

## Softmax regression

$C = \# \text{classes}$ ,  $n^{[L]} = C$ ,  $\hat{y}$  is  $(C, 1)$

$$\hat{y} \rightarrow \begin{cases} P(\text{class1}|x) \\ P(\text{class2}|x) \\ \dots \\ P(\text{classC}|x) \end{cases}$$

$$z^{[L]} = W^{[L]} a^{[L-1]} + b^{[L]}$$

Activation function :  $g^{[L]}(z^{[L]})$

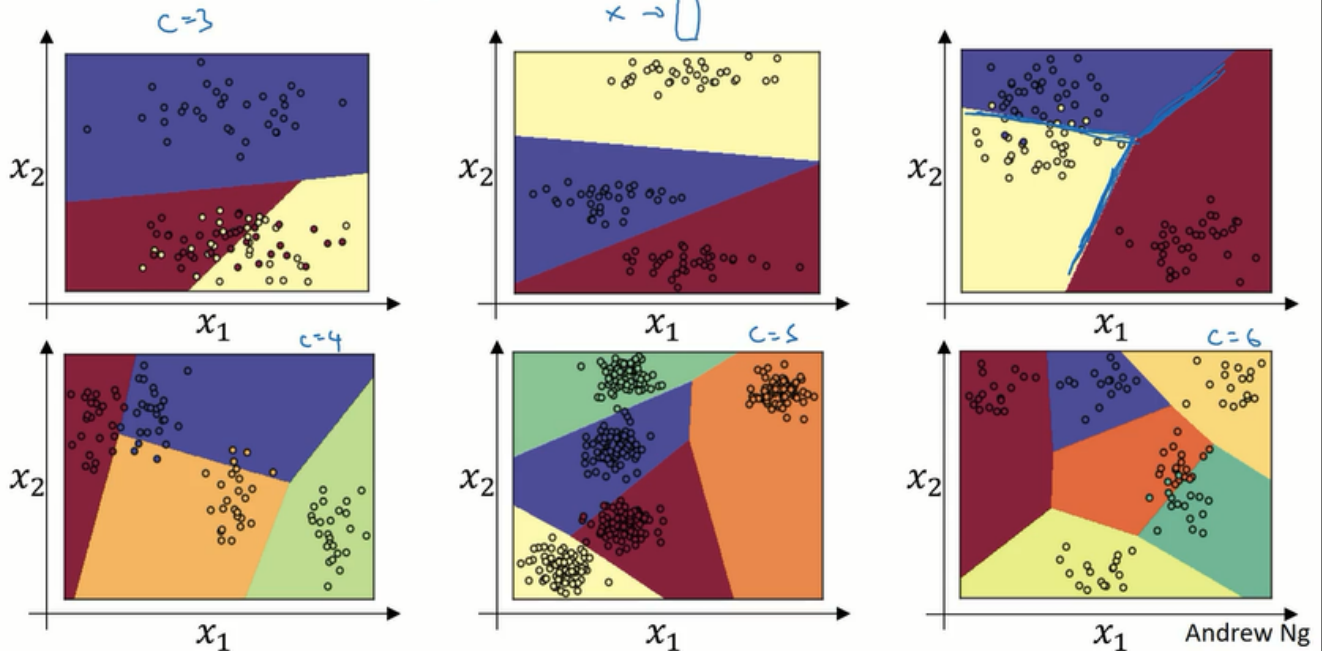
$$t = e^{(z^{[L]})}$$

$$a^{[L]} = \frac{e^{z^{[L]}}}{\sum_i t_i}, a_i^{[L]} = \frac{t_i}{\sum_i t_i}$$

$a^{[L]} = g^{[L]}(z^{[L]})$ ,  $a^{[L]}$  and  $z^{[L]}$  are  $(C, 1)$

## Softmax examples

$$\begin{aligned} x_1 &\rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \hat{y} \\ x_2 &\rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\ z^{[L]} &= W^{[L]}x + b^{[L]} \\ a^{[L]} &= \hat{y} = g(z^{[L]}) \end{aligned}$$



## training a softmax classifier

- loss function

$$\mathcal{L}(\hat{y}, y) = - \sum_{j=1}^C y_j \log(\hat{y}_j)$$

it looks at wherever the ground true class in the training set is, and tries to make the corresponding probabilities as high as possible

- $\mathcal{J}(W^{[1]}, b^{[1]}, \dots) = \frac{1}{m} \sum_i \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$
- $Y = [y^{(1)} \dots y^{(m)}], Y \in \mathbb{R}^{(C, m)}$

- $\hat{Y} = \begin{bmatrix} \hat{y}^{(i)} & \dots & \hat{y}^{(m)} \end{bmatrix}, \hat{Y} \in \mathbb{R}^{(C,m)}$
- **Backprop**
  - $\frac{\partial \mathcal{J}}{\partial z^{[L]}} = dz^{[L]} = \hat{y} - y, dz^{[L]} \in \mathbb{R}^{(C,1)}$