Exploring dynamic neural field self-organizing maps for dimensionality reduction, visualization, and classification

Paulo Barbosa[1], **Flora Ferreira**[1], **Estela Bicho**[2], **Wolfram Erlhagen**[1]

[1]Centre of Mathematics, School of Sciences, University do Minho, [2] Algoritmi Centre, School of Engineering, University of Minho.

Introduction

Traditional Machine Learning methods such as K-NN (for classification) and PCA (for visualization) often **require the tuning of free parameters** to achieve optimal performance.

Classical Self-Organizing Maps (SOMs), while effective, rely on decaying learning rates and dynamic neighborhood functions to maintain stability. As a result, SOMs **cannot adapt to varying input distributions and**, in practice, **can only be realistically implemented offline**.

We propose a biologically realistic model of neuronal activity that can address these limitations, offering an adaptive, online approach capable of performing both classification and visualization.

Dynamic Neural Fields

Dynamic Neural Fields (DNFs) [1] are **Recurrent Neural Networks** that describe the coarse-grained activity of populations of interacting neurons **organized in a continuous feature space** $\Omega \subset \mathbb{R}^q$, $\forall q \in \mathbb{N}_0$.

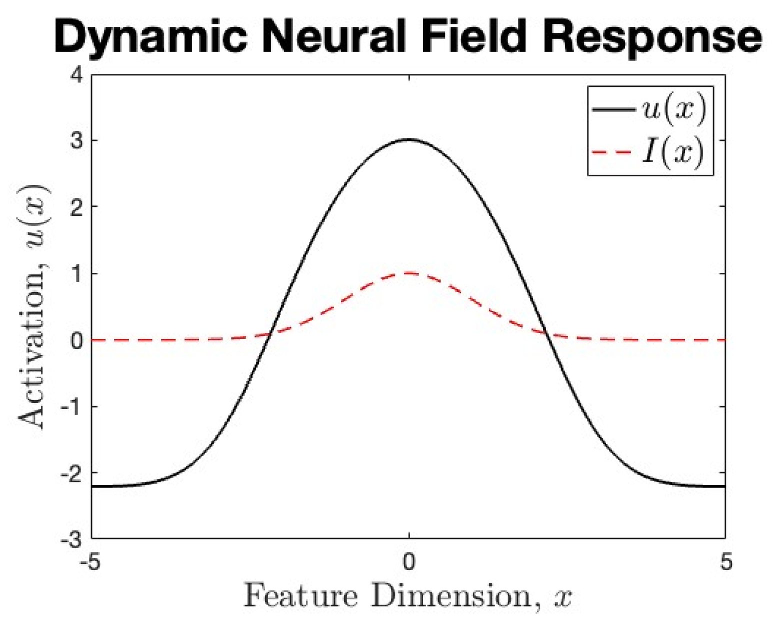
Information in DNFs **is represented by supra-threshold, localized regions of neural activity**, commonly referred to as bumps.

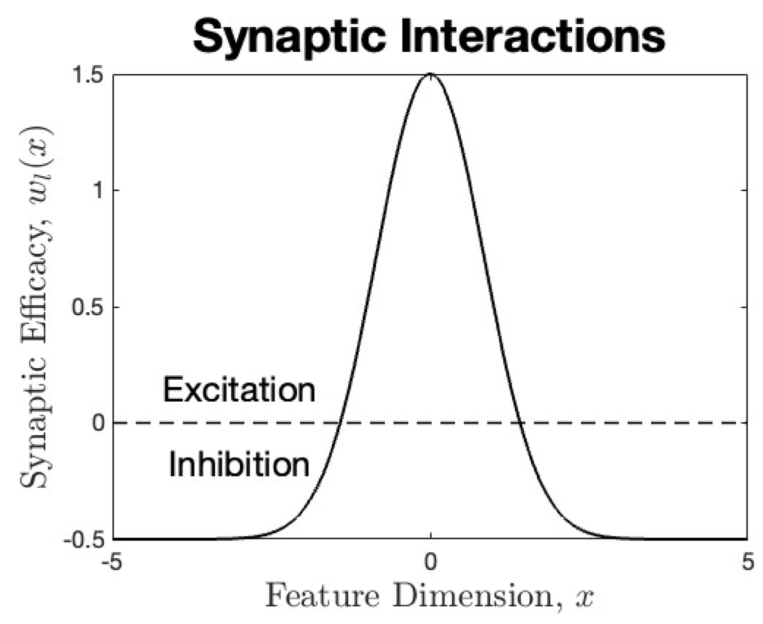
A common mathematical formulation of DNFs is given by the **integro-differential equation**:

$$\tau \frac{\partial u(x,t)}{\partial t} = -u(x,t) + I(x,t) + \int_{\Omega} w_l(|x-y|)\mathrm{rect}[u(y,t)]\, dy,$$

where:

- $u(x,t)$: Neural activation at site $x \in \Omega$ and time $t \geq 0$;
- $\tau > 0$: Time constant;
- $I(x,t)$: Localized external stimulus;
- $w_l(x)$: Synaptic interactions, separated into excitatory and inhibitory, i.e., $w_l(x) = w_e(x) - w_i(x)$.





[1] Shun-Ichi Amari. Dynamics of pattern formation in lateral-inhibition type neural fields. Biological cybernetics, 27(2):77–87, 1977

Dynamic Neural Field Self-Organizing Map

Consider a dataset $\mathcal{D} = \{(x_i, y_i)\}_{i=1}^N$, where $x_i \in \mathbb{R}^m$ for $1 \leq i \leq N$, i.e., each feature x_i has m attributes, and y_i represents the corresponding label.

The DNF-SOM [2] initializes each neuron's codeword to have the same number of attributes, i.e., $w_f \in \Omega \times \mathbb{R}^m$.

At each epoch, a random stimulus s is drawn from $\mathcal{D}$.

The input for the DNF is given by the following function:

$$I(x,t) = 1.0 - \frac{|w_f(x) - s|_1}{m}.$$

To ensure $I(x,t) \in [0,1]$, the input data must be normalized beforehand.

The neural activity is simulated, and the codebooks are updated according to:

$$\frac{\partial w_f(x,t)}{\partial t} = \gamma\left(s - w_f(x,t)\right) \int_{\Omega} w_e(|x-y|)\mathrm{rect}[u(y,t)]\, dy,$$

where $\gamma > 0$ is the learning rate.

[2] Detorakis, G. I., & Rougier, N. P. (2014). Structure of receptive fields in a computational model of area 3b of primary sensory cortex. Frontiers in Computational Neuroscience, 8, 76.

Classification and Prediction

The dataset is partitioned into training and testing sets, denoted as $\mathcal{D}_{\text{train}} = (x_{\text{train}}, y_{\text{train}})$ and $\mathcal{D}_{\text{test}} = (x_{\text{test}}, y_{\text{test}})$, respectively.

After training, classification and prediction are performed as follows:

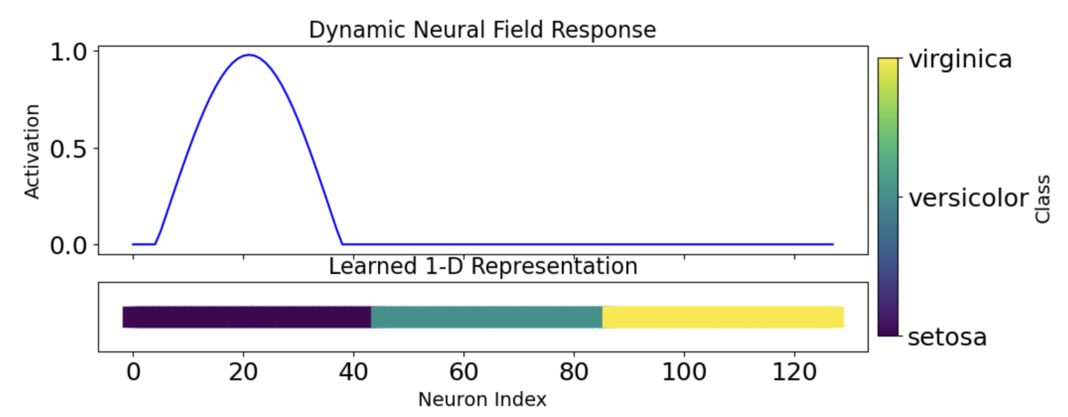
Classification Algorithm:
Input: Training set $\mathcal{D}_{\text{train}}$; Codebook $w_f(x)$.
Output: Assigned labels for each neuron $x \in \Omega$.
for each $x \in \Omega$ **do**
  idx $\leftarrow \arg\min_i |w_f(x) - x_{\text{train},i}|_1$
  classf$[x] \leftarrow y_{\text{train}}[idx]$
end for
return classf

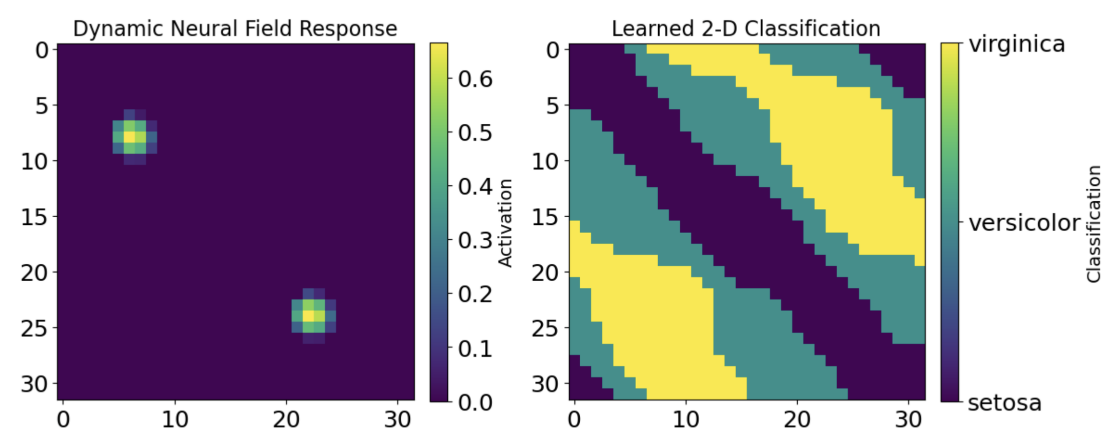
Prediction Algorithm:
Input: Sample $s \in x_{\text{test}}$; Codebook $w_f(x)$; Neuron classifications classf.
Output: Predicted class $\hat{y}$.

$I(x,t) \leftarrow 1.0 - \frac{|w_f(x)-s|_1}{m}$.
$u(x) \leftarrow$ Simulate neural dynamics until convergence.
idx $\leftarrow \arg\max_{x \in \Omega} u(x)$
return classf[idx]

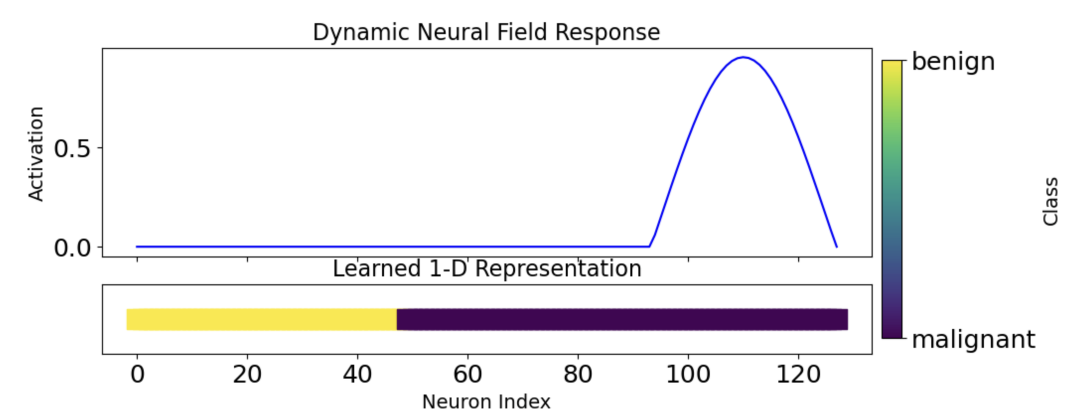
Results

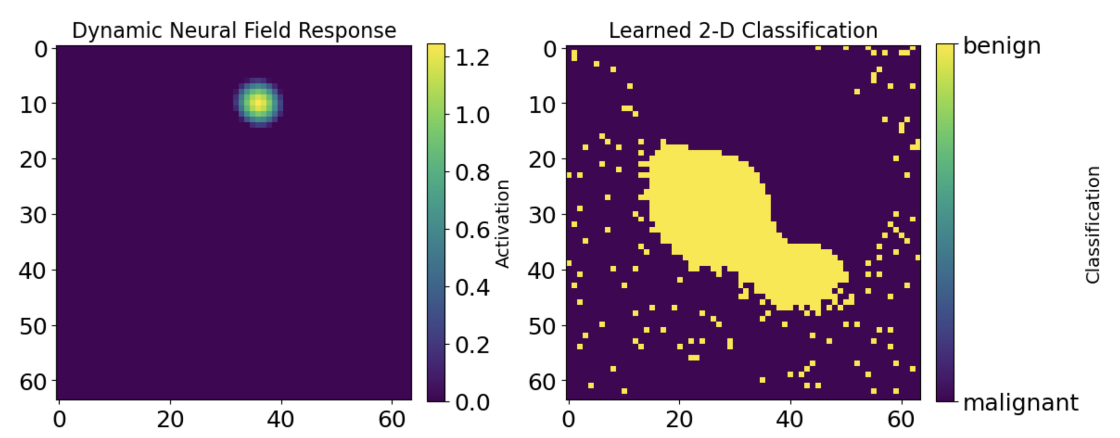
Iris Dataset Accuracy:
1-D: 90.0%, 2-D: 96.7%.





Breast Cancer Dataset Accuracy:
1-D: 89.5%, 2-D: 95.6%.





Conclusion

The DNF-SOM combines Dynamic Neural Fields and Self-Organizing Maps into a biologically inspired framework for **classification** and **visualization**, enabling **online learning**, **parameter-free adaptation**, and **robust feature organization**.

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