流行病學與生物統計計算 Homework 11
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 # Homework7

#EX 22-1

 # n = 30

```
    # n = 30
n <- 30
norep <- 1000
beta <- c(-6, 1, 0.005)
y <- c()
mle30 <- matrix(NA, norep, 3)
for (i in 1 : norep) {
   set.seed(i)
   gpa <- rnorm(n = n, mean = 3.1, sd = 0.3)
   gre <- rnorm(n = n, mean = 580, sd = 80)
   linear <- beta[1] + beta[2] * gpa + beta[3] * gre
   pii <- exp(linear) / (1 + exp(linear))
   x <- cbind(rep(1, n), gpa, gre)
   for (k in 1 : n) {
      y[k] <- sample(c(0, 1), 1, c(1 - pii[k], pii[k]), replace = FALSE)
   }
   ftn <- function(betacoef) {
      pi1 <- exp(x %*% betacoef) / (1 + exp(x %*% betacoef))
      gradient <- t(x) %*% (y - pi1)
      hessian <- -t(x) %*% diag(c(pi1 * (1 - pi1)), n) %*% x
      return(list(gradient, hessian))
   }

   highnew <- function(ftn, x0, tol, maxiter) {
      x <- x0
      fx <- ftn(x)
      iter <- 0

      while ((max(abs(fx[[1]])) > tol) && (iter < maxiter)) {
```

```
            x <- x - (solve(fx[[2]]) %*% fx[[1]])
            fx <- ftn(x)
            iter <- iter + 1
        }

        if (max(abs(fx[[1]])) > tol) {
            cat("algorithm failed to converge\n")
            return(NA)
        } else {
            cat("algorithm converges\n")
            return(x)
        }
    }
    mle30[i, ] <- highnew(ftn, x0 = c(0, 0, 0), tol = 1e-9, maxiter = 100)
}

mle30
meanmle30 <- colSums(mle30) / norep
(bias30 <- meanmle30 - beta)
```

MLE of regression coefficient

Bias of MLE of regression coefficient

```
> (meanmle30 <- colSums(mle30) / norep)
[1] -7.25114905  1.18544476  0.00620335
> (bias30 <- meanmle30 - beta)
[1] -1.25114905  0.18544476  0.00120335
```

n = 230

```r
# n = 230
n <- 230
norep <- 1000
beta <- c(-6, 1, 0.005)
y <- c()
mle230 <- matrix(NA, norep, 3)
for (i in 1 : norep) {
   set.seed(i)
   gpa <- rnorm(n = n, mean = 3.1, sd = 0.3)
   gre <- rnorm(n = n, mean = 580, sd = 80)
   linear <- beta[1] + beta[2] * gpa + beta[3] * gre
   pii <- exp(linear) / (1 + exp(linear))
   x <- cbind(rep(1, n), gpa, gre)
   for (k in 1 : n) {
      y[k] <- sample(c(0, 1), 1, c(1 - pii[k], pii[k]), replace = FALSE)
   }
   ftn <- function(betacoef) {
      pi1 <- exp(x %*% betacoef) / (1 + exp(x %*% betacoef))
      gradient <- t(x) %*% (y - pi1)
      hessian <- -t(x) %*% diag(c(pi1 * (1 - pi1)), n) %*% x
      return(list(gradient, hessian))
   }

   highnew <- function(ftn, x0, tol, maxiter) {
      x <- x0
      fx <- ftn(x)
      iter <- 0

      while ((max(abs(fx[[1]])) > tol) && (iter < maxiter)) {
         x <- x - (solve(fx[[2]]) %*% fx[[1]])
         fx <- ftn(x)
         iter <- iter + 1
      }

      if (max(abs(fx[[1]])) > tol) {
```

```
            cat("algorithm failed to converge\n")
            return(NA)
        } else {
            cat("algorithm converges\n")
            return(x)
        }
    }
    mle230[i, ] <- highnew(ftn, x0 = c(0, 0, 0), tol = 1e-9, maxiter = 100)
}

mle230
meanmle230 <- colSums(mle230) / norep
(bias230 <- meanmle230 - beta)
```

MLE of regression coefficient

Bias of MLE of regression coefficient

```
> (meanmle230 <- colSums(mle230) / norep)
[1] -6.130837632  1.022874614  0.005112127
> (bias230 <- meanmle230 - beta)
[1] -0.1308376319  0.0228746136  0.0001121271
```

n = 430

```r
# n = 430
n <- 430
norep <- 1000
beta <- c(-6, 1, 0.005)
y <- c()
mle430 <- matrix(NA, norep, 3)
for (i in 1 : norep) {
   set.seed(i)
   gpa <- rnorm(n = n, mean = 3.1, sd = 0.3)
   gre <- rnorm(n = n, mean = 580, sd = 80)
   linear <- beta[1] + beta[2] * gpa + beta[3] * gre
   pii <- exp(linear) / (1 + exp(linear))
   x <- cbind(rep(1, n), gpa, gre)
   for (k in 1 : n) {
      y[k] <- sample(c(0, 1), 1, c(1 - pii[k], pii[k]), replace = FALSE)
   }
   ftn <- function(betacoef) {
      pi1 <- exp(x %*% betacoef) / (1 + exp(x %*% betacoef))
      gradient <- t(x) %*% (y - pi1)
      hessian <- -t(x) %*% diag(c(pi1 * (1 - pi1)), n) %*% x
      return(list(gradient, hessian))
   }

   highnew <- function(ftn, x0, tol, maxiter) {
      x <- x0
      fx <- ftn(x)
      iter <- 0

      while ((max(abs(fx[[1]])) > tol) && (iter < maxiter)) {
         x <- x - (solve(fx[[2]]) %*% fx[[1]])
         fx <- ftn(x)
         iter <- iter + 1
      }

      if (max(abs(fx[[1]])) > tol) {
```

```
            cat("algorithm failed to converge\n")
            return(NA)
        } else {
            cat("algorithm converges\n")
            return(x)
        }
    }
    mle430[i, ] <- highnew(ftn, x0 = c(0, 0, 0), tol = 1e-9, maxiter = 100)
}

mle430
meanmle430 <- colSums(mle430) / norep
(bias430 <- meanmle430 - beta)
```

MLE of regression coefficient

Bias of MLE of regression coefficient

```
> (meanmle430 <- colSums(mle430) / norep)
[1] -6.09723871  1.01552158  0.00509303
> (bias430 <- meanmle430 - beta)
[1] -9.723871e-02  1.552158e-02  9.303015e-05
```

 # n = 630

```r
# n = 630
n <- 630
norep <- 1000
beta <- c(-6, 1, 0.005)
y <- c()
mle630 <- matrix(NA, norep, 3)
for (i in 1 : norep) {
   set.seed(i)
   gpa <- rnorm(n = n, mean = 3.1, sd = 0.3)
   gre <- rnorm(n = n, mean = 580, sd = 80)
   linear <- beta[1] + beta[2] * gpa + beta[3] * gre
   pii <- exp(linear) / (1 + exp(linear))
   x <- cbind(rep(1, n), gpa, gre)
   for (k in 1 : n) {
      y[k] <- sample(c(0, 1), 1, c(1 - pii[k], pii[k]), replace = FALSE)
   }
   ftn <- function(betacoef) {
      pi1 <- exp(x %*% betacoef) / (1 + exp(x %*% betacoef))
      gradient <- t(x) %*% (y - pi1)
      hessian <- -t(x) %*% diag(c(pi1 * (1 - pi1)), n) %*% x
      return(list(gradient, hessian))
   }

   highnew <- function(ftn, x0, tol, maxiter) {
      x <- x0
      fx <- ftn(x)
      iter <- 0

      while ((max(abs(fx[[1]])) > tol) && (iter < maxiter)) {
         x <- x - (solve(fx[[2]]) %*% fx[[1]])
         fx <- ftn(x)
         iter <- iter + 1
      }

      if (max(abs(fx[[1]])) > tol) {
```

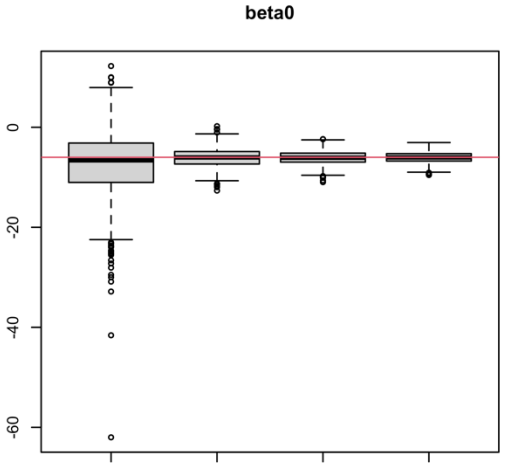
```
            cat("algorithm failed to converge\n")
            return(NA)
        } else {
            cat("algorithm converges\n")
            return(x)
        }
    }
    mle630[i, ] <- highnew(ftn, x0 = c(0, 0, 0), tol = 1e-9, maxiter = 100)
}

mle630
meanmle630 <- colSums(mle630) / norep
(bias630 <- meanmle630 - beta)
```

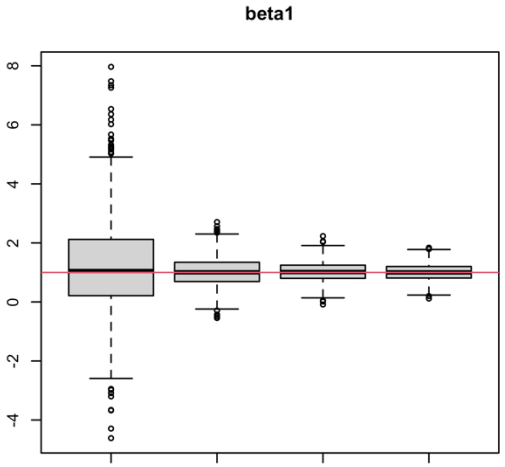
MLE of regression coefficient

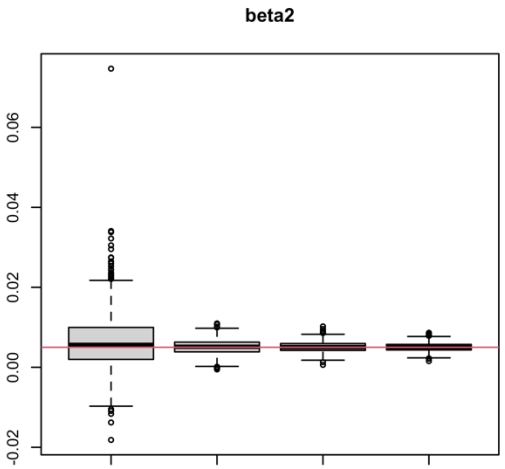
Bias of MLE of regression coefficient

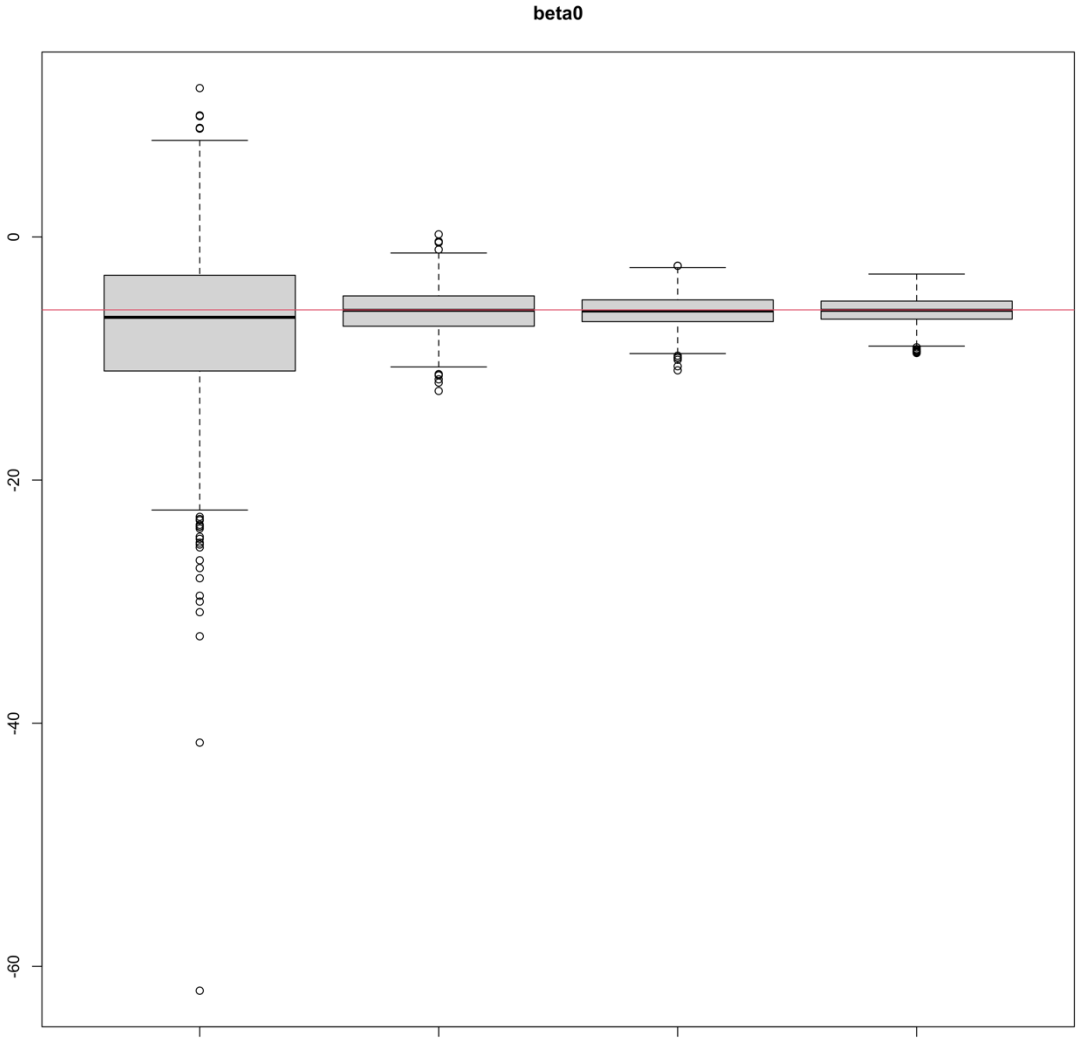
```
> (meanmle630 <- colSums(mle630) / norep)
[1] -6.038478311  1.007797278  0.005027782
> (bias630 <- meanmle630 - beta)
[1] -3.847831e-02  7.797278e-03  2.778161e-05
```

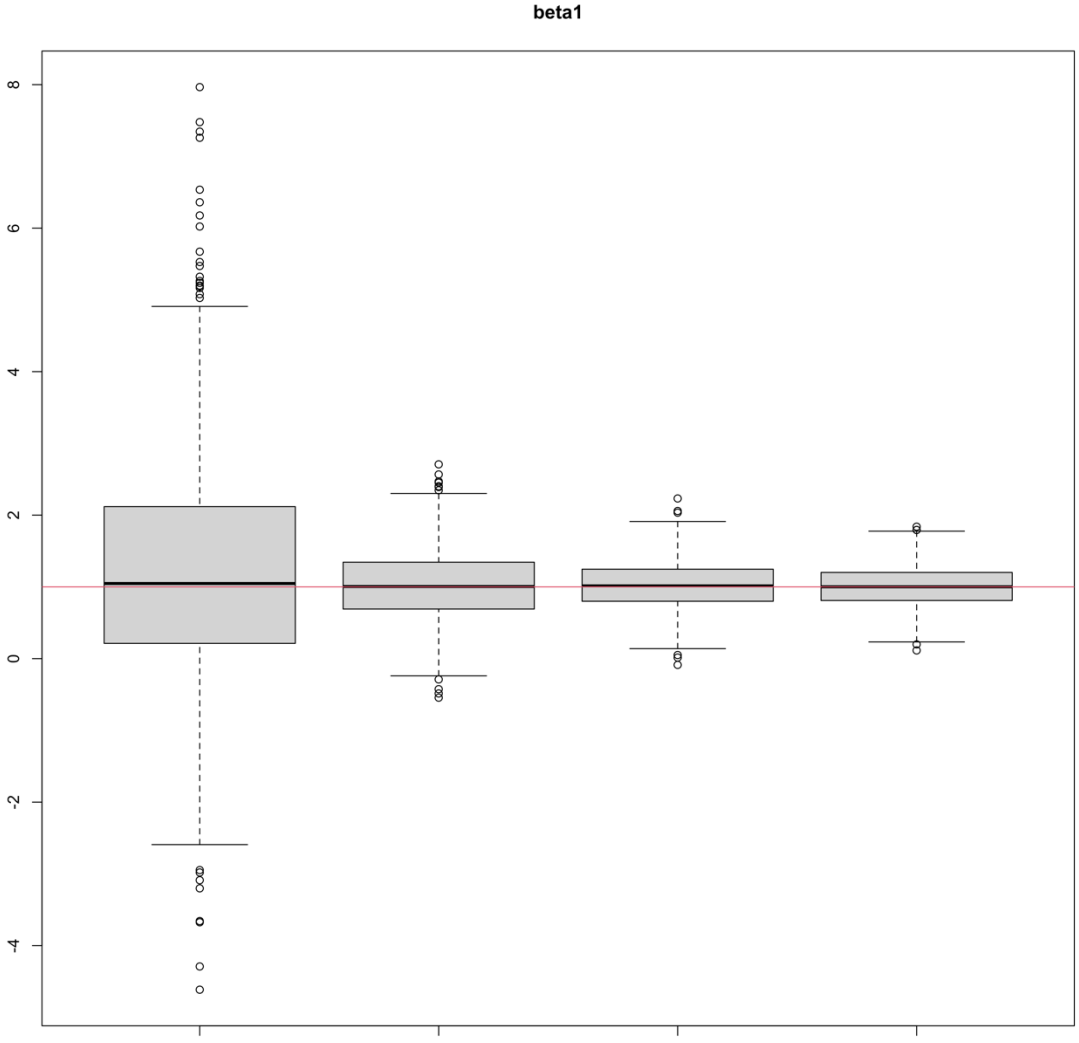
```
# plotting
setwd("/Users/raymond/Desktop/")
png(filename = "logistic1.png", width = 2000, height = 6000, res = 500)
par(mfrow = c(3, 1))
boxplot(mle30[, 1], mle230[, 1], mle430[, 1], mle630[, 1], main = "beta0")
abline(h = beta[1], col = 2)
boxplot(mle30[, 2], mle230[, 2], mle430[, 2], mle630[, 2], main = "beta1")
abline(h = beta[2], col = 2)
boxplot(mle30[, 3], mle230[, 3], mle430[, 3], mle630[, 3], main = "beta2")
abline(h = beta[3], col = 2)
dev.off()

png(filename = "logistic2.png", width = 6000, height = 6000, res = 500, main =
"beta0")
par(mfrow = c(1, 1))
boxplot(mle30[, 1], mle230[, 1], mle430[, 1], mle630[, 1])
abline(h = beta[1], col = 2)
dev.off()
png(filename = "logistic3.png", width = 6000, height = 6000, res = 500, main =
"beta1")
boxplot(mle30[, 2], mle230[, 2], mle430[, 2], mle630[, 2])
abline(h = beta[2], col = 2)
dev.off()
png(filename = "logistic4.png", width = 6000, height = 6000, res = 500, main =
"beta2")
boxplot(mle30[, 3], mle230[, 3], mle430[, 3], mle630[, 3])
abline(h = beta[3], col = 2)
dev.off()
```

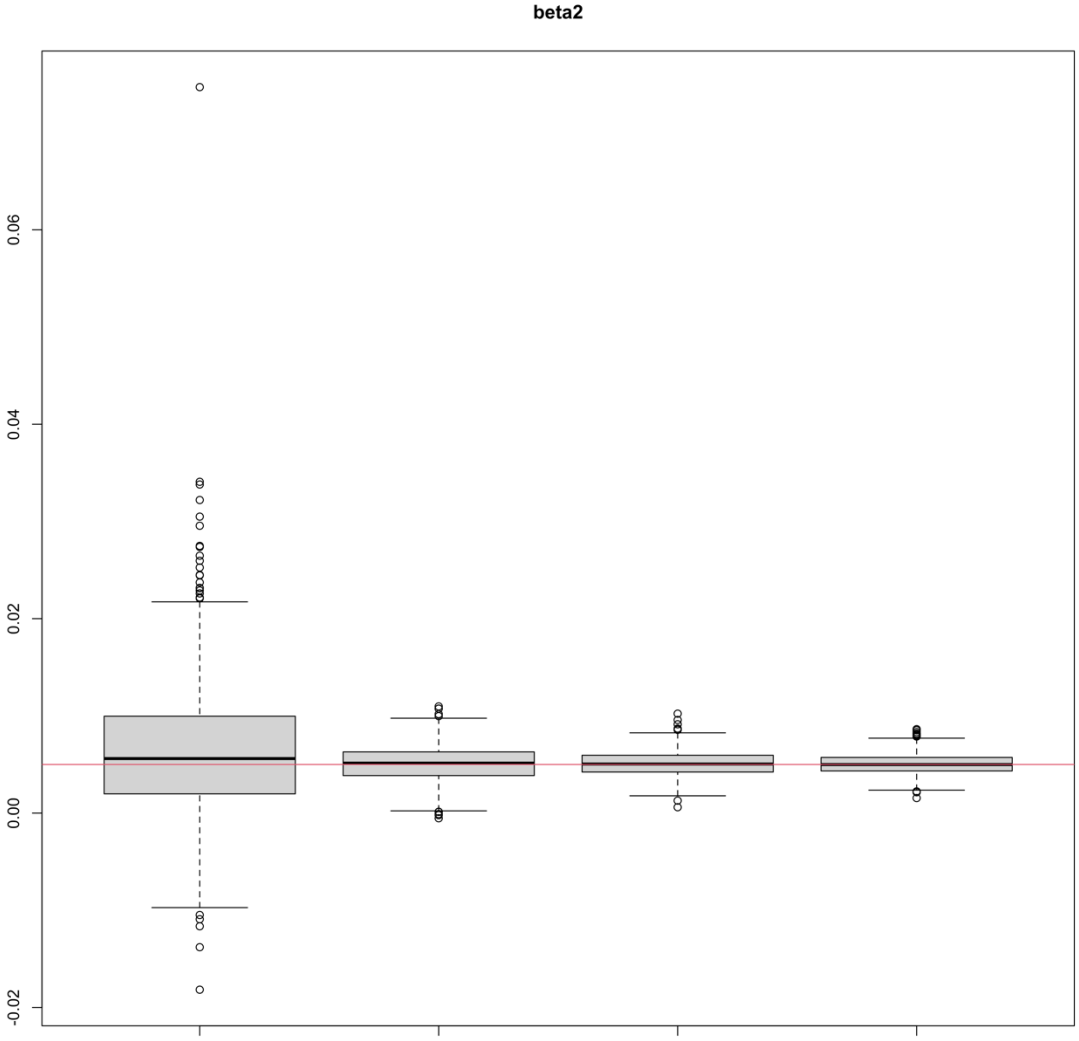

beta0
0
-20
-40
-60


beta1
8
6
4
2
0
-2
-4


beta2
0.06
0.04
0.02
0.00
-0.02


beta0
0
-20
-40
-60


beta1
8
6
4
2
0
-2
-4


beta2
0.06
0.04
0.02
0.00
-0.02