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## Explaining and Preventing Delay at a Bottleneck

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| <b>Abstract:</b>  | <p>This study tests the feasibility of using a Kalman filter feedback control system at a bottleneck where drivers are forced to merge. Since the bottleneck is assumed to be operating at capacity, traffic delay is expected owing to high traffic density. The roadway geometry, a lane drop where two traffic lanes merge into one, plays a critical role. Individual and group driver behavior is known to cause traffic delay and to be the sole contributor to traffic noise. A driver behavior or stochastic car-following model is developed to study the effects of roadway geometry, traffic density and traffic noise on performance. The model features Brownian motion speed models that mimic driver behavior and simulates real-world bottleneck conditions where drivers start from a standing position, comfortably accelerate to a cruising speed, and then jockey for position when entering the bottleneck. All drivers are assumed to keep a safe headway. Since the model allows individual drivers to adjust their speed according to changing conditions, the model predictions help explain queue formation and performance. The feedback control system, which is assumed to use V2X (vehicle-to-everything) technology, uses an entirely different strategy. It assists drivers: (1) by managing vehicle speed and (2), by arranging vehicles in a ``zipper'' mode formation. A reference signal or tracking function, which proves to be vitally important in arranging vehicles for a ``zipper'' mode, is derived with a non-linear acceleration model. The impacts of deploying this scheme with V2X technology in a ``Smart City'' are discussed.</p> |
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# EXPLAINING AND PREVENTING DELAY AT A BOTTLENECK

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## ABSTRACT

This study tests the feasibility of using a Kalman filter feedback control system at a bottleneck where drivers are forced to merge. Since the bottleneck is assumed to be operating at capacity, traffic delay is expected owing to high traffic density. The roadway geometry, a lane drop where two traffic lanes merge into one, plays a critical role. Individual and group driver behavior is known to cause traffic delay and to be the sole contributor to traffic noise. A driver behavior or stochastic car-following model is developed to study the effects of roadway geometry, traffic density and traffic noise on performance. The model features Brownian motion speed models that mimic driver behavior and simulates real-world bottleneck conditions where drivers start from a standing position, comfortably accelerate to a cruising speed, and then jockey for position when entering the bottleneck. All drivers are assumed to keep a safe headway. Since the model allows individual drivers to adjust their speed according to changing conditions, the model predictions help explain queue formation and performance. The feedback control system, which is assumed to use V2X (vehicle-to-everything) technology, uses an entirely different strategy. It assists drivers: (1) by managing vehicle speed and (2), by arranging vehicles in a “zipper” mode formation. A reference signal or tracking function, which proves to be vitally important in arranging vehicles for a “zipper” mode, is derived with a non-linear acceleration model. The impacts of deploying this scheme with V2X technology in a “Smart City” are discussed.

**Keywords:** Traffic breakdown, car following, congestion mitigation, Kalman Filter, stochastic processes, stochastic differential equations

25 **INTRODUCTION**

26 A study was conducted and a case was made for using smart technology to prevent traffic delay  
27 and improve traffic flow in a “ring road” experiment (Ossenbruggen 2019b). The basic idea is  
28 straightforward: provide driver assistance using Kalman Filter feedback control. The study was  
29 motivated by a carefully conducted field experiment where traffic breakdown was observed. Drivers  
30 of more than twenty passenger cars were instructed to drive at the same *target speed* (Sugiyama  
31 et al. 2008). At the start of the experiment, all vehicles were evenly positioned around a single-  
32 lane circular roadway. When signaled, the vehicles were observed to move simultaneously at a  
33 comfortable acceleration rate. After a few seconds, moving queues were observed and vehicles  
34 momentarily stopped, i.e., traffic breakdown. The author’s car following model was developed  
35 to simulate these conditions and to develop and test a feedback control system. With it, a traffic  
36 breakdown condition can be detected and its speed adjusted in a manner to prevent breakdown.

37 In this paper, the same approach is used to analyze a “bottleneck.” Here, the drivers are  
38 forced to merge. Before describing the bottleneck study, we will highlight the key conditions and  
39 findings from the previous study. Much was learned from this simple controlled experiment. The  
40 conditions were ideal. All drivers were motivated to follow instructions. Distractions, such as,  
41 passing vehicles, were eliminated. Only passenger cars were used. Granted, individual reaction  
42 times are affected by gender, age, alcohol use, temperament, etc. These factors are considered  
43 immaterial given the nature of the experiment.

44 Four factors explain breakdown in the “ring road” experiment: (1) individual behavior, (2) the  
45 behavior of other road users (Ou et al. 2018), (3) traffic density (Mahnke et al. 2005), and (4) traffic  
46 volatility (Kish and Bezrukov 2000). Evans (2004) is “overwhelming” convinced that the first two  
47 factors determine individual risk associated with traffic safety. One-third of severe car accidents are  
48 related to drowsiness (Friedrichs et al. 2010). Since safety is principal concern here, it is reassuring  
49 that individual and group behavior was identified as key factors in the “ring road” experiment.  
50 Third, the vehicles in the experiment were traveling too close together. By reducing the number of  
51 vehicles on the road, i.e., reducing the traffic density, an individual driver could operate a vehicle

52 over a range of speeds without affecting others. By reducing traffic density, vehicle interaction  
53 is lessened and the chance of breakdown becomes less likely. Fourth, if the drivers were able to  
54 maintain the *target speed*, then the behavior of the other road users and traffic density would have  
55 been irrelevant. *Traffic volatility*, measured as the variability in speed that varies over time, is a  
56 necessary condition for breakdown. It is also called *traffic noise*, which the author prefers because  
57 the latter term implies unwanted.

58 To explain breakdown at a bottleneck, a fifth factor is added to the above list of items: (5) road  
59 geometry or lane drop (Cassidy and Bertini 1999). A bottleneck where two parallel freeway lanes  
60 merge into one lane, as shown in Figure 1, is studied. Three zones are shown: *upstream*, *merge* and  
61 *downstream*. To explain driver behavior in each zone, mathematical models are developed for each  
62 zone and then are linked together to simulate traffic behavior in a case study consisting of twenty  
63 vehicles. The vehicles must align themselves and be traveling a speed that allows them to safely  
64 proceed to the downstream location at  $x(t_3)$ .

65 To achieve a safe merge, drivers must make critical decisions at or before entering the merge  
66 zone at location  $x(t_2)$ . Consider two drivers prior to reaching point  $x(t_2)$ ; one vehicle on the  
67 *left-lane* and the other one on the *right-lane*. Their relative positions are important. They are  
68 called “side-by-side” and “zipper” merges. Traffic engineers and managers have a strong opinion  
69 that a “zipper” merge is more efficient than a “side-by-side” merge (Johnson 2008). The reason is  
70 simple. When two vehicles are traveling side-by-side on parallel lanes, one driver must yield to the  
71 driver of the other vehicle to avoid a crash. If two vehicles are staggered, then there is no conflict.  
72 The author estimates that the majority of drivers prefer to travel side-by-side. Why? It deals with  
73 self interest. Simply, an individual wants to minimize travel time and get an advantage “by getting  
74 ahead of the next guy.” Instinctively, drivers choose the latter option, the “side-by-side” merge.

75 A Kalman Filter (KF) feedback control system or *Driver-Assistant System* (DAS) has the po-  
76 tential to eliminate this driver instinct and to effectively deal with delay causing factors: roadway  
77 geometry, high traffic density, and traffic noise. The acronyms KF and DAS can be used inter-  
changeably. The author chooses to use KF when emphasizing its use as a mathematical tool;

79 whereas, DAS when emphasizing its potential as a congestion mitigation tool. A two-step strategy  
80 is used to demonstrate DAS: (1) Explain delay by simulating it without driver assistance. (2) Prevent  
81 delay with a KF feedback control system. A worse case is used. The traffic flow at  $x(t_3)$  equals the  
82 capacity of a single-lane freeway lane. Performance is evaluated in terms of speed primarily where  
83 safety (crash avoidance) acts as a constraint.

84 The material in this paper is presented step-by-step. The **METHODS** section contains three  
85 sections. Each section deals with model building and each has its own focus area. The **Basic**  
86 **Concepts** section focuses on roadway performance and safety. Targets are established and used as  
87 benchmarks of success. The **Explaining Traffic Delay** section focuses on driver behavior, drivers  
88 who are faced with the task of safely driving through a bottleneck at capacity. A driver behavior  
89 or stochastic car-following model is derived and used to describe the obstacles faced by these  
90 drivers. The **Preventing Traffic Delay** section focuses on overcoming these obstacles with state  
91 space modeling and a Kalman filter. The **DISCUSSION** section focuses on implementing the DAS  
92 system in a “Smart City” committed to using ITS technology.

### 93 METHODS

94 A few words about the mathematical notation will help clarify the paper. Drivers must respond  
95 to various stimuli at different times; thus location, speed and acceleration are important variables.  
96 Driver behavior is described with time-series functions:  $x(t)$ ,  $\dot{x}(t)$  and  $\ddot{x}(t)$ , respectively. Consider  
97 the schematic diagram of Figure 1 again and a simulation consisting of twenty vehicles,  $n = 20$   
98 vehicles over a time frame of 60 seconds,  $0 \leq t \leq 60$  s. At  $t = 0$ ,  $n_{left} = n_{right} = 10$  vehicles.  
99 All vehicles start at rest, accelerate, reach a cruise speed, and merge. Thus,  $n = n_{left} + n_{right} = 20$   
100 vehicles starting at time  $t_3 = 42$  s. Each driver will enter the merge zone at location  $x(t_2)$  at speed  
101  $\dot{x}(t_2)$  at time  $t_2$  and enter the downstream zone at location  $x(t_3)$  at speed  $\dot{x}(t_3)$  at time  $t_3 = 42$  s.  
102 Each early arrival driver will spend approximately twelve seconds merging,  $t_2 = 30 < t < t_3 = 42$   
103 s. Late arrivals will spend more time merging when  $t_2 < 30$  s. Early and late arrivals are defined  
104 presently.

105      **Basic Concepts**

106      The material given in this section is used throughout the paper. Topics include: *Acceleration*,  
107      *Merging Protocols, Risk Aversion and Safety* and *Capacity of a Single Freeway Lane*.

108      *Acceleration*

109      A non-linear acceleration model plays a critical role for describing merging behavior:

110       $\ddot{x}(t) = a - bt \quad (1)$

111      The parameters  $a$  and  $b$  are the acceleration rate and correction factor, respectively. For  $a > 0$  and  
112       $b > 0$ , the acceleration capability of a vehicle decreases as the speed increases (Elefteriadou 2014).  
113      Speed  $\dot{x}(t)$  and location  $x(t)$  functions are determined by integration.

114      In the **Explaining Traffic Delay** section, a driver behavior or stochastic car-following model is  
115      used to describe a situation where a vehicle decelerates and then accelerates again. This situation  
116      typically occurs over time period  $t_2 \leq t < t_3$ , the *merge zone*, when a following vehicle must change  
117      speed to avoid crashing into its lead vehicle. Knowing the speeds,  $\dot{x}(t_2)$  and  $\dot{x}(t_3)$ , and locations  
118       $x(t_2)$ , and  $x(t_3)$ , the model parameters of  $a$  and  $b$  can be determined and speed  $\dot{x}(t)$ , a quadratic  
119      equation, and location  $x(t)$ , a third-order equation, can be estimated over time  $t_2 \leq t < t_3$ .

120      The acceleration model plays equally important roles for describing traffic behavior over time  
121      periods  $0 \leq t < t_1$ , *start-up free-flow traffic zone*. Each vehicle is assumed to comfortably accelerate  
122      from a standing position to the *target speed*,  $u^*$  at  $t_1$ . In this case, the acceleration parameter is  
123      equal to  $a = u^*/t_1$  and  $b = 0$ . Here, the vehicle speed  $\dot{x}(t)$  and location  $x(t)$  equations are linear  
124      and quadratic equations, respectively.

125      There are two cruise zones:  $t_1 \leq t < t_2$ , *cruise free-flow traffic zone* and  $t_3 \leq t < t_4$  *cruise*  
126      *congested traffic zone*. In these cases, the acceleration parameter is equal to  $a = b = 0$  with a  
127      constant speed equation,  $\dot{x}(t)$  and linear location equation  $x(t)$ .

128      *Merging Protocols*

129      The efficiencies of the “side-by-side” and “zipper” merges can be quantified using the mod-  
130      els described in the previous subsection on *Acceleration*. Consider the reactions of any two  
131      drivers traveling in parallel upstream lanes who must avoid conflict (crashing) when they reach  
132      the bottleneck. Assume when entering the merge zone that they are traveling at same speed at  $t_2$ ,  
133       $\dot{x}_{left}(t_2) = \dot{x}_{right}(t_2) = u^*$ .

134      First consider a “side-by-side” merge where a pair of drivers are assumed to be at the same  
135      location,  $x_{left}(t_2) = x_{right}(t_2)$  at  $t_2$ . At time  $t_3$ , however, both vehicles are traveling on the same  
136      single-lane roadway where one vehicle is positioned behind the other one. For example, say,  
137       $x_{left}(t_3) > x_{right}(t_3)$ . The drivers have transitioned from a *free-flow traffic* to a *congested traffic*  
138      condition. The vehicle on the left-lane becomes a *leader* and the vehicle on the right-lane becomes a  
139      *follower*. Since both drivers want to minimize travel time, both vehicles are assumed to be traveling  
140      at the same speed at  $t_3$ , such that  $\dot{x}_{leader}(t_3) = \dot{x}_{follower}(t_3) = u^*$  and  $x_{leader}(t_3) > x_{follower}(t_3)$ . Of  
141      course, there are inefficiencies in this maneuver. One or both vehicles must change their speeds to  
142      achieve this result.

143      Consider a “zipper” merge where similar assumptions are made but this pair of drivers are  
144      assumed to be located at different positions,  $x_{left}(t_2) > x_{right}(t_2)$  at  $t_2$ . Again, both are traveling at  
145      the same speed,  $\dot{x}_{left} = \dot{x}_{right} = u^*$ . It is not beyond the imagination a merge where neither driver  
146      must change speed. In this ideal situation, there is no conflict and no delay. Speed  $\dot{x}(t)$  and location  
147       $x(t)$  equations are defined by a constant and first-order equations described above.

148      *Risk Aversion and Safety*

149      Regardless of the merge type, all traffic merges are done safely. That means the *gap* between the  
150      *lead* and *following* vehicles must be sufficiently wide when the vehicle cross the bottleneck at time  
151       $t_3$ . What is a “sufficiently wide gap?” There is no clear-cut answer. It depends on the point of view  
152      of the individual being asked. A driver will most likely answer differently than a traffic manager  
153      or police officer. There is a delicate balance between the quest for good roadway performance and  
154      acceptable risk. The following discussion is an attempt to quantify, codify safety and establish a

155 “safe” gap.

156 According to the *rule-of-the-road* (ROR) definition advocated in American driving schools, a  
157 *safe driving distance* is one car length for each 10 mph. For speed  $u$ , a “safe” ROR gap is calculated  
158 as  $ul/10$  where  $l$  is a passenger car length. A “safe” gap, now expressed as a “*safe*” space headway,  
159 is calculated and denoted as  $s_{ROR} = ul/10 + l$ , where  $l$  is measured in feet.

160 The distance headway at time  $t_3$  is straightforwardly calculated as:  $s = s(t_3) = x_{leader}(t_3) -$   
161  $x_{follower}(t_3)$ . If  $s > s_{ROR}$ , then a merge is classified as a “safe, acceptable driver risk” merge;  
162 otherwise, it is not.

163 *Capacity of a Single Freeway Lane*

164 To trigger traffic breakdown, a *worse case scenario* is assumed where the bottleneck, a single  
165 freeway lane, is assumed to be operating at capacity  $c$ . At  $c$ , roadway performance is unstable  
166 and breakdown is expected. Breakdown is typically associated with a dramatic decrease in speed,  
167 queue formation and capacity drop (Castillo 2001) (Cassidy and Bertini 1999).

168 The Highway Capacity Manual (TRB 2010) guideline lists the capacity as  $c = 2,380 \text{ v/h/lane}$   
169 (*vehicles per hour per lane*) at a traffic density equal to  $k = 28 \text{ v/km}$  (45 v/mile) and a speed equal to  
170  $u = 23.7 \text{ m/s}$  (53.3 mph). Given  $q = ku$  and  $s = 1/k$ , capacity relationship is rewritten as  $c = u^*/s$ .  
171 Since  $s$  has a different numerical value than  $s_{ROR}$ , it is denoted as  $s_{HCM}$ . If  $s > s_{HCM}$ , then a merge  
172 is classified as a “safe” merge; otherwise, it is not.

173 We have two “safe” headway values:  $s_{HCM} = 36.2 \text{ m}$  (117 feet) and  $s_{ROR} = 26.9 \text{ m}$  (82 feet), and  
174 a discrepancy that is easily resolved. Unstable traffic and breakdown are expected to occur when  
175 the traffic flow equals the roadway capacity, which is before a “safe, acceptable driver risk” merge  
176 can be achieved. Thus,  $s_{HCH} = 36 \text{ m}$  takes precedence and is declared the “safe” headway target  
177 and is denoted as  $s_{safe}$ .

178 Table 1 contains two sets of information, one set associated with a traffic manager, whose goal  
179 is to optimize roadway performance and a set associated with society’s perception of roadway risk  
180 and acceptance. It is interesting that neither goal is met. In the case of a traffic manager, when the  
181 traffic reaches an optimum, it becomes unstable and delay occurs. In the case of the driving public,

182 drivers, who feel comfortable driving close together, cannot do so because traffic becomes unstable  
183 before their “acceptable risk averse” merge can occur.

## 184 Explaining Traffic Delay

185 A principle aim of this paper is to develop a model that simulates reality. The material presented  
186 above contains models and assumptions for achieving this aim in a realistic manner. *Traffic noise*,  
187 a driver’s inability to maintain a target speed  $u^*$ , is now addressed.

### 188 Stochastic Differential Equations

189 To stimulate driver behavior, stochastic differential equations (SDE) models are used (Iacus  
190 2008) (Galtier and Touboul 2012) (Weits 1992). The general form of a SDE model is  $dX =$   
191  $f(t, X)dt + g(t, X)dW$  where  $X$  and  $W$  are random variables and where  $W$  refers to Brownian motion.  
192 For the purposes of simulation,  $dW$  is estimated as  $W(\Delta T) \sim \sqrt{\Delta T}\sigma N(0, 1)$  with a time step equal  
193 to  $\Delta t = 0.1$  s and  $\sigma$  denoting *traffic noise*.  $N(0, 1)$  is a standard normal probability distribution.  
194 The introduction of  $W$  and  $\sigma$  value are critical in realistically simulating field conditions with a  
195 stochastic car-following model and simulation (Kendziorra et al. 2016) (Wagner 2012).

196 The following models, which were introduced above as deterministic models, are now treated  
197 as stochastic differential equations:

$$198 d\dot{X} = a^*dt + \sigma dW \quad \text{for start-up free-flow traffic, } 0 \leq t < t_1; \quad (2)$$

$$199 d\dot{X} = \sigma dW \quad \text{for cruise free-flow traffic, } t_1 \leq t < t_2; \quad (3)$$

$$200 d\dot{x} = (a - b(t - t_2))dt \quad \text{for merge zone traffic, } t_2 \leq t \leq t_3; \quad (4)$$

$$201 d\dot{X} = \sigma dW \quad \text{for cruise congested traffic, } t_3 \leq t < t_4 \quad (5)$$

202 where  $a^*$ ,  $a$  and  $b$  are acceleration rate parameters defined above. Predicted speeds are denoted  
203 as  $\hat{x}(t)$ . The estimates of location  $\hat{x}(t)$  are estimated using these speed predictions and numerical  
204 integration.

205 The *merge zone traffic* model, a deterministic model, requires clarification. It is invoked only  
206 when a driver of a following vehicle violates the safe headway rule,  $s < s_{safe}$ . When a driver needs

207 to take action, decelerate, the act is deliberate and is assumed to be executed with no traffic noise.  
 208 At other times when a driver not violate the safe driving rule, traffic noise plays a role. This will be  
 209 illustrated in the following subsection using different examples.

210 *Car-following*

211 Now, attention turns to describing how vehicles interact with one another and when necessary, a  
 212 driver will make a correction or corrections. The simplest example deals with the driver of vehicle  
 213 1, who is never a driver of a following vehicle, thus this individual is immune from correction.  
 214 The simulation of this driver uses models (2) and (3) only for time ranges  $0 \leq t \leq t_1 = 10$  and  
 215  $t_1 = 10 \leq t \leq t_2 = 60$ , respectively. The initial condition for model (3) is  $\dot{X}(t_1) = \hat{x}(t_1)$ , the last  
 216 predicted value from model (2). The predicted speeds for  $0 \leq t \leq 60$  are used to predict locations  
 217  $\hat{x}(t)$  for vehicle 1 over this time range.

218 This scheme is used for vehicles 2 through  $n = 20$ . Vehicle spacing  $s(t)$  for all lead and following  
 219 pairs is now important. The mathematical notation for a merge is tricky and can divert attention  
 220 away from obtaining a basic understanding of a bottleneck operating at capacity. To simplify the  
 221 notation, a *lead* vehicle is denoted as  $i$  and a *following* vehicle is denoted as  $i + 1$ . Vehicle spacing  
 222 is estimated as  $s(t) = \hat{x}_i(t) - \hat{x}_{i+1}(t)$ .

223 Consider a *cruise congested traffic merge* for  $t_3 = 42 \leq t < t_4 = 60$  for drivers 1 and 2. Precisely  
 224 at time  $t_3 = 42$  s, the safe headway condition is known with certainty,  $s(t_3) \geq s_{safe}$ . After this point  
 225 in time where  $t_3 > 42$  s,  $s(t)$  can be observed to increase with time or can initially increase and  
 226 then decrease with time. Suppose in the former case, the headway  $s(t)$  is observed to grow over  
 227 time. It suggests the lead vehicle is speeding up,  $\dot{x}_i(t) > \dot{x}_{i+1}(t)$ . The latter case, there are three  
 228 possibilities:  $s(t) < 0$ , indicating a crash,  $s(t) < s_{safe}$ , indicating a safe headway rule violation,  
 229 and  $s(t) \leq s_{safe}$ , indicating no violation. When either  $s(t) < 0$  or  $s(t) < s_{safe}$  is encountered, the  
 230 car-following model alters the speed and location of the following vehicle:

$$\begin{aligned}
 \hat{x}_{i+1}(t) &= \hat{x}_i(t) \\
 \hat{x}_{i+1}(t) &= \hat{x}_i(t) - s_{safe}
 \end{aligned} \tag{6}$$

233 The following speed is set equal to the lead vehicle speed,  $\hat{x}_{i+1}(t) = \hat{x}_i(t)$ , and is located a safe  
234 distance behind the the lead vehicle.

235 Model (6) is used to explain the formation of moving and stationary queues for *cruise free-flow*  
236 *traffic* for  $t_1 \leq t < t_2$ . Even though the traffic on the left and right traffic lanes are classified  
237 as free-flow,  $s(t) < 0$  or  $s(t) < s_{safe}$  violations can be encountered and a driver must make a  
238 correction. Model (6) is also used for *cruise congested traffic* for  $t_3 \leq t \leq 60$ .

239 When dealing with a stochastic process, surprises occur. It is tacitly assumed above that vehicles  
240 will not back up. If  $\hat{x}_{i+1}(t) < 0$ , then the vehicle is backing up. The situation can occur when *merge*  
241 *zone traffic* model (4) is applied. It is corrected by assuming that a driver will detect the traffic  
242 has slowed and will begin decelerating in a timely fashion before time  $t_2$ . In other words, a shock  
243 wave has formed and a queue is moving in the upstream direction. This impact of this restriction is  
244 illustrated in the following section where queue formation is inspected for each of the four merge  
245 phases with graphics.

246 *Model Synthesis*

247 Bottleneck breakdown and delay is proving to be a complicated process to analyze as a stochastic  
248 process. The use of time-speed  $t - \dot{x}$  and time-space  $t - x$  trajectories brings clarity. Begin by  
249 inspecting Figures 2 through 5 where the  $t - \dot{x}$  and  $t - x$  trajectories are shown in the upper and  
250 lower panels, respectively. Since  $t_2$  and  $t_3$  are important transition times, they are shown. In order  
251 to appreciate the importance of each phase of the stochastic car-following algorithm and how they  
252 interact, each of these figures are discussed in turn.

253 *Start-up Free-Flow Traffic:* The  $t - \dot{x}$  trajectories shown in the upper panels of Figure 2 may  
254 suggest the drivers are out of control. The reader must be cognizant of the fact that these traces  
255 are based on predictions derived from a far from perfect mathematical description of reality. No  
256 attempt is made to smooth these trajectories. The simulation uses a time-step of  $\Delta t = 0.1$  s giving  
257 the impression the drivers are accelerating and braking in quick succession. Looking at these  
258 trajectories over the 60 second duration of the experiment show different driver trends where most  
259 maintain a constant average speed over time. Significantly, there is a wide range of speeds at  $t = 10$

260  $s$ , suggesting the condition for a successful “zipper” merge is highly unlikely.

261 The time-space  $t - x$  trajectories are smoother than the  $t - \dot{x}$  trajectories. Since many of the  
262 trajectories cross one another, indicating a crash, suggests that some drivers are reckless. At this  
263 point in the discussion, the drivers are acting independently and the constraints discussed above  
264 have not been imposed as yet.

265 *Cruise Free-Flow Traffic:* Figure 3 shows the  $t - \dot{x}$  and  $t - x$  trajectories after the restrictions  
266 of model (6) are imposed. The drivers are now following one another safely over the entire study  
267 period. Over the long term extending past  $t_2$ , clusters form. Two form in the left-lane and three  
268 form in the right-lane. This is interesting from the point of view that if no bottleneck exists, the  
269 drivers would theoretically proceed in this manner.

270 *Merge Zone Traffic:* The left panels of Figure 4 show the consequence of superposing the  $t - \dot{x}$   
271 and  $t - x$  trajectories. The lower panel shows the  $t - x$  trajectories crossing. For  $0 \leq t < t_2$ , this is  
272 fine because the drivers are traveling in separate lanes, thus there are no conflicts. For the merge for  
273  $t_2 \leq t < t_3$ , the situation is more complicated. At  $t_2$ , the  $t - x$  trajectories may cross, but at  $t_3$ , the  
274 safe headway rule is enforced. The lower-right panel shows its effect. The upper-right panel with  
275 smooth quadratic-shaped  $t - \dot{x}$  trajectories show the vehicles being affected. The smooth trajectories  
276 and the irregular shaped trajectories extending from  $t_2 = 30$  to  $t_3 = 42$  s are early arrivals. The  
277 smooth trajectories extending from  $t = 18$  to  $t_3 = 42$  s are late arrivals. The drivers of the late  
278 arrivals see a traffic jam forming and begin to comfortably decelerate at time  $t = 18$  s.

279 *Cruise Congested Traffic:* The lower-right panel of Figure 4 shows large discontinuities between  
280 the  $t - x$  trajectories at  $t_3$  to draw attention to the impact associated with the steps to mitigate  
281 congestion. The discontinuities are removed by generating a new set speed predictions for  $t_3 \leq t \leq$   
282 60.

283 Figure 5 shows the complete simulation run. It shows a few drivers can affect the group and  
284 trigger cluster formation. Figure 6 gives a different perspective. Here, the merge is more orderly  
285 especially after time  $t_3$  where the drivers operate their vehicles with military precision. Instead  
286 of a few drivers affecting the group, two groups of early and late arrivals form. Two vehicles are

287 slowed to a speed approaching zero in comparison to one driver being momentarily slowed to a  
288 speed approaching zero in Figure 5 demonstrating how traffic noise  $\sigma$  can affect an outcome.

289 The average speed, headway and flow at the bottleneck shown in Table 2 for  $\sigma/u^* = 0.00$  and  
290 0.03 show neither predictions meet target condition listed in Table 1. A sensitivity analysis puts  
291 these predictions of performance in perspective.

292 *Sensitivity Analysis*

293 Before selecting assignment of  $\sigma/u^* = 0.03$  or  $\sigma = 0.72 \text{ m/s}$  for discussion, a sensitivity analysis  
294 was performed. Hundreds of simulation runs where the information compiled in tables and figures  
295 like Table 2 and Figure 6 were inspected. While an effort to identify a representative Brownian  
296 motion value of  $\sigma$  was abandoned, the sensitivity analysis brings insight to the role of traffic noise,  
297 i.e., driver behavior.

298 First, focus on the small values of  $\sigma/u^*$  ratio. At  $\sigma/u^* = 0.0$ , the stochastic car-following model  
299 is transformed into a deterministic model. Regardless of the fact that this model is considered to  
300 be poor representation of reality, it clearly shows the roles that high traffic density, measured as a  
301 headway, and road geometry affect performance. Since the average flow  $\bar{q}(t_3)$  is a function of these  
302 two values and  $\bar{q}(t_3) < c$ , these two factors in combination negatively affect performance.

303 As the  $\sigma/u^*$  ratio increases leads to a decrease in flow,  $\sigma/u^* \uparrow \Rightarrow \bar{q}(t_3) \downarrow$ . The decrease is  
304 particularly pronounced for  $\sigma/u^* \geq 0.03$  where  $\bar{q}(t_3) \ll c$ . The decline in flow is due to decline in  
305 average speed and increase average headway. As the traffic noise increases, average speed decreases  
306 and average headway widens as shown in Table 2. Here, the combination of high traffic density,  
307 road design and traffic noise - driver behavior - produce a decline in capacity.

308 In summary, the *stochastic car-following* model or if you prefer a *driver behavior* model, deals  
309 with drivers, who wish and act as if free-flow traffic conditions prevailed. According to the model,  
310 drivers will act independently until they realize their actions are constrained. When reality finally  
311 prevails, the model attempts to mimic the actions of drivers faced with potential crash risk situations.

312      **Preventing Traffic Breakdown**

313      Now, imagine driver assistance, technical assistance, is offered to all drivers entering the  
314      bottleneck. The goal is to allow drivers to pass through the bottleneck in “zipper” mode formation.  
315      A KF feedback control system, which offers assistance of all  $n = 20$  drivers, is contemplated.

316      *A Kalman Filter Model*

317      A KF model is a linear state space model of the following form:

$$\frac{d}{dt}\dot{x} = -\frac{f_d}{m}\dot{x} + \frac{1}{m}f + w_d \quad (7)$$

$$y = \dot{x} + w_d + w_n \quad (8)$$

320      where the symbol  $w$  denotes white noise (MATLAB@9.2 2019). The state equation (7) describes  
321      vehicle acceleration  $\ddot{x} = d\dot{x}/dt$  as a function of vehicle speed  $\dot{x}$ , thrust  $f$ , drag  $f_d$ , and process  
322      noise  $w_d$ . The measurement equation (8) describes sensor measurement  $y$  as the sum of vehicle  
323      speed  $\dot{x}$ , process  $w_d$  and measurement  $w_n$  noise.

324      The state equation is derived from Newton’s Law,  $F = m\ddot{x}$ , where the force on the vehicle is  
325       $F = f - f_d$  ( $N$ ),  $m$  = vehicle mass ( $kg$ ) and  $\ddot{x}$  = vehicle acceleration ( $m/s^2$ ). The drag force is  
326      estimated as  $f_d = \rho C_d A u^*$  where  $\rho$  = air density ( $kg/m^3$ ),  $C_d$  = drag coefficient for a passenger  
327      vehicle,  $A$  = frontal area of the vehicle ( $m^2$ ) and  $u^*$  is the target speed. The drag force  $f_d$  is  
328      approximately equal to  $250N$  for a standard-size American passenger car traveling at  $u^* = 23.7$   
329       $m/s$ .

330      The KF controller is constructed around minimizing process error,  $\epsilon = w_r - y$  where  $w_r$  denotes  
331      a reference signal (Duriez et al. 2017). For this study,  $w_r$  is defined to be a tracking function  $u^*(t)$   
332      for the time period  $0 < t = 60$ . The  $u^*(t)$  function is distinguishable from the target speed  $u^*$ . The  
333      tracking function is constructed in a manner to assure that vehicles are aligned properly to achieve  
334      a “zipper” merge.

335      To minimize  $\epsilon$ , not only does the  $w_r = u^*(t)$  function need to be specified but the values of  $w_d$   
336      and  $w_n$  need to be assigned.

337     *The Tracking Function*

338     To make a convincing argument for feedback control, it is important to appreciate the challenges  
339     that drivers face when driving at capacity  $c$ . Let's set the stage again. All vehicles start from a  
340     standing position, comfortably accelerate and then try to maintain a constant speed. These drivers  
341     are assumed to be motivated the same way the drivers in the "ring road" experiment were. They  
342     aim to follow instructions but are unable to do so. With feedback control, the operation of the  
343     vehicles is taken out of the hands of the drivers and vehicle are controlled by external means for  
344      $0 \leq t \leq 60$ .

345     A reference signal  $w_r$  is designed to fulfill these drivers' needs by taking advantage of a "zipper"  
346     merge. The modeling strategy used here takes advantage of models (2) through (5) assuming  $\sigma = 0$   
347     for all these models. In other words, driver behavior as reflected by traffic noise  $\sigma$  - driver behavior  
348     - is eliminated from consideration. Thus, the  $w_r$  speeds  $\dot{x}(t)$  and locations  $x(t)$  are derived from  
349     deterministic models.

350     The  $w_r$  modeling building effort is straight forward for all merge phases with the exception of  
351     the *merge zone traffic*. The left-lane and right-lane vehicles are cruising side-by-side at the same  
352     speed  $u^*$  and are at the same location  $x_{left}(t) = x_{right}(t)$  for times  $t > 10$ . At time  $t_2 = 20$ , the  
353     controller with the aid of the  $w_r$  will instruct the left-lane vehicle to become a leader  $i$  and the  
354     right-lane vehicle to become a follower  $i + 1$ . To achieve this positioning, the left-lane vehicle will  
355     accelerate and then decelerate and right-lane vehicle will decelerate and then accelerate over the  
356     time interval  $t_2 = 20 < t < t_3 = 30$ . These maneuvers are achieved with the aid of model (4). At  
357      $t_2$ , the speeds and locations of the vehicles are:

$$\begin{aligned} 358 \quad \dot{x}_i(t_2) &= \dot{x}_{left}(t_2) = u^* \\ 359 \quad x_i(t_2) &= x_{left}(t_2) = x_{right}(t_2) \\ 360 \quad \dot{x}_{i+1}(t_2) &= \dot{x}_{right}(t_2) = u^* \\ 361 \quad x_i(t_2) &= x_{right}(t_2) = x_{left}(t_2) \end{aligned} \tag{9}$$

362 At  $t_3$ , they are:

$$\begin{aligned} 363 \quad \dot{x}_i(t_3) &= u^* \\ 364 \quad x_i(t_3) &= x_i(t_2) + u^*(t_3 - t_2) + \frac{s_{safe}}{2} \quad \text{on left-lane} \\ 365 \quad \dot{x}_{i+1}(t_3) &= u^* \\ 366 \quad x_{i+1}(t_3) &= x_{i+1}(t_2) + u^*(t_3 - t_2) - \frac{s_{safe}}{2} \quad \text{on right-lane} \end{aligned} \quad (10)$$

367 The  $w_r$  reference signal for a vehicle traveling on a left-lane is shown by the broken line of the  
368 upper panel of Figure 7. Note that the horizontal axis uses the sample  $\kappa$  for plotting and the  $t_2$  and  
369  $t_3$  refer to sample numbers corresponding to clock times 30 and 42 s, respectively.

### 370 Model Calibration

371 As stated above in the *Kalman Filter Model* subsection, the reference signal  $w_r$ , traffic noise  
372  $w_d$  and measurement noise  $w_n$  must be specified. The following rationale is used to make these  
373 assignments.

374 Since traffic noise  $\sigma$  is a reflection of a driver's inability to follow instructions and the sensitivity  
375 analysis results given in Table 2 indicate a poor level of performance at  $\sigma = 0.72 \text{ m/s}$ , it seemed  
376 appropriate to assign it to  $w_d$ .

377 Since speed measurement data are assumed to be collected on-board the vehicle, transmitted to  
378 a central controller, analyzed and transmitted back to the vehicle, it seemed appropriate to make a  
379 conservative assignment of measurement noise.  $w_n = \sigma = 0.72 \text{ m/s}$  was chosen. The measurement  
380 noise of equation (8) is the sum,  $w_d + w_n = 1.42 \text{ m/s}$ .

### 381 Model Performance

382 The KF controller delivers three responses: *true*,  $y(t)$ ; *filtered*  $y_F(t)$  and *measured*,  $y_M(t)$ .  
383 The  $t - \dot{x}$  trajectories of  $y_F(t)$  and  $y_M(t)$ , shown in the upper panel of Figure 7, track  $w_r$  well.  
384 The trajectory is delayed but it is of no concern. More importantly, the bottom panel shows the  
385 effectiveness of the filter in dealing with noise  $w_d$  and  $w_n$ ,  $\epsilon_F(t) < \epsilon_M(t)$ .

386 To test the reliability of the controller, fifty simulations were run. The average values and

standard deviations of  $y(t)$ ,  $y_F(t)$  and  $y_M(t)$  are given in Table 3. The  $\bar{u}(t_3)$  predictions are precise and inaccurate, they fail to match the target speed,  $u^* = 23.7 \text{ m/s}$ . For reasons of safety, speed precision is critically important because precisely located vehicles of  $\hat{x}(t)$  offers assurance the safe headway rule,  $s(t) < s = s_{safe}/2 = 18.1 \text{ m}$  is not violated. The speed  $t - \dot{x}$  and location  $t - x$  trajectories of Figure 8 show  $n = 20$  vehicles that start “side-by-side” are rearranged in a “zipper” merge formation prior to reaching the *merge zone* and proceed through the bottleneck with precision.

The feedback control system has two basic aims: to track  $w_r$  and to minimize the process  $w_d$  and measurement noise  $w_n$ . The obstacles associated roadway geometry, operating at roadway capacity, and driver behavior are theoretically overcome with this technique. The benefits are clear. Its performance can be declared optimum. Since the predictions are precise, it suggests the vehicles will proceed through the bottleneck “safely.”

## DISCUSSION

The author used care in calling DAS a “Driver Assistant System.” DAS is considered to be in the same class as ACC, an adaptive cruise control system where V2V (vehicle-to-vehicle) wireless communication is used. In a sense, DAS is similar to the operation of an autonomous vehicle, a self-driving car where the vehicle is capable of sensing the environment with no human input. But unlike an autonomous vehicle, a driver of a DAS vehicle, classified as a Level 1 autonomous car, must steer the vehicle and most importantly, remain alert for reasons of safety. The Green Book (AASHTO 2011) stopping sight distance is calculated to be 152 m or 500 ft with a 2.5 second driver reaction time. The merge length is assumed to be over 280 m or 930 ft, therefore a driver, it can be argued, has ample space to perceive a dangerous situation and respond.

To implement DAS at a bottleneck, real-time vehicle location and speed data are needed. These data can be collected and shared using ITS Cellular-V2X (vehicle-to-everything) communication. For DAS, V2V and V2N (vehicle-to-network) communication are needed where vehicle control instructions and data will be shared using V2N wireless communication.

While Cellular-V2X technology is not fully implemented, no proof of concept can be offered

414 with road testing. According to 5GAA, a partnership of automobile manufacturers and communica-  
415 tion companies, steady progress is being made (5G Automotive Association 2018). Success with  
416 radar, GPS, Lidar, tire, light sensors and video sensors using Kalman filtering has been reported  
417 in the literature. These papers have one thing in common. They track one vehicle. There are  
418 exceptions. Ponsa et al. (2005), Luling et al. (2009), Nemati and Astrand (2014), Mathew and  
419 Asari (2013) and Yuan et al. (2015) have tracked people and vehicles. In the interim, prototype  
420 testing can be pursued where DAS is operated under different weather conditions, times of the day,  
421 etc.

422 DAS meets the requirements of a “Smart City” mobility device. It is aimed at (1) creating  
423 less congestion by using cutting edge ITS technology and (2) improving infrastructure by wisely  
424 managing traffic data. The “Smart City” concept, which take advantage of improvements in  
425 technology for improving lives, has been endorsed by large companies, start-up companies, banks,  
426 university-city partnerships (Network Metrolab 2019), the federal government (U.S. Department  
427 of Transportation 2019) and the (American Society of Civil Engineering 2019).

428 Public perception is important. Critics may claim that traveling in a DAS vehicle through  
429 a bottleneck with a precisely controlled speed of  $u^*$  and headway of  $s = 18.1 \text{ m}$ , is not “safe.”  
430  $s < s_{ROR} = 26.9 \text{ m}$ . Public opinion polls indicate that the majority of people would not travel  
431 in a Level 5 autonomous car. Convincing the critics that a DAS vehicle is a Level 1 autonomous  
432 car will be a difficult task. It uses the same technology as an autonomous vehicle. Finally, there  
433 is issue of false security. Hand-held phone use is risky and illegal. Research has shown that  
434 “handsfree” devices, despite the claim otherwise, make drivers prone to error including failures in  
435 visual perception and in the inability to detect and react to hazards. A DAS vehicle will need more  
436 precise speed and a safe stopping distance. The good news is that technologies are being designed  
437 and laws are being passed to require new vehicles to install equipment to alert drowsy and distracted  
438 drivers (Schalit 2019).

## 439 **SUMMARY**

440 The paper has two major components devoted to:

441     • Explaining Delay: A stochastic car-following or driver behavior model, a collection of dif-  
442       ferent stochastic differential equations, was derived and used to explain delay at a bottleneck  
443       operating at capacity.

444     • Preventing Delay: A Driver Assistance System (DAS) or Kalman Filter (KF) feedback  
445       control system was derived and used to prevent delay at a bottleneck operating at capacity.

446       Both model types describe a traffic merge by phases: *start-up*, *cruise* and *merge*. Phase models  
447       are derived and then linked together to form model structures that predict speed and location  
448       as vehicles proceed through the bottleneck. Both model types consider the effects of individual  
449       driver behavior, group driver behavior, traffic density, roadway geometry and traffic noise (speed  
450       volatility).

451       The stochastic car-following or driver behavior model features a Brownian motion models that  
452       mimic driver behavior under various conditions. The model also features constraints to assure  
453       drivers conform to a safe driving rule, i.e. maintains safe gaps between vehicles. Sensitivity  
454       analyses help explain the effects roadway geometry, operating at high traffic density and driver  
455       behavior have on queue formation and performance.

456       The KF feedback control system is designed to assist drivers. It manages vehicle speed and  
457       arranges vehicles in a “zipper” mode formation to efficiently travel through the bottleneck. The  
458       critically important feedback reference signal or tracking function, the mechanism for arranging  
459       vehicles while merging, is derived with a non-linear acceleration model.

460       The challenges associated with the driving public adopting autonomous cars and DAS technol-  
461       ogy in the future was discussed.

462       **DATA AVAILABILITY STATEMENT**

463       Some or all data, models, or code generated or used during the study are available in a repository  
464       online in accordance with data retention policies (Ossenbruggen 2019a).

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| Traffic Conditions at Roadway Capacity<br>Targets for Optimum Performance |       |            |
|---|-------|------------|
| $c = u^*/s_{safe}$  | $u^*$ | $s_{safe}$ |
| (v/h)   | (m/s) | (m)        |
| 2380  | 23.7  | 36.2       |
| Risk Aversion and Safety Measures   |       |            |
| $s_{ROR} = 26.9$  |       |            |
| $s_{HCM} = 36.2$  |       |            |

**TABLE 1.** Targets for Optimum Performance

| Sensitivity Analysis          |                   |   |                                  |                                  |
|-------------------------------|-------------------|---|----------------------------------|----------------------------------|
| Traffic Noise<br>$\sigma/u^*$ | $\sigma$<br>(m/s) | Flow<br>$\bar{q}(t_3) = \bar{u}(t_3)/\bar{s}(t_3)$<br>(v/h) | Speed<br>$\bar{u}(t_3)$<br>(m/s) | Headway<br>$\bar{s}(t_3)$<br>(m) |
| 0.00                          | 0                 | 2110  | 18.9                             | 34.4                             |
| 0.01                          | 0.24              | 2090  | 18.8                             | 34.4                             |
| 0.02                          | 0.48              | 2030  | 18.8                             | 34.4                             |
| 0.03                          | 0.72              | 1700  | 17.9                             | 37.8                             |
| 0.04                          | 0.96              | 1380  | 15.6                             | 40.7                             |
| 0.05                          | 1.20              | 1120  | 13.6                             | 46.2                             |

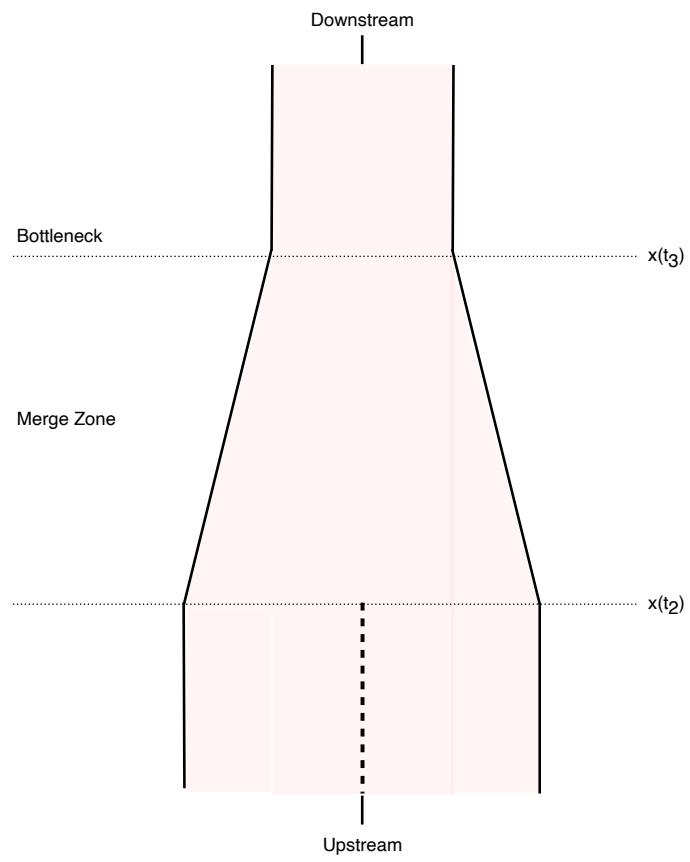
**TABLE 2.** SCF Model Predictions

| Predictions    |   |                                    |
|----------------|---|------------------------------------|
| Speed Response | Speed<br>$\bar{u}(t_3) \pm sd$<br>(m/s) | Headway<br>$s = s_{safe}/2$<br>(m) |
| True           | $24.0 \pm 0.14$                         | 18.1                               |
| Filtered       | $24.0 \pm 0.05$                         | 18.1                               |
| Measured       | $24.3 \pm 0.74$                         | 18.1                               |

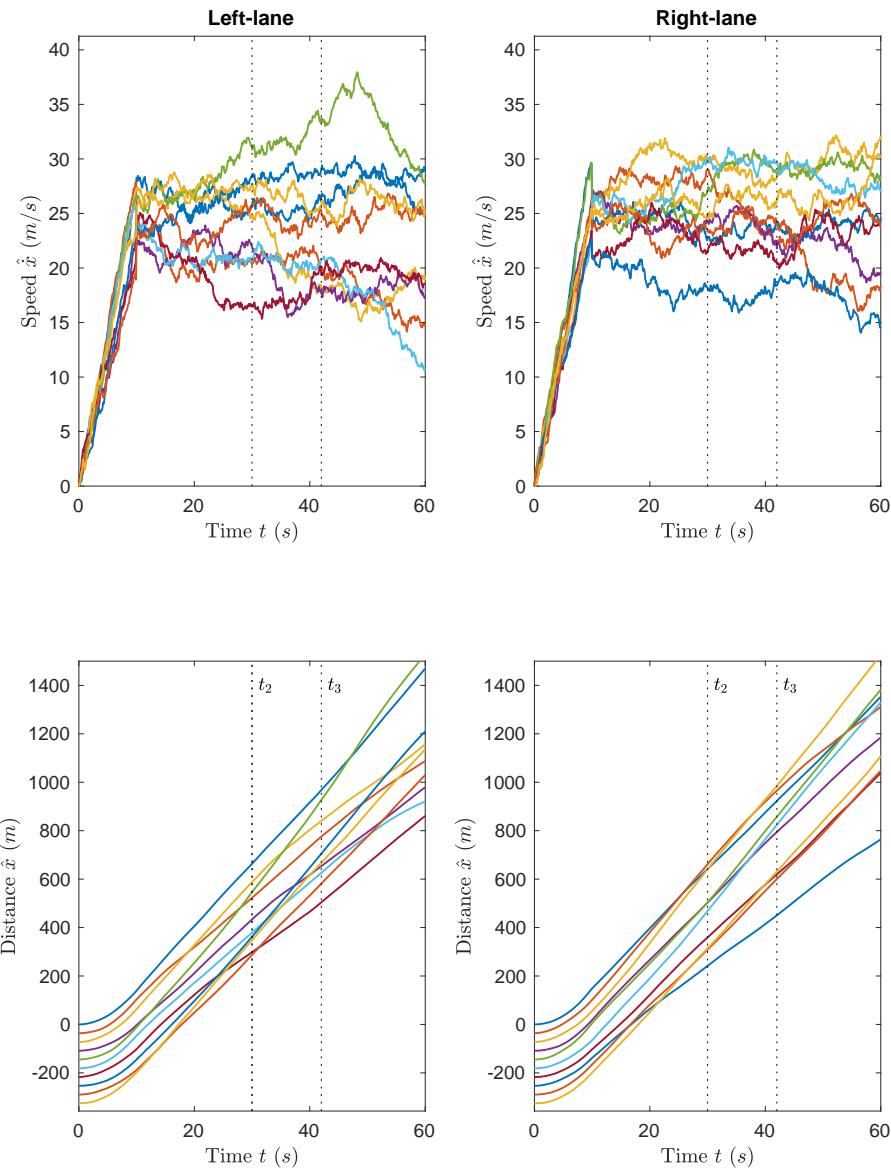
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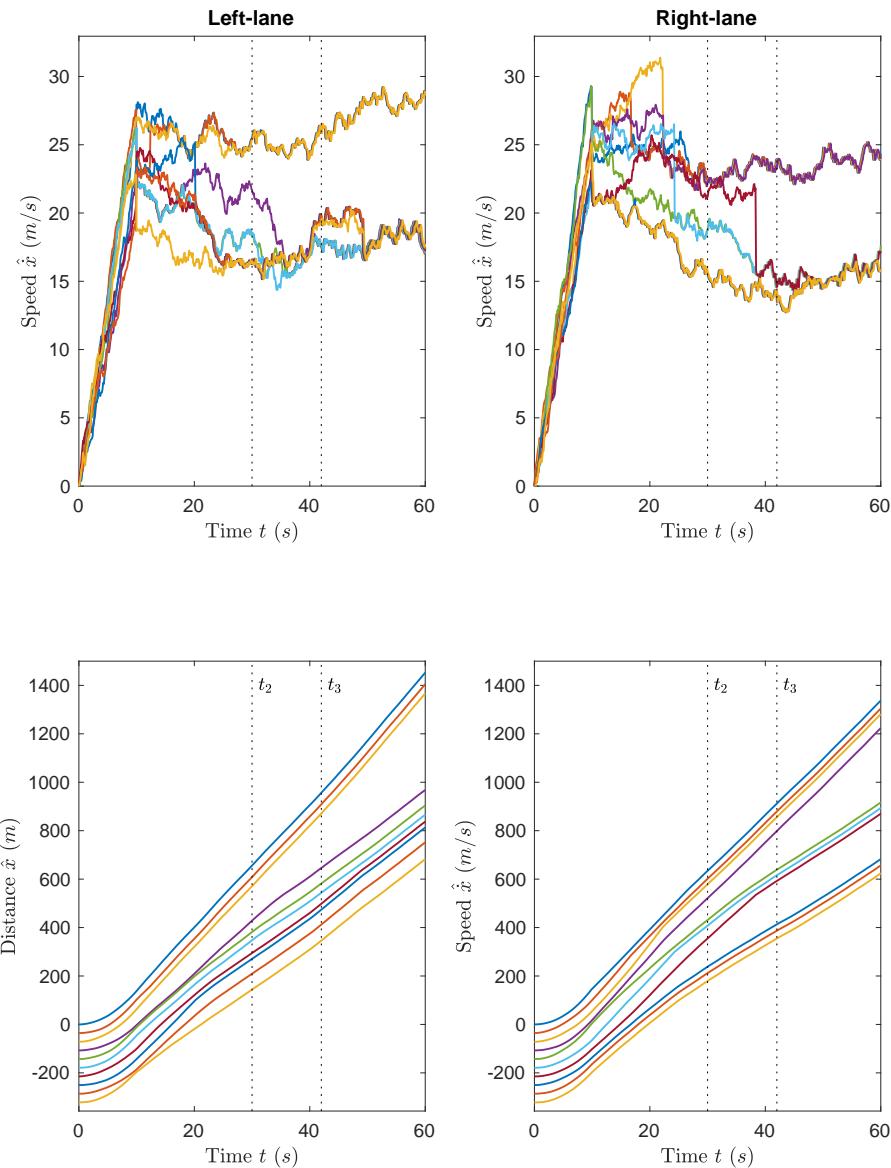
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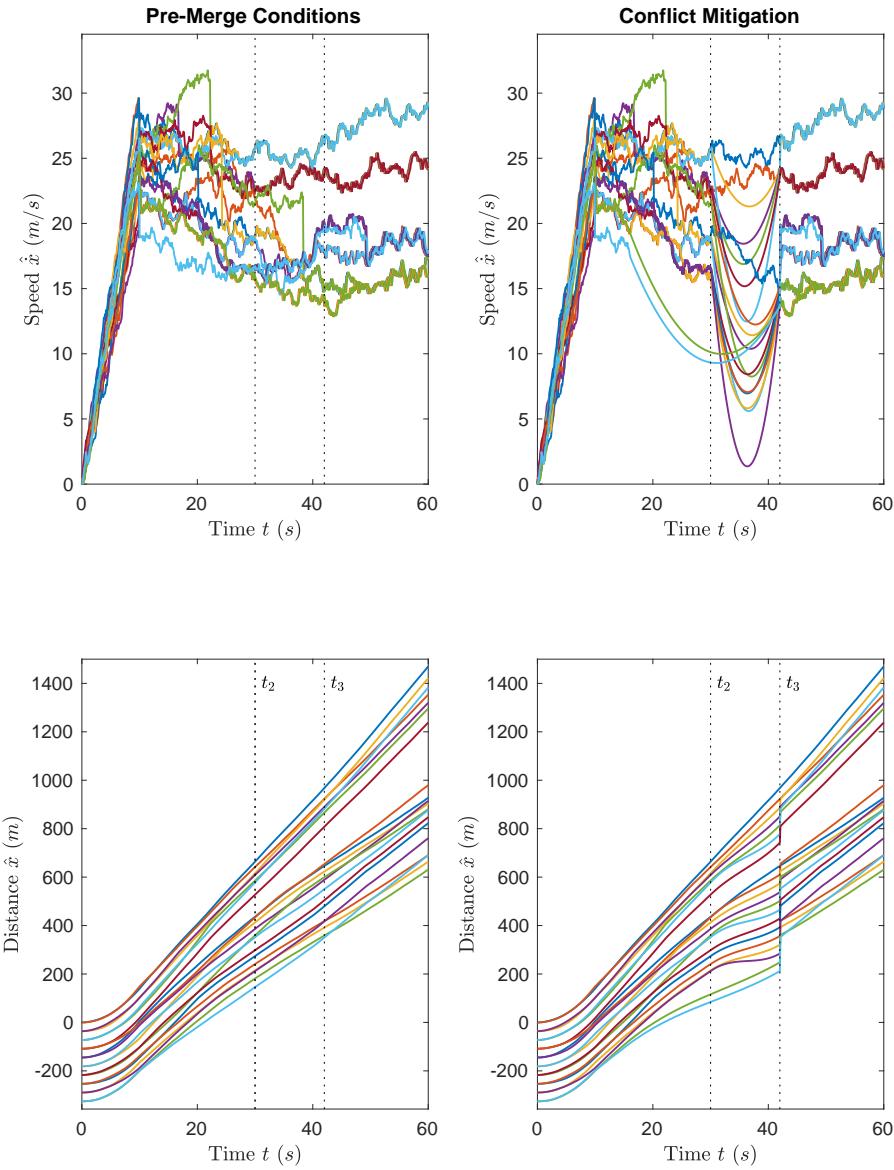
**Fig. 1.** Bottleneck schematic diagram.



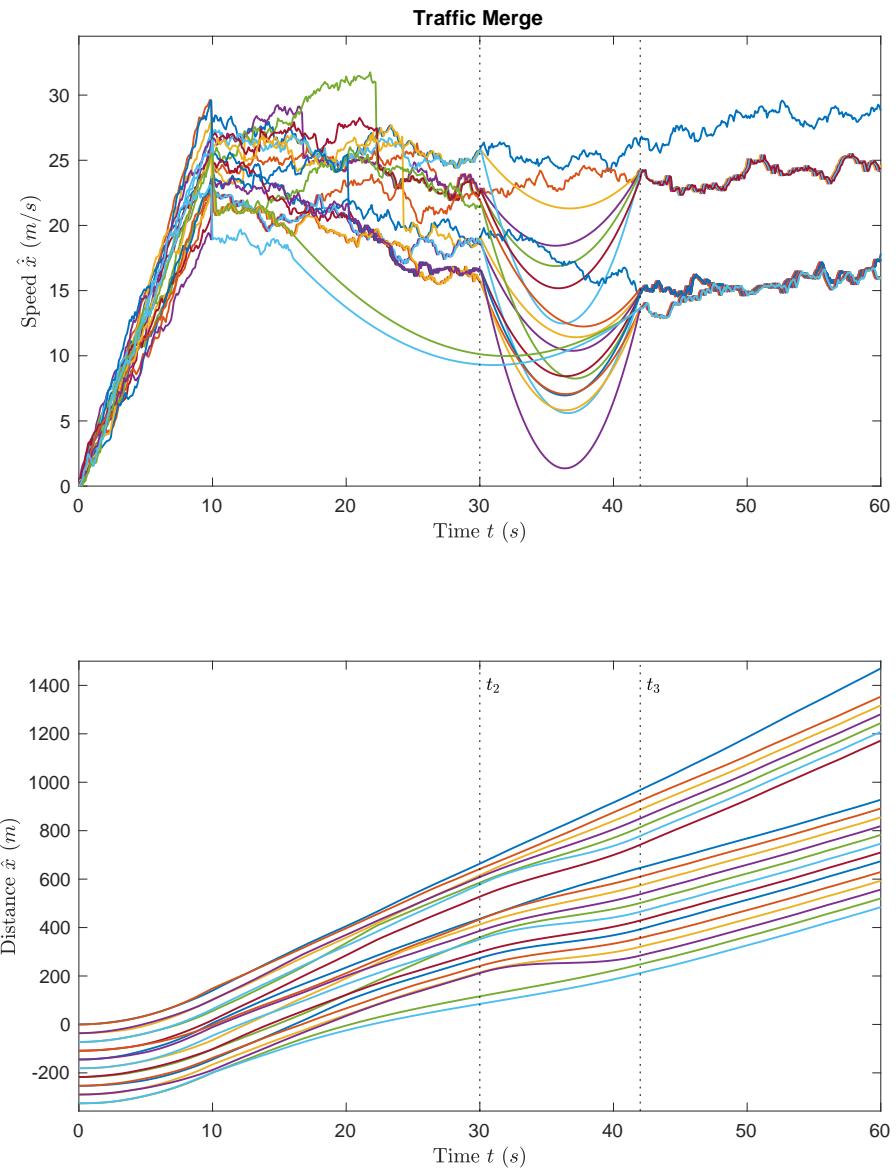
**Fig. 2.** “Unassisted” driver behavior by start-line for  $0 \leq t < t_2$  and  $\sigma/u^* = 0.03$ .



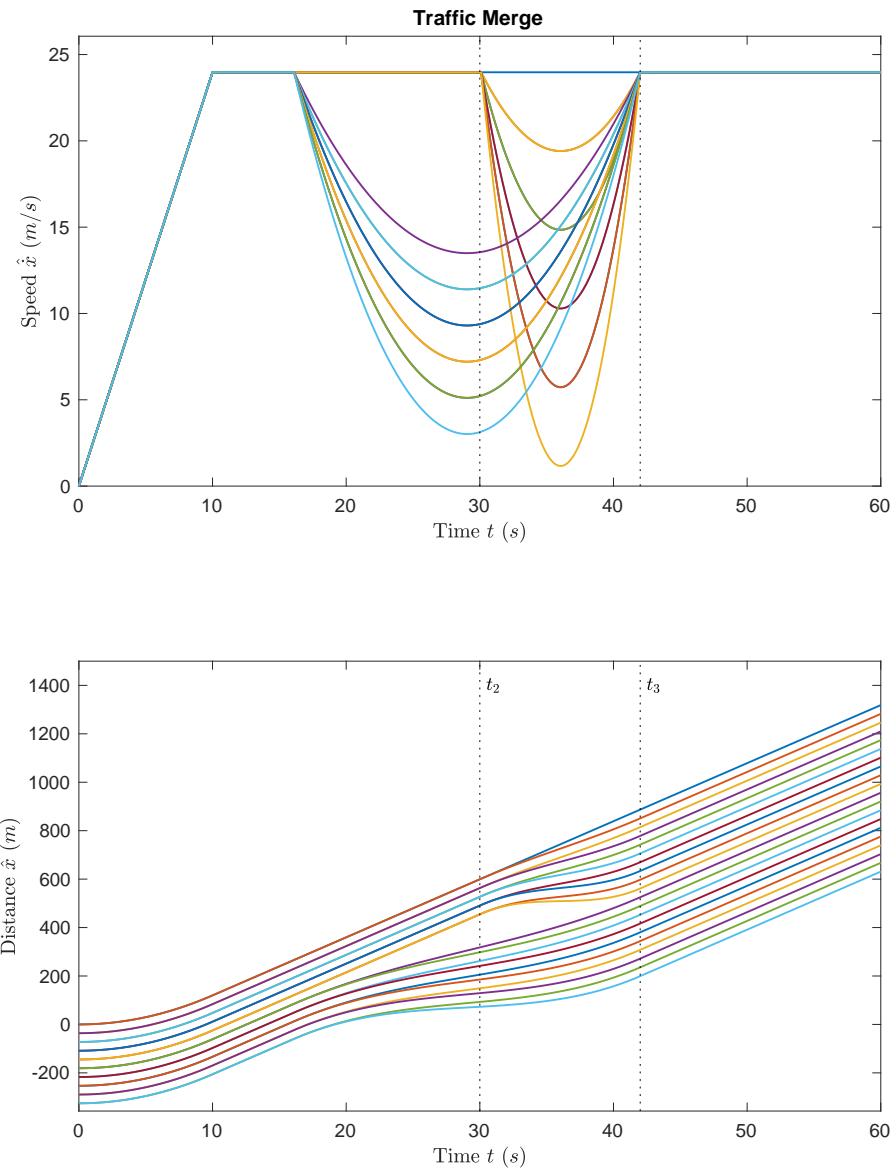
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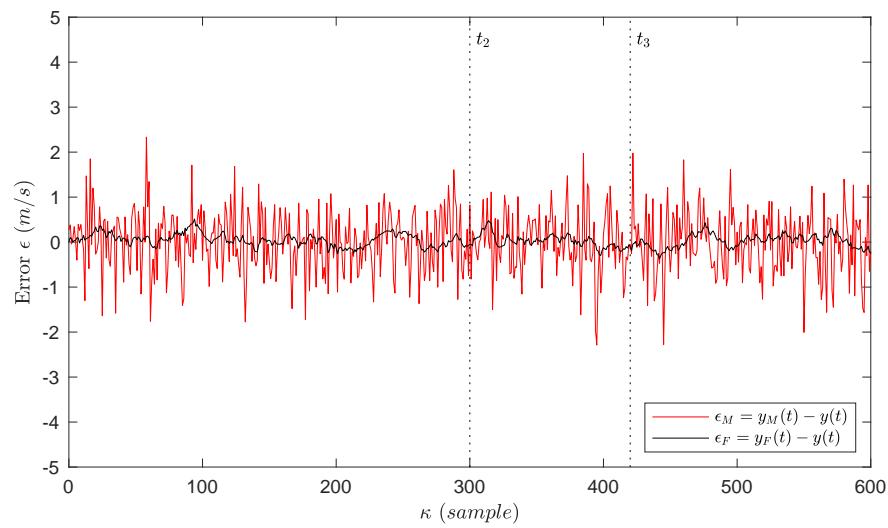
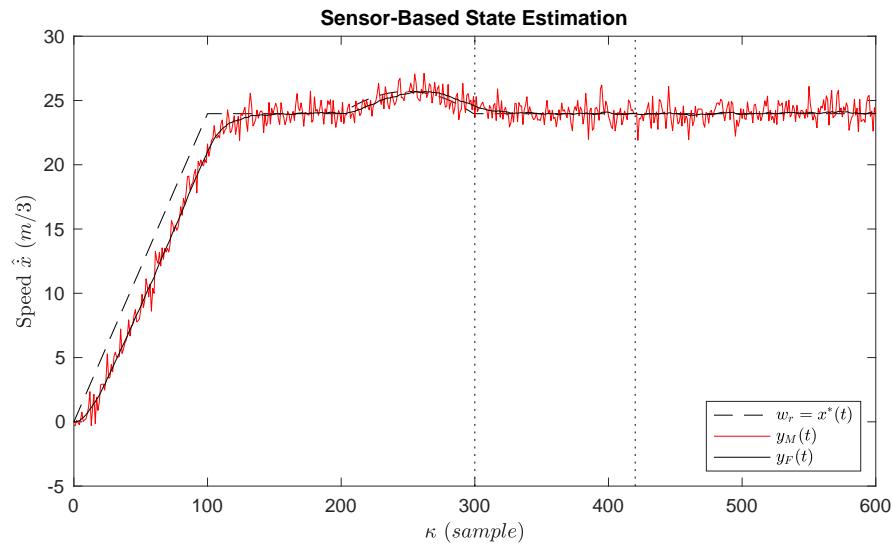
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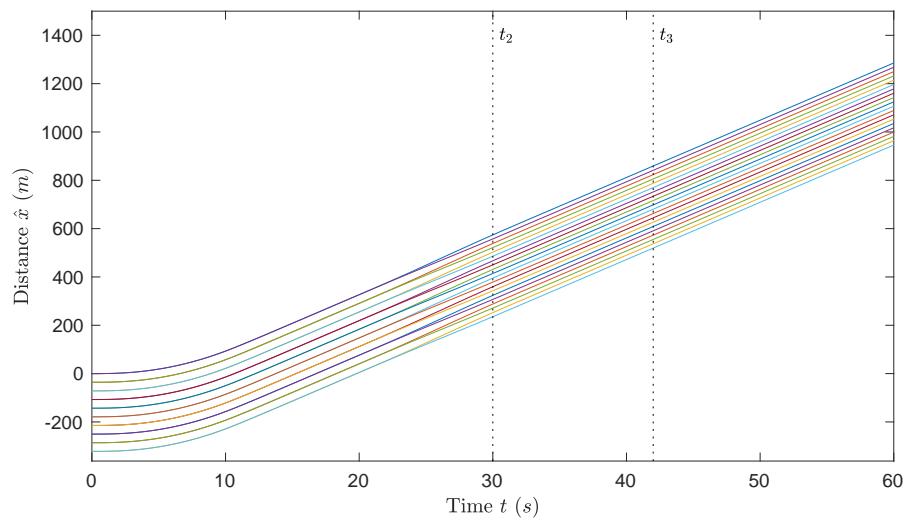
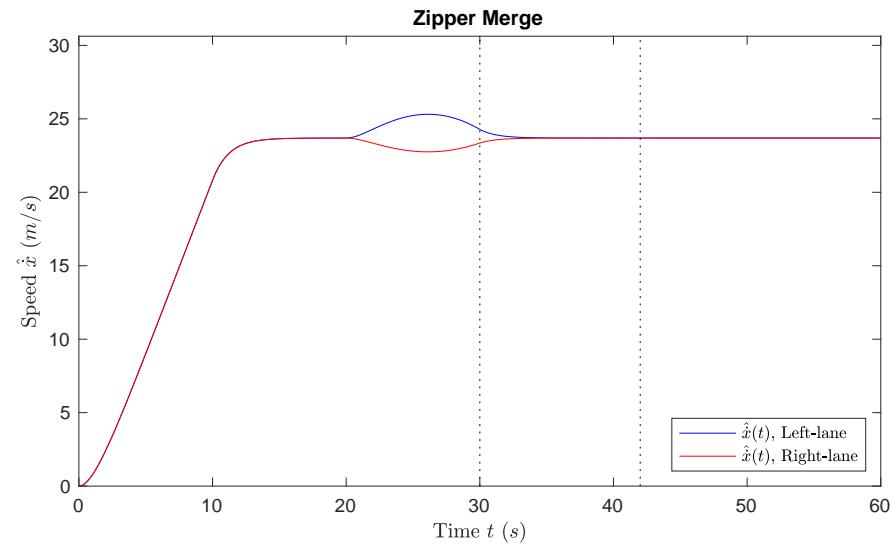
**Fig. 5.** Stochastic car-following model predictions for  $\sigma/u^* = 0.03$ .



**Fig. 6.** Deterministic car-following model predictions for  $\sigma/u^* = 0.00$ .



**Fig. 7.** State estimation for left-lane traffic using  $w_r$ :  $w_d = 0.72$  (m/s);  $w_n = 0.72$  (m/s).



**Fig. 8.** A “zipper” merge with feedback control.

## ABSTRACT

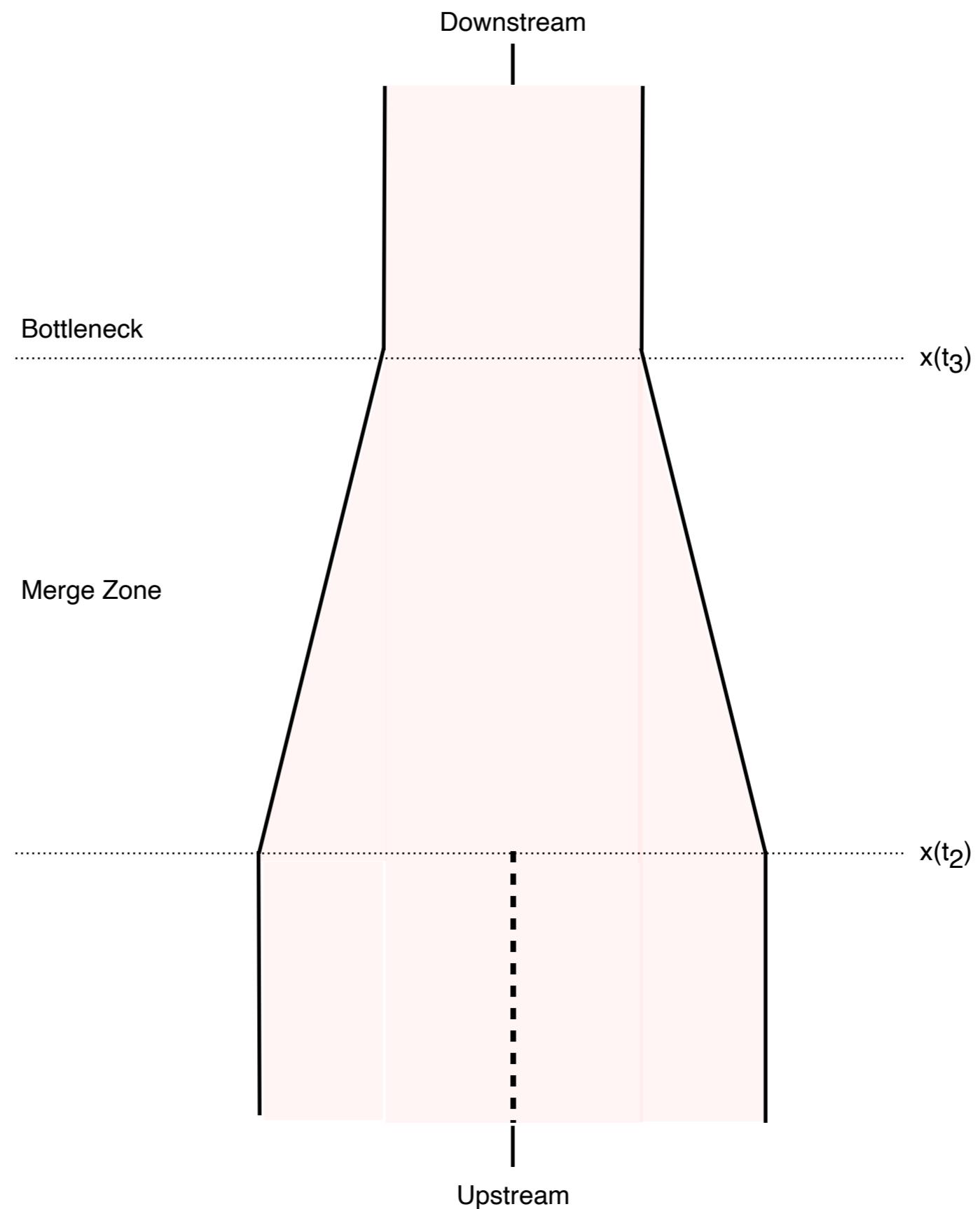
This study tests the feasibility of using a Kalman filter feedback control system at a bottleneck where drivers are forced to merge. Since the bottleneck is assumed to be operating at capacity, traffic delay is expected owing to high traffic density. The roadway geometry, a lane drop where two traffic lanes merge into one, plays a critical role. Individual and group driver behavior is known to cause traffic delay and to be the sole contributor to traffic noise. A driver behavior or stochastic car-following model is developed to study the effects of roadway geometry, traffic density and traffic noise on performance. The model features Brownian motion speed models that mimic driver behavior and simulates real-world bottleneck conditions where drivers start from a standing position, comfortably accelerate to a cruising speed, and then jockey for position when entering the bottleneck. All drivers are assumed to keep a safe headway. Since the model allows individual drivers to adjust their speed according to changing conditions, the model predictions help explain queue formation and performance. The feedback control system, which is assumed to use V2X (vehicle-to-everything) technology, uses an entirely different strategy. It assists drivers: (1) by managing vehicle speed and (2), by arranging vehicles in a "zipper" mode formation. A reference signal or tracking function, which proves to be vitally important in arranging vehicles for a "zipper" mode, is derived with a non-linear acceleration model. The impacts of deploying this scheme with V2X technology in a "Smart City" are discussed.

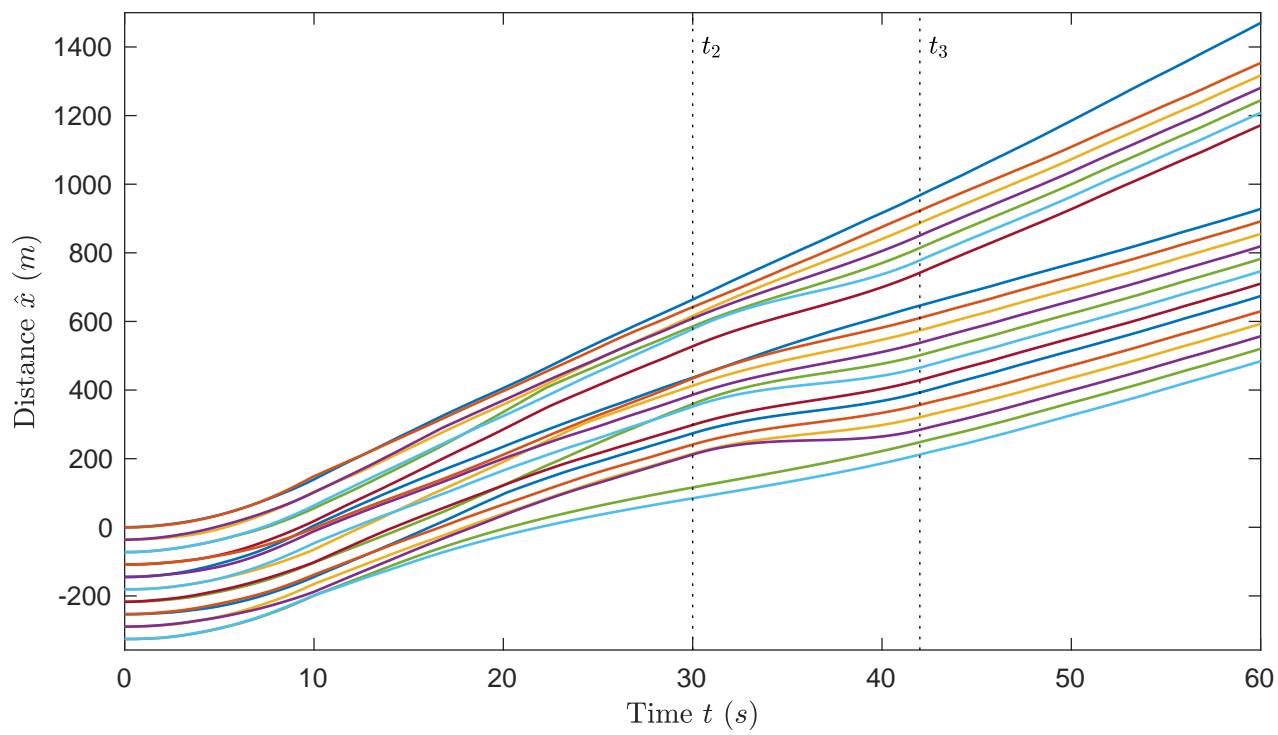
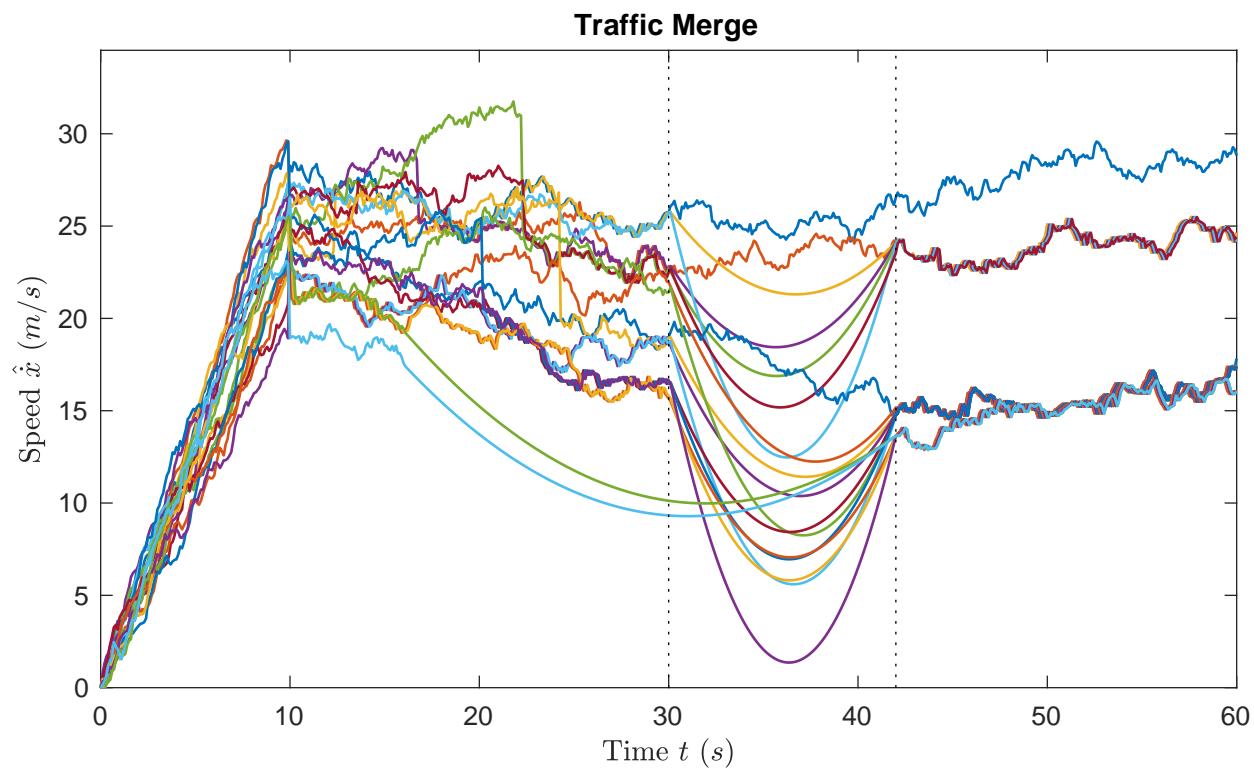
**Keywords:** Traffic breakdown, car following, congestion mitigation, Kalman Filter, stochastic processes, stochastic differential equations

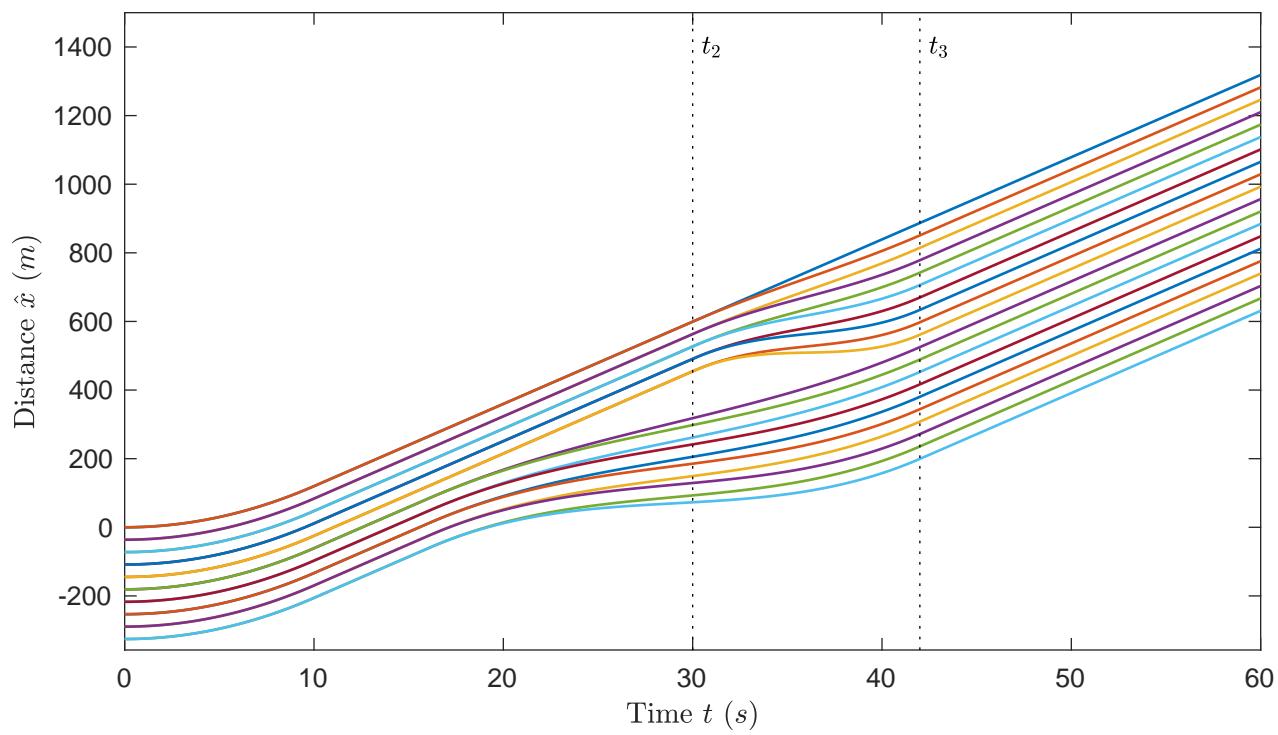
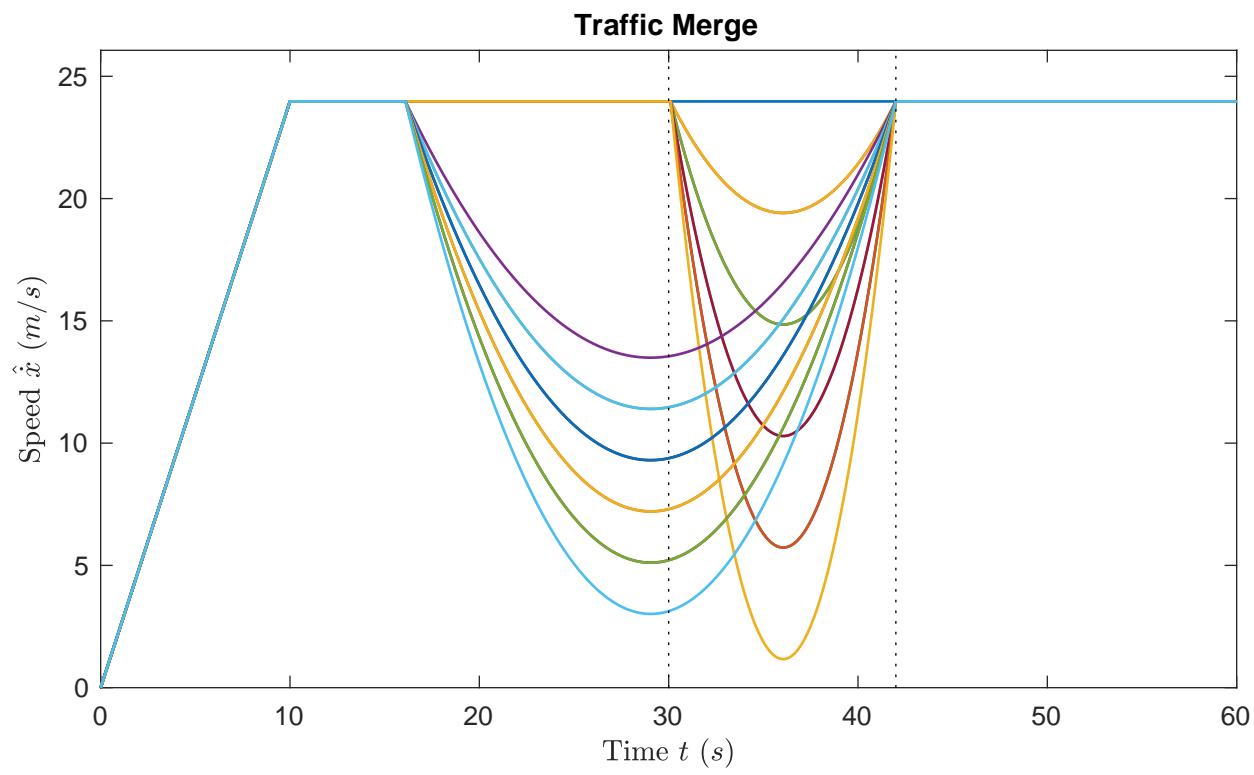
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|--|----------------|-------------------|
| $c = u^*/s_{safe}$<br>(v/h)  | $u^*$<br>(m/s) | $s_{safe}$<br>(m) |
| 2380   | 23.7           | 36.2              |
| Risk Aversion and Safety Measures                                      |                |                   |
| $SROR = 26.9$  |                |                   |
| $s_{HCM} = 36.2$   |                |                   |

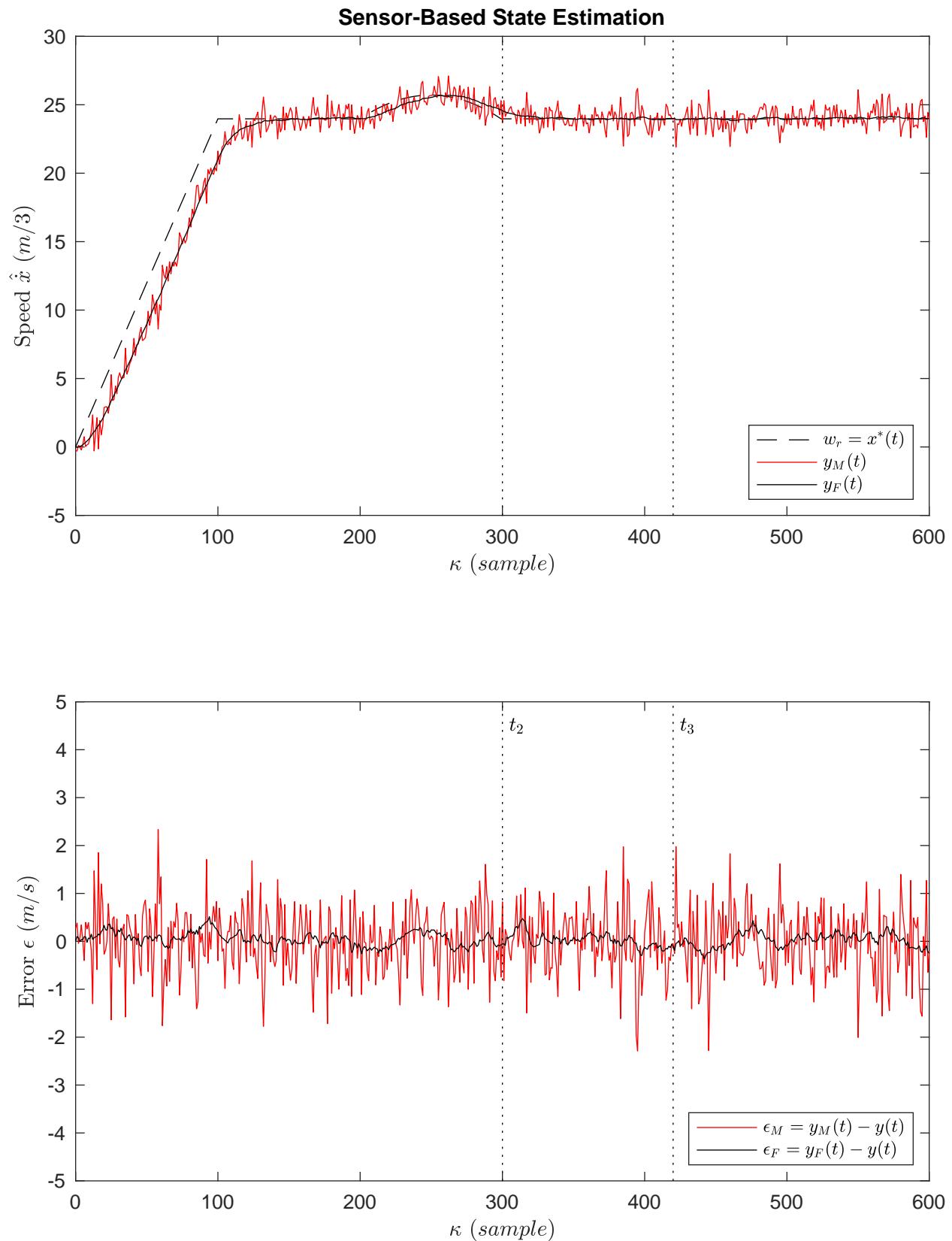
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| 0.00                          | 0                 | 2110   | 18.9                             | 34.4                             |
| 0.01                          | 0.24              | 2090   | 18.8                             | 34.4                             |
| 0.02                          | 0.48              | 2030   | 18.8                             | 34.4                             |
| 0.03                          | 0.72              | 1700   | 17.9                             | 37.8                             |
| 0.04                          | 0.96              | 1380   | 15.6                             | 40.7                             |
| 0.05                          | 1.20              | 1120   | 13.6                             | 46.2                             |

| Predictions    |   |                                    |
|----------------|---|------------------------------------|
| Speed Response | Speed<br>$\bar{u}(t_3) \pm sd$<br>(m/s) | Headway<br>$s = s_{safe}/2$<br>(m) |
| True           | 24.0 ± 0.14                             | 18.1                               |
| Filtered       | 24.0 ± 0.05                             | 18.1                               |
| Measured       | 24.3 ± 0.74                             | 18.1                               |









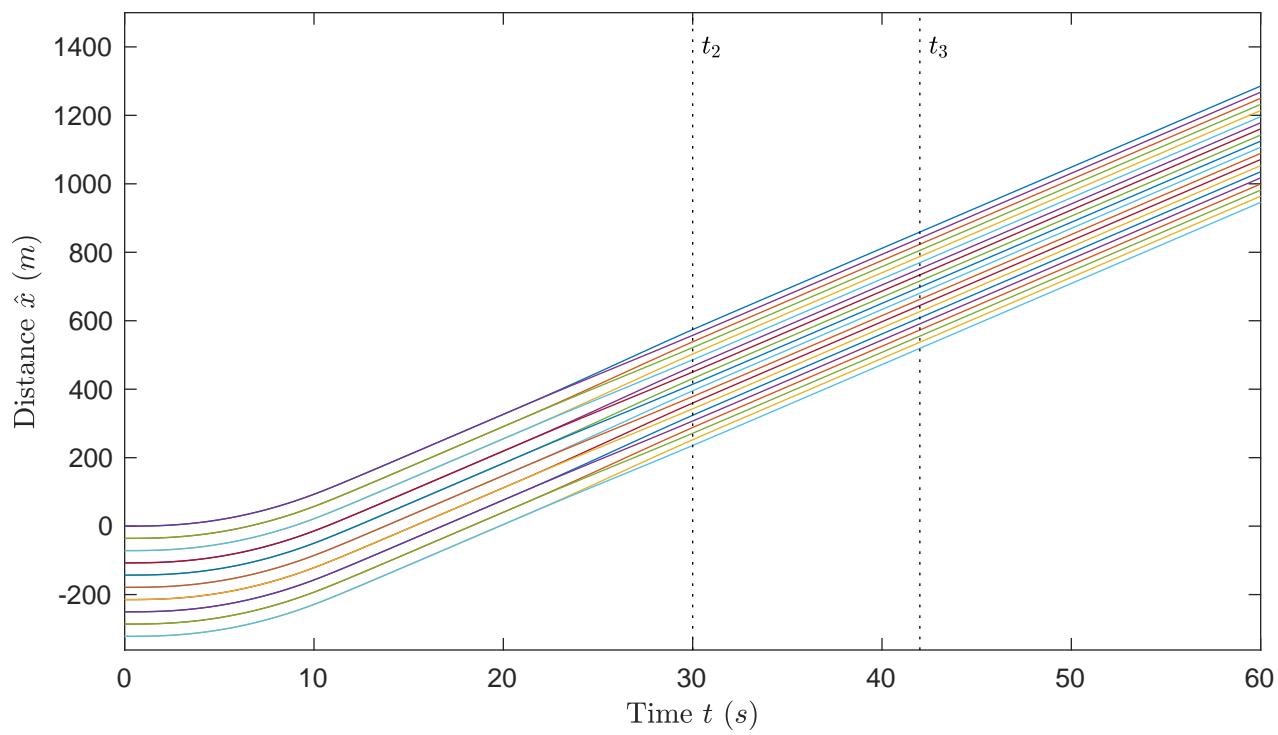
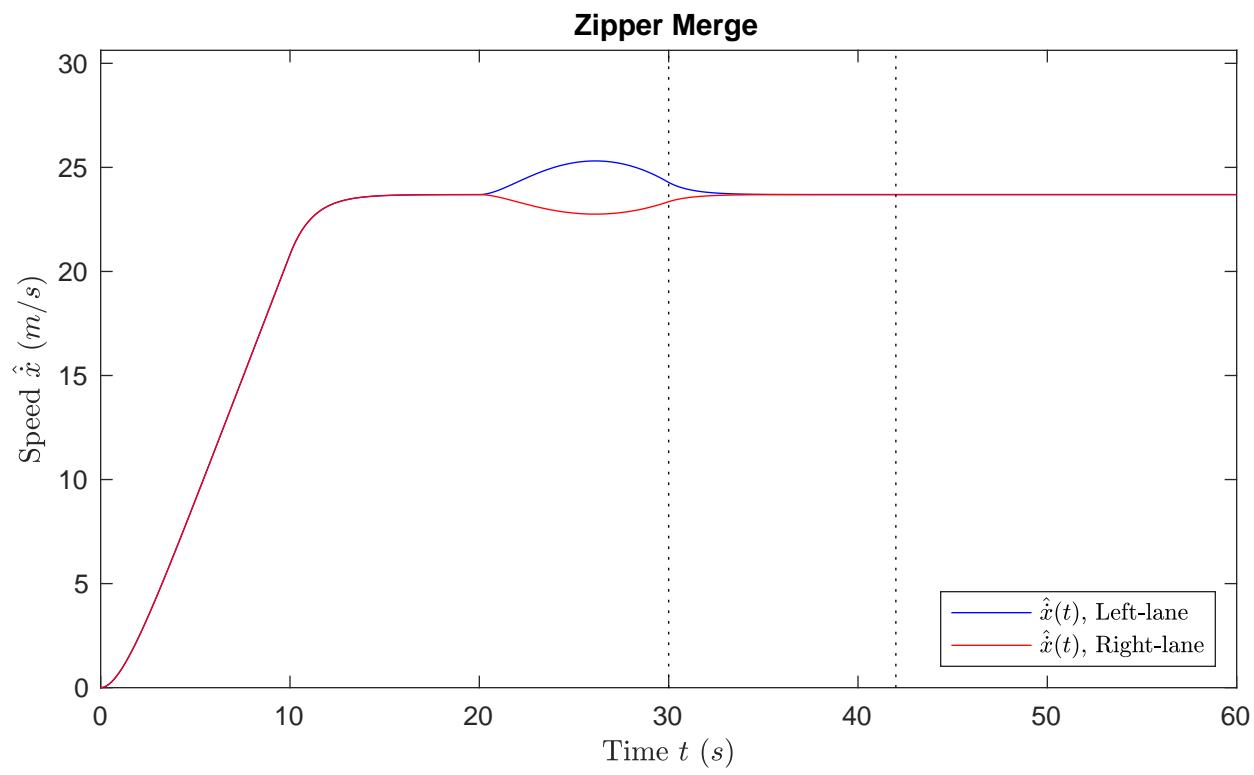
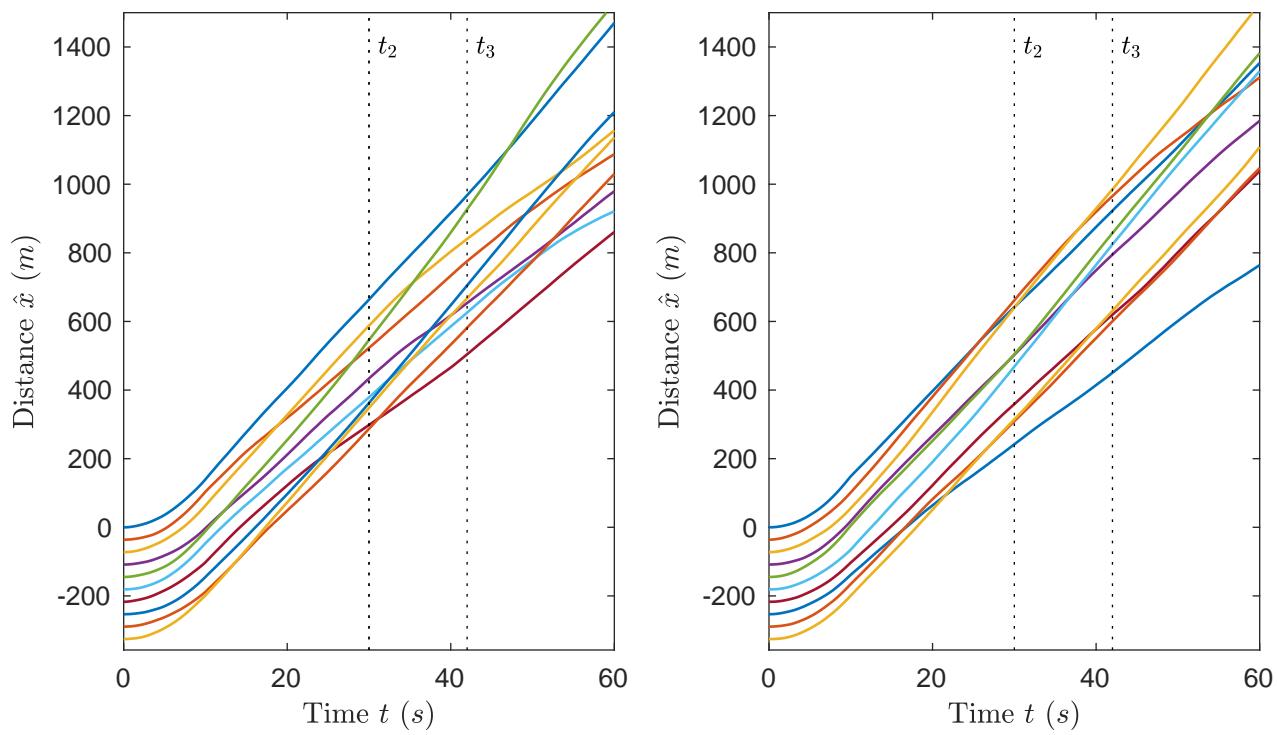
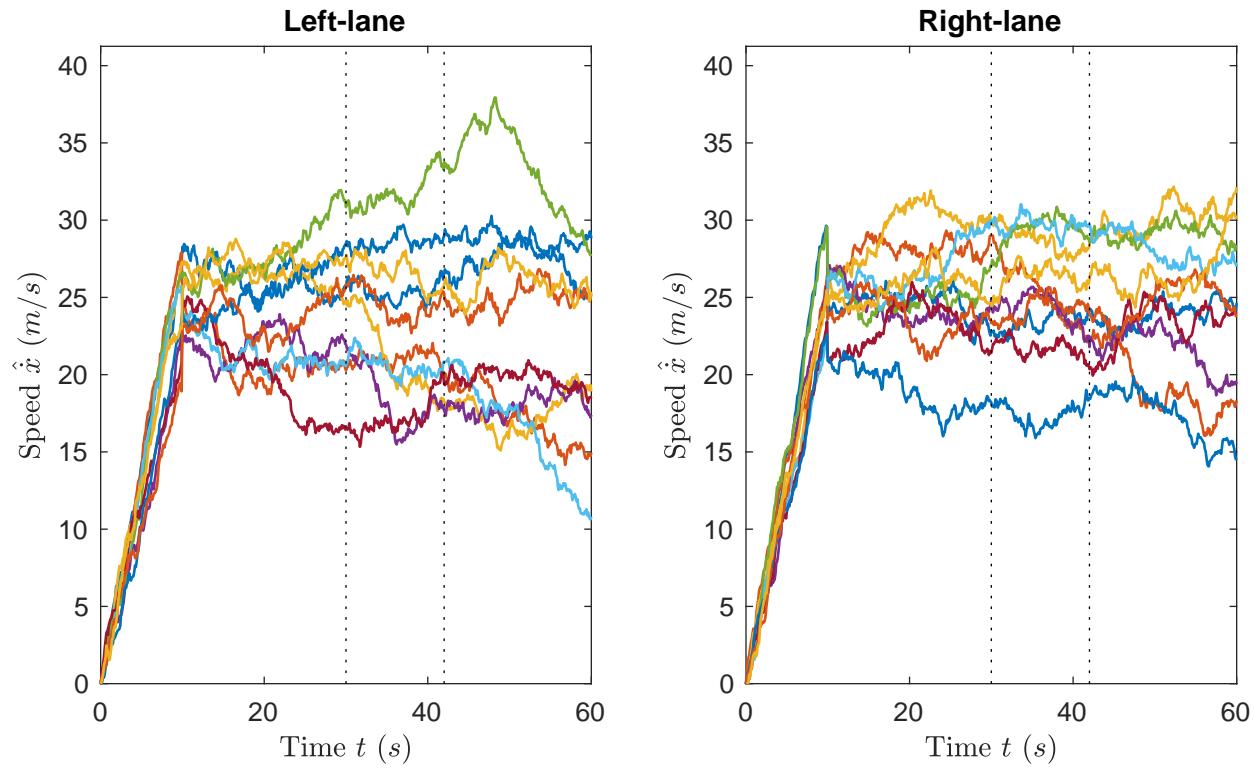


Figure2

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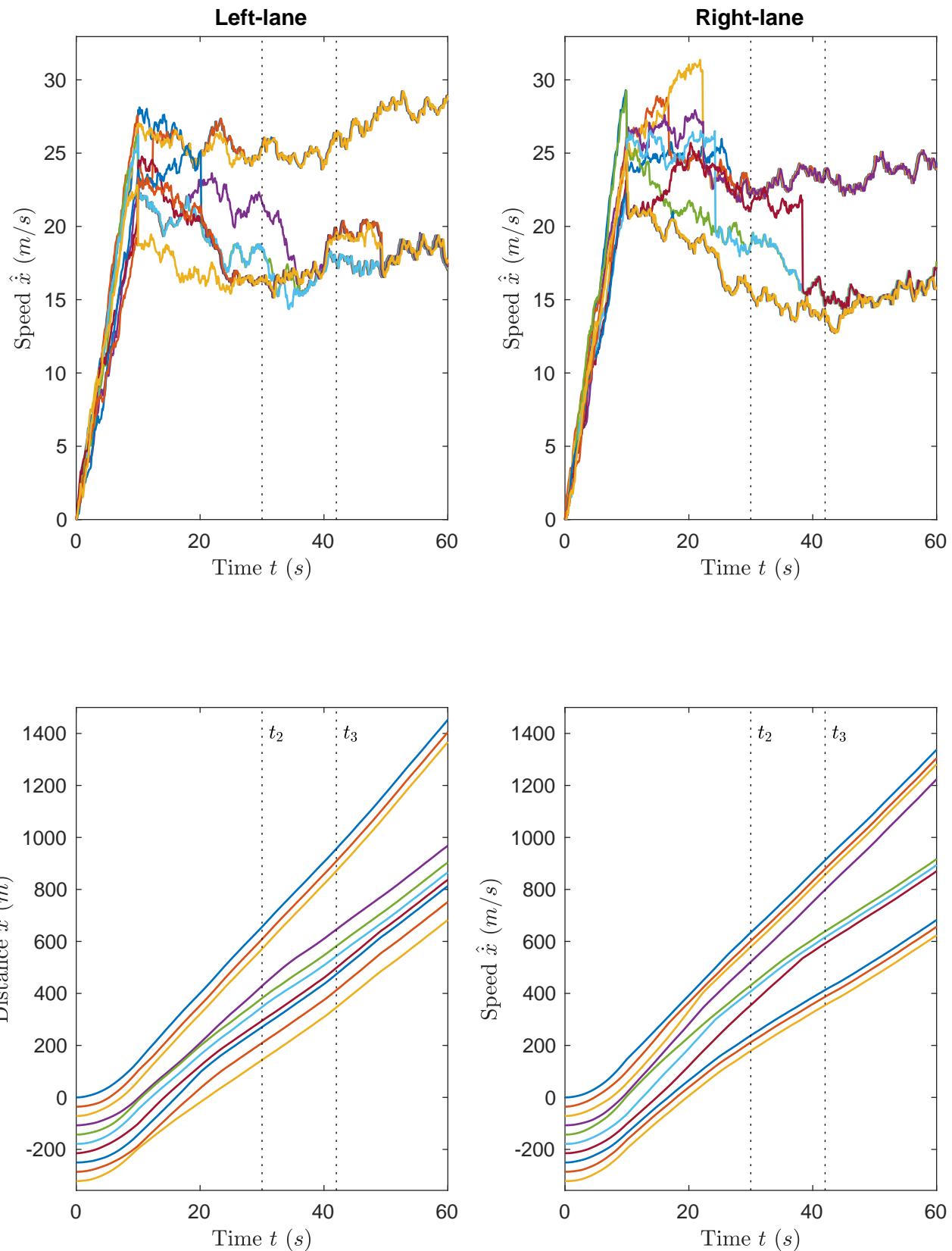
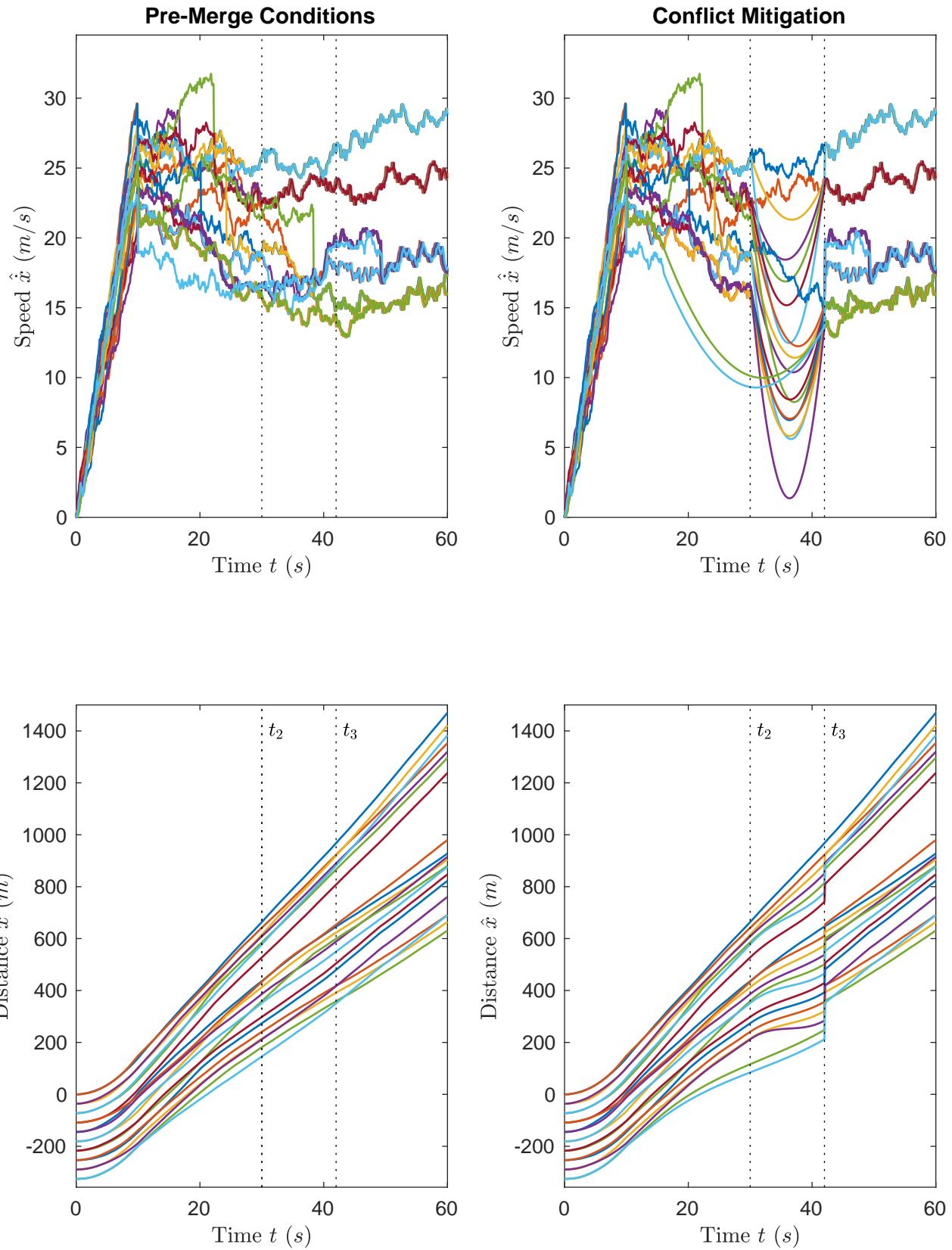


Figure4

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**Reviewers' comments:**

Reviewer #1: This paper introduces a set of SDE models to explain breakdown of a bottleneck operating at capacity and derived a DAS to prevent breakdown at such bottlenecks.

The topic is meaningful while the contribution on the methodology and performance analysis is vague.

The reviewer has below comments:

I agree with the reviewer's critique. The last comment asking why the paper is limited to 4 vehicles. It is an excellent question.

Limiting the analysis to 4 vehicles resulted into a weak paper. I expanded to analysis and discussion to 20 vehicles traveling for 60 seconds. These changes required that my computer code to be improved and my paper to be reorganized and parts rewritten. Figures in the original paper were replaced.

1. It should be clearly stated in the abstract and title that this paper is focused on bottleneck caused by lane drop, not a more general concept of breakdown.

The abstract has been rewritten. In lieu of focusing on breakdown, the paper focuses on delay, driver behavior and safety.

2. Before "merge option and safety" section, t\_1, t\_2 and t\_3 are assigned values. Why are these time points regulated here? How is the duration for each phase estimated or calculated?

See the **Basic Concepts** Section. I define four zones: start-up, cruise free-flow, merge and cruise congested traffic zones. The merge zone extends from 30 to 42 seconds, which corresponds to a vehicle entering the merge zone and entering the downstream zone where owing to high density, the traffic is in a congested state.

3. Similar to the previous comment, how is the target speed calculated? Since the target speed significantly impacts the occurrence of the breakdown and capacity of the merging point, its value should be determined before completely introducing the formulation.

I added two subsections in the **METHODS** sections: *RiskAversion and Safety* and *Capacity of a Single Freeway Lane*. The target consists of a speed and a safe headway.

4. Since the analysis is conducted with a set of predetermined values for all the

parameters, the obtained results cannot serve as a general conclusion. To show sensitivity of some factors and effectiveness of breakdown identification model and the DAS, qualitative analysis and some expected results should be first introduced, followed by a comprehensive case study with different sets of values for the parameters. Using a single set of values for merging time, speed, and other factors cannot enable the model to simulate "realistic" conditions.

I added a section entitled *Sensitivity Analysis* in the **Explaining Traffic Delay** section. The traffic noise parameter is varied from zero to a value where the model simulation produces nonsensical output. The stochastic model becomes a deterministic model when the traffic noise parameter is assigned zero. It offers an interesting comparison deterministic world and my simulation of the real world. Figures 5 and 6.

5. At the end of "merge option and safety" section, the safe driving distance is derived based on capacity. However, such distance should not be utilized in the bottleneck analysis since vehicles under congestion or near a bottleneck may keep a shorter distance. This value is critical to following analysis and it should be derived with more careful formulation, if not from data collected on real-world segments.

I considered this suggestion. The computer, which now considers queue formation, was not addressed with 4 vehicles. The set of rules that a driver follows is more clearly presented in the revised paper. Equations 2 through 5 describes driver behavior for each zone. The material that follows explains how a driver changes speed to avoid crashing.

6. Does the assumption at the top of page 11 imply that all vehicles do not change speed between t\_start and t\_end?

No. The vehicles change speed. See # 5.

7. Why are 4 vehicles discussed in the DAS section? Does the proposed DAS has a limit for vehicles under consideration?