Report:

Summary

Introducion

They are made up of multiple smaller decisions that we hope are correct and optimal. It can be represented as (X1, X2 … Xn) which is also called the decision variables within the model. We also looked at how these optimisations may be faced with constraints for example the limitations on a factories ability to manufacture products. This also introduced how we are going to look at solving these constraints for linear constraints.

Graphical approach

This part discussed initially how to find the constraints and conditions for a problem when given a possible example. From there the chapter introduced the concepts of boundary lines and constraints and how they can limit the feasibility region. These can all be visualised on a two dimensional graph. Then the chapter discusses how we find the optimal solution by taking the right-hand-side value and the furthest point of the feasibility region. After the chapter looked at some issues found in linear programming. The first two mentioned alternate optimal solutions and redundant constraints dont stop you from solving a linear programming problem. This is because alternate optimal solutions just means there isn't one clear answer and redundant constraints don't prevent you from finding the answer, it's just unnecessary information. Whereas unbound solutions are either infinitely small or infinitely large and are unbound so the optimal solution cannot be found. The infeasible problems are ones that also cannot be solved. Their constraints might prevent an answer being available or then limit the results too much.

Standard and slack forms

This part of the chapter looked at different ways we can look at how a linear programming problem can be written. It listed elements that are not in standard form and how to convert a problem into standard form. This can be done by enforcing non negativity on some constraints by adding a Xj’ and Xj” to replace it. Then it looked at slack form which allowed us to add in new variables and we can apply the same trick to standard form. Meaning we can add new variables like X4, X5 and X6.

Gaussian Elimination

This is a technique to solve a linear equation by using y = Ax. This chapter initially introduces how systems of linear equations work, as they are needed to know how LU-decomposition works. We looked at the slack form and we can convert it into a system of linear equations. When we look for an LU decomposition the L is for a lower triangular matrix where the U is for the upper triangular matrix.

The process is to begin with Ax = b. First step is to find A by getting the L and U. Then from LUx=b is to find y = Ux so you can have Ly = b. Then we can know b through the process of backwards and forwards substitution through Ly = b and Ux = y respectively.
The unit page then covered how we get both L and U and apply the LU decomposition algorithm. This also covered how to find x when we have A and B since A = LU and LUx=b.

Simplex Algorithm

The simplex algorithm is another way we can solve an LU decomposition problem. However it aims to solve this problem using Gaussian elimination. This chapter went through how we can convert from a standard form into a set of linear equations and convert that into a simplex tableau. From that we can use gaussian elimination for our first and second X values and then we can look at the result.

Tasks:

1. I have included the answers attached in the IPYNB file

Max: $18X_1 + 12.5X_2$

Subject to: $X_1 + X_2 \leq 20$

$X_1 \leq 12$

$X_2 \leq 16$

$X_1, X_2 \geq 0$

$18X_1 + 12.5X_2 = Z$

$X_1 + X_2 + X_3 = 20$

$X_1 + X_4 = 12$

$X_2 + X_5 = 16$

$X_1, X_2, X_3, X_4, X_5 \geq 0$

Convert

Simplex tableau

	x_1	x_2	x_3	x_4	x_5	Z	C
R_1	1	1	1	0	0	0	20
R_2	$\boxed{1}$ Pivot	0	0	1	0	0	12
R_3	0	1	0	0	1	0	16
R_4	-18	-12.5	0	0	0	1	0

		x_1	x_2	x_3	x_4	x_5	Z	C
$-1R_2$	R_1	0	$\boxed{1}$ Pivot	1	-1	0	0	8
	R_2	1	0	0	1	0	0	12
	R_3	0	1	0	0	1	0	16
$+18R_2$	R_4	0	-12.5	0	18	0	1	216

		x_1	x_2	x_3	x_4	x_5	Z	C
	R_1	0	1	1	-1	0	0	8
	R_2	1	0	0	1	0	0	12
$+R_1$	R_3	0	0	-1	1	1	0	8
$+12.5R_1$	R_4	0	0	12.5	5.55	0	1	316

$x_1 = 12$

$x_2 = 8$

$Z = 316$

2.

Second part is included in the ipynb file