Test Flight Q7

Prove that for any natural number n,
$2 + 2^2 + 2^3 + ... + 2^n = 2^{n+1} - 2$

Proof by the method of induction:
(a) Check that the equation holds for n=1

$$\begin{aligned} 2^1 &= 2^{1+1} - 2 \\ 2 &= 2^2 - 2 \\ 2 &= 4 - 2 \\ 2 &= 2 \end{aligned}$$

Which is true.

(b) assume the equation holds for any n
(c) prove the equation is true for n+1. Substitute n+1 for n and perform algebraic manipulations to arrive at final form:

$$\begin{aligned} 2 + 2^2 + 2^3 + ... + 2^n + 2^{n+1} &= 2^{n+1+1} - 2 \\ &= 2^{n+2} - 2 \\ 2(1 + 2^1 + 2^2 + ... + 2^n) &= 2(2^{n+1} - 1) \\ 1 + 2^1 + 2^2 + 2^3 + ... + 2^n &= 2^{n+1} - 1 \\ 2^1 + 2^2 + 2^3 + ... + 2^n &= 2^{n+1} - 1 - 1 \\ 2 + 2^2 + 2^3 + ... + 2^n &= 2^{n+1} - 2 \end{aligned}$$

Which is the equation for n and is assumed true is step (b). Thus the statement for $n + 1$ is true.

By the method of induction the statement is true for any n.