Random Variables

Lecture 4

Random Variable

A **random variable** is a **function** that associates a **real number** to each **element** of the sample space of an experiment.

Random variable is usually denoted by uppercase letters.

Example 1

In tossing a coin twice, let X be a random variable that indicates the number of heads. What are the possible values of X?

Example 2

In tossing a coin twice, let Y be a random variable that is 1 if the result is a double and 0 otherwise. What are the possible values of Y?

Example 3

In a deck of cards, there are numbers, Aces, and faces. If 10 cards are randomly selected, and the random variable X is defined as the number of faces, what are the possible values of X?

Example 4

In a deck of cards, there are numbers, Aces, and faces. If 10 cards are randomly selected, and the random variable Y is defined as the number of aces, what are the possible values of Y?

Example 5. Identify if r.v. or not r.v. ?

W= the number of students who are absent from the class

X = the weight (in kilograms) of randomly selected student from the class.

X_1= the name of mayors from each city of NCR

Example 5. Identify if r.v. or not r.v. ?

X_2= daily temperature (in Celsius) of Japan for a week

Z= the square root of daily temperature (in Celsius) of Japan for a week

Discrete Random Variable

A random variable is said to be discrete if it could take on only a finite or countably infinite number of possible values.

Continuous Random Variable

A random variable is said to be continuous if it could take on any value within a given finite or infinite range of possible values.

Example 6 Discrete or Continuous

W= the number of students who are absent from the class

X= the number of wrong answers in a multiple-choice test

Y= the amount of time (in minutes) it took a student to complete a test

Z= the weight (in kilograms) of a randomly selected student from the class

X_1= the number of raffle tickets sold for a fund drive

X_2=the number of missed shots before a successful shot in a basketball game

X_3= the age(in years of randomly selected employee) of a large supermarket chain

X_4=the number of coin tosses until a tail shows up.

Probability Distribution function

Probability mass function (pmf)

Probability density function (pdf)

Probability Mass Function (pmf)

The *probability mass function* (pmf) of a discrete random variable X is a listing of all possible values of X and their corresponding probabilities.

Formula form

$$p(x) = \begin{cases} P(X = x), if\ x = x_1, x_2, x_3, \ldots, x_k \\ 0, otherwise \end{cases}$$

Tabular form

x	$p(x)$
x_1	$p(x_1)$
x_2	$p(x_2)$
x_3	$p(x_2)$
$\cdots$	$\cdots$
x_k	$p(x_k)$

Properties of pmf

1. Nonnegative property

$$p(x) \geq 0$$

2. Norming property

$$\sum_{i=1}^{k} p(x_i) = 1$$

Example 7 .A fair die is rolled once. Let X be the number of dots on the upturned face

a. Give the pmf of X in formula form
b. Give the pmf of X in tabular form
c. Draw the probability histogram of X

Example 8. A box contains three red marbles and four green marbles. Jayvee picks two marbles at random from this box.

a. Give the pmf of the number of red marbles picked from the box.
b. Give the corresponding probability histogram.

Example 9. Tweet, a quality control engineer of a large computer firm, inspects a large shipment of printed circuit boards (PCB). The shipment of 1000 PCBs were inspected for defects such as misplaced components or the application of too much solder paste. Tweet found that 750 of the PCBs have no defects, 100 with a defect each, 75 have two defects each, 50 have three defects each, and the rest have 5 defects each. Give the probability mass function of X, the number of defects found in a PCB, in tabular form.

Example 10. Find the missing value for P(X=7) in the probability distribution of X=no. of bell pepper from one plant.

Probability distribution of X	
x	Probability $p(x)$
2	0.5
4	0.4
7	
8	0.07

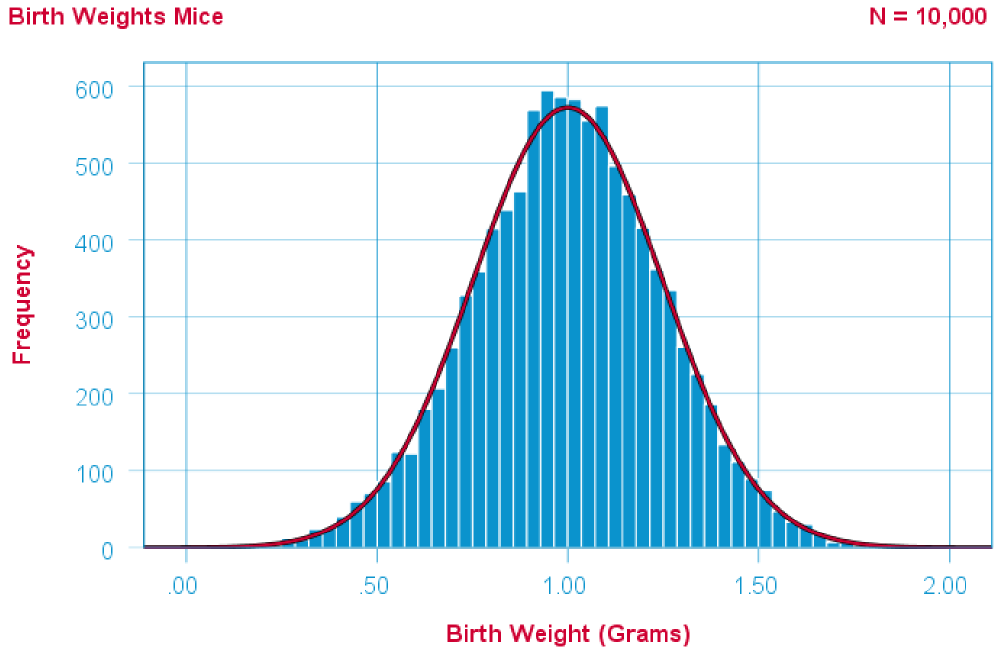
Example 11. Let X be the number of children ages 13 and below in a household . Find the value of c?

x	1	2	3	4	5
p(x)	c	0.20c	0.30	0.05	0.10

Probability Density Functions (pdf)

Probability Density Function is the probability distribution for continuous random variables.

The curve of pdf lies above the horizontal axis and bounds an area equal to 1 above the said axis.



Properties of pdf

1. Non negativity property

The graph of $f(x)$ lies above the x-axis.

2. Norming property

The total area under the curve of $f(x)$ above the x-axis is 1.

Example 10. Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} x - 1, & if\ 1.5 \leq\ x \leq 2.5 \\ 0, & otherwise \end{cases}$$

a. Verify that $f(x)$ satisfies the properties of a pdf
b. Find $P(1.5 \leq X \leq 2)$
c. Find $P(X = 2)$