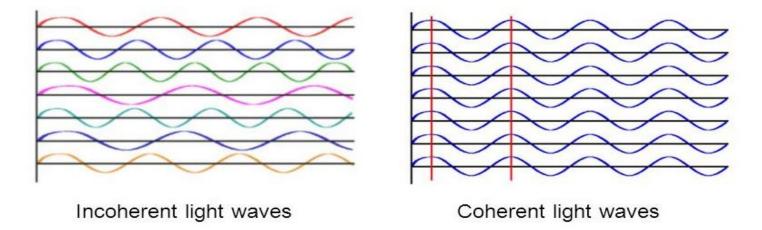
Chapter 5- Coherence and Optical Fibre

Coherence: Coherence is a measure of the correlation between the phases measured at different points on a wave.

There are two types of optical coherence first one is Spatial coherence correlates the phases at different points in space at a single moment in time, whereas temporal coherence correlates the phases at a single point in space over a period of time.



There are two criteria for coherence

- 1. The criterion of time: The wave must appear to be a pure wave for infinitely large period of time.
- 2. The criterion of space: The wave must appear to be a pure wave in an infinitely extended space.

This gives rise to two types of coherence

- 1. Temporal coherence (coherence in time related to wavelength)
- 2. Spatial coherence (coherence in space related to finite size of the source)

Temporal coherence: This is the coherence which refers to the correlation between field at a point and the field at the same point at a later time. If the phase difference between the two fields is constant during the period normally covered by observations the wave has temporal coherence.

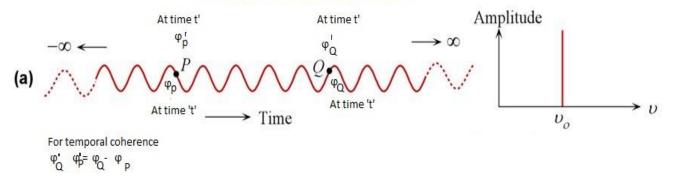
OR

Temporal coherence is a measure of the correlation between the phases of a light wave at different pints along the direction of propagation. It is related to the emitted line width.

Temporal coherence tells us how monochromatic a source is. It is also known as longitudinal coherence.

Perfectly coherent light wave would have a constant amplitude of vibration at any time, while its phase would vary linearly with time. However no actual source of light produces an ideal sinusoidal field for all values of time. When an excited atom returns to the initial state, it jumps pulse of short duration of the order of 10^{-10} second. Thus the field remains sinusoidal for time intervals of the order of 10^{-10} second, after which the phase changes abruptly.

Temporal Coherence



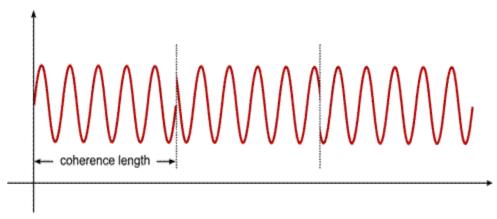
Spatial Coherence: Spatial coherence is a measure of the correlation between the phases of a light wave at different points at perpendicular/transverse to the direction of propagation in space. Spatial coherence tells us how uniform the phase of the wave front is. It describes the correlation between signals at different points in space. It is also known as transverse or lateral coherence.

Coherence Length:

The average distance for which the field remains sinusoidal is called the coherence length of the light beam and it is represented by Lc. If c is the velocity of light then coherent length is given by

$$L_c = c\tau_c$$

Where τ_c is coherent time



Coherent Time: The average time interval during which the field remains sinusoidal i.e. there exists a definite phase relationship, is known as 'coherent time' and is denoted by τ_c .

Q Factor or Spectral purity: The measure of the purity of spectral lines is given by Q factor expressed as

$$Q = \frac{\lambda}{\Delta \lambda} = \frac{\Delta \nu}{\nu}$$

Where λ is wavelength and υ is frequency of the spectral line and $\Delta\lambda$ and $\Delta\upsilon$ are the spread of line in terms of wavelength and frequency.

Thus Q factor gives the degree of monochromaticity of spectral line

For an ideal monochromatic source $\Delta \lambda = 0$

Or
$$Q = \infty$$

Q decreases as band width of wave increases.

Coherence length in terms of wavelength:

The time of coherence is of the order of reciprocal of the frequency range

$$\tau_c = \frac{1}{\Delta v}$$

This shows that longer is the coherence time, thinner is the frequency width of the wave

Coherent length

$$L_c = c\tau_c$$

$$\therefore L_c = \frac{c}{\Delta v}$$

Also for light waves

$$v = \frac{c}{\lambda}$$

On differentiating this equation

$$\Delta v = -\frac{c}{\lambda^2} \Delta \lambda$$
$$|\Delta v| = \frac{c}{\lambda^2} \Delta \lambda$$
$$L_c = \frac{c\lambda^2}{c\Delta \lambda}$$
$$L_c = \frac{\lambda^2}{\Delta \lambda}$$

Thus coherent length is inversely proportional to the bandwidth of the wave packet. The width of a spectral line is a measurement of coherent length.

Further the quality factor for light can be related to coherent length as

$$Q = \frac{\lambda}{\Delta \lambda}$$

$$\therefore L_c = \frac{\lambda^2}{\Delta \lambda} = \frac{\lambda \times \lambda}{\Delta \lambda} = Q\lambda$$

 $Coherence\ length = Purity\ factor \times wavelength$

Coherence time, Coherence Length and Spectral Purity Factor

The average time interval for which definite phase relationship exists in knowledge is known as coherence time τ

The distance Lofor which the wave field remains sinusoidal is given by coherence Length Lc

$$L = c \tau_c$$
 -----Eq 1

& Spectral Purity Factor is given by

Coherence Length In Terms of Frequency

$$L_{\rm c} = c \, \tau_{\rm c}$$

$$\tau_{\rm c} = \Delta t = \frac{1}{\Delta v}$$

$$v = \frac{c}{\lambda}$$
-------Eq 3

So
$$Q = \frac{\lambda}{\Delta \lambda} \qquad ---- Eq 5$$

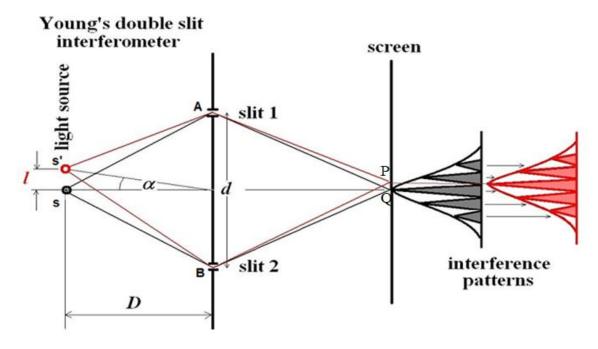
$$L_c = c \tau_c = \frac{c}{\Delta v} = \frac{\lambda^2}{\Delta \lambda} = \lambda Q$$

By Differentiate equation 3 we got

$$\Delta \upsilon = -\frac{c}{\lambda^2} \Delta \lambda = \left| \frac{c}{\lambda^2} \Delta \lambda \right| - - - - Eq^{-c}$$

$$\tau_{\rm c} = \frac{L_{\rm c}}{c} = \frac{\lambda Q}{c}$$
 -----Eq 8

Spatial Coherence and size of source:



Spatial coherence of light is related with the dimension i.e. size of the source. Let S be a point source of light and PQ be two points lying on line joining them. The phase relationship between P and Q depends on (i) Coherence length (*l*) of the light beam and (ii) distance between PQ.

If PQ<<*l* there is high coherence between p and Q

If $PQ \gg l$ there is no coherence between them.

This waves produced by an ideal point source will show spatial coherence as all points equidistant from the source will have the same phase. But an extended source will show lesser spatial coherence. This concept of spatial coherence and size of the source can be understood by Young's double slit experiment.

In fig S be a point source placed in front of a screen with two pinholes A and B such that interference pattern is observed on the screen.

Now consider S' placed near S and assume the waves from S and S' have no phase relationship. Thus the interference pattern formed on the screen will be a superposition of the intensity distributions of the interference pattern due to S and S'. For a particular separation between S and S' the interference pattern on the screen disappears and only uniform illumination is observed.

Let
$$AB = d$$
 and $SB = D$

For disappearance of fringes assuming S and O to be equidistant from A and B, The path difference S'B and S'Awill be $\lambda/2$. If we assume SS'= l

Then

$$S'A = \left[D^2 + \left(\frac{d}{2} - l\right)^2\right]^{1/2} \cong D + \frac{1}{2D}\left(\frac{d}{2} - l\right)^2$$
 using binomial expansion

and

$$S'B = \left[D^2 + \left(\frac{d}{2} + l\right)^2\right]^{1/2} \cong D + \frac{1}{2D}\left(\frac{d}{2} + l\right)^2$$
 using binomial expansion

Where it is assumed that d and *l*<<D. Thus from the condition for dis appearance of fringes i.e. destructive interference the path difference is given by

$$S'B - S'A = \frac{\lambda}{2} \cong \frac{1}{2D} \left[\left(\frac{d}{2} + l \right)^2 - \left(\frac{d}{2} - l \right)^2 \right]$$
$$= \frac{1}{2D} \left[\frac{d^2}{4} + l^2 + \frac{2dl}{2} - \frac{d^2}{4} - l^2 + \frac{2dl}{2} \right]$$

OR

$$\frac{\lambda}{2} = \frac{ld}{D}$$

$$l = \frac{\lambda D}{2d}$$

The maximum value of d for which the sources A and B remain coherent is defined spatial coherence length l.

And $l = d_{max}$

Thus for observing good interference fringes for an extended source of width l

We will get

$$l << \frac{\lambda D}{d}$$

in other words for a given source of width *l*, interference fringes of good contrast will be fromed by interference of light from two points A and B separated by distance d

$$d < < < \frac{\lambda D}{I}$$

Since l/D is the angle (say α) subtended by the source at the slit can be written as

$$d \ll \frac{\lambda}{\alpha}$$

It is obvious from above equation that spatial coherence length can be increased by decreasing l or α .

The area over which pinhole can be moved and interference fringes still be seen is called the coherent area of light wave. The distance d between pinholes for which the fringes disappear is called transverse or spatial coherence length and is given by

Spatial coherence length =
$$d = \frac{\lambda}{\alpha} = \frac{\lambda D}{l}$$

Where $\alpha = l/D$ (in radian)

Visibility as a measure of coherence: Let us consider two interfering waves, each of intencity I_0 . Both interfering waves consists of coherent part Ic and the incoherent part Iinc. If C is the degree of coherence in both waves then

$$I_C = CI_0$$

$$I_{inc} = (1 - C)I_0$$

When both waves superimposed each other, the coherent part will interfere each other by adding their amplitudes, whereas the incoherent part will interfere each other by adding their intensities.

For coherent part, the resultant intensity

$$I_{RC} = (a_C + a_C)^2 = 4a_C^2 = 4I_C = 4CI_0$$

Where a_C is the amplitude of the coherent interfering waves

For incoherent part the resultant intensity

$$I_{Rinc} = I_{inc} + I_{inc} = 2I_{inc}$$

Hence the resultant intensity for constructive interference

$$I_{max} = I_{RC} + I_{Rinc} = 4CI_0 + 2(1 - C)I_0$$

And the resultant intensity for destructive interference

$$I_{min} = 0 + I_{inc} = 2(1 - C)I_0$$

Now fringe visibility

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

$$= \frac{4CI_0 + 2(1 - C)I_0 - 2(1 - C)I_0}{4CI_0 + 2(1 - C)I_0 + 2(1 - C)I_0}$$

$$= \frac{4CI_0}{4I_0} = C$$

Hence the fringe visibility is a measure of a degree of coherence between two interfering waves of equal intensity.

Optical Fibre

Introduction:

The development of lasers and optical fibre has brought about a revolution in the field of communication systems. Experiments on the propagation of information – carrying light waves through an open atmosphere were conducted. The atmospheric conditions like rain, fog etc affected the efficiency of communication through light waves.

To have efficient communication systems, the information carried by light waves should need a guiding medium through which it can be transmitted safely.

This guiding mechanism is optical fibre. The communication through optical fibre is known as light wave communication or optical communication.

A light beam acting as a carrier wave is capable of carrying more information than that of radio waves and microwaves due to its larger bandwidth.

Currently in most part of the world, fibre optics is used to transmit voice, video and digital data signals using light waves from one place to other place.

Optical Fibre:

The optical fibre is a wave guide.

It is made up of transparent dielectrics (SiO2), (glass or plastics).

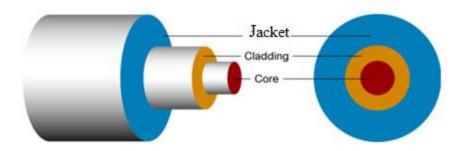
Principle:

The basic principle of optical fibre in the transmission of optical signal is total internal reflection.

Construction of Optical Fibre:

It consists of an inner cylinder made of glass or plastic called core. The core has high refractive index n_1 . This core is surrounded by cylindrical shell of glass or plastic called cladding.

The cladding has low refractive index n_2 . This cladding is covered by a jacket which is made of polyurethane. It protects the layer from moisture and abrasion.



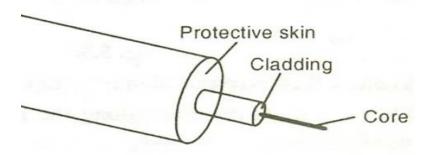
The light is transmitted through this fibre by total internal reflection. The fibre guides light waves to travel over longer distance without much loss of energy.

Core diameters range from 5 to 600µm while cladding diameters vary from 125 to 750µm.

Core transmits the light waves. The cladding keeps the light waves within the core by total internal reflection.

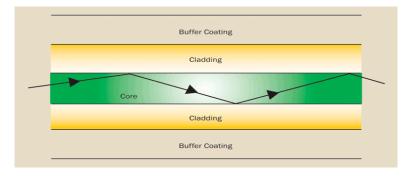
Refractive Index:

The *refractive index or index of refraction* of a substance is a measure of the speed of light in that substance. It is expressed as a ratio of the speed of light in vacuum relative to that in the considered medium.



Principle of propagation of light in an optical fibre:

The light launched inside the core at one end of the fibre propagates to the other end due to total internal reflection at the core and cladding interface.

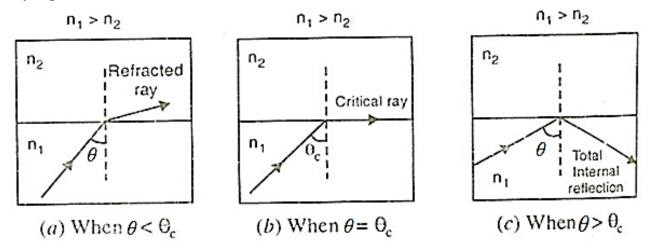


Total internal reflection at the fibre wall can occur only if two conditions are satisfied.

- 1. The refractive index of the core material n1 must be higher than that of the cladding n2 surrounding it.
- 2. At the core cladding interface, the angle of incidence (between the ray and normal to the interface) must be greater than the critical angle defined as

$$\sin \theta_c = \frac{n_2}{n_1}$$

Let the light ray travel from core of refractive index n_1 to cladding of refractive index n_2 , for total internal reflection $n_1 > n_2$.



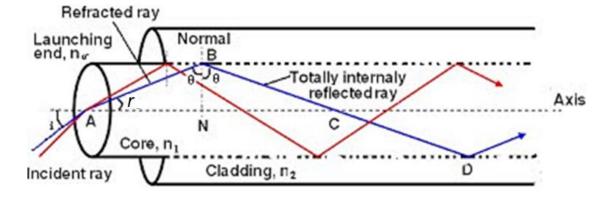
- When $(\theta < \theta_C)$, then the ray refracts into the rarer medium
- When $(\theta = \theta_C)$, then the ray travels along the interface so that the angle of refraction is 90°
- When $(\theta > \theta_C)$, then the ray totally reflects back into the same medium When $(\theta = \theta_C)$, then from Snell's law

$$n_1 \sin \theta_C = n_2 \sin 90^\circ$$
$$\therefore \sin \theta_C = \frac{n_2}{n_1}$$

Propagation of light through fibre (Acceptance Angle):

Consider an optical fibre through which the light is being sent. The end at which light enters is called launching end. Let the refractive indices of the core and cladding be n_1 and n_2 respectively; $n_1 > n_2$. Let the refractive index of the medium from which the light is launched be n_0 .

Let the light ray enter at an angle I to the axis of the fibre



The ray refracts at an angle r.

The ray strikes the core – cladding interface at an angle θ . If θ is greater than the critical angle θ c, the ray undergoes total internal reflection at the interface.

Let us now find out up to what maximum value of i at A total internal reflection at B is possible.

In triangle ABN

$$r = 90^{\circ} - \theta \dots \dots \dots \dots (1)$$

From Snell's law

$$n_o \sin i = n_1 \sin r$$

$$\frac{\sin i}{\sin r} = \frac{n_1}{n_o} \dots \dots \dots \dots \dots (2)$$

$$\sin i = \frac{n_1}{n_o} \sin r$$

Putting the value of angle r

$$\sin i = \frac{n_1}{n_o} \sin(90^o - \theta)$$
$$\sin i = \frac{n_1}{n_o} \cos \theta$$

If θ is less than the critical angle θc , the ray will be lost by refraction. Therefore, limiting value for containing the beam inside the core by total internal reflection is θc . Let i_m be the maximum possible angle of incidence at the fibre end face A for which $\theta = \theta c$.

$$\sin i_m = \frac{n_1}{n_o} \cos \theta_C \dots \dots (3)$$

$$\sin i_m = \frac{n_1}{n_o} \sqrt{(1 - \sin^2 \theta_C)}$$

But $\sin \theta_c = \frac{n_2}{n_1}$

Then

$$\sin i_m = \frac{n_1}{n_o} \sqrt{\left(1 - \frac{n_2^2}{n_1^2}\right)}$$

$$\sin i_m = \frac{n_1}{n_o} \sqrt{\left(\frac{n_1^2 - n_2^2}{n_1^2}\right)}$$

$$\sin i_m = \sqrt{\left(\frac{n_1^2 - n_2^2}{n_o^2}\right)}$$

or

$$i_m = sin^{-1} \sqrt{\left(\frac{n_1^2 - n_2^2}{n_0^2}\right) \dots \dots (4)}$$

If lunching end is air then $n_0=1$, Then

$$i_m = \sin^{-1} \sqrt{(n_1^2 - n_2^2)}$$

This angle i_m is called the maximum acceptance angle of the optical fibre

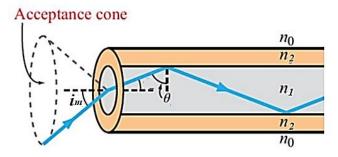
Definition: Acceptance angle is defined as the maximum angle of incidence at the interface of air medium and core medium for which the light ray enters into the core and travels along the interface of core and cladding.

or

Acceptance angle is defined as the maximum angle that a light ray can have relative to the axis of the optical Fibre and propagate down the optical Fibre.

Acceptance cone:

An optical fibre accepts only those rays which are incident within a cone having a semi angle im.



The light rays contained within the cone having a full angle $2i_m$ are accepted and transmitted along the fibre. Therefore, the cone is called the acceptance cone.

Light incident at an angle beyond i_m refracts through the cladding and the corresponding optical energy is lost. It is obvious that the larger the diameter of the core, the larger the acceptance angle.

Numerical Aperture:

The numerical aperture (NA) is defined as the sine of the acceptance angle.

$$NA = \sin i_m = \sqrt{\left(\frac{n_1^2 - n_2^2}{n_0^2}\right)}$$

If lunching end is air then $n_0=1$, Then

$$NA = \sqrt{(n_1^2 - n_2^2)}$$

Definition: Numerical aperture determines the light gathering ability of the fibre. It is a measure of amount of light that can be accepted by a fibre. NA depends only on the refractive indices of the core and cladding materials. A large NA implies that a fibre will accept large amount of light from the source.

Fractional Index change:

It is the ratio of refractive index difference in core and cladding to the refractive index of the core.

$$\Delta = \frac{n_1 - n_2}{n_1}$$

Relation between NA and Fractional Index change (Δ):

We have

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$n_1\Delta = n_1 - n_2$$

We know that

$$NA = \sqrt{n_1^2 - n_2^2}$$

$$NA = \sqrt{(n_1 - n_2)(n_1 + n_2)}$$

Putting the value of $(n_1 - n_2)$

$$NA = \sqrt{(n_1 - n_2)n_1\Delta}$$

If
$$n_1 = n_2$$
 then $NA = \sqrt{2n_1^2} \Delta$

Therefore
$$NA = n_1 \sqrt{2\Delta}$$

Classification of Fibres:-

Based on the number of modes, they are classified as

- 1. Single mode fibre
- 2. Multimode fibre

Based on the refractive index profile, they are classified as

- 1. Step- index fibre
- 2. Graded index fibre

Modes of Propagation:

Light propagates as electromagnetic waves through an optical fibre. All waves, having ray directions above the critical angle will be trapped within the fibre due to total internal reflection. However, all such waves do not propagate through the fibre. Only certain ray directions are allowed to propagate. The allowed directions correspond to the modes of the fibre.

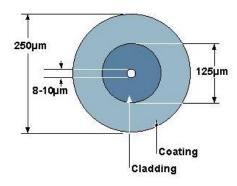
In simple terms, modes can be visualized as the possible number of paths of light in an optical fibre. The paths are all zigzag paths excepting the axial direction. Accordingly, light rays travelling through a fibre are classified as axial rays or zigzag rays. As a ray gets repeatedly reflected at the walls of the fibre, phase shift occurs. Waves travelling along the certain zigzag paths will be in phase and intensified. Waves travelling along certain other paths will be out of phase and diminish due to destructive interference. The light rays path along which the waves are in phase inside the fibre are called modes. The number of modes that a fibre will support depends upon the ratio of d/λ where d is the diameter of the core and λ is the wavelength of the wave being transmitted.

Modes are designated by an 'order' number 'm'. In a fibre of fixed thickness, the higher order modes propagate at smaller angles than the lower order modes.

Axial ray that travels along the axis of the Fibre is called zero order rays.

Single Mode Fibres:

In general, the single mode fibres are step – index fibres. These types of fibres are made from doped silica. It has a very small core diameter so that it can allow only one mode of propagation and hence called single mode fibres. The cladding diameter must be very large compared to the core diameter. Thus in the case of single mode fibre, the optical loss is very much reduced. The structure of a single mode fibre as shown.



Structure:

Core diameter : 5-10µm

Cladding diameter : Generally around 125µm

Protective layer : $250 \text{ to } 1000 \mu\text{m}$

Numerical aperture : 0.08 to 0.10

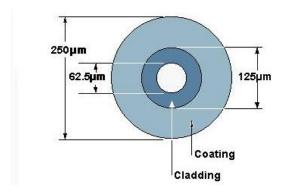
Band width : More than 50MHz km.

Application:

Because of high bandwidth, they are used in long haul communication systems.

Multi- Mode Fibres

The multi mode fibres are useful in manufacturing both for step – index and graded index fibres. The multi-mode fibres are made by multi-component glass compounds such as Glass – Clad Glass, Silica – Clad – Silica, doped silica etc. Here the core diameter is very large compared to single mode fibres, so that it can allow many modes to propagate through it and hence called as Multi mode fibres. The cladding diameter is also larger than the diameter of the single mode fibres. The structure of the multimode fibre is as shown in the figure.



Structure:

Core diameter : 50-350µm

Cladding diameter : 125µm - 500µm

Protective layer : 250 to 1100µm

Numerical aperture : 0.12 to 0.5

Band width: Less than 50MHz km.

The total number of modes possible for such an electromagnetic wave guide is

$$N_{step} = 4.9 \left(\frac{d \times NA}{\lambda}\right)^2$$

Where-

d →Diameter of core

NA→ Numerical Aperture

 $\lambda \rightarrow$ Wavelength of Light

Application:

Because of its less band width it is very useful in short haul communication systems.

Differences between Single and Multimode Fibre

Single mode Fibre	Multi mode Fibre	
1.In single mode optical Fibres onlyone mode of	1. In multi-mode optical Fibres many mummer	
propagation is possible	ofmodes of propagation are possible.	
2. In case of single mode Fibre the diameter of	2. In case of multi-mode Fibre the diameter of	
core is about 10 micrometers	core is 50 to 200 micrometers.	
3. The difference between the refractive indices of core and cladding is very small.	3. The difference between the refractive indices of core and cladding is also large compared to the single mode Fibres.	
4. In single mode Fibres there is no dispersion, so	4. Due to multi-mode transmission, the dispersion	
these are more suitable for communication.	is large, so these Fibres are not used for	
	communication purposes.	
5. The process of launching of light into single mode Fibres is very difficult	5. The process of launching of light into	
	singlemode Fibres is very easy.	
6. The condition for single mode	6. The condition for multi-mode propagation is	
operation is	$(d \times NA)^2$	
$V=rac{\pi d}{\lambda}n_1\sqrt{2\Delta}$	$N_{step} = 4.9 \left(\frac{d \times NA}{\lambda} \right)^2$	
7. Fabrication is very difficultand the Fibre is	7. Fabrication is very easy and the Fibre is	
costly.	cheaper.	

Refractive Index Profile:

In any Optical Fibre, the whole material of the cladding has a uniform refractive index value. But the refractive index of the core material may either remain constant or subjected to various in a particular way.

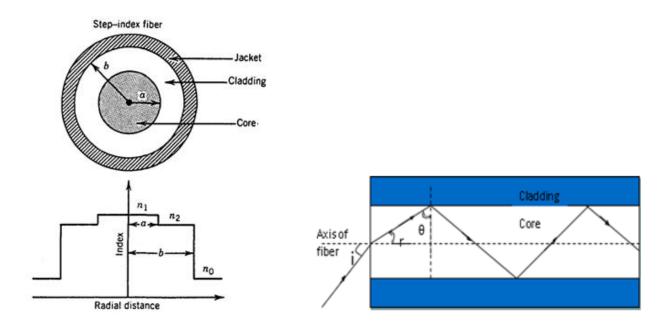
The curve which represents the variation of refractive index (along the vertical axis) with respect to the radial distance (along the horizontal axis) from the axis of the Fibre is called the refractive index profile. Depending on refractive index profile, there are two types of Fibres:

- 1. Step index Fibre
- 2. Graded index Fibre

Step index (SI) Fibre:

A step index Fibre has a central core with a uniform refractive index. The core is surrounded by an outside cladding with a uniform refractive index less than that of core. The light ray propagate through it in the form of meri-diagonal rays which cross the Fibre axis during every reflection at the core cladding boundary i.e. rays propagate in zig-zag manner inside the core.

From the figure it is seen that there is an abrupt change in the refractive index at the core cladding interface and hence the refractive index profiles takes the shape of a step. Hence the name step index Fibre.

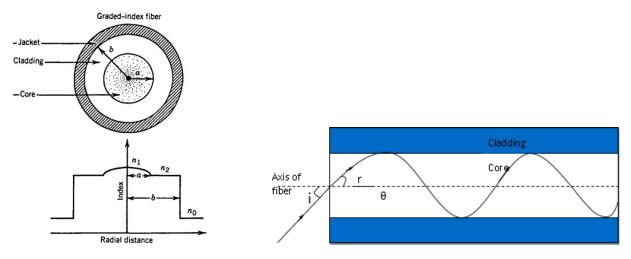


Refractive Index Profile for Step Index Fibre

Propagation of Light signals through Step Index Fibre

Graded Index (GRIN) Fibre:

In a graded index Fibre the refractive index of the core is non uniform, it is maximum at the center and decreases gradually with distance towards the outer edge. The light rays propagate through it in the form of helical rays. They never cross the Fibre axis.

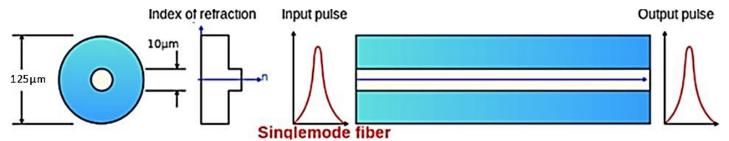


Refractive Index Profile for Graded Index Fibre

Propagation of Light signals through Graded Index Fibre

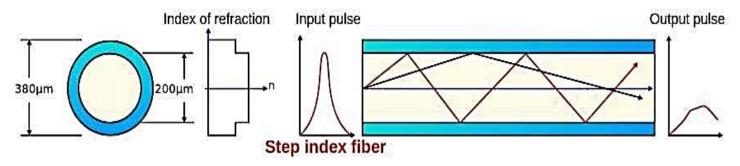
Single Mode Step Index Fibre

A single mode step index fibre consists of a very thin core of uniform refractive index surrounded by a cladding of refractive index lower than that of core. The refractive index abruptly changes at the core cladding boundary. Light travels along a side path, i.e., along the axis only. So zero order modes is supported by Single Mode Fibre.



Multimode Step Index Fibre

A multimode step index fibre consists of a core of uniform refractive index surrounded by cladding of refractive index lower than that of the core. The refractive index abruptly changes at the core cladding boundary. The core is of large diameter. Light follows zigzag paths inside the fibre. Many such zigzag paths of propagation are permitted in Multi Mode Fibre. The Numerical Aperture of a Multi mode fibre is larger as the core diameter of the fibre is larger.



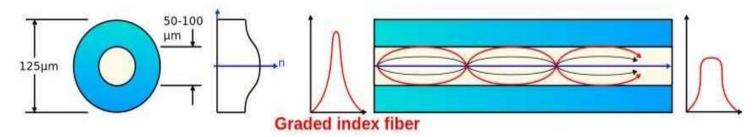
STEP INDEX FIBRE

1. The refractive index of the core is uniform throughout and undergoes on abrupt change at the core cladding boundary

- 2. The diameter of the core is about 50-200μm in the case of multimode fibre and 10μm in the case of single mode fibre
- 3. The path of light propagation is zig- zag in manner
- 4. Attenuation is more for multimode step index fibre but for single mode it is very less. Explanation: When a ray travels through the longer distances there will be some difference in reflected angles. Hence high angle rays arrive later than low angle rays causing dispersion resulting in distorted output.
- 5. This fibre has lower bandwidth
- 6. The light ray propagation is in the form of meridional rays and it passes through the fibre axis.

GRADED INDEX FIBRE

GRIN fibre is one in which refractive index varies radially, decreasing continuously in a parabolic manner from the maximum value of n1, at the center of the core to a constant value of n2 at the core cladding interface.



In graded index fibre, light rays travel at different speeds in different parts of the fibre because the refractive index varies throughout the fibre. Near the outer edge, the refractive index is lower. As a result, rays near the outer edge travel faster than the rays at the center of the core. Because of this, rays arrive at the end of the fibre at approximately the same time. In effect light rays arrive at the end of the fibre are continuously refocused as they travel down the fibre. All rays take the same amount of time in traversing the fibre. This leads to small pulse dispersion.

The pulse dispersion is given by
$$\Delta \tau = \tau_{\text{max}} - \tau_{\text{min}} = \frac{n_2 L}{2c} \Delta^2$$

Here
$$\Delta = \frac{n_1 - n_2}{n_2}$$

For a parabolic index fibre, the pulse dispersion is reduced by a factor of about 200 in comparison to step index fibre. It is because of this reason that first and second generation optical communication systems used near parabolic index fibres.

The total number of modes possible for such an electromagnetic wave guide is

$$N_{grad} = \frac{N_{step}}{2} = \frac{4.9 \left(\frac{d \times NA}{\lambda}\right)^2}{2}$$

Where-

 $d \rightarrow \text{Diameter of core}$

NA→ Numerical Aperture

 $\lambda \rightarrow$ Wavelength of Light

Advantages of Graded Index Fibre

- It can transmit a large amount of information.
- The distortion is comparatively small than step index Fibre.

Disadvantages of Graded-index Fibre

- These Fibres possess low light coupling efficiency.
- It is costly when compared to step index Fibre.

Graded Index Fibre

- 1. The refractive index of the core is made to vary gradually such that it is maximum at the center of the core.
- 2. The diameter of the core is about 50 µm in the case of multimode fibre
- **3.** The path of light is helical in manner
- **4.** Attenuation is less.

Explanation:

Here the light rays travel with different velocity inn different paths because of their variation in their refractive indices. At the outer edge it travels faster than near the center. But almost all the rays reach the exit at the same time due to helical path. Thus, there is no dispersion.

- 5. This fibre has higher bandwidth
- **6.** The light propagation is in the form of skew rays and it will not cross fibre axis.

V-Number and No. of Modes in Fibre:

V-Number or Normalized Frequency

V – number determines how many modes a fibre can support, It is given by,

$$V = \frac{\pi d}{\lambda} NA$$

Where d is the diameter of the core, l is the wavelength of light used and NA is the numerical aperture of the fibre.

$$V = \frac{\pi d}{\lambda} \sqrt{n_1^2 - n_2^2}$$

Or

$$V = \frac{\pi d}{\lambda} n_1 \sqrt{2\Delta}$$

If $V \le 2.405$, then the fibre is single mode fibre (SMF)

If V > 2.405, then the fibre is multimode fibre (MMF)

Number of Modes traveling in Fibre

The total number of mode traveling in a fibre depends on the V – Number and is related as:

For Step Index Fibre:

$$N = \frac{V^2}{2}$$

For Graded Index Fibre:

$$N = \frac{V^2}{4}$$

Differences between Step Index Fibre and Graded Index Fibre:

S. NO	STEP INDEX FIBER	GRADED INDEX FIBER
1.	The refractive index of the core is uniform throughout and undergoes on abrupt change at the core cladding boundary	The refractive index of the core is made to vary gradually such that it is maximum at the center of the core.
2.	The diameter of the core is about 50-200μm in the case of multimode fiber and 10μm in the case of single mode fiber	The diameter of the core is about 50µm in the case of multimode fiber
3.	The path of light propagation is zig- zag in manner	The path of light is helical in manner
3.	Attenuation is more for multimode step index fiber but for single mode it is very less. Explanation: When a ray travels through the longer distances there will be some difference in reflected angles. Hence high angle rays arrive later than low angle rays causing dispersion resulting in distorted output.	Attenuation is less. Explanation: Here the light rays travel with different velocity inn different paths because of their variation in their refractive indices. At the outer edge it travels faster than near the center. But almost all the rays reach the exit at the same time due to helical path. Thus, there is no dispersion.
4.	This fiber has lower bandwidth	This fiber has higher bandwidth
5.	No of modes of Propagation: $N_{skp} = 4.9 \left(\frac{d \times NA}{\lambda}\right)^2 = \frac{V^2}{2}$ Where d= diameter of the fiber core λ = wavelength NA = Numerical Aperture V- V-number is less than or equal to 2.405 for single mode fibers and greater than 2.405 for multimode fibers.	No of modes of Propagation: $N_{Graded} = \frac{4.9 \left(\frac{d \times NA}{\lambda}\right)^{2}}{2} = \frac{v^{2}}{4}$ Or $N_{graded} = \frac{N_{step}}{2}$

Applications of optical Fibres:

- 1. **In communication:** A very large number of information can be sent through the optical Fibre with least attenuation. It has a capacity of transmitting 35000 channels at a time to a distant place.
- 2. **In medical:** As optical Fibre can be bent in any shape and no obstruction is found in the propagation of light through the Fibre, it can be used to a variety of medical needs.
 - a) The instrument based on optical Fibre used for searching an ulcer in the stomach of a patient is called Fibroscope. Such **fibroscopes** are employed widely in **endoscopic applications**.
 - **b)** A laser beam guided by the Fibres is used to reattach-detached retinas and to correct defects in vision.
 - c) In cardiology, laser angioplasty is expected to do away with the balloon angioplasty and bypass surgery. A special Catheter is developed for the removal of blocks in the veins.
 - **d**) For delivering high power laser energy in laser surgery.

3. In optical Fibre sensing:

- a) In a thermometer (of range 80° to 7000) Fibre is used as heat sensing element.
- **b)** A smoke detector and pollution detector can be built using Fibres.
- 4. In optical face plates on CRT (cathode ray tube) and IR (infrared) imaging device
- 5. For delivering light from a source to a remote or inaccessible point.
- 6. **In military applications**: An aircraft, a ship or a tank reduces much weight; and also maintains true communication silence to the enemy using optical Fibre. Fibre-guided missiles are used during the recent wars.

Besides this, there are multiple application of optical Fibre. Optical Fibres are also used in industrial application, used as a decorative material, in civil applications and many more.