

## Test Flight Q2

Say whether the following is true or false and support your answer by a proof: The sum of any five consecutive integers is divisible by 5 (without remainder).

This statement is true. Show that  $\sum_{k=1}^{k+4} n_k = 5a$  (with no remainder).

Proof is by induction.

(a) show this statement is true for  $n=1$

$$\begin{aligned}\sum_{k=1}^{k+4} n_k &= 5a \\ 1 + 2 + 3 + 4 + 5 &= 15 \\ 5a &= 15 \\ a &= 3\end{aligned}$$

The statement for  $n=1$  is true.

(b) Assume the statement is true for any  $n$ .

(c) Show the statement is true for  $n+1$ .

$$\begin{aligned}5a &= \sum_{k=k+1}^{k+1+4} n_k \\ 5a &= (n+1) + (n+1+1) + (n+1+2) + (n+1+3) + (n+1+4) \\ 5a &= (n+1) + (n+2) + (n+3) + (n+4) + (n+5) \\ 5a &= 5n + 15 \\ 5a &= 5(n+1) \\ a &= (n+1)\end{aligned}$$

$n$  is an integer, therefore  $n+1$  is also an integer. The statement is true for case of  $n+1$  (divisible by 5 without remainder) and by the method of induction, it is true for any  $n$ .