Mean and Variance of a Discrete Random Variable

Lecture 2

Mean of X

expected value

"myu"

$$\mu_X = E(X) = \sum_{i=1}^{k} x_i \cdot p(x_i)$$

Variance of X

"sigma"

$$\sigma_X^2 = Var(X) = E[(X - \mu_X)^2] = \sum_{i=1}^{k} (x_i - \mu_X)^2 \cdot p(x_i)$$

Standard deviation of random variable X

$$\sigma_X = \sqrt{Var(X)}$$

Points to remember

The mean of a random variable X(that is, the expected value) is a measure of the center of the possible values of X.

It is actually the weighted mean of these values of X, with $p(x_i)$ as the weight assigned to the value x_i.

The mean of a random variable X need not be one of its mass points.

Standard deviation of random variable X

$$\sigma_X = \sqrt{Var(X)}$$

Point to remember

The variance σ_X^2, as well as the standard deviation σ_X, of a random variable X is a measure of the degree of scattering (or spread) of the different possible values of the random variable X about is mean μ_X.

The unit of σ_X^2 is the same as the unit of the values of X.

Example 1

Let X be a discrete random variable with probability mass function given by the table below.

x	-1	0	2	4
$p(x)$	1/7	?	2/7	1/7

a. Determine $P(X = 0)$
b. Find the mean of X
c. Find the variance of X
d. Give the standard deviation of X
e. Give the probability that X assumes a positive value
f. Draw the probability histogram of X.

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f. Draw the probability histogram of X.

x	$p(x)$	$x \cdot p(x)$	$x - E(X)$	$[x - E(X)]^2$	$[x - E(X)]^2 \cdot p(x)$
-1	1/7	-1/7	-2	4	4/7
0	3/7	0	-1	1	3/7
2	2/7	4/7	1	1	2/7
4	1/7	4/7	3	9	9/7
		$E(X) = 1$			$Var\ (X) = \frac{18}{7}$

Alternative formula for variance

$$\sigma_X^2 = Var\,(X) = E[(X - \mu_X)^2] = E(X^2) - (\mu_X)^2$$

x	$p(x)$	$x \cdot p(x)$	x^2	$x^2 \cdot p(x)$
-1	1/7	-1/7	1	1/7
0	3/7	0	0	0
2	2/7	4/7	4	8/7
4	1/7	4/7	16	16/7
		$E(X) = 1$		$E(X^2) = 25/7$

Alternative formula for variance

$$Var\,(X) = \sigma_X^2 = E(X^2) - (\mu_X)^2$$

Example 2. Powee found a damaged die that is twice as likely to give a prime number than a nonprime number. Let X be the outcome when this damaged die is tossed once.

a. Give the probability mass function of X in tabular form

x	1	2	3	4	5	6
$p(x)$						

Example 2. Powee found a damaged die that is twice as likely to give a prime number than a nonprime number. Let X be the outcome when this damaged die is tossed once.

b. Compute the mean and variance of X.

Example 3. The age distribution of the students in a basic statistics course is given in the table below

x	16	17	18	20
$p(x)$	0.55	0.25	0.15	0.05

a. Give the mean and variance of X
b. Give the standard deviation of X
c. If the minimum voting age is 18, what proportion of the students are of voting age?

Example 4

Let X be the number of letters in a word chosen at random from the nine words of the following statement from the movie

The Renaissance Man:

"The choices you make dictate the life you lead"

Give the following:

a. The probability mass function of X
b. The expected value of X
c. The variance of X
d. The probability of randomly selecting a word from the given statement with at most five letters
e. The probability of randomly selecting a word from the given statement with exactly five letters

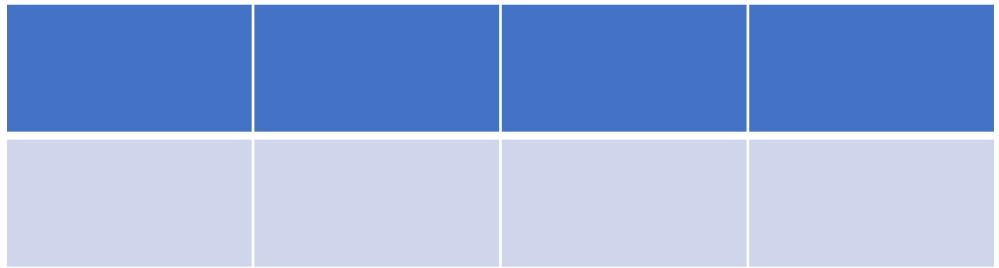
Example 4. Let X be the number of letters in a word chosen at random from the nine words of the following statement from the movie

The Renaissance Man:

"The choices you make dictate the life you lead"

Give the following:

a. The probability mass function of X



b. The expected value of X

c. The variance of X

Example 4. Let X be the number of letters in a word chosen at random from the nine words of the following statement from the movie

The Renaissance Man:

"The choices you make dictate the life you lead"

Give the following:

a. The probability mass function of X

x	3	4	7
p(x)	4/9	3/9	2/9

d. The probability of randomly selecting a word from the given statement with at most five letters

e. The probability of randomly selecting a word from the given statement with exactly five letters

Applications of Expected Value

Games of Chance

Decision Making

Game of Chance

A game is said to be fair if the expected amount of winnings is equal to the ante, bet, or paid out amount.

Example 5. A student organization is raising funds by selling 300 raffle tickets worth P20 each. The sole lucky winner of this raffle will receive a prepaid cell phone card worth P1000. What is your expected net gain if you buy one raffle ticket?

Let X be the amount to be won

Example 5. A student organization is raising funds by selling 300 raffle tickets worth P20 each. The sole lucky winner of this raffle will receive a prepaid cell phone card worth P1000. What is your expected net gain if you buy one raffle ticket?

Try another random variable

Let Y be the amount of net gain, Y= X-20

Analysis

If you compare your expected winning of P3.33 with the P20 price of each raffle ticket, you can say that the raffle is not fair since you expect to lose P16.67 for each raffle ticket you buy.

Example 6. A biased die is twice as likely to show an even number than an odd number. A game of tossing this biased die gives you a reward of P50 if it shows an even prime number, P 20 if it shows a perfect square, and you pay a penalty of P10 if the outcome is any of the other numbers on the die. What is the expected amount that you could receive in this game?

1	2	3	4	5	6

Analysis

If you play this game several times, you can expect to receive an average of P13.33 for each toss of the biased die. This amount is also the fair price that you would pay for each toss of the biased die.

Decision Making

Example 7. Peter and Sheila are deciding where to invest the cash gifts they received form their godparents during their recent wedding. They were presented with two investment options by their financial adviser. The amounts to be gained and the corresponding probability mass functions of these two investment options are shown in the table below.

Investment A		Investment B	
Profit (in pesos), x	p(x)	Profit (in pesos) , y	p(y)
7000	0.70	10 000	0.5
3000	0.25	5000	0.2
1000	0.04	2000	0.2
-500	0.01	-3000	0.1

Compare the profits from these two investment options.

Investment A		Investment B	
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Analysis

Comparing the means and standard deviations of the variables X and Y, you could see that both investments promise almost the same expected profit. Although, a slightly higher profit under Investment B, this has higher standard deviation than A. In other words, Investment B is a higher yielding but more risky investment, while Investment A is a lower yielding but less risky investment. Thus, if Peter and Sheila are risk-averse investors, they should choose Investment A over Investment B.

$$E(X) = 5685 \quad \sigma(X) = 2067.19$$
$$E(Y) = 6100 \quad \sigma(Y) = 4414.75$$