Flight Test Q10

Given an infinite collection $A_n, n = 1, 2, ...$ of intervals of the real line, their intersection is defined to be

$$\bigcap_{n=1}^{n=\infty} A_n = \{x | (\forall n)(x \in A_n)\}$$

Give an example of a family of intervals $A_n, n = 1, 2, ...$, such that $A_{n+1} \subset A_n$ for all n and $\bigcap_{n=1}^{n=\infty} A_n$ consists of a single real number. Prove that your example has the stated property.

Let A_1 be the interval of all real numbers greater $\geq$ 0, i.e. the interval $[l_1, r_1) = [0, \infty)$. Let subsequent intervals be reduced by a value of ϵ : For A_n, $[l_n, r_n) = [l_{n-1}, r_{n-1} - \epsilon)$. At $n = \infty, A_\infty = [0]$

Graphically, this looks like the diagram below.

a_1 $[0$ ———————————————— $\infty)$

a_2 $[0$ —————————————— $)$

a_3 $[0$ ———————————— $)$

$\vdots$

$a_{n=\infty}$ $[0]$

Every interval A_{n+1} is a subset of the previous interval A_n, a requirement of the family of intervals: $A_{n+1} \subset A_n$ for all n.

From the definition of set intersection,

$$\bigcap_{n=1}^{n=\infty} A_n = \{x | (\forall n)(x \in A_n)\}$$

there must be at least one $x \in A_n$ for all n. In the family of intervals defined above, there is one element $x = 0$ that is satisfies this requirement.

The family of intervals as constructed meet the requirements as stated.