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1. The logical framework of the methods

Section 4 presents methods for solving the model. Section 4.1 restructures the network to demonstrate optimal integer solutions for decision variables \mathbf{x} , using KKT conditions to convert the bi-level model into a single-level formulation. Section 4.2 introduces a deterministic equivalent program solvable by commercial solvers like Gurobi and CPLEX. In Section 4.3, we propose an integer L-shaped algorithm with both continuous and integer optimality cuts, along with a polynomial-time algorithm using network flow techniques. In summary, Figure 1 illustrates the logical framework of the methods.

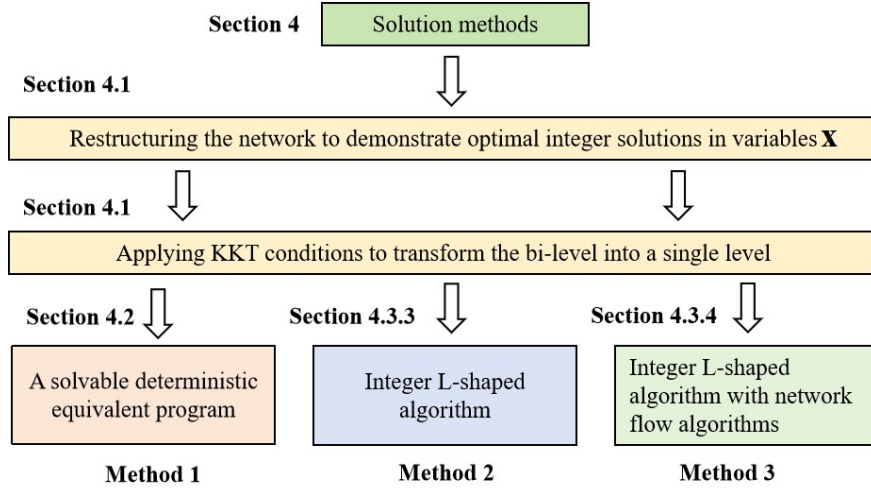


Figure 1: Logical framework of the proposed methods.

2. The flowchart for the algorithmic procedure

Section 4 proposes the integer L-shaped algorithm with continuous and integer optimality cuts, which can effectively solve each od pair in each scenario (s, t) separately. To obtain the continuous optimality cut, we relax the lower level to a linear programming problem and derive the cut using duality theory. To obtain the integer optimality cut, instead of directly solving the mixed integer linear programming problem, we propose **Algorithm 2**, which repeatedly employs polynomial-time network flow algorithms to solve each od pair in each scenario. In summary, the algorithmic procedure for solving the model is illustrated in the flowchart depicted in Figure 2.

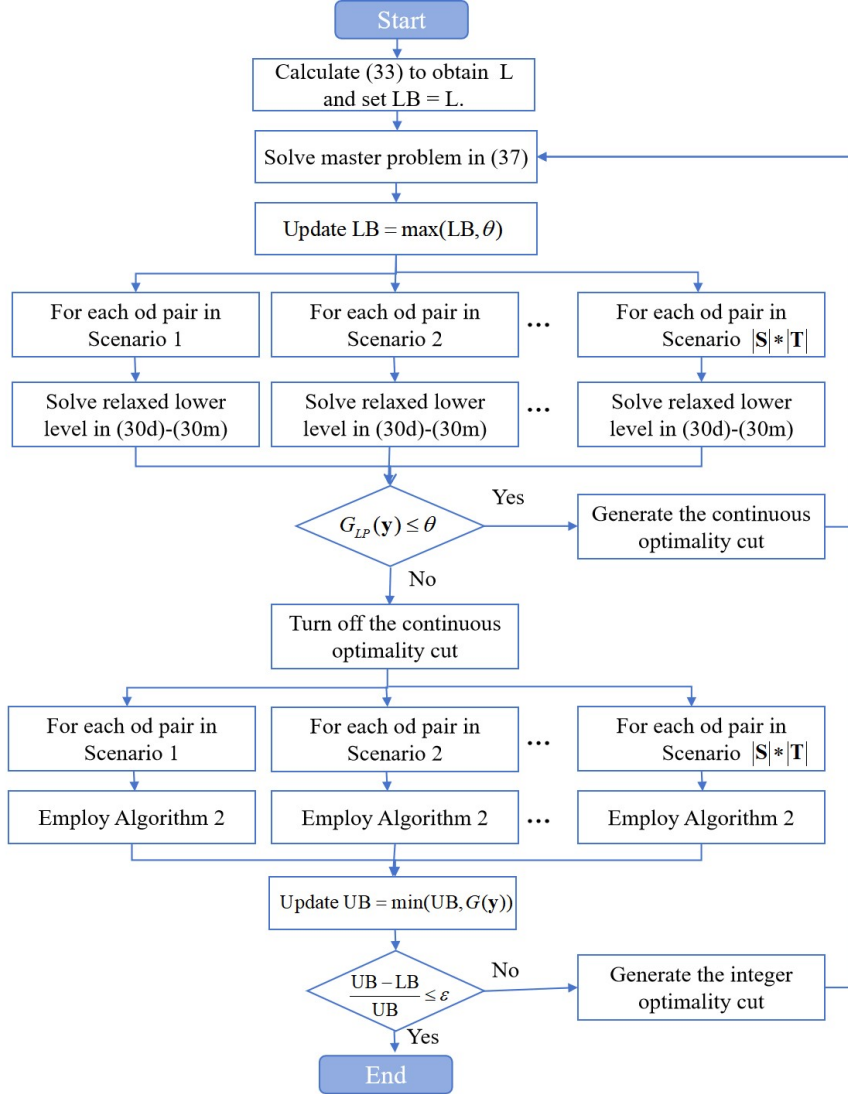


Figure 2: Flowchart of the integer L-shaped algorithm with continuous and integer optimality cuts.

3. The fitted regression equations

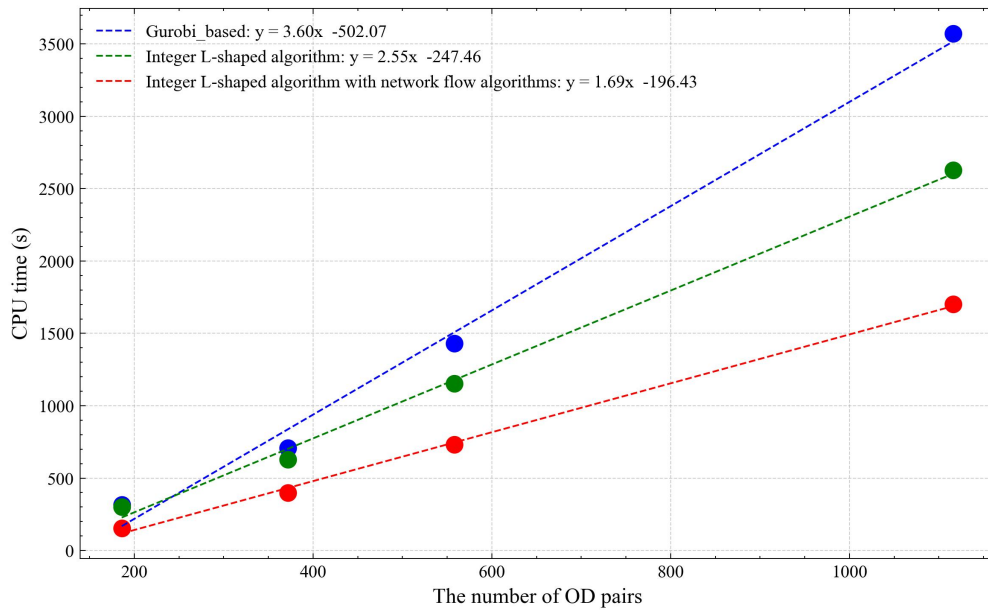


Figure 3: The fitted regression equations for the three methods with increasing OD pair numbers.

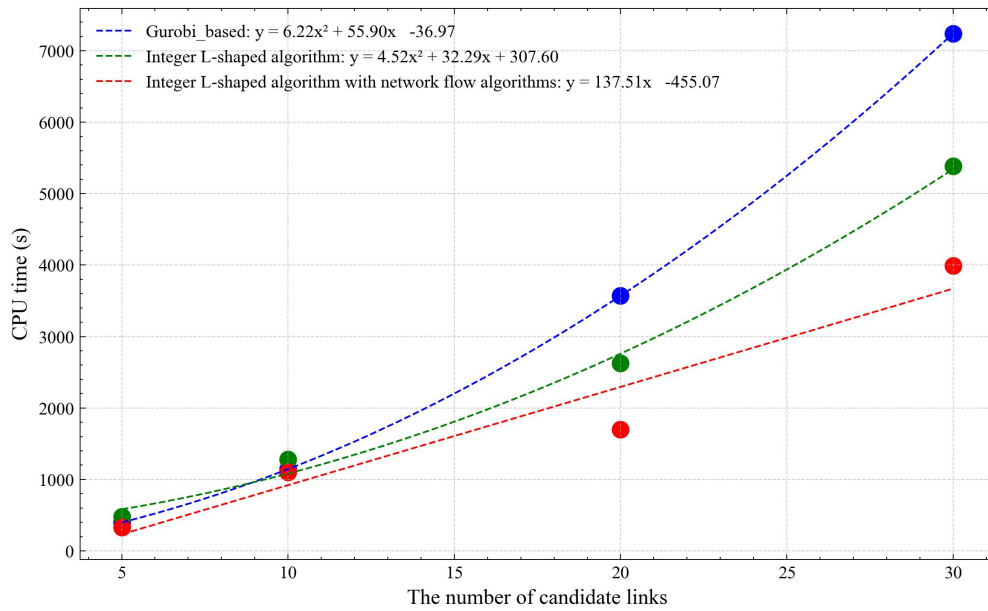


Figure 4: The fitted regression equations for the three methods with increasing candidate link numbers.

4. Appendix A: Proof of Proposition 2

Proof. When $x_{ij} = 0$, Equation (25) can be expressed as follows:

$$\left. \begin{array}{l} \mu_{ij} \leq 0 \\ -1 \leq 0 \\ \mu_{ij} \geq 0 \end{array} \right\} \Rightarrow \mu_{ij} (x_{ij} - 1) = 0, \forall (i, j) \in A$$

When $x_{ij} = 1$, Equation (25) can be expressed as follows:

$$\left. \begin{array}{l} \mu_{ij} \leq M \\ x_{ij} - 1 = 0 \\ \mu_{ij} \geq 0 \end{array} \right\} \Rightarrow \mu_{ij} (x_{ij} - 1) = 0, \forall (i, j) \in A$$

When $x_{ij} = 0$, Equation (26) can be expressed as follows:

$$\left. \begin{array}{l} \xi_{ij} \leq M \\ \xi_{ij} \geq 0 \end{array} \right\} \Rightarrow \xi_{ij} x_{ij} = 0, \forall (i, j) \in A$$

When $x_{ij} = 1$, Equation (26) can be expressed as follows:

$$\left. \begin{array}{l} \xi_{ij} \leq 0 \\ \xi_{ij} \geq 0 \end{array} \right\} \Rightarrow \xi_{ij} x_{ij} = 0, \forall (i, j) \in A$$

This completes the proof. □

5. Appendix B: Proof of Proposition 3

Proof. By temporarily ignoring constraint 38e, the lower level can be viewed as a minimum-cost maximum-flow problem pertaining to the determination of distinct paths without acceptable travel time. Let ρ be the distinct path with the longest travel time in the distinct path set. Define $\hat{Y} (n_{od}(s, t) - 1) = Y (n_{od}(s, t)) - \rho$. Note that $\hat{Y} (n_{od}(s, t) - 1)$ comprises $n_{od}(s, t) - 1$ distinct paths, making it a feasible solution for the model in equations 38f-38h when $n_{od}(s, t) := n_{od}(s, t) - 1$. Since this model is a minimization problem, we have:

$$\hat{Y} (n_{od}(s, t) - 1) \geq Y (n_{od}(s, t) - 1) \quad (1)$$

Due to the boundedness of $\sum_{(i,j) \in A} (r_{ij}(t) + (1 - \beta_{ij} y_{ij}) r_{ij}(s)) x_{ij}^*$ by definition, the travel time of each path in this path set is also bounded. Moreover, as ρ represents the path with the longest travel time among the path set, it must be greater than or equal to the average travel time of the other $n_{od}(s, t) - 1$ paths. Therefore, we can express this relationship as follows:

$$\rho \geq \frac{1}{n_{od}(s, t) - 1} \hat{Y} (n_{od}(s, t) - 1) \quad (2)$$

By combining equations (1) and (2), we have:

$$\begin{aligned} \frac{1}{n_{od}(s, t)} Y (n_{od}(s, t)) &= \frac{1}{n_{od}(s, t)} (\hat{Y} (n_{od}(s, t) - 1) + \rho) \\ &\geq \frac{1}{n_{od}(s, t)} \left(\hat{Y} (n_{od}(s, t) - 1) + \frac{1}{n_{od}(s, t) - 1} \hat{Y} (n_{od}(s, t) - 1) \right) \\ &= \frac{1}{n_{od}(s, t) - 1} \hat{Y} (n_{od}(s, t) - 1) \geq \frac{1}{n_{od}(s, t) - 1} Y (n_{od}(s, t) - 1) \end{aligned} \quad (3)$$

This completes the proof. □