Test Flight Q4

Prove that every odd natural number is of one of the forms 4n + 1 or 4n + 3, where n is an integer.

Prove by the method of induction.

(a) Prove the statement is true for n=1. If n=1,

$$4n + 1 = 4(1) + 1 = 5$$
$$4n + 3 = 4(1) + 3 = 7$$

Since 5 and 7 are both odd, the statement is true for n=1

(b) Assume the statement is true for any n.

(c) Prove the statement is true for n+1.

First case 4n+1

$$\begin{aligned} 4(n+1)+1 &= \\ 4n+4+1 &= \\ 4n+5 &= \\ 2(2n)+5 & \end{aligned}$$

- 2n is an even integer by definition, and by extension 2(2n) is also an even integer by definition.
- 5 is an odd integer.
- By integer arithmetic, an odd integer plus an even integer is odd.

The statement is true for n+1 for case 1 by the method of induction.

Second case 4n + 3

$$
\begin{aligned}
4(n+3)+1 &= \\
4n+12+1 &= \\
4n+13 &= \\
2(2n)+13
\end{aligned}
$$

Using the same logic from case 1, the result for case 2 is the sum of an even integer and an odd integer which is an odd integer.

The statement is true for n+1 for case 2 by the method of induction.

Since the choice of n is arbitrary, the statement is true for all $n \in \mathbb{Z}$