

# CAS CS 350 HW2

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TOTAL POINTS

**87 / 90**

## QUESTION 1

1 Q1 32 / 34

- 0 pts Correct

a)

- 2 pts Minor Error

- 4 pts Incorrect or No Work Shown

b)

- 2 pts Minor Error

- 4 pts Incorrect or No Work Shown

c)

- 2 pts Minor Error

- 4 pts Incorrect or No Work Shown

d)

- 2 pts Minor Error

- 5 pts Incorrect or No Work Shown

e)

✓ - 2 pts Minor Error

- 5 pts Incorrect or No Work Shown

f)

- 3 pts Minor Error

- 6 pts Incorrect or No Work Shown

g)

- 3 pts Minor Error

- 6 pts Incorrect or No Work Shown

- 3.75 pts Late

- 7.75 pts Late

- 7.25 pts Late

## QUESTION 2

2 Q2 32 / 32

part a)

✓ - 0 pts correct -- 0.01 sub per sec

- 2 pts incorrect with work shown

- 4 pts incorrect/missing

part b)

✓ - 0 pts correct --  $(0.01)e^{(-0.01t)}$

- 2 pts incorrect with work shown

- 4 pts incorrect/missing

part c)

✓ - 0 pts correct -- 0.3

- 2 pts incorrect with work shown

- 4 pts incorrect/missing

part d)

✓ - 0 pts correct -- 100

- 2 pts incorrect with work shown

- 4 pts incorrect/missing

part e)

✓ - 0 pts correct --  $e^{(-1.5)*(((1.5)^x)/x!)}$

- 2 pts incorrect with work shown

- 4 pts incorrect/missing

part f)

✓ - 0 pts correct -- 0.0662

- 2 pts incorrect with work shown

- 4 pts incorrect/missing

part g)

✓ - 0 pts correct -- 0.2

- 2 pts incorrect with work shown

- 4 pts incorrect/missing

part h)

✓ - 0 pts correct -- 0.3

- 2 pts incorrect with work shown

- 4 pts incorrect/missing

- 8 pts LATE PENALTY

QUESTION 3

3 Q3 23 / 24

part a)

✓ - 0 pts correct - 0.5 seconds

- 2 pts incorrect answer with work shown

- 4 pts incorrect/missing

part b)

✓ - 0 pts correct - 5

- 2 pts incorrect with work shown

- 4 pts incorrect/missing

part c)

✓ - 0 pts correct -  $p = 6.4$  so needs 7 requests in parallel, at least 7 CPUs are needed

- 2 pts incorrect with work shown

- 4 pts incorrect/missing

part d)

✓ - 0 pts correct - 0.91

- 2 pts incorrect with work shown

- 4 pts incorrect/missing

part e)

✓ - 0 pts correct - 9.1 connections per second

- 2 pts incorrect with work shown

- 4 pts incorrect/missing

part f)

- 0 pts correct - load is equally balanced among CPUs and CPUs are identical

✓ - 1 pts missing a key assumption

- 2 pts incorrect but showed effort

- 4 pts incorrect/missing

- 6 pts LATE PENALTY

# Problem 1

a)

$$\text{Availability} = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}} \quad \left. \begin{array}{l} \text{MTBF} = 8 \text{ mins} \\ \text{MTTR} = 2 \text{ mins} \end{array} \right\} \text{plug in values!}$$

$$\left. \begin{array}{l} \text{MTBF} = 8 \text{ mins} \\ \text{MTTR} = 2 \text{ mins} \end{array} \right\} \frac{8}{8+2} = \frac{8}{10} = \frac{4}{5}$$

To find  $\text{Pr}(\text{new request will be lost}) = 1 - \text{Availability}$

$$\text{Pr}(\text{new req will be lost}) = 1 - \frac{4}{5}$$

plug in availability!

$$\boxed{\text{Pr}(\text{new req will be lost}) = \frac{1}{5}}$$

b) Geometric distribution

↳ request complete  $\Rightarrow$  logged in disk

$$\left. \begin{array}{l} \text{Pr}(\text{success}) = \frac{4}{5} \rightarrow \text{availability} \rightarrow p \\ \text{Pr}(\text{loss}) = \frac{1}{5} \rightarrow \text{part a.} \rightarrow 1-p \end{array} \right\} M = \frac{p}{1-p} = \frac{\frac{4}{5} \times \frac{5}{1}}{\frac{1}{5}} \Rightarrow$$

$$\frac{20}{5} = 4$$

$$\boxed{M = 4}$$

c) get reliability

$$\left. \begin{array}{l} R(t) = e^{-t/\text{MTBF}} \\ R(3) = e^{-(3)/(8)} \end{array} \right\} \text{plug in values!}$$

$$\boxed{= 0.687 \text{ of correct processing on first attempt}}$$

d) 1 - min of 99% prob (total)

-  $\text{Pr}(\text{correct processing on first attempt}) = 0.687$

↳ not processed correctly =  $1 - 0.687 = 0.313 = 31.3\%$

↳ not processed correctly (N times) =  $[1 - 0.313]^N$

↳ processed correctly (N times) =  $1 - [1 - 0.313]^N$

put into equation & solve!

Problem 1 continued...

$$d) 1 - [0.313]^N \geq 0.99$$

$$\frac{1}{1} - [0.313]^N \geq \frac{0.99}{1}$$

$$[0.313]^N \leq 0.01$$

↳ for this to be true, N has to be 4.

$$\boxed{N=4}$$

e) M of geometric distribution

$$M = \frac{1}{1-p}, \quad p = R(3) = 0.687$$

$$M = \frac{1}{1-(0.687)}$$

$$M = \frac{1}{0.313} = 3.19$$

$$\boxed{M=3.19}$$

f) get reliability of new strategy:

$$R(t) = e^{-t/MTBF}$$

$$R(1.5) = e^{-1.5/8} = 0.829 \quad (1st \ 1/2)$$

$$R(1.5) = e^{-1.5/8} = 0.829 \quad (2nd \ 1/2)$$

multiply the 2 to get total reliability:

$$0.829 \times 0.829$$

$$0.687$$

↳ Since both strategies are 0.687 when  $N=1$ , then we can say that they are identical.

Problem 1 continued...

g) reliability =  $R(t) = e^{-t/MTBF}$

when  $t=1$   $\left\{ \begin{array}{l} R(1) = e^{-1/8} = 0.882 \end{array} \right.$

$$1 - [1 - R(1.5)]^2 = 1 - [1 - 0.829]^2 \\ = 1 - [0.171]^2 = 0.971^2 = \underline{0.943}$$

$$R(3) = e^{-3/8} = 0.687$$

$$1 - [1 - R(3)]^3 = 1 - [1 - 0.687]^3 = 1 - [0.313]^3 = \underline{0.969}$$

Since the two values are different, then we can conclude that it is NOT possible to set  $N=2$  & receive the same quality of service that we would in the initial retrial strategy &  $N=3$ .



## Problem 2

- a) At the steady-state, the throughput is equivalent to the rate of the request arrival ( $\lambda$ ).

$$\lambda = \frac{1}{\text{interarrival time}} = \frac{1}{100} = \boxed{0.01 \text{ req/sec}}$$

- b) assuming it is an M/M/1 system (exponential interarrival time)

PDF =

$$f(x) = \lambda \exp^{-\lambda x} \quad \text{plug in } \lambda!$$

$$\boxed{f(x) = 0.01 \exp^{-0.01 x}}$$

- c)  $P(t > 120 \text{ sec})$

↳ We can do this by subtracting 1 from the probability that the two subsequent interarrival times are NOT more than 120 sec apart ( $1 - P(t \leq 120 \text{ sec})$ )

$$\hookrightarrow e^{-\lambda t} \quad \text{plug in!}$$

$$e^{-0.01(120)} = 0.3012 = \boxed{30.12\%}$$

- d) Since the interarrival times are exponentially distributed, then the std. dev. is always equal to the mean?

Therefore,

$$\mu = 1/\lambda = 1/0.01 = \boxed{100}$$

- e) Poisson Distribution formula:

$$P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$\frac{0.01(150)^x e^{-(0.01 \times 150)}}{x!} = \boxed{\frac{1.5^x e^{-1.5}}{x!}}$$

Problem 2 continued

f)  $\Pr(X > 3)$  received within 150 sec time period

↳ equivalent to  $1 - \Pr(X \leq 3)$

$$1 - (\Pr(X=0) + \Pr(X=1) + \Pr(X=2) + \Pr(X=3))$$

↳ plug in the #'s ( $X$ ) to Poisson distribution formula

$$1 - (0.223 + 0.335 + 0.251 + 0.126)$$

$$1 - 0.934$$

$$\boxed{0.066}$$

g)  $\rho = \lambda * T_s$

$$= 0.01 * 20$$

$$= 0.2$$

$$\rightarrow \boxed{\rho = 0.2}$$

h)  $\omega = \lambda * T_w$  }  $\omega = \lambda (T_q - T_s)$  } plug in values given!  
 $T_w = T_q - T_s$

$$\omega = \lambda (T_q - T_s)$$

$$\omega = 0.01 (50 - 20)$$

$$\omega = 0.01 (30)$$

$$\omega = 0.3 \text{ requests}$$

### Problem 3

- a) 32 connections on avg.

↳ rate = 64/sec

↳ 100 ms per request  $\rightarrow T_s$

$$\frac{32}{64} = \boxed{\frac{1}{2} \text{ seconds}}$$

Since there are 32 starting connections, and 64 take 1 second to be serviced, then each connection will be in the system for .5 seconds.

b) slow down =  $\frac{T_q}{T_s} = \frac{\frac{1}{2}}{100 \text{ ms}} = \frac{.5}{.1} = \boxed{5 \text{ sec}}$

c)  $\rho = \lambda \cdot T_s$

$$\rho = \frac{64}{\text{sec}} \cdot 0.1 = 6.4$$

In order for this to work, we need to have a higher number of CPUs than the utilization of the system.

Therefore, the server must have

at least  $\boxed{7 \text{ CPUs.}}$

- d) Since the utilization of the whole system is 6.4, and the total amount of CPU's is 7, then to get the utilization of each CPU we have to divide the total utilization by 7. Therefore,  $6.4/7 = \boxed{0.91}$

e)  $\frac{6.4 \text{ total}}{7 \text{ CPUs}} = \boxed{9.14 \text{ requests/sec per CPU}}$

f) Assumptions made:

① It is an M/M/1 system

- Poisson distributed arrival times

- exponentially distributed interarrival times