CAS CS 350 HW4

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TOTAL POINTS

95.5 / 100

QUESTION 1

1 Q1 **30 / 30**

✓ **- 0 pts** Correct

part a)

- **1 pts** Minor error
- **5 pts** Incorrect $$\lambda$$ or $$T_q$$
- **5 pts** Incorrect capacity

part b)

- **1 pts** Minor error
- **2 pts** Incorrect $$\rho$$
- **5 pts** Incorrect

part c)

- **2 pts** Minor error
- **5 pts** Partially correct
- **10 pts** Incorrect

part d)

- **1 pts** Minor error
- **2.5 pts** Incorrect part 1
- **2.5 pts** Incorrect part 2

QUESTION 2

2 Q2 **30 / 30**

✓ **- 0 pts** Correct

- **30 pts** Missing

part a)

- **2 pts** Incorrect or Missing

part b)

- **3 pts** Incorrect $$q_1$$
- **3 pts** Incorrect $$q_2$$
- **2 pts** Incorrect $$q_{tot}$$

part c)

- **2 pts** Incorrect $$P(S_5)$$
- **2 pts** Incorrect computation time

part d)

- **2 pts** Incorrect $$T_{q, 1}$$
- **3 pts** Incorrect $$\lambda_{admit}$$
- **3 pts** Incorrect $$T_{q, 2}$$
- **2 pts** Incorrect $$T_{q, admit}$$

part e)

- **2 pts** Partially correct
- **4 pts** Incorrect or Missing

Late

- **3.25 pts** Click here to replace this description.
- **3.75 pts** Click here to replace this description.
- **4.25 pts** Click here to replace this description.
- **4.5 pts** Click here to replace this description.
- **5 pts** Click here to replace this description.
- **5.5 pts** Click here to replace this description.
- **6 pts** Click here to replace this description.
- **7 pts** Click here to replace this description.
- **7.5 pts** Click here to replace this description.

QUESTION 3

3 Q3 **35.5 / 40**

part a)

✓ **- 0 pts** Correct

- **6 pts** Missing/Incomplete
- **1 pts** Error in Diagram
- **1 pts** No mention of Jackson's Theorem
- **1 pts** Missing a key assumption
- **3 pts** Partially Correct
- **1 pts** Does not show what type of queuing model is used
- **1 pts** Missing probabilities
- **3 pts** missing diagram

part b)

✓ - **0 pts** **Correct**

- **6 pts** Missing/Incomplete

- **3 pts** Incorrect with work shown

part c)

✓ - **0 pts** **Correct**

- **6 pts** Missing/Incomplete

- **3 pts** Incorrect with work shown

part d)

✓ - **0 pts** **Correct**

- **8 pts** Missing/Incomplete

- **4 pts** Incorrect with work shown

part e)

- **0 pts** Correct

- **9 pts** Missing/Incomplete/Incorrect with no work

✓ - **2 pts** **Incorrect Tq idle/missing**

✓ - **2 pts** **Incorrect Tq tot/missing**

✓ - **0.5 pts** **Incorrect slowdown**

part f)

✓ - **0 pts** **Correct**

- **5 pts** Missing/Incomplete/Incorrect with no work

- **2.5 pts** Incorrect with work shown

- **10 pts** Late Penalty

- **40 pts** Missing

Problem 1

a) - $\lambda = 10$ req / sec $\times 1500$ (Poisson)

- request rejected/dropped $\rightarrow$ client disconnects
- per-req latency avg > 0.5 ms $\rightarrow$ everyone experiences lags (T_q)
- need to find T_s

① convert ms $\rightarrow$ sec

$0.5 \text{ ms} = 0.0005 \text{ sec}$

new variables:

- $\lambda = 15{,}000$ req/sec
- $T_q = 0.0005$ sec

② solve for q

$q = \lambda \cdot T_q$ plug in!

$q = 15{,}000 \times 0.0005$

$q = 7.5$

③ solve for p

$p = \frac{q}{q+1}$ plug in!

$p = \frac{7.5}{7.5+1}$

$p = \frac{7.5}{8.5} = 0.882$

$1-p \left(q = \frac{p}{1-p} \right) 1-p$

$q - \cancel{pq} = p$

$+\cancel{pq} \quad +pq$

$q = p + pq$

$\frac{q = p(\cancel{1+q})}{1+q \quad \cancel{1+q}}$

$p = \frac{q}{1+q}$

④ solve for Ts

$T_q = \frac{T_s}{1-p}$

$0.0005 = \frac{T_s}{1-0.882}$

$.118\left(\cancel{0.0005} = \frac{T_s}{0.1\cancel{18}} \right) \cancel{0.118}$

$T_s = 0.000059$

Problem 1 continued...

(5) solve for capacity

$$\text{capacity} = \frac{1}{T_s}$$ plug in!

$$\text{capacity} = \frac{1}{0.000059}$$

$\boxed{\text{capacity} = 16{,}949.1525 \text{, which is about } 17{,}000}$

b) $T_s = 0.06$ ms $\rightarrow$ 0.00006 secs

$p = 0.81$

$p = \lambda \cdot T_s$

↳ 100

(1) plug in & solve!

$$\frac{0.81}{0.00006} = \frac{100 \cdot 0.00006}{0.00006}$$

$$\frac{10U}{10} = \frac{13{,}500}{10}$$ $\boxed{U = 1{,}350 \text{ active users}}$

c) avg T_s (per request) = 0.07 ms

1500 active users

(K) capacity of queue: 10 - 15

} ~~guarantees constraint~~ on latency (< 0.5 ms)

M/M/1/K

(10-15)

rejecting < 11% of requests

↳ Pr(rejection) = 0.11

$$Pr(s_i) = \frac{(1-p)\,p^i}{1-p^{k+1}}$$

(1) solve for p

$p = \lambda \cdot T_s$

$p = 15 \cdot 0.07 = 1.05$

(2) solve using diff k's & see if conditions hold for any

k = 10

$$0.11 \geq \frac{(1-1.05)(1.05^{10})}{1-1.05^{(10)+1}}$$

$$0.11 \geq \frac{(-0.05)(1.6289)}{-0.7103} = \frac{-0.0814}{-0.7103} = 0.1146 \quad X$$

Not possible for k = 10

Problem 1 continued...

c) $K = 11$

$$0.11 \geq \frac{(1-1.05)(1.05^{11})}{1-1.05^{(11)+1}}$$

$$0.11 \geq \frac{(-0.05)(1.7103)}{-0.7959} = \frac{-0.0855}{-0.7959} = 0.1075$$ ✓ (less than 11%)

2.1 Check if latency is < 0.5 ms

$$q = \frac{p}{1-p} - \frac{(K+1) \cdot p^{K+1}}{(1-p^{K+1})} = 6.08$$

$$Tq = \frac{q}{\lambda} = \frac{6.08}{15} = 0.4 \text{ ms}$$ ✓ (less than 0.5 ms)

Yes, when K=11 the rejection is 10.75% (which is less than 11%) and the latency is 0.4 ms (which is less than 0.5 ms). Therefore, it is possible to to have 1500 active users, with a cap of 10-15 requests to guarantee less than 0.5 ms latency and less than 11% rejection.

d) (1) no queuing

$$P(S_{i=0}) = \frac{(1-p)(p^i)}{1-p^{K+1}}$$ plug in!

$p = 1.05$

$$P(S_0) = \frac{(1-1.05)(1.05)^0}{1-(1.05^{(10)+1})} = \frac{-0.05}{-0.7103} = \boxed{0.0704 \rightarrow 7.04\%}$$

(2) 5 requests in the queue

$$P(S_{i=5}) = \frac{(1-p)(p^i)}{1-p^{K+1}}$$

$$P(S_5) = \frac{(1-1.05)(1.05)^5}{1-1.05^{(10)+1}} = \frac{(-.05)(1.2763)}{1-1.7103} = \frac{-0.0638}{-0.7103} = \boxed{0.0898}$$

↳ 8.98%

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part a)

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- **- 5 pts** Incorrect $$\lambda$$ or $$T_q$$
- **- 5 pts** Incorrect capacity

part b)

- **- 1 pts** Minor error
- **- 2 pts** Incorrect $$\rho$$
- **- 5 pts** Incorrect

part c)

- **- 2 pts** Minor error
- **- 5 pts** Partially correct
- **- 10 pts** Incorrect

part d)

- **- 1 pts** Minor error
- **- 2.5 pts** Incorrect part 1
- **- 2.5 pts** Incorrect part 2

Problem 2

a) $\lambda = 10$ requests/sec ← authentication server

$\lambda = 12$ requests/sec ← backend

↳ cap at 5

$\boxed{10 \text{ requests / sec}}$

b) ① get T_s

- Authentication server

$T_s(10 = \frac{1}{T_s})T_s$

$\frac{10T_s}{10} = \frac{1}{10}$

$T_s = \frac{1}{10}$

- Backend server

$T_s(12 = \frac{1}{T_s})T_s$

$\frac{12T_s}{12} = \frac{1}{12}$

$T_s = \frac{1}{12}$

② solve for ρ

Authentication server

$\rho = \lambda \cdot T_s$

$\rho = 8 \cdot \frac{1}{10}$

$\rho = 8/10 = 4/5$

Backend server

$\rho = \lambda \cdot T_s$

$\rho = 8 \cdot \frac{1}{12}$

$\rho = 8/12 = 4/6 = 2/3$

③ solve for q

Authentication server

$q = \frac{4/5}{1 - 4/5}$

$q = \frac{.8}{.2} = 4$

Backend server

$\frac{\rho}{1-\rho} - \frac{(K+1)\rho^{K+1}}{1-\rho^{K+1}} = q$

$\frac{2/3}{1-2/3} - \frac{(5+1)((2/3)^{(5)+1})}{1-(2/3)^{(5)+1}} = q$

$q = 1.4226$

$\boxed{q = 5.4226}$

c) $\lambda = 8$ req / sec $\times 100$ sec $= 800$

$$Pr(\text{rejection}) = \frac{(1-\rho)\rho^i}{1-\rho^{k+1}} = \frac{(1-2/3)(2/3)^5}{1-2/3^{(5+1)}} = 0.0481$$

$$80(0.0481) = \boxed{3.848 \text{ seconds}}$$

d) $\lambda' = \lambda \cdot (1 - Pr(\text{rejection}))$

$\lambda' = 8 \cdot (1 - 0.0481)$

$\lambda' = 7.6152$

$Tq = \frac{q}{\lambda'}$ plug in!

$Tq = \frac{1.4226}{7.6152}$

$Tq = 0.1868 + 0.5$

$\boxed{Tq = 0.6868}$

e) ① Find Tq of other server

$Tq = Tq_{S_1} \cdot P(S_k) + Tq_{success}(1 - P(S_k))$

$Tq = 0.5(0.0482) + 0.686(0.9518)$

$\boxed{Tq = 0.677}$

2 Q2 **30 / 30**

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part a)

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part c)

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part e)

- **- 2 pts** Partially correct
- **- 4 pts** Incorrect or Missing

Late

- **- 3.25 pts** Click here to replace this description.
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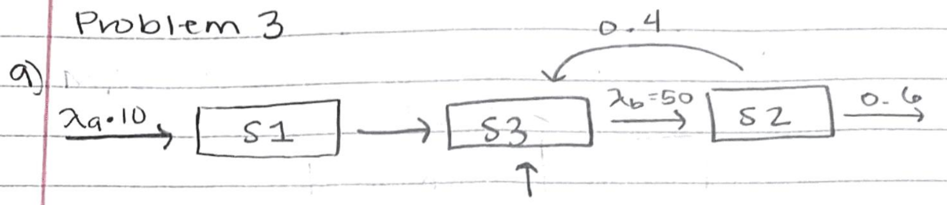
Problem 3

a)



$\lambda_a = 10$ req/sec (new users)

$\lambda_b = 50$ req/sec (well-known users)

$T_s(S_2) = 9$ ms $\rightarrow 0.009$ sec

$T_s(S_3) = 19$ ms $\rightarrow 0.019$ sec

$T_q(S_1) = 525$ ms $\rightarrow .525$ sec

Assumptions:

- use Jackson's theorem
- Poisson arrival times
- exponential interarrival times
- queue size is infinite
- can use little's law
- steady-state

b) $T_s(S_1)$

$T_q = T_w + T_s$

$-T_w \quad -T_w$

$T_s = T_q - T_w \qquad T_w = \frac{\rho T_s}{1-\rho}$

① ~~solve for ρ~~

$\rho = \lambda \cdot T_s \qquad q = \lambda \cdot T_q \qquad w = \lambda \cdot T_w$

$q = 10 \cdot 0.525 \qquad w = \frac{\rho^2}{1-\rho}$

$q = 5.25$

② ~~solve for q~~

$q = w + \rho \rightarrow \rho = \frac{q}{1+q} = \frac{5.25}{1+5.25} = \frac{5.25}{6.25} = 0.84$

③ solve for w

$w = q - \rho$

$w = 5.25 - 0.84$

$w = 4.41$

④ solve for T_w

$T_w = \frac{w}{\lambda} \rightarrow 4.41/10 = 0.441$

Problem 3 continued...

b) (5) solve for Ts

$$T_w = \frac{\rho T_s}{1-\rho}$$ plug in!

$$.441 = \frac{(0.84)T_s}{1-0.84} = \frac{0.84 T_s}{0.16}$$

$$0.16\left(.441 = \frac{0.84 T_s}{\cancel{0.16}}\right)\cancel{0.16}$$

$$\frac{0.0706}{0.84} = \frac{0.\cancel{84} T_s}{0.\cancel{84}}$$

$$\boxed{T_s = 0.084}$$

c) (1) find ρ (of S_2)

$$\rho = \lambda \cdot T_s$$

$$\rho = 100 \times 0.009 = \boxed{0.9}$$

d) (1) find q

$$q = \frac{\rho}{1-\rho}$$

$$\rho = \lambda \cdot T_s$$

<u>Server 1:</u>

$$\rho = \lambda \times T_q$$ plug in!

$$\rho = 10 \times 0.084 = 0.84$$

$$q = \frac{\rho}{1-\rho}$$ plug in!

$$q = \frac{0.84}{1-0.84} = 5.25$$ $\leftarrow S_1$

<u>Server 3:</u>

$$\rho = 50 \times 0.019 = 0.95$$

$$q = \frac{\rho}{1-\rho}$$

$$q = \frac{0.95}{1-0.95} = 19 \leftarrow S_2$$

Problem 3 continued...

Server 2:

$\lambda_{s_2} = 60 + 0.4(\lambda_{s_2})$

$1\lambda_{s_2} - 0.4\lambda_{s_2} = 60$

$\frac{0.6\lambda_s}{0.6} = \frac{60}{0.6} = 100 \leftarrow \lambda_{s_2}$

$\rho = \lambda \cdot T_s$ plug in!

$\rho = 100 \cdot 0.009$

$\rho = 0.9$

$q = \frac{\rho}{1-\rho}$ plug in!

$q = \frac{0.9}{1-0.9}$

$q = 9$

Total q of system:

$q_{s_1} + q_{s_2} + q_{s_3} = Q$

$5.25 + 19 + 9 = 33.25$

$\boxed{\text{total number of requests in the system is } 33.25}$

e) calculate slowdown $\left(\frac{T_q}{T_s}\right)$

$T_q = T_{q1} + T_{q2} + T_{q3} + n\,(T_{q2} + T_{q3})$

$T_s = T_{s1} + T_{s2} + T_{s3} + n\,(T_{s2} + T_{s3})$

① calculate T_{q1}, T_{q2} & T_{q3}

$T_{q1} = \frac{q}{\lambda} = \frac{5.25}{10} = 0.525$

$T_{q2} = \frac{q}{\lambda_b} = \frac{9}{100} = 0.09$

$T_{q3} = \frac{q}{\lambda_b} = \frac{19}{50} = 0.38$

e) 2 calculate T_q

$T_q = T_{q1} + T_{q2} + T_{q3} + n\,(T_{q^2} + T_{q^3})$

$T_q = 0.525 + 0.09 + 0.38 + \left(\frac{0.4}{1-0.4}\right)(0.09 + 0.38)$

$T_q = 0.525 + 0.09 + 0.38 + 0.67\,(0.09 + 0.38)$

$\boxed{T_q = 1.308}$

f) 1 find utilization of our bottleneck resource (T_3)

$\rho_{T_3} = \lambda_{T_3} \cdot T_{s_3}$, $\rho_{T_3} = 1$ plug in!

$\frac{1}{0.019} = \frac{\lambda_{T_3} \cdot \cancel{0.019}}{\cancel{0.019}}$

$\lambda_{T_3} = \frac{1}{0.019}$

$\lambda_{T_3} = 52.63$

2 make it in terms of λ_a

$\lambda_{T_3} = \lambda_a + \lambda_b$, $\lambda_b = \lambda_3 + (5 \cdot \lambda_a)$

$\lambda_{T_3} = \lambda_a + 0.4\,(\lambda_b)$

$\lambda_{T_3} = \lambda_a + 0.4\,(\lambda_3 + (5 \cdot \lambda_a))$

3 plug in and solve!

$52.63 = \lambda_a + 0.4\,(52.63 + (5 \cdot \lambda_a))$

$52.63 = \lambda_a + 0.4\,(52.63 + 5\lambda_a)$

$52.63 = \lambda_a + 21.052 + 2\lambda_a$

$52.63 = 3\lambda_a + \cancel{21.052}$

$-21.052 \qquad -\cancel{21.052}$

$\frac{31.578}{3} = \frac{\cancel{3}\lambda_a}{\cancel{3}}$

$\lambda_a = 10.526$

4 solve for λ_b

$\lambda_b = 5 \times \lambda_a$

$\lambda_b = 5 \times 10.526$

$\lambda_b = 52.63$

5 solve for λ_{T_3}

$\lambda_{T_3} = \lambda_a + \lambda_b$

$\lambda_{T_3} = 10.526 + 52.63$

$\lambda_{T_3} = 63.16$

$\boxed{\text{capacity} = 63.16}$

3 Q3 **35.5 / 40**

part a)

✓ **- 0 pts** **Correct**

- 6 pts Missing/Incomplete

- 1 pts Error in Diagram

- 1 pts No mention of Jackson's Theorem

- 1 pts Missing a key assumption

- 3 pts Partially Correct

- 1 pts Does not show what type of queuing model is used

- 1 pts Missing probabilities

- 3 pts missing diagram

part b)

✓ **- 0 pts** **Correct**

- 6 pts Missing/Incomplete

- 3 pts Incorrect with work shown

part c)

✓ **- 0 pts** **Correct**

- 6 pts Missing/Incomplete

- 3 pts Incorrect with work shown

part d)

✓ **- 0 pts** **Correct**

- 8 pts Missing/Incomplete

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part e)

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- 9 pts Missing/Incomplete/Incorrect with no work

✓ **- 2 pts** **Incorrect Tq idle/missing**

✓ **- 2 pts** **Incorrect Tq tot/missing**

✓ **- 0.5 pts** **Incorrect slowdown**

part f)

✓ **- 0 pts** **Correct**

- 5 pts Missing/Incomplete/Incorrect with no work

- 2.5 pts Incorrect with work shown

- 10 pts Late Penalty

- 40 pts Missing