

CS350fall20-HW3-Solution

easyabi

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1 Question 1

1.1 a

First, we need to quantify the utilization:

$$Ts = \frac{1}{\mu} = \frac{1}{200}$$

$$\lambda = 60$$

$$\rho = \lambda \times Ts = 60/200 = 3/10$$

Next, we calculate the average response time:

$$Tq = \frac{Ts}{1 - \rho} = \frac{1/200}{1 - 3/10} = 1/140sec = 7.1ms$$

the average response time is 7.1ms which is less than the desired response time (10ms), therefore, we guarantee the QoS terms.

1.2 b

$$Ts = \frac{1}{\mu} = \frac{1}{200}$$

$$\lambda = 30$$

$$\rho = \lambda \times Ts = 30/200 = 15/100$$

Next, we calculate the average response time:

$$Tq = \frac{Ts}{1 - \rho} = \frac{1/200}{1 - 15/100} = 1/170sec = 5.9ms$$

the average response time is 5.9 ms which is higher than the desired response time (5ms), therefore, we cannot guarantee the QoS terms.

1.3 c

$$Ts = \frac{1}{\mu} = \frac{1}{200}$$

$$\lambda = 100$$

$$\rho = \lambda \times Ts = 100/200 = 0.5$$

Next, we calculate the average response time:

$$Tq = \frac{Ts}{1 - \rho} = \frac{1/200}{1 - 0.5} = 1/100 \text{sec} = 10 \text{ms}$$

the average response time is 10 ms which is less than the desired response time (20ms), therefore, we can guarantee the QoS terms.

1.4 d

for costumer in part a : The desired Tq is 10ms= 0.01 sec

$$Tq = \frac{Ts}{1 - \rho} \rightarrow Ts = \frac{Tq}{1 + \lambda Tq} = \frac{0.01}{1 + 0.01 \times 60} = 0.00625 \text{sec} = 6.25 \text{ms}$$

So, we need 6.25ms as service time and we already know that 1GHz gives us 1sec/200= 5ms service time. Therefore, the new frequency must be :

$$\frac{5}{6.25} \times 1 \text{GHz} = 0.8 \text{GHz}$$

for costumer in part c :

The desired Tq is 20ms= 0.02 sec

$$Tq = \frac{Ts}{1 - \rho} \rightarrow Ts = \frac{Tq}{1 + \lambda Tq} = \frac{0.02}{1 + 0.02 \times 100} = 0.00667 \text{sec} = 6.67 \text{ms}$$

So, we need 6.67 ms as service time and we already know that 1GHz gives us 1sec/200= 5ms service time. Therefore, the new frequency must be :

$$\frac{5}{6.67} \times 1 \text{GHz} = 0.75 \text{GHz}$$

1.5 e

For customer in part a:

$$\rho = \lambda Ts = 60 \times 0.00625 = 0.375$$

The average number of requests in the queue:

$$w = \frac{\rho^2}{1 - \rho} = 0.225$$

For customer in part c:

$$\rho = \lambda Ts = 100 \times 0.00667 = 0.67$$

The average number of requests in the queue:

$$w = \frac{\rho^2}{1 - \rho} = 1.33$$

1.6 f

for costumer in part a:

$$p(\textit{immediately_serving}) = 1 - \rho = 1 - 0.375 = 0.625$$

for costumer in part c:

$$p(\textit{immediately_serving}) = 1 - \rho = 1 - 0.67 = 0.33$$

2 2

2.1 a

First, we calculate Ts

$$Ts = \frac{1.2 \times 1024 \times 8}{10^8} = 0.00001024 = 0.000098304$$

Second, we find the utilization of the system:

$$\rho = \lambda Ts = 8200 \times 0.000098304 = 0.81$$

Finally, we find the average number of requests in the system:

$$q = \frac{\rho}{1 - \rho} = \frac{0.81}{1 - 0.81} = 4.26 \textit{packets}$$

The amount of memory needed is as follows:

$$4.26 \times 1.2 \times 1024 = 5235 \textit{bytes}$$

2.2 b

$$Tw = \frac{\rho}{\mu(1 - \rho)} = \frac{0.81}{\frac{1}{0.000098304} \times (1 - 0.81)} = 0.00042 \textit{sec}$$

2.3 c

Yes, because the utilization (0.81) is higher than 0.7

2.4 d

The system is a M/M/1 system

2.5 e

old response time:

$$Tq_{old} = \frac{Ts}{1 - \rho} = \frac{0.000098304}{1 - 0.81} = 0.00052$$

New Response time: First, we calculate Ts

$$Ts = \frac{1.2 \times 1024 \times 8}{10^9} = 0.00001024 = 0.0000098304$$

Second, we find the utilization of the system:

$$\rho = \lambda Ts = 8200 \times 0.0000098304 = 0.081$$

$$Tq_{new} = \frac{Ts}{1 - \rho} = \frac{0.0000098304}{1 - 0.081} = 0.0000107$$

Finally; the speedup is as follows:

$$Speed_{up} = \frac{Tq_{old}}{Tq_{new}} = \frac{0.00052}{0.0000107} = 48$$

2.6 f

$$q = \frac{\rho}{1 - \rho} = \frac{0.081}{1 - 0.081} = 0.088$$

The amount of memory needed using new switch:

$$new_memory = 0.088 \times 1.2 \times 1024 = 108$$

Therefore, we need (5235 bytes - 108 bytes) 5127 bytes less memory

3 Question 3

3.1 a

In steady state the throughput of the system is equal to the arrival rate

$$q = 20Ts = 35$$

$$q = \frac{\rho}{1 - \rho} = \frac{\lambda Ts}{1 - \lambda Ts} \implies 20 = \frac{\lambda 35}{1 - \lambda 35} \implies \lambda = 0.0272req/sec$$

3.2 b

The capacity (i.e., maximum achievable throughput) of the system is equal to service rate:

$$Capacity = \frac{1}{Ts} = \frac{1}{35} = 0.029req/sec$$

3.3 c

Adding 8% to the traffic, the new arrival rate is:

$$1.08 \times 0.0272 = 0.029376req/sec$$

And the new utilization will be:

$$\rho = \lambda Ts = 0.029376 \times 35 = 1.03$$

The utilization is higher than one. Therefore, the system cannot sustain this increase to the traffic

3.4 d

$$Ts = 25$$

The capacity:

$$capacity = \mu = 1/25 = 0.04$$

$$\frac{0.04}{0.029} = 1.4$$

3.5 e

New Ts:

$$new_Ts = (1 - 0.2)old_Ts = 28s$$

New Capacity:

$$\frac{1}{Ts} = \frac{1}{28} = 0.036$$

Old income:

$$0.029 \times 1\$ = 0.029$$

New income

$$0.036 \times 0.5 = 0.018$$

The new configuration will not bring any benefit in terms of cash flow