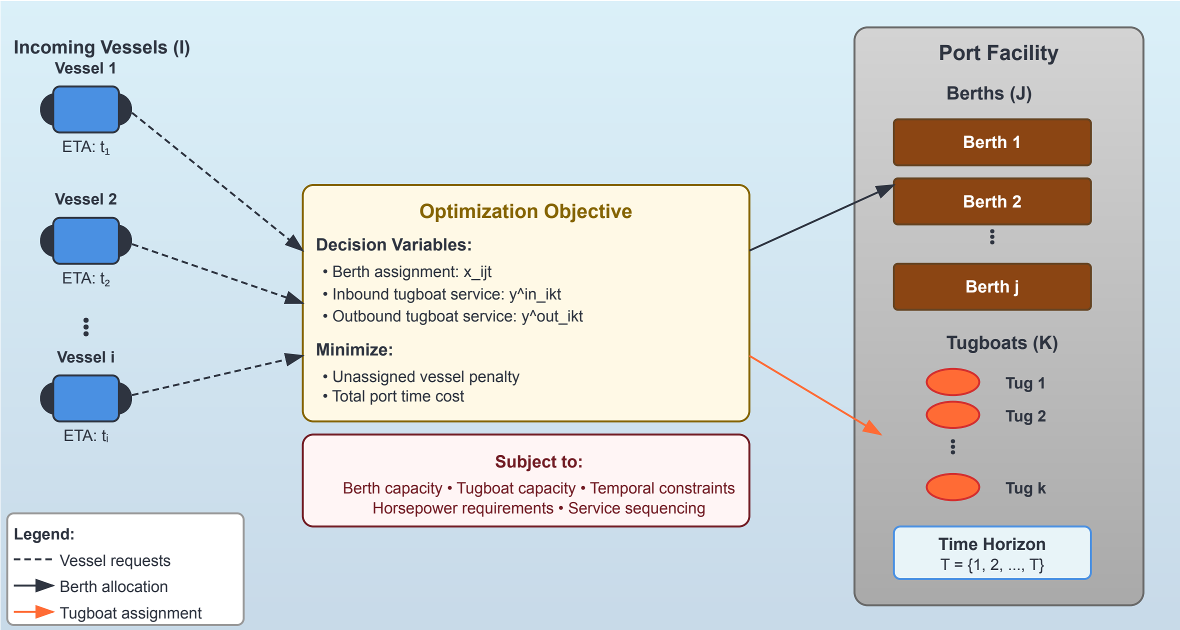

Figure 1: Schematic overview of PSP. The problem requires simultaneous allocation of berths and tugboats to incoming vessels over a discrete time horizon, subject to capacity constraints, temporal constraints, and service sequencing requirements. Decision variables determine berth assignments (x_{ijt}), inbound tugboat services (y^{in}_{ikt}), and outbound tugboat services (y^{out}_{ikt}), with the objective of minimizing unassigned vessel penalties and total port time costs.

1. Port Scheduling Problem (PSP): Mathematical Formulation

1.1. Problem Description

Maritime transportation serves as the backbone of global trade, with over 80% of world merchandise transported by sea (UNCTAD, 2023). Port operations represent critical bottlenecks in the global supply chain, where efficient resource allocation directly impacts vessel turnaround times, port throughput, and economic competitiveness (Bierwirth and Meisel, 2010). The coordination of berth allocation, vessel scheduling, and tugboat assignment poses substantial operational challenges for port authorities, as suboptimal decisions can lead to vessel queuing, berth underutilization, and increased operational costs (Wei et al., 2025).

PSP represents a novel combinatorial optimization challenge that integrates berth allocation, vessel scheduling, and tugboat assignment under temporal constraints, resulting in tightly coupled spatial–temporal–resource interactions that preclude straightforward decomposition. As illustrated in Fig. 1, a port operates with $|\mathcal{J}|$ berths and $|\mathcal{K}|$ tugboats over a discrete scheduling horizon $\mathcal{T} = \{1, 2, \dots, T\}$. Within this period, $|\mathcal{I}|$ vessels request berthing services. The objective is to assign each vessel a berth, berthing time, and tugboat services for both arrival and departure operations, while minimizing total scheduling costs. Effective solution methods for PSP enable port operators to reduce vessel waiting times, improve resource utilization, and enhance overall port efficiency, directly contributing to supply chain resilience and economic productivity (Zhang et al., 2019).

1.2. Notation

1.2.1. Sets and Indices

- $\mathcal{I} = \{1, \dots, m\}$: Set of vessels, indexed by i
- $\mathcal{J} = \{1, \dots, n\}$: Set of berths, indexed by j
- $\mathcal{K} = \{1, \dots, K\}$: Set of tugboats, indexed by k
- $\mathcal{T} = \{1, \dots, T\}$: Set of time periods, indexed by t

1.2.2. Parameters

Vessel-Berth Compatibility:

- S_i: Size class of vessel i
- C_j: Capacity class of berth j

Temporal Parameters:

- ETA_i: Expected time of arrival (ETA) for vessel i
- D_i: Berthing operation duration for vessel i (number of consecutive time periods)
- τ_i^{in}: Inbound tugboat service duration for vessel i
- τ_i^{out}: Outbound tugboat service duration for vessel i
- ρ^{in}: Tugboat preparation time after completing inbound service
- ρ^{out}: Tugboat preparation time after completing outbound service
- Δ_i^{early}: Maximum allowable early arrival periods for vessel i
- Δ_i^{late}: Maximum allowable late arrival periods for vessel i

Cost Parameters:

- α_i: Priority weight for vessel i
- β_i: Unit waiting cost for vessel i
- γ_i: Unit cost for just-in-time (JIT) deviation for vessel i
- c_k: Unit time period usage cost for tugboat k

Horsepower Parameters:

- P_k: Horsepower of tugboat k
- P_i^{req}: Minimum required tugboat horsepower for vessel i

System Parameters:

- $H_{\max}$: Maximum number of tugboats allowed per service operation
- ϵ_{time}: Maximum allowable time deviation for sequencing constraints
- M: Large penalty parameter for constraint relaxation
- $\lambda_1, \lambda_2, \lambda_3, \lambda_4$: Weight coefficients in the objective function

1.2.3. Decision Variables

- $x_{ijt} \in \{0, 1\}$: Binary variable equal to 1 if vessel i begins berthing at berth j at time period t (with $C_j \geq S_i$), and 0 otherwise
- $y_{ikt}^{\text{in}} \in \{0, 1\}$: Binary variable equal to 1 if tugboat k initiates inbound service for vessel i at time period t, and 0 otherwise
- $y_{ikt}^{\text{out}} \in \{0, 1\}$: Binary variable equal to 1 if tugboat k initiates outbound service for vessel i at time period t, and 0 otherwise
- $z_{it}^{\text{in}} \in \{0, 1\}$: Auxiliary binary variable equal to 1 if vessel i begins inbound tugboat service at time period t, and 0 otherwise

- $z_{it}^{\text{out}} \in \{0, 1\}$: Auxiliary binary variable equal to 1 if vessel i begins outbound tugboat service at time period t, and 0 otherwise
- $u_i^{\text{early}} \geq 0$: Continuous variable representing the early arrival time of vessel i relative to ETA_i
- $u_i^{\text{late}} \geq 0$: Continuous variable representing the late arrival time of vessel i relative to ETA_i

1.3. Mathematical Formulation

1.3.1. Objective Function

The objective function balances service coverage, operational efficiency, schedule adherence, and tugboat utilization through a weighted combination of cost components. The objective minimizes the weighted sum of four cost components:

$$\min Z = \lambda_1 Z_1 + \lambda_2 Z_2 + \lambda_3 Z_3 + \lambda_4 Z_4 \tag{1}$$

where Z_1 penalizes vessels that cannot be assigned to any berth:

$$Z_1 = \sum_{i \in \mathcal{I}} M \cdot \alpha_i \left(1 - \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} x_{ijt}\right) \tag{2}$$

The term Z_2 captures the total time each vessel spends in the port system, from inbound tugboat service initiation to outbound service completion:

$$Z_2 = \sum_{i \in \mathcal{I}} \alpha_i \beta_i \left[\sum_{t \in \mathcal{T}} (t + \tau_i^{\text{out}}) z_{it}^{\text{out}} - \sum_{t \in \mathcal{T}} t \cdot z_{it}^{\text{in}}\right] \tag{3}$$

The term Z_3 penalizes deviations from the expected arrival time, where u_i^{early} and u_i^{late} are linearization variables for absolute deviation:

$$Z_3 = \sum_{i \in \mathcal{I}} \alpha_i \gamma_i \left(u_i^{\text{early}} + u_i^{\text{late}}\right) \tag{4}$$

The term Z_4 accounts for the operational cost of tugboat deployment across all services:

$$Z_4 = \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} c_k \left(\tau_i^{\text{in}} y_{ikt}^{\text{in}} + \tau_i^{\text{out}} y_{ikt}^{\text{out}}\right) \tag{5}$$

1.3.2. Constraints

Each vessel is assigned to at most one berth at one time period:

$$\sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} x_{ijt} \leq 1, \quad \forall i \in \mathcal{I} \tag{6}$$

Vessels can only be assigned to berths with sufficient capacity:

$$x_{ijt} = 0, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}, t \in \mathcal{T} : C_j < S_i \tag{7}$$

Inbound tugboat service must be provided if and only if the vessel is assigned to a berth:

$$\sum_{t \in \mathcal{T}} z_{it}^{\text{in}} = \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} x_{ijt}, \quad \forall i \in \mathcal{I} \tag{8}$$

Outbound tugboat service must be provided if and only if the vessel is assigned to a berth:

$$\sum_{t \in \mathcal{T}} z_{it}^{\text{out}} = \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} x_{ijt}, \quad \forall i \in \mathcal{I} \tag{9}$$

The total horsepower of tugboats assigned to inbound service must meet vessel requirements:

$$\sum_{k \in \mathcal{K}} P_k y_{ikt}^{\text{in}} \geq P_i^{\text{req}} z_{it}^{\text{in}}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{10}$$

The total horsepower of tugboats assigned to outbound service must meet vessel requirements:

$$\sum_{k \in \mathcal{K}} P_k y_{ikt}^{\text{out}} \geq P_i^{\text{req}} z_{it}^{\text{out}}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{11}$$

The number of tugboats assigned to inbound service cannot exceed system limits:

$$\sum_{k \in \mathcal{K}} y_{ikt}^{\text{in}} \leq H_{\max} \cdot z_{it}^{\text{in}}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{12}$$

The number of tugboats assigned to outbound service cannot exceed system limits:

$$\sum_{k \in \mathcal{K}} y_{ikt}^{\text{out}} \leq H_{\max} \cdot z_{it}^{\text{out}}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{13}$$

The auxiliary variable z_{it}^{in} can only be active if at least one tugboat is assigned:

$$z_{it}^{\text{in}} \leq \sum_{k \in \mathcal{K}} y_{ikt}^{\text{in}}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{14}$$

The auxiliary variable z_{it}^{out} can only be active if at least one tugboat is assigned:

$$z_{it}^{\text{out}} \leq \sum_{k \in \mathcal{K}} y_{ikt}^{\text{out}}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{15}$$

Each berth can accommodate at most one vessel at any time, accounting for operation duration:

$$\sum_{i \in \mathcal{I}} \sum_{\tau=\max(1, t-D_i+1)}^{t} x_{ij\tau} \leq 1, \quad \forall j \in \mathcal{J}, t \in \mathcal{T} \tag{16}$$

Each tugboat can handle at most one service operation at any time, including service duration and preparation time:

$$\sum_{i \in \mathcal{I}} \left(\sum_{\tau=\max(1, t-\tau_i^{\text{in}}-\rho^{\text{in}}+1)}^{t} y_{ik\tau}^{\text{in}} + \sum_{\tau=\max(1, t-\tau_i^{\text{out}}-\rho^{\text{out}}+1)}^{t} y_{ik\tau}^{\text{out}} \right) \leq 1, \quad \forall k \in \mathcal{K}, t \in \mathcal{T} \tag{17}$$

Tugboat services can only begin within the allowable time window around the expected arrival time:

$$y_{ikt}^{\text{in}} = 0, \quad \forall i \in \mathcal{I}, k \in \mathcal{K}, t < \text{ETA}_i - \Delta_i^{\text{early}} \text{ or } t > \text{ETA}_i + \Delta_i^{\text{late}} \tag{18}$$

Inbound service must complete before or shortly after berthing begins, with allowable deviation ϵ_{time}:

$$0 \leq \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} t \cdot x_{ijt} - \sum_{t \in \mathcal{T}} (t + \tau_i^{\text{in}}) \cdot z_{it}^{\text{in}} \leq \epsilon_{\text{time}} \cdot \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} x_{ijt}, \quad \forall i \in \mathcal{I} \tag{19}$$

Outbound service must begin at or shortly after berthing operations complete, with allowable deviation ϵ_{time}:

$$0 \leq \sum_{t \in \mathcal{T}} t \cdot z_{it}^{\text{out}} - \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} (t + D_i) \cdot x_{ijt} \leq \epsilon_{\text{time}} \cdot \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} x_{ijt}, \quad \forall i \in \mathcal{I} \tag{20}$$

The following constraint linearizes the absolute deviation from ETA by decomposing it into early and late components:

$$\sum_{t \in \mathcal{T}} t \cdot z_{it}^{\text{in}} = \text{ETA}_i + u_i^{\text{late}} - u_i^{\text{early}}, \quad \forall i \in \mathcal{I} \tag{21}$$

Finally, the variable domains are defined as:

$$\begin{aligned} x_{ijt}, y_{ikt}^{\text{in}}, y_{ikt}^{\text{out}}, z_{it}^{\text{in}}, z_{it}^{\text{out}} &\in \{0, 1\}, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K}, t \in \mathcal{T} \\ u_i^{\text{early}}, u_i^{\text{late}} &\geq 0, \quad \forall i \in \mathcal{I} \end{aligned} \tag{22}$$

1.4. Problem Characteristics

The PSP exhibits several distinctive characteristics that distinguish it from classical combinatorial optimization problems:

1. **Multi-resource Coordination**: The problem requires simultaneous coordination of spatial resources (berths), mobile resources (tugboats), and temporal resources (scheduling slots), creating complex interdependencies absent in simpler allocation problems.

2. **Dual Service Requirements**: Each vessel requires two distinct tugboat service operations (inbound and outbound) with different timing constraints, resource requirements, and preparation times, introducing asymmetric temporal dependencies.

3. **Horsepower Aggregation**: Multiple tugboats can be assigned to a single vessel to meet horsepower requirements, requiring discrete combinatorial decisions about tugboat coalition formation.

These structural features create a problem class that is unlikely to appear in standard optimization benchmarks or LLM training corpora, making PSP an ideal test case for evaluating zero-shot generalization capabilities of automated algorithm design frameworks. Due to its scale and combinatorial complexity, PSP instances are not intended to be solved to proven optimality; instead, they serve as a testbed for evaluating algorithmic feasibility, robustness, and quality-time trade-offs under practical computational constraints.

1.5. Instance Generation

Test instances were generated with the following systematic approach to ensure realistic operational characteristics.

Instance naming convention: `{vessels}_{berths}_{tugboats}_T{horizon}`, where the horizon parameter determines the temporal granularity (T=24: 1-hour periods; T=48: 30-minute periods).

Vessel characteristics: Four vessel types (container, bulk, tanker, general cargo) with sizes distributed according to realistic port traffic patterns. Vessel sizes range from 1 to 5, with corresponding horsepower requirements from 800–6000 HP. Expected arrival times follow clustered distributions (70% in early periods, 30% distributed through remaining horizon) to reflect typical traffic patterns.

Berth configuration: Berth capacities range from 2 to 5, with approximately 50% of berths capable of accommodating large vessels (capacity ≥ 4) to ensure sufficient matching options for heterogeneous vessel sizes.

Tugboat fleet: Tugboat horsepower ranges from 800–5000 HP, distributed across four categories: small (800–1500 HP), medium (1500–2500 HP), medium-large (2500–3500 HP), and large (3500–5000 HP). Fleet composition ensures that high-horsepower vessels can be serviced through multi-tugboat combinations (maximum 2–3 tugboats per service).

Cost parameters: Penalty parameter M and objective weights $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are automatically calibrated to balance the four objective components, ensuring that unassigned penalties, port time costs, ETA deviation costs, and tugboat utilization costs contribute comparably to the optimization landscape.

Scale categories: Instances span three scales—Small (100–200 vessels), Medium (250–300 vessels), and Large (400–500 vessels)—with time horizons of 24–48 periods and proportional resource allocation to maintain consistent capacity ratios across problem sizes.

All test instances are available in the supplementary materials repository.

References

Bierwirth, C., Meisel, F., 2010. A survey of berth allocation and quay crane scheduling problems in container terminals. European Journal of Operational Research 202, 615–627.

UNCTAD, 2023. Review of Maritime Transport 2023. Technical Report. United Nations Conference on Trade and Development. Geneva.

Wei, X., Lau, H.C., Xiao, Z., Fu, X., Zhang, X., Qin, Z., 2025. Bi-objective dynamic tugboat scheduling with speed optimization under stochastic and time-varying service demands. Transportation Research Part E: Logistics and Transportation Review 193, 103876.

Zhang, Q., Zheng, X., Diabat, A., 2019. A model for tugboat scheduling in container terminals. Maritime Policy & Management 46, 991–1009.