Flight Test Q8

Prove (from the definition of a limit of a sequence) that if the sequence $\{a_n\}_{n=1}^{\infty} \to L$ as $n \to \infty$, then for any fixed number $M > 0$, the sequence $\{Ma_n\}_{n=1}^{\infty} \to ML$ where L, ML are limits.

The definition of a limit of a sequence:

$a_n \to a$, as $n \to \infty$
$(\forall \epsilon > 0)(\exists n \in \mathbb{N})(\forall m \geq n)(|a_m - a| < \epsilon)$

Restating for this problem:

$$\lim_{n\to\infty} a_n = L$$
$$M(\lim_{n\to\infty} a_n) = ML$$
$$\lim_{n\to\infty} Ma_n = ML$$

Select a positive real number $M\epsilon > 0$. $(\forall M\epsilon > 0)(\exists n \in \mathbb{N})(\forall m \geq n)$:

$$|Ma_m - ML| < M\epsilon$$
$$M|a_m - L| < ML$$
$$|a_m - L| < L$$

which is given as true in the statement of the problem. Therefore the statement that the sequence

$\{Ma_n\}_{n=1}^{\infty} \to ML$

is true.