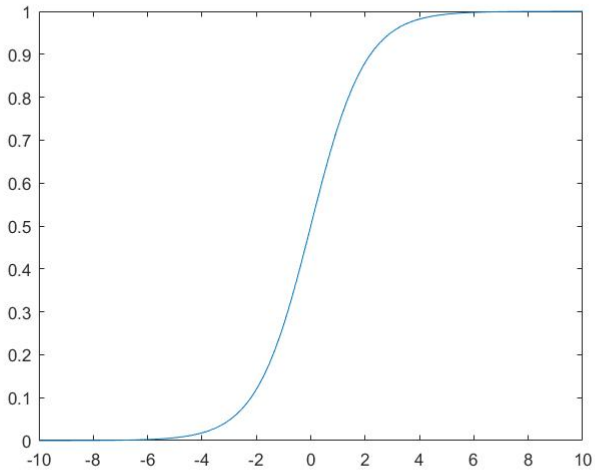
Logistic Regression

- Given x, want $\hat{y} = P(y=1|x)(0 \leq \hat{y} \leq 1)$ and $x \in \mathbb{R}^{\mathrm{n_x}}$
- Parameters: $w \in \mathbb{R}^{\mathrm{n_x}}, b \in \mathbb{R}$
- Output $\hat{y} = \sigma(w^T x + b)$
- Sigmoid function: $\sigma(z) = \frac{1}{1+e^{-z}}$



Logistic Regression cost function

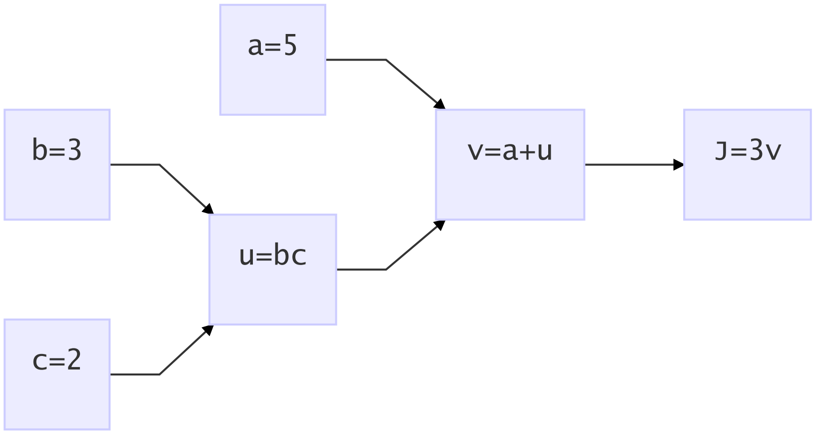
- Given $\{(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})\}$, want $\hat{y}^{(i)} \approx y^{(i)}$
- Loss function: $\mathscr{L}(\hat{y}, y) = -ylog\hat{y} - (1-y)log(1-\hat{y})$
- Cost function: $\mathscr{J}(w,b) = \frac{1}{m}\sum_{n=1}^{N} \mathscr{L}(\hat{y}^{(i)}, y^{(i)})$

Gradient Descent

- Want to find w, b that minimize $\mathscr{J}(w,b)$
- Repeat

$$w := w - \alpha \frac{\partial \mathscr{J}(w,b)}{\partial w} \tag{1}$$
$$b := b - \alpha \frac{\partial \mathscr{J}(w,b)}{\partial b}$$

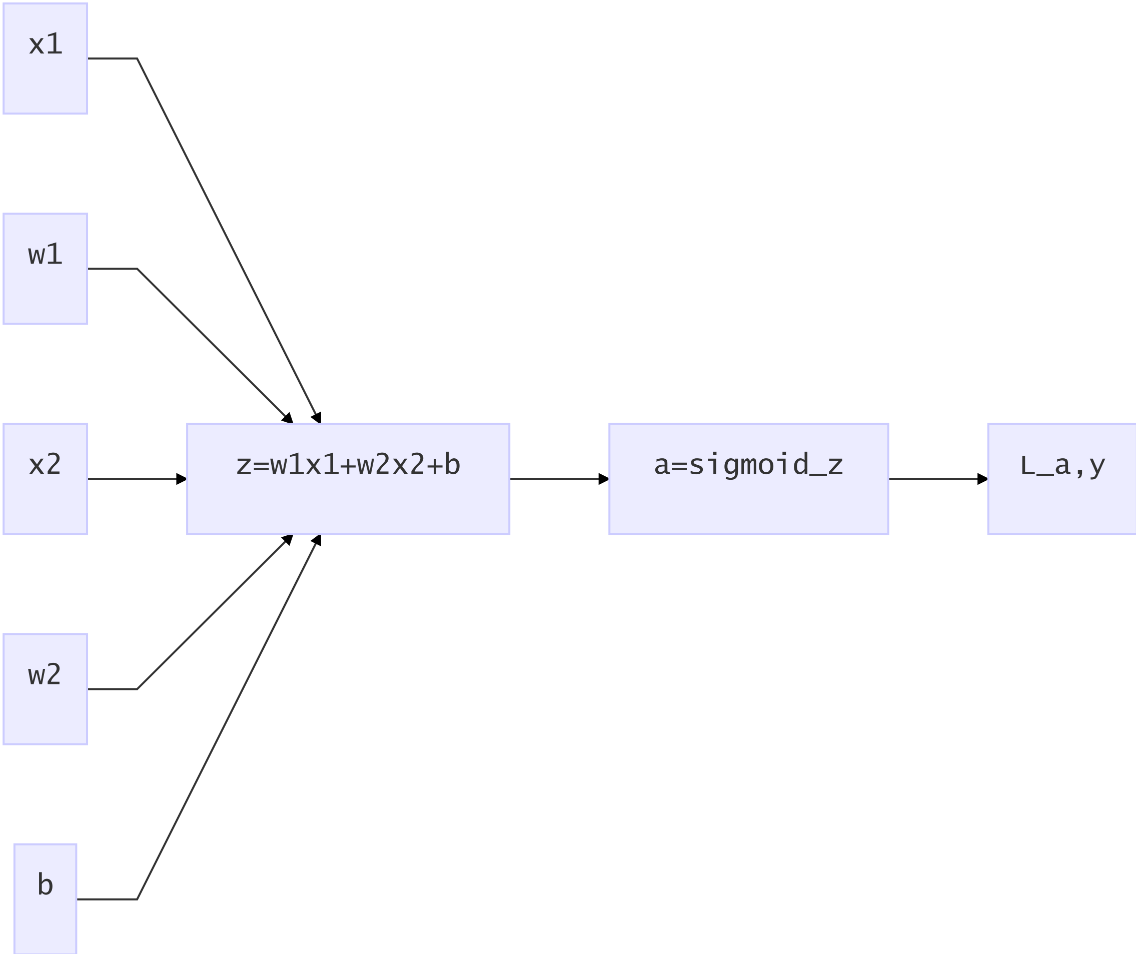
Computational Graph



- In *python* a way to name a derivative of $\frac{dFinalOutput}{dvar}$ simply as dvar, because we already acknowledge the $dFinalOuput$ is the derivative of cost functions.
- Chain rule of derivative

$$
\begin{aligned}
da &= \frac{dJ}{dv}\frac{dv}{da} \\
db &= \frac{dJ}{dv}\frac{dv}{du}\frac{du}{db} \\
dc &= \frac{dJ}{dv}\frac{dv}{du}\frac{du}{dc}
\end{aligned} \tag{2}
$$

Logistic Regression derivatives



$$\begin{aligned} da &= \frac{d\mathscr{L}(a,y)}{da} = -\frac{y}{a} + \frac{1-y}{1-a} \\ dz &= \frac{\mathscr{J}(a,y)}{dz} = \frac{d\mathscr{L}}{da}\frac{da}{dz} = a - y \\ dw_1 &= \frac{\partial \mathscr{J}}{\partial w_1} = x_1 dz \\ dw_2 &= \frac{\partial \mathscr{J}}{\partial w_2} = x_2 dz \\ db &= \frac{\partial \mathscr{J}}{\partial b} = dz \end{aligned} \tag{3}$$

Logistic Regression on m examples

- In examples $(x^{(i)}, y^{(i)})$, calculate $dw_1^{(i)}, dw_2^{(i)}, db^{(i)}$

$$\frac{\partial}{\partial w_i}\mathscr{J}(w,b) = \frac{1}{m}\sum_{i=1}^{m}\underbrace{\frac{\partial}{\partial w_i}\mathscr{L}(a^{(i)},y^{(i)})}_{dw_i^{(i)}} \tag{4}$$