Test Flight Q3

Say whether the following is true or false and support your answer by a proof: For any integer n, the number $n^2 + n + 1$ is odd.

The proof will be by the method of induction.

(a) Prove the statement is true for n=1.

$$1^2 + (1) + 1 = 3$$

Three is an odd integer, so the statement is true for n=1.

(b) Assume that the statement is true for n.

(c) Prove the statement is true for n+1.

$$(n + 1)^2 + (n + 1) + 1$$
$$n^2 + 2n + 1 + (n + 1) + 1$$
$$\underbrace{(n^2 + n + 1)}_{\text{term 1}} + \underbrace{(2n + 2)}_{\text{term 2}}$$

Term 1 is the given statement for n and is assumed to be true in step (b), therefore it is odd.

Term 2 is even: 2n is even integer - it is in fact the definition of an even integer; 2 is an even integer. It is well known from integer arithmetic that the sum of two even integers is also even. Therefore the second term is even.

It is also well known from integer arithmetic that the sum of an odd integer plus an even integer is odd (term 1 is odd and term 2 is even). Therefore, by the method of induction the statement is proven to be true.