

Homework4

#Q1 : exact 95% CI

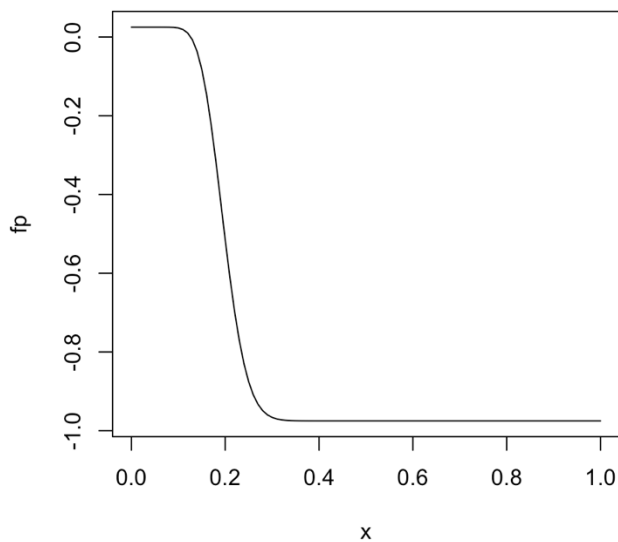
PI

```
# exact 95% CI

# PI :  $P(X = 0) + P(X = 1) + \dots + P(X = 19) = 0.975$ 

# step1 : find initial value
p <- seq(0, 1, by = 0.001)
fp <- function(p) {
  y <- -0.975
  for (k in 0:19) {
    y <- y + choose(100, k) * p^k * (1 - p)^(100 - k)
  }
  return(y)
}

png(filename = "hwk6.2.png", width = 1000, height = 1000, res = 200)
plot(fp)
dev.off()
```



```

# step 2 : find  $f(p)$ ,  $f'(p)$ 
ftn <- function(p) {
  y <- -0.975
  dydp <- 0

  for (k in 0:19) {
    y <- y + choose(100, k) * (p^k) * ((1 - p)^(100 - k))

    dydp <- dydp + choose(100, k) * ((k * p^(k - 1) * (1 - p)^(100 - k))
    - (p^k * (100 - k) * (1 - p)^(99 - k)))
  }

  return(c(y, dydp))
}

```

```

# step 3 : Newton-Raphson function
root <- function(ftn, x0, tol, max_iter) {
  x <- x0
  y <- ftn(x)
  iter <- 0
  while ((abs(y[1]) > tol) && (iter < max_iter)) {
    x <- x - y[1] / y[2]
    y <- ftn(x)
    iter <- iter + 1
    cat("at iteration", iter, "value of x is", x, "\n")
  }
  if (abs(y[1] > tol)) {
    cat("algorithm failed to converge\n")
    return(NULL)
  } else {
    cat("algorithm converges to \n")
    return(x)
  }
}

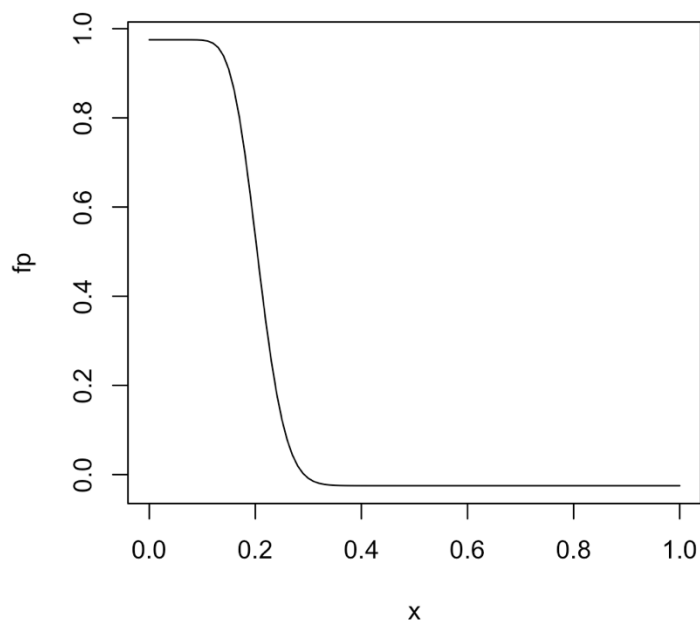
```

```
root(ftn, x0 = 0.1, tol = 1e-09, max_iter = 100)
# Pl = 0.1266556
```

```
> root(ftn, x0 = 0.1, tol = 1e-09, max_iter = 100)
at iteration 1 value of x is 0.1982993
at iteration 2 value of x is 0.1485413
at iteration 3 value of x is 0.1340764
at iteration 4 value of x is 0.1279234
at iteration 5 value of x is 0.1267005
at iteration 6 value of x is 0.1266556
at iteration 7 value of x is 0.1266556
algorithm converges to
[1] 0.1266556
```

Pu

```
# Pu :  $P(X = 0) + P(X = 1) + \dots + P(X = 20) = 0.025$ 
# find initial value
p <- seq(0, 1, by = 0.001)
fp <- function(p) {
  y <- -0.025
  for (k in 0:20) {
    y <- y + choose(100, k) * p^k * (1 - p)^(100 - k)
  }
  return(y)
}
png(filename = "hwk6.2.png", width = 1000, height = 1000, res = 200)
plot(fp)
dev.off()
```



```

# step2 : find  $f(p)$ ,  $f'(p)$ 
ftn <- function(p) {
  y <- -0.025
  dydp <- 0

  for (k in 0:20) {
    y <- y + choose(100, k) * (p^k) * ((1 - p)^(100 - k))

    dydp <- dydp + choose(100, k) * ((k * p^(k - 1) * (1 - p)^(100 - k))
    - (p^k * (100 - k) * (1 - p)^(99 - k)))
  }

  return(c(y, dydp))
}

```

```

# step3 : Newton-Raphson function
root <- function(ftn, x0, tol, max_iter) {
  x <- x0
  y <- ftn(x)
  iter <- 0
  while ((abs(y[1]) > tol) && (iter < max_iter)) {
    x <- x - y[1] / y[2]
    y <- ftn(x)
    iter <- iter + 1
    cat("at iteration", iter, "value of x is", x, "\n")
  }
  if (abs(y[1] > tol)) {
    cat("algorithm failed to converge\n")
    return(NULL)
  } else {
    cat("algorithm converges to \n")
    return(x)
  }
}

```

```
root(ftn, x0 = 0.3, tol = 1e-09, max_iter = 100)
```

```
# Pu = 0.2918427
```

```
# exact 95%CI : 0.1266556-0.2918427
```

```
> root(ftn, x0 = 0.3, tol = 1e-09, max_iter = 100)
at iteration 1 value of x is 0.2901394
at iteration 2 value of x is 0.2917824
at iteration 3 value of x is 0.2918426
at iteration 4 value of x is 0.2918427
algorithm converges to
[1] 0.2918427
```

#Q1 : asymptotic 95% CI

```
# asymptotic 95% CI
```

```
n <- 100
```

```
phat <- (20 / n)
```

```
pl <- phat - (qnorm(0.975) * sqrt(((phat) * (1 - phat)) / n))
```

```
pu <- phat + (qnorm(0.975) * sqrt(((phat) * (1 - phat)) / n))
```

```
(ci <- c(pl, pu))
```

```
# asymptotic 95%CI : 0.1216014-0.2783986
```