Flight Test Q9

Given an infinite collection $A_n, n = 1, 2, ...$ of intervals of the real line, their intersection is defined to be

$\bigcap_{n=1}^{n=\infty} A_n = \{x | (\forall n)(x \in A_n)\}$

Give an example of a family of intervals $A_n, n = 1, 2, ...$, such that $A_{n+1} \subset A_n$ for all n and $\bigcap_{n=1}^{n=\infty} A_n = \emptyset$

Let A_1 be the interval of all real numbers greater than 0, i.e. the interval $(l_1, r_1) = (0, \infty)$. Let subsequent intervals be reduced by a value of 2ϵ : For A_n, $(l_n, r_n) = (l_{n-1} + \epsilon, r_{n-1} - \epsilon)$. At $n = \infty, A_\infty = \emptyset$

Graphically, this looks like the diagram below.

$$\begin{array}{ll}
a_1 & (0\rule[0.5ex]{9cm}{0.4pt}\infty) \\
a_2 & \quad (\rule[0.5ex]{8.5cm}{0.4pt}) \\
a_3 & \qquad (\rule[0.5ex]{7.5cm}{0.4pt}) \\
\vdots & \\
a_{n=\infty} & \qquad\qquad\qquad\qquad \emptyset
\end{array}$$

Every interval A_{n+1} is a subset of the previous interval A_n, a requirement of the family of intervals: $A_{n+1} \subset A_n$ for all n.

From the definition of set intersection,

$\bigcap_{n=1}^{n=\infty} A_n = \{x | (\forall n)(x \in A_n)\}$\, there must be at least one $x \in A_n$ for all n. In the family of intervals defined above, the last interval $A_\infty = \emptyset$ has no members. Therefore there does not exist an x which is member of all intervals A_n for $n = 1, 2, ...\infty$.

The family of intervals as constructed meet the requirements in the statement.