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A cubic unsharp masking technique for contrast enhancement

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Abstract

The cubic unsharp masking method is introduced in this paper. It is demonstrated, through both a statistical study and some computer simulations, that the proposed method has a much reduced noise sensitivity with respect to the linear unsharp masking technique and it permits to obtain perceptually pleasant results. The proposed operator also compares favourably with other algorithms which recently have been studied to improve the behaviour of the unsharp masking approach. © 1998 Elsevier Science B.V. All rights reserved.

Zusammenfassung

In diesem Artikel wird die *Cubic Unsharp Masking* Methode vorgestellt. Sowohl durch statistische Untersuchungen als auch mit Hilfe von Computersimulationen wird gezeigt, daß die vorgeschlagene Methode eine stark herabgesetzte Rauschempfindlichkeit in Bezug auf die linear unscharfe Maskierungstechnik besitzt und daß sie wahrnehmbar zufriedenstellende Resultate liefert. Der vorgeschlagene Operator ist vorteilhaft im Vergleich zu anderen Algorithmen, die kürzlich zur Verbesserung des Verhaltens des unscharfen Maskierungsansatzes untersucht wurden. © 1998 Elsevier Science B.V. All rights reserved.

Résumé

Nous introduisons dans cet article la méthode de masquage à contraste non marqué cubique (cubic unsharp masking) Il est montré, à la fois par une analyse statistique et quelques simulations sur ordinateur, que la méthode proposée présente une sensibilité au bruit beaucoup plus faible vis-à-vis de la technique de masquage à contraste non marqué linéaire et qu'elle permet d'obtenir des résultats visuellement plaisants. L'opérateur proposé soutient également favorablement la comparaison avec d'autres algorithmes récemment étudiés pour améliorer le comportement de l'approche de masquage à contraste non marqué. © 1998 Elsevier Science B.V. All rights reserved.

Keywords: Image processing; Image enhancement; Unsharp Masking; Nonlinear filtering

1. Introduction

A large number of approaches have been devised to improve the perceived quality of an image [4], using point operators (for example, histogram modification), spatial operators (high emphasis filtering), or transform techniques (α-rooting). Many of them aim at different objectives, such as to exploit the whole available dynamic range or to improve the local contrast; a wide class of algorithms aim at increasing the maximum luminance gradients which are achieved

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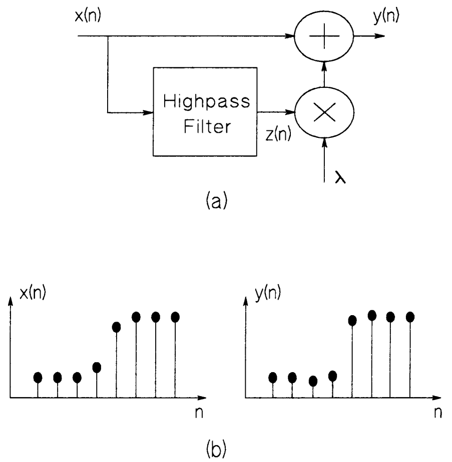


Fig. 1. (a) Block structure and (b) typical behaviour of the UM technique.

at the border between different objects or within textured areas. The algorithm proposed in the present paper belongs to this class, and is based on the well-known unsharp masking (UM) technique.

The fundamental idea of unsharp masking is to subtract from the input signal a lowpass filtered version of the signal itself. The same effect can however be obtained by *adding* to the input signal a processed version of the signal in which high-frequency components are enhanced; we shall refer to the latter formulation, schematically shown in Fig. 1(a). The correction signal z_n in Fig. 1(a) introduces an emphasis which makes the signal variations more sharp. Its effect is exemplified in Fig. 1(b). On the other hand, an undesired side–effect of z_n is also the amplification of noise, which often makes this method not usable in practice.

To reduce the sensitivity to noise, Lee and Park [5] suggest to use a modified Laplacian in the UM scheme. They propose the order statistic (OS) Laplacian, an operator the output of which is proportional to the difference between the local average and the median of the signal; they demonstrate that the resulting UMOS filter introduces a much smaller noise amplification than the conventional UM filter, with comparable edge-enhancement characteristics. A different approach is taken by Mitra et al. in [6]: they replace the Laplacian filter with a very simple and effective operator based on a generalisation of the so-called Teager's algorithm. A quadratic operator is used which, under simple hypotheses, approximates the behaviour of a local-mean-weighted highpass filter, having reduced high-frequency gain in dark image areas. According to Weber's law [4], the sensitivity of the human visual system is higher in dark areas; hence the proposed filter introduces a perceptually smaller noise amplification, without diminishing the edge-enhancing capability of the UM method. Another quadratic filter is defined in [3], where the filter coefficients at a given position in the image are calculated by taking into account the grey level distribution of the surrounding pixels.

An UM-based technique which satisfies Weber's law uses the logarithmic image processing model. In [1], Deng et al. introduce a method which is able to enhance the details in the dark areas of an image and improve the overall contrast. The algorithm is highly sensitive to noise, but it can be modified using multi-scale processing to reduce noise enhancing. A logarithmic transformation is proposed also by De Vries [2]; the transformation is applied to the input image, and from the resulting image (also used for dynamic range control) the contrast image is derived. Contrast in bright and dark areas is treated in the same manner. In this paper, we propose to use a cubic operator in the sharpening branch of the UM technique. As it will be demonstrated, the proposed method has a much reduced noise sensitivity and permits to obtain perceptually pleasant results.

2. The cubic unsharp masking operator

The formal definition of the UM method is given first, considering for simplicity only one-dimensional signals but keeping in mind that this technique is to be applied to images. In the 1-D case, the output signal y_n is obtained from the input x_n through the relation

$$y_n = x_n + \lambda z_n \,, \tag{1}$$

where λ is a positive factor which permits to adjust the intensity of the correction. In the conventional UM method, z_n is derived as the output of a linear filter, which can be a simple Laplacian filter:

$$z_n = 2x_n - x_{n-1} - x_{n+1} \,. \tag{2}$$

The amplitude response of this filter is a monotonically increasing function of the frequency; hence, it is used in Eq. (1) to introduce the required emphasis on the signal variations. As mentioned above, however, an effect of this highpass filter is also the amplification of noise. We propose in this paper to cope with this problem by *modulating* the sharpening signal using another very simple function dependent on the local gradient of the data. The result is a cubic unsharp masking (CUM) technique, in which a discrimination between signal and noise is attempted. To this purpose, two implicit hypotheses are made on the significant image details: first, that they are represented by local gradient values which are larger than those introduced by noise; and second, that they contribute to the data spectrum mainly in its mid-frequency range. Of course, the formal validity of these hypotheses is questionable; on the other hand, they are supported by a couple of practical observations. The former is that detail sharpening techniques which do not permit noise smoothing, as is the case of the UM method, are applicable only when the noise amplitude is small (i.e., in cases in which the first hypothesis is more reasonably satisfied); the latter is that such techniques are needed just when the high-frequency content of the original data is scarce (second hypothesis).

The correction term z_n of the CUM filter may be expressed, again in the 1-D case, as

$$z_n = (x_{n-1} - x_{n+1})^2 (2x_n - x_{n-1} - x_{n+1}) . \tag{3}$$

The modulation component $(x_{n-1} - x_{n+1})^2$ can be thought of as a quadratic function of an estimate of the local gradient. Being quadratic, this function tends to privilege high-gradient areas and it is less sensitive to slow signal variations. Furthermore, the gradient estimate is performed using the bandpass filter $(x_{n-1} - x_{n+1})$, the response of which has a maximum at radian frequency $\pi/2$ and decreases for higher frequencies, rather than the highpass filters $(x_n - x_{n-1})$ or $(x_{n+1} - x_n)$. Both these facts make the overall sharpening signal z_n much more robust to noise than its linear counterpart used in conventional UM, as it will be demonstrated in the following sections.

The price to be paid for achieving this result is a reduced sensitivity to low-contrast image details, which may be insufficiently amplified. If such is the case, a compromise can be suggested between the operator in Eq. (3) and the linear UM method by defining

$$z_n = [(x_{n-1} - x_{n+1})^2 + c]\,(2x_n - x_{n-1} - x_{n+1}) . \tag{4}$$

A positive value should be chosen for the offset factor c which is added to the modulating function in Eq. (4); its effect is to give the highpass term an appropriate amplification even when $x_{n-1} \simeq x_{n+1}$. Of course, the sensitivity to the noise increases with c. In fact, the choice of $c > 0$ moves the behavior of the proposed operator towards the one of a conventional linear unsharp masking filter. In principle, if the maximum luminance value of the data is L, a value of $c \gg L^2$ (and a suitable choice for λ in Eq. (1)) would lead to an approximation of the linear UM filter. In the following, we will not give further explicit mention of the offset factor in Eq. (1); it can be used whenever the specific characteristics of the processed data require it. We shall keep $c = 0$ in the experiments presented in Section 5.

Finally, we observe that, if the input signal has the typical maximum luminance $L = 255$, the dynamic range of the correction term for the cubic operator is much larger than the one for the linear UM filter. In order to reduce the overshoots in the output signal, which should in any case be truncated before displaying the data, and to avoid the introduction of streaking effects, we simply limit all the correction values to make them stay within the range $[-T, T]$. $T = 50$ is used in the following.

3. 2-D CUM filters

Once the main formulation of the CUM technique is given, we can look for the most convenient way to apply it to multidimensional data. For the 2-D case, we shall select among the possible filtering configurations the two which have demonstrated through computer experiments to be able to yield the best results in terms of visual appearance of the processed images.

The general expression of the UM method now is

$$y_{m,n} = x_{m,n} + \lambda\, z_{m,n} , \tag{5}$$

and we first observe that, in the linear case, the correction signal $z_{m,n}$ can be defined as

$$z_{m,n}(\mathrm{LIN}) = 4x_{m,n} - x_{m-1,n} - x_{m+1,n} - x_{m,n-1} - x_{m,n+1}. \tag{6}$$

The first and most obvious 2-D formulation of the CUM filter is deduced by adding the effects of the 1-D filter of Eq. (3) (or Eq. (4)) applied in two orthogonal directions. We shall call this approach 'separable' (S-CUM) in analogy with linear filtering, even if of course this filter is nonlinear and hence the superposition principle does not hold. If the horizontal and vertical directions are chosen, one obtains the expression

$$\begin{aligned} z_{m,n}&(\text{S-CUM}) \\ &= (x_{m-1,n} - x_{m+1,n})^2(2x_{m,n} - x_{m-1,n} - x_{m+1,n}) \\ &\quad + (x_{m,n-1} - x_{m,n+1})^2(2x_{m,n} - x_{m,n-1} - x_{m,n+1}). \end{aligned} \tag{7}$$

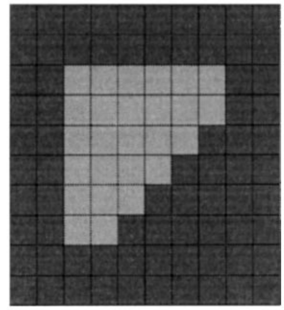
A similar relation could be obtained by selecting the two diagonal directions.

The second formulation of the CUM filter we propose is "nonseparable" (NS-CUM) and is obtained from

$$\begin{aligned} z_{m,n}&(\text{NS-CUM}) \\ &= (x_{m-1,n} + x_{m+1,n} - x_{m,n-1} - x_{m,n+1})^2 \\ &\quad \times (4x_{m,n} - x_{m-1,n} - x_{m+1,n} - x_{m,n-1} - x_{m,n+1}). \end{aligned} \tag{8}$$

Here we use the same Laplacian filter as in the $z_{m,n}$(LIN) expression; we modulate it with the square of a 2-D filter, the response of which in turn is a Laplacian in the horizontal and vertical directions, null along diagonal directions. The reason of this choice can be made clear by some observations about the behaviour of the operators defined by Eqs. (6)–(8) when the input is a simple synthetic signal .

Let us suppose that we want to enhance the image formed by a small, bright and uniform object on a dark background, shown in Fig. 2(a). Here, each square represents a pixel. It should be observed that, since the proposed operator is nonlinear, the numerical values of the pixel luminances significantly affect the amplitude of the output; nevertheless, we are interested only in the gross behavior of the filter and we suppose that a convenient choice



(a)

0	0	0	0	0	0	0	0	0	0
0	-1	-1	-1	-1	-1	-1	-1	-1	0
0	-1	1	1	1	1	1	1	-1	0
0	-1	1	0	0	0	0	1	-1	0
0	-1	1	0	0	0	1	-1	0	0
0	-1	1	0	0	1	-1	0	0	0
0	-1	1	0	1	-1	0	0	0	0
0	-1	1	1	-1	0	0	0	0	0
0	-1	-1	-1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

(b)

0	0	0	0	0	0	0	0	0	0
0	0	-1	-1	-1	-1	-1	-1	0	0
0	-1	2	1	1	1	1	2	-1	0
0	-1	1	0	0	0	0	2	-1	0
0	-1	1	0	0	0	2	-2	0	0
0	-1	1	0	0	2	-2	0	0	0
0	-1	1	0	2	-2	0	0	0	0
0	-1	2	2	-2	0	0	0	0	0
0	0	-1	-1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

(c)

0	0	0	0	0	0	0	0	0	0
0	0	-1	-1	-1	-1	-1	-1	0	0
0	-1	0	1	1	1	1	0	-1	0
0	-1	1	0	0	0	0	0	-1	0
0	-1	1	0	0	0	0	0	0	0
0	-1	1	0	0	0	0	0	0	0
0	-1	1	0	0	0	0	0	0	0
0	-1	0	0	0	0	0	0	0	0
0	0	-1	-1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

(d)

0	0	0	0	0	0	0	0	0	0
0	0	-1	-1	-1	-1	-1	-1	0	0
0	-1	1	1	1	1	1	1	-1	0
0	-1	1	0	0	0	0	1	-1	0
0	-1	1	0	0	0	1	-1	0	0
0	-1	1	0	0	1	-1	0	0	0
0	-1	1	0	1	-1	0	0	0	0
0	-1	1	1	-1	0	0	0	0	0
0	0	-1	-1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

(e)

Fig. 2. (a) A synthetic image; (b) the luminance correction values used to enhance it in the ideal case and yielded (c) by a linear or S-CUM filter, (d) by the NS-CUM filter, (e) by the SNS-CUM filter.

of the coefficient λ will allow us to take into account the signal scale. We can reasonably hypothesise that an ideal UM technique should yield the correction signal $z_{m,n}$ reproduced in Fig. 2(b): the luminance of the image pixels which are adjacent to the object border shall be either increased by a given amount (set to a value of one for simplicity) if the pixel belongs to the object, or decreased by the same amount if the pixel belongs to the background. In all the other cases, $z_{m,n} = 0$. Fig. 2(c) shows the values of $z_{m,n}$ which are output from the S-CUM operator in Eq. (7). As it can be easily verified, they are identical to those which would be yielded by the linear UM operator in Eq. (6). Fig. 2(c) clearly indicates that the required sharpening effect is achieved, but its intensity is a function of the orientation of the object details: the sharpening is stronger along the diagonal border and at the object corners. It is known that the human visual system is more sensitive to horizontal and vertical high-frequency components than to diagonal ones; in this sense, the S-CUM filter can be considered acceptable because it tends to compensate for this effect. On the other hand, if we want to adhere more strictly to the model introduced in Fig. 2(b), the NS-CUM expression in Eq. (8) should be examined. Fig. 2(d) shows the values of the $z_{m,n}$(NS-CUM) signal, and we can observe that the opposite happens with respect to the previous case: the sharpening action is present only along horizontal and vertical borders, and is absent along diagonal borders and at the corners. This motivates the definition of a combined SNS-CUM filter, the correction signal of which is simply the average (or, more simply, the sum) of the two expressions in Eqs. (7) and (8). The $z_{m,n}$ output in this case becomes the one reproduced in Fig. 2(e), which is very close to the ideal case of Fig. 2(b).

As a consequence, we propose the use either of the S-CUM filter represented by Eq. (7) or of the SNS-CUM filter, the correction of which is expressed as the sum of the relations in Eqs. (7) and (8). The study which is presented in the following section will further highlight the properties of such two filters, and it will give a basis for selecting one of them according to the characteristics of the input image.

4. Statistical properties

The effects on noise of the proposed filters are examined in this section. To this purpose, we determine the output variance of the CUM operators when only noise is present at their input. We start by examining the 1-D operator, and we suppose that zero mean i.i.d. Gaussian noise having variance σ_x^2 feeds the filter. The conventional UM technique is studied first, to establish a reference. When the linear operator of Eq. (2) is used, the variance of the correction signal is $\sigma_z^2 = 6\sigma_x^2$, and the overall filter output has variance

$$\sigma_y^2(\mathrm{LIN}) = (6\lambda^2 + 4\lambda + 1)\sigma_x^2 . \tag{9}$$

If the CUM filter of Eq. (3) is used, some manipulations are needed to determine the output variance, since the random variables in this equation are no longer independent one of each other. The details of this calculation are reported in Appendix A. The result is $\sigma_z^2 = 72\sigma_x^6$, and the overall output variance becomes

$$\sigma_y^2(\mathrm{CUM}) = 72\lambda^2\sigma_x^6 + 8\lambda\sigma_x^4 + \sigma_x^2 . \tag{10}$$

We are interested in comparing the expressions in Eqs. (9) and (10), observing first that in both cases the amplification of noise is unavoidable, since it is always $\sigma_y^2 > \sigma_x^2$. At first glance, it would appear that the CUM filter is more sensitive to noise, according to the fact that σ_x^6 and σ_x^4 terms appear in the expression of its variance, but it should be observed that in a real application the values of λ used are very different in the UM and in the CUM cases: due to its nonlinear response, the CUM filter needs a much smaller λ value in order to obtain the desired enhancement effect, and its noise amplification is in fact smaller in all practical cases. To better demonstrate this fact, we need to examine the 2-D operators.

Applying the same mechanism used in the 1-D case, the relations between input and output noise variances can be derived for the various operators above defined. More precisely, linear 2-D UM filtering, as expressed in Eq. (6), yields $\sigma_z^2 = 20\sigma_x^2$, while the S- and NS-CUM filters, respectively, yield $\sigma_z^2 = 176\sigma_x^6$ and $\sigma_z^2 = 960\sigma_x^6$. The overall output noise variance in the cases of interest is

$$\sigma_y^2(\mathrm{LIN}) = (20\lambda^2 + 8\lambda + 1)\sigma_x^2 \tag{11}$$

in the linear case, while it becomes

$$\sigma_y^2(\text{S-CUM}) = 176\lambda^2\sigma_x^6 + 16\lambda\sigma_x^4 + \sigma_x^2 \tag{12}$$

for the separable CUM filter. If the combined SNS-CUM operator is used, we have

$$\sigma_y^2(\text{SNS-CUM}) = 1456\lambda^2\sigma_x^6 + 48\lambda\sigma_x^4 + \sigma_x^2 . \tag{13}$$

In order to compare the noise response of these three filters, we must select a proper value for the coefficient λ in each case. This can be done by resorting to a practical case; indeed, the nonlinear response of the proposed filter makes a comparison based on synthetic signals such as sinusoids or unit steps infeasible. Hence, we take a typical test image (we use the central portion of 'Lenna', but similar results are found by choosing other images) and we look for the values of λ which make the various operators yield comparable sharpening effects. Then, we compare the noise amplifications.

To this purpose we use two methods: the former gives us a very approximate idea of the different filters' behaviour, the latter permits to refine the comparison.

First, we evaluate the average $\hat{z}$ of the modulus of the correction term $z_{m,n}$ over the whole image for the different UM methods. We could hypothesize that equal $\hat{z}$ values indicate equal achieved enhancement; due to the strongly nonlinear behaviour of the CUM operators this is not true, but it can give us at least a hint about the typical λ values which can be selected. For the linear UM filter we choose $\lambda(\mathrm{LIN}) = 1$, which is a practically reasonable value. For our test image, we obtain $\hat{z}(\mathrm{LIN}) \simeq 20$. To get the same $\hat{z}$ from the CUM filters, the values of λ we need can be easily determined by trial and error, and are respectively $\lambda(\text{S-CUM}) \simeq 0.0006$, $\lambda(\text{SNS-CUM}) \simeq 0.0004$. These λ values permit a rough comparison of the different UM techniques. Suppose that i.i.d. Gaussian noise having variance $\sigma_x^2 = 50$ is present at the input of the operators; substituting in Eqs. (11)–(13), respectively, we obtain $\sigma_y^2(\mathrm{LIN}) = 1450$, $\sigma_y^2(\text{S-CUM}) = 82$ and $\sigma_y^2(\text{SNS-CUM}) = 127$. The advantage of the CUM operators in terms of noise amplification is evident, and it remains significant even if the input variance is larger. Only when σ_x^2 is much larger (a few hundreds) the noise amplification of the CUM method becomes too high.

It should be mentioned that the result we have just obtained is optimistic. In fact, we have neglected here the effect of the limiting function which is used in the CUM filters. This limiting function permits the choice of a higher value of λ in practical cases, in order to obtain a more effective enhancement, at the expense of a somewhat larger noise amplification. A more precise comparison results from a slightly more complex approach. We shall exploit the *detail variance* (DV), a parameter which has already been used to assess the performance of enhancement operators [8], finding that it is in good agreement with the perceived image quality. The DV will be more thoroughly illustrated and used also later in this paper; suffice it now to say that we can estimate it by first selecting the portions of the original image in which the local signal variance is high. Only in these detailed zones we evaluate the local variance after processing. Averaging these values yields the DV of the processed image. We assume that high values of DV indicate enhancement of visible image details, and that two processed images wih similar DV values have undergone similar detail enhancement.

We choose in this case $\lambda = 0.7$ for the UM filter, which yields a visually pleasant result (as it will be shown below). Then, selecting $\lambda = 0.0015$ for the S-CUM filter and $\lambda = 0.00073$ for the SNS-CUM filter, we reach the same DV in the three cases, indicating that the sharpening effect achieved is very similar. Some experimental results presented in the next section, in which the same parameter values are used, will corroborate this statement. We are now able to compare the noise amplification yielded by the different techniques, substituting the values of λ in Eqs. (11)–(13), respectively, and plotting the resulting formulae with respect to the input noise variance σ_x^2. The result is shown in Fig. 3; it should be mentioned that the maximum input variance $\sigma_x^2 = 150$ indicated is beyond the typical noise limits of images the details of which can be enhanced without prior noise attenuation. From this diagram, we can observe that:

- the noise amplification of the S-CUM filter is much smaller than the one of the linear UM filter for practical values of σ_x^2. This property makes the use of the CUM filter possible with noise levels at which the linear UM filter would yield unacceptable results. For example, when $\sigma_x^2 = 50$, we have $\sigma_y^2(\mathrm{LIN}) = 820$ and $\sigma_y^2(\text{S-CUM}) = 160$.
- the S-CUM filter is less sensitive to noise than the SNS-CUM filter; hence, the latter can be conveniently used only if the input data are corrupted with a very limited amount of noise; in this case its better deterministic response, as

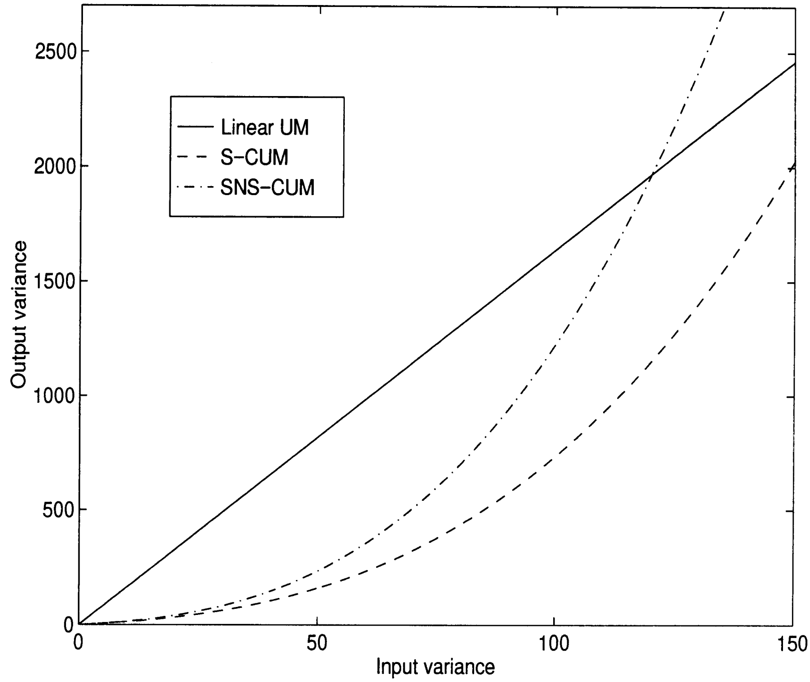


Fig. 3. Output versus input noise variances for different UM operators.

shown in the previous section, makes it the best choice.

It should be further observed that the mean value of the correction signal $z_{m,n}$ is zero for all the linear and nonlinear operators discussed. Moreover, as it can be verified by repeating the procedure described in Appendix A, the formulae for the output variance of the different operators remain valid even if the noise is not zero mean. This is a determining factor for the practical significance of the statistical study presented above. Indeed, the presence of noise is particularly annoying in smooth areas of the input image; such areas can be adequately modelled (as it is often done in image restoration methods) by a locally uniform luminance plus zero mean noise, i.e. by a nonzero mean i.i.d. process. Hence, Eqs. (11)–(13) describe the actual noise amplification in smooth image areas.

As a comparison, we can examine the noise behaviour of the Teager-based UM filter proposed in [6]. We deal for simplicity only with a 1-D version of the operator. Using the same mechanism as above, it can be shown that the output variance when the input is a Gaussian noise with mean μ_x and standard deviation σ_x is given by

$$\sigma_y^2 = 3\lambda^2\sigma_x^4 + (6\mu_x^2\lambda^2 + 4\mu_x\lambda + 1)\sigma_x^2 \,. \tag{14}$$

This relation is in accordance with the criterion used in the design of this operator: noise amplification is a direct function of μ_x, i.e. it is smaller in dark regions than in bright regions, to comply with the Weber effect. Since a μ_x^2 term is present, and as it will be verified in the section devoted to computer experiments, the increase of noise amplification with the local luminance is strong.

5. Computer simulations

The CUM technique has been tested on a variety of different images, obtaining stable and homogeneous performance. An example of the results is presented here. Fig. 4(a) shows the 256×256 central portion of the 'Lenna' image. The effect of the conventional UM method on this image is shown in Fig. 4(b) ($\lambda = 0.7$); it can be seen that the image is

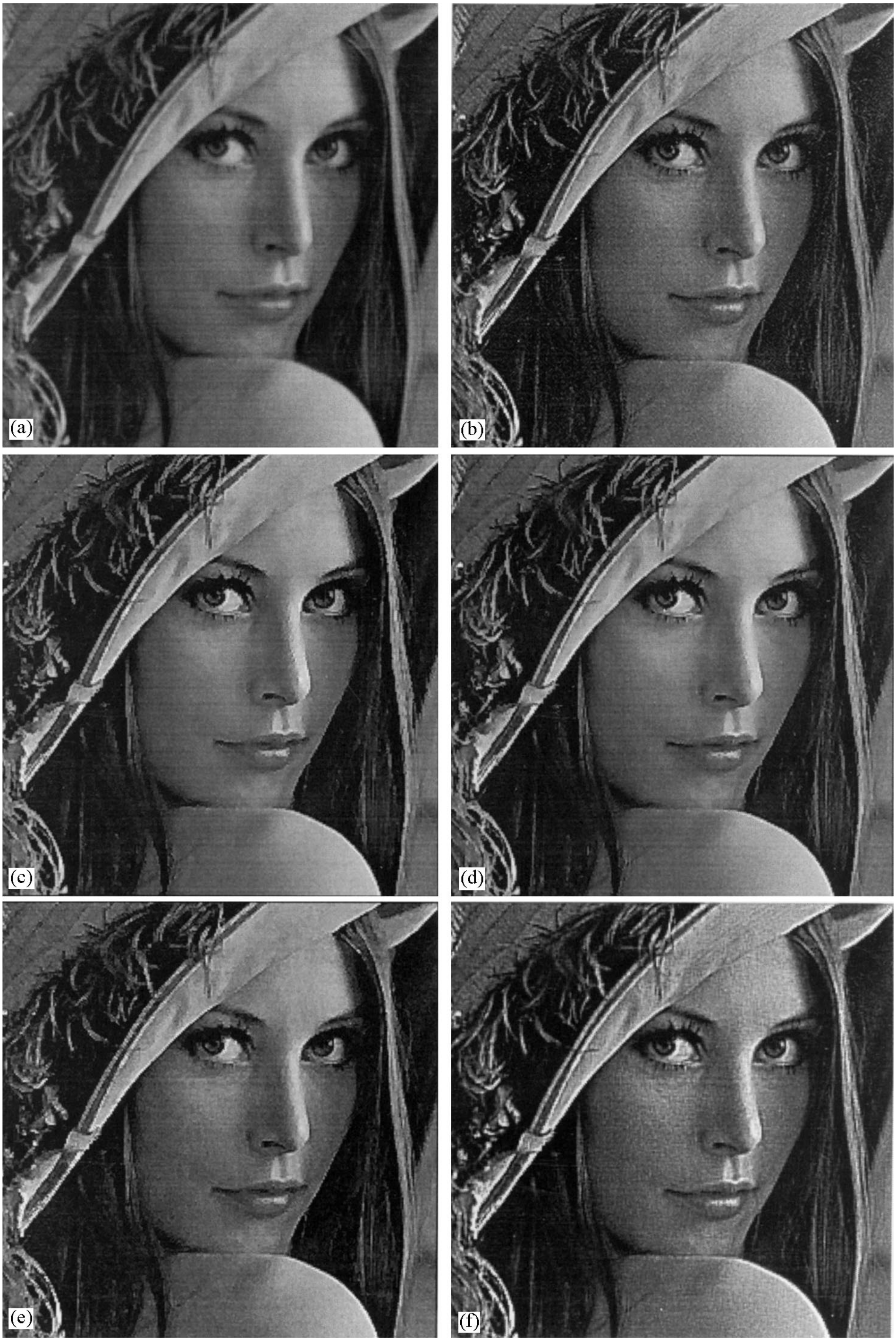


Fig. 4. (a) Original and (b–f) processed images: (b) linear UM, (c) S-CUM, (d) SNS-CUM, (e) UMOS and (f) Type 1B operators.

much sharper, but noise is well visible in the uniform areas. The same original image has been processed using the S-CUM filter of Eqs. (5) and (7), with $\lambda = 0.0015$ (Fig. 4(c). In this image the enhancement of the significant details is very easily perceived, while quantization noise effects are negligible. Very similar results can be obtained from the same operator, applied along the two diagonal directions; in this case, a smaller value for λ should be used (say, $\lambda = 0.001$) due to the wider spacing of the pixels involved in the filtering.

The effects of the SNS-CUM operator are then presented; due to the fact that the correction term for this operator yields a stronger action with respect to the S-CUM filter (it is formed by three rather than two components), and that its behaviour is almost isotropic, the value of λ can be reduced. This is an advantage with respect to noise amplification. When $\lambda = 0.00073$ the result reported in Fig. 4(d) is obtained. This image is similar to the one in Fig. 4(c); a better visual quality has however been achieved in some details: observe for example the better appearance of the ridge of the nose.

It should be observed that the most convenient value of the parameters may change depending on how the image is finally displayed (it may be printed on glossy or opaque paper and at different resolutions, output to a cathode ray or a liquid crystal monitor, and viewed from a variable distance and in differently illuminated environments). For example, to reduce the overshoot effects in high-quality reproductions, the value of T mentioned in Section 2 could be reduced to, say, 30.

To permit a comparison, the same original image has been filtered using two other enhancement algorithms. The first one is the already cited UMOS technique; the algorithm proposed in [5] has been slightly modified to allow the user to control the amount of introduced sharpening: while in [5] the OS Laplacian is directly subtracted from the original data, we introduced a λ coefficient which scales the correction term before subtraction. The result is presented in Fig. 4(e) ($\lambda = 1.25$). The noise amplification is significantly larger than the one of the CUM filter, but it is however smaller than the one of the simple UM method. On the other side, an effect of the OS filter used is the introduction of some visible noise structures in the uniform image areas (observe the forehead and the cheek). Finally, the Type 1B filter [6] has been used, with a scale factor of 190 (Fig. 4(f)). The enhancement of the details in this case is satisfactory, but noise is more visible; in particular, due to the fact that the effect of this operator increases with the local mean luminance, bright image areas appear noisy.

From a quantitative point of view, we can also resort to a measure of the quality of the enhancement which has recently been proposed [8]. It is based on the evaluation of the local image variance using a 3×3 or 5×5 window scanning the image. To this purpose, first of all, each pixel of the original image is assigned a label according to the local measured variance: if this variance is below a threshold, the pixel is deemed to belong to a background area, otherwise to a detail. In this way, a binary reference map is created. Then, the local variance is measured again in the processed images (and even in the original one, if desired). Each variance sample is accumulated as 'detail' variance (DV) or 'background' variance (BV) according to the label assigned to that pixel, and the two average values for DV and BV are obtained for each image. Qualitatively, it can be stated that an ideal enhancement method should yield a DV value significantly larger than the one measured on the original data, while the BV should remain almost constant to indicate unitary noise amplification. Of course, this is a general remark which does not permit to deduce absolute performance measures for a given technique, but DV and BV are a useful tool so as to *compare* different techniques.

For the images we are examining the results presented in Table 1 are obtained. By the way, we verify that the particular values of λ used in the different examples produce in all cases the same DV values (i.e. very similar subjective detail sharpening) and hence allow us to compare the respective noise amplifications through the value taken by the BV parameter.

The values in Table 1 confirm the qualitative observations reported above: with respect to the conventional UM filter and to the UMOS technique, the two proposed CUM methods permit to obtain identical sharpening effects but are much less sensitive to noise (the BV value is four times larger for the linear method and two times larger

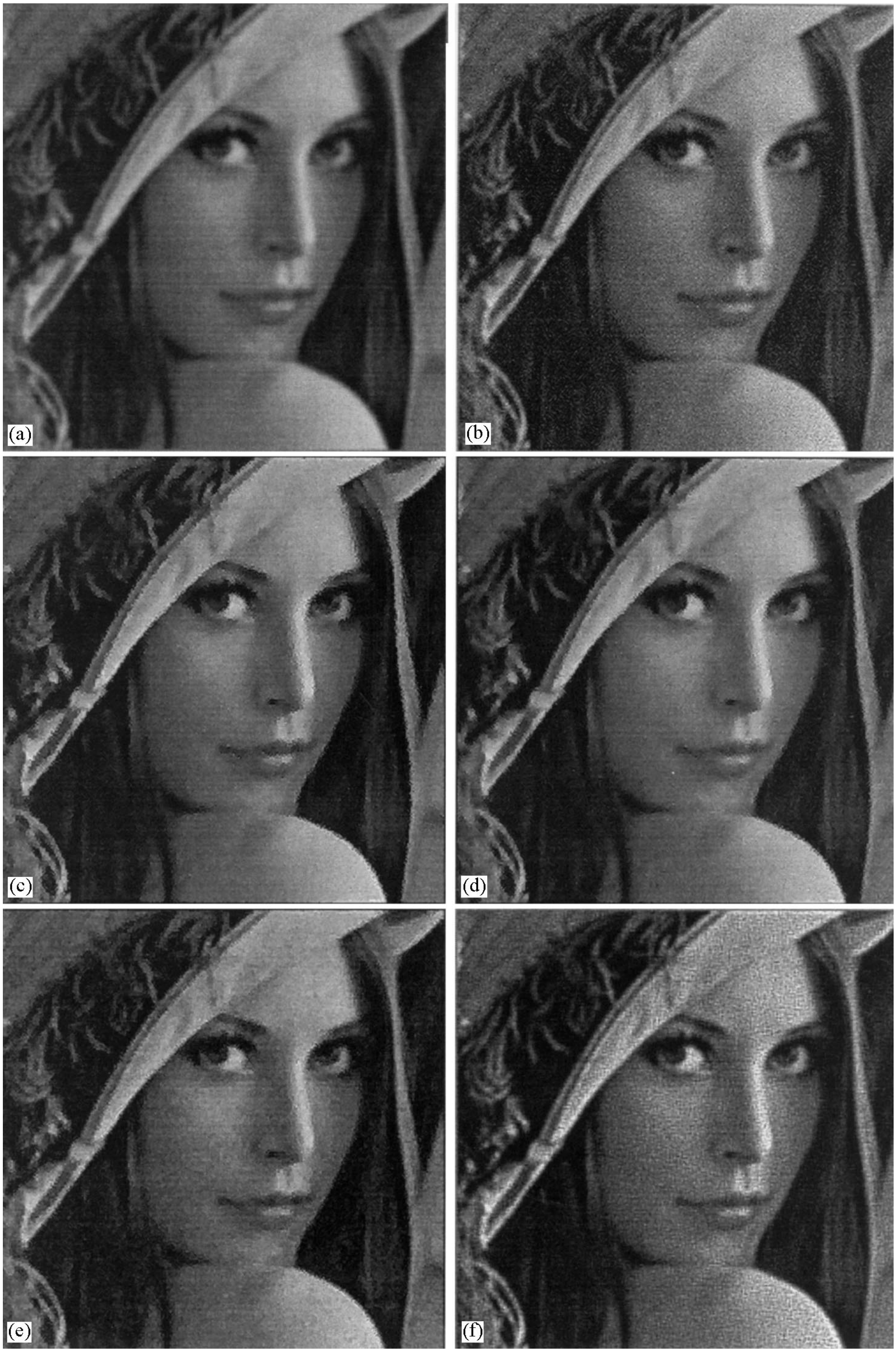


Fig. 5. (a) Blurred and noisy original image and (b–f) processed images: (b) linear UM, (c) S-CUM, (d) SNS-CUM, (e) UMOS and (f) Type 1B operators.

Table 1
DV and BV of the images in Fig. 4

Filter	DV	BV
None	450	19
S-CUM	1738	44
SNS-CUM	1743	46
Linear UM	1738	195
UMOS	1739	98
Type 1B	1734	148

Table 2
DV and BV of the images in Fig. 5

Filter	DV	BV
None	186	32
S-CUM	733	77
SNS-CUM	570	81
Linear UM	675	389
UMOS	610	129
Type 1B	547	286

for the UMOS method than the one for the CUM techniques). The increase of noise which was demonstrated in Section 4 for the SNS-CUM filter with respect to the S-CUM filter is in this case neglectable: this is due to the fact that the input noise is very small (only the quantization noise of the original 'Lenna' data is present). The Type 1B filter ranks approximately halfway between the UMOS and the linear UM filters with respect to noise sensitivity. For this filter, we can also observe that a significant noise amplification in bright areas had been predicted in Section 4.

The advantages of the proposed schemes are very relevant even if the original image is more noisy and blurred. Fig. 5(a) shows the previous test image after blurring with a 5×5 lowpass filter of approximately Gaussian shape and addition of a small amount of white, Gaussian–distributed noise ($\sigma_n^2 = 30$). The image in Fig. 5(a) has been processed with the same operators as above, using for simplicity also the same parameter setting; other values of λ can of course be chosen if a different compromise is preferred between noise amplification and edge enhancement. When we use the conventional linear UM, the superimposed noise makes the result unacceptable, as it is clearly seen in Fig. 5(b). On the contrary, the proposed nonlinear operators perform much better (Fig. 5(c,d)); the noise amplification of the S-CUM method is slightly better than the one of the SNS-CUM method in homogeneous areas such as the shoulder and the cheek. This is the practical demonstration of the diminished noise robustness of the SNS-CUM filter, which had been illustrated in the diagram of Fig. 3. However, the output of the S-CUM filter makes some artifacts more visible, and some sparse white and black spots appear in the image. It should be observed, anyway, that the CUM approach can be slightly modified in order to make it able to better cope with noisy data [9].

The same figures of merit as those used above have been evaluated for the new set of images; they are reported in Table 2 and again confirm the qualitative observations made above. In particular, it is seen that the linear UM technique is extremely sensitive to noise, while progressively better results are yielded by the Type 1B and by the UMOS methods; the CUM methods permit to obtain the best compromise between contrast enhancement and noise sensitivity.

6. Conclusions and future work

In this paper, the Cubic Unsharp Masking technique has been presented. From a statistical analysis of its behaviour, we have been able to demonstrate its reduced sensitivity to noise; the computer experiments performed have validated this analysis, and have practically shown the improvement of the visual quality which can be achieved. A figure of merit has also been used which permits to compare the performances of this and other different enhancement techniques.

An advantage of the CUM method is also its simplicity. This is an important reason for its further development, in particular in the area of video sequence processing, where real-time and cost constraints require simple and effective algorithms to be devised. The 2-D CUM filters presented above can rather successfully be used also for the enhancement of video sequences: it is sufficient to apply the proposed operators to each frame of the sequence. However, we can expect better results if we also take into account the temporal correlation of the data. A 3-D version of the CUM filter is presently under study.

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Appendix A.

In order to find the output variance of the proposed CUM filters when only Gaussian i.i.d. noise of known variance σ_x^2 is present at the input, we use the general formula dealing with linear combinations of random variables X_i:

$$\mathrm{var}[\textstyle\sum a_i X_i] = \sum a_i^2 \mathrm{var}[X_i] + 2\sum\sum_{i<j} a_i a_j \mathrm{cov}[X_i, X_j]\,, \qquad (15)$$

where

$$\mathrm{cov}[X_i, X_j] = E[X_i X_j] - E[X_i]\,E[X_j]\,.$$

Then, we develop Eq. (3) and substitute the result in Eq. (15) [7]; 28 terms are obtained. Four are of the form $\mathrm{var}[X_i^2 X_j]$, with X_i and X_j independent; hence,

$$\mathrm{var}[X_i^2 X_j] = \sigma^2 E[X^4] = 3\sigma^6\,.$$

Two more terms are of the form

$$\mathrm{var}[X^3] = E[X^6] = 15\sigma^6\,,$$

a single term of the type

$$\mathrm{var}[X_i X_j X_k] = \sigma^6$$

(remember that the r.v. are independent) finally exists. All the covariance terms are zero, with the exception of one term of the type

$$\mathrm{cov}[X_i^2 X_j, X_j X_k^2] = \sigma^6$$

and of two terms of the kind

$$\mathrm{cov}[X_i^2 X_j, X_j^3] = 3\sigma^6\,.$$

Adding the different contributions we obtain $\sigma_z^2 = 72\sigma_x^6$ for the correction term defined in Eq. (3). Finally, using again Eq. (15), the overall output variance expression for the CUM filter reported in Eq. (10) is found.

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