

# Flight Test Q10

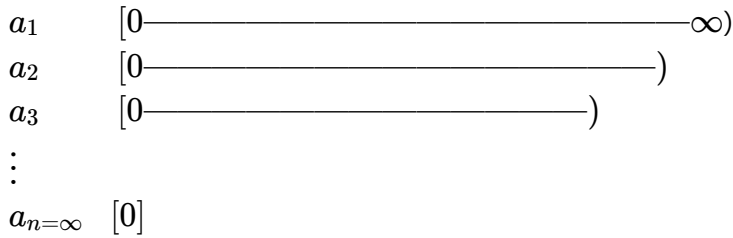
Given an infinite collection  $A_n, n = 1, 2, \dots$  of intervals of the real line, their intersection is defined to be

$$\bigcap_{n=1}^{n=\infty} A_n = \{x | (\forall n)(x \in A_n)\}$$

Give an example of a family of intervals  $A_n, n = 1, 2, \dots$ , such that  $A_{n+1} \subset A_n$  for all  $n$  and  $\bigcap_{n=1}^{n=\infty} A_n$  consists of a single real number. Prove that your example has the stated property.

Let  $A_1$  be the interval of all real numbers greater  $\geq 0$ , i.e. the interval  $[l_1, r_1) = [0, \infty)$ . Let subsequent intervals be reduced by a value of  $\epsilon$  : For  $A_n, [l_n, r_n) = [l_{n-1}, r_{n-1} - \epsilon)$ . At  $n = \infty, A_\infty = [0]$

Graphically, this looks like the diagram below.



Every interval  $A_{n+1}$  is a subset of the previous interval  $A_n$ , a requirement of the family of intervals:  $A_{n+1} \subset A_n$  for all  $n$ .

From the definition of set intersection,

$$\bigcap_{n=1}^{n=\infty} A_n = \{x | (\forall n)(x \in A_n)\}$$

there must be at least one  $x \in A_n$  for all  $n$ . In the family of intervals defined above, there is one element  $x = 0$  that satisfies this requirement.

The family of intervals as constructed meet the requirements as stated.