

Flight Test Q9

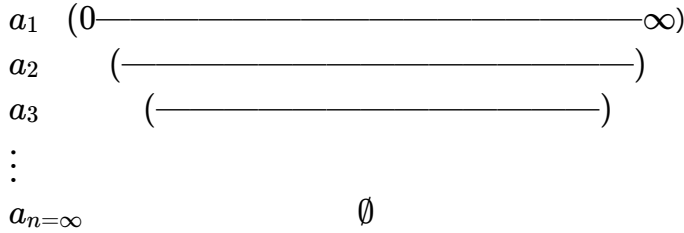
Given an infinite collection $A_n, n = 1, 2, \dots$ of intervals of the real line, their intersection is defined to be

$$\bigcap_{n=1}^{n=\infty} A_n = \{x | (\forall n)(x \in A_n)\}$$

Give an example of a family of intervals $A_n, n = 1, 2, \dots$, such that $A_{n+1} \subset A_n$ for all n and $\bigcap_{n=1}^{n=\infty} A_n = \emptyset$

Let A_1 be the interval of all real numbers greater than 0, i.e. the interval $(l_1, r_1) = (0, \infty)$. Let subsequent intervals be reduced by a value of 2ϵ : For $A_n, (l_n, r_n) = (l_{n-1} + \epsilon, r_{n-1} - \epsilon)$. At $n = \infty, A_\infty = \emptyset$

Graphically, this looks like the diagram below.



Every interval A_{n+1} is a subset of the previous interval A_n , a requirement of the family of intervals: $A_{n+1} \subset A_n$ for all n .

From the definition of set intersection,

$\bigcap_{n=1}^{n=\infty} A_n = \{x | (\forall n)(x \in A_n)\}$, there must be at least one $x \in A_n$ for all n . In the family of intervals defined above, the last interval $A_\infty = \emptyset$ has no members. Therefore there does not exist an x which is member of all intervals A_n for $n = 1, 2, \dots, \infty$.

The family of intervals as constructed meet the requirements in the statement.