

Test Flight Q1

Say whether the following is true or false and support your answer by a proof.

$$(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m + 5n = 12)$$

Proof by contradiction:

$$3m + 5n = 12 \quad \text{Given in statement of problem.}$$

$$3m = 12 - 5n$$

$$m = 4 - \frac{5}{3}n.$$

$$4 - \frac{5}{3}n \geq 1 \quad \text{Since } m \text{ is a natural number.}$$

n must be a multiple of 3 in order for $\frac{5}{3}n$ to be a natural number. But if $n = 3$,
 $4 - \frac{5}{3}3 = -1 < 1$ Which is a contradiction of $n \in \mathbb{N}$ All n that are greater multiples of 3 also results in a negative integer for m which is a contradiction of $n \in \mathbb{N}$.

This proves by contradiction that $(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m + 5n = 12)$ is false.