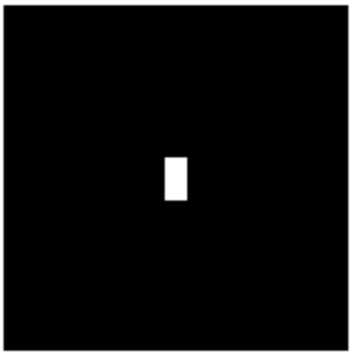
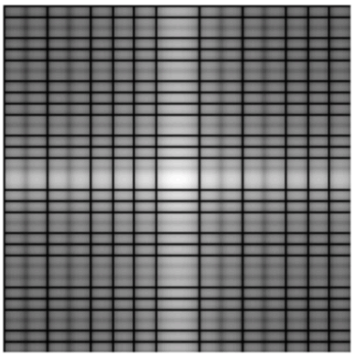
Assignment 3

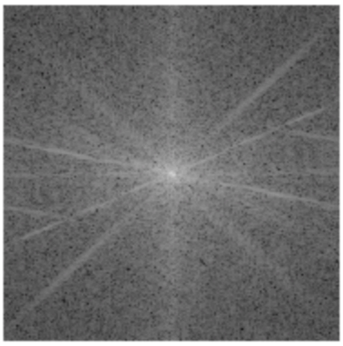
Praffullitt Jain
20171142

Q1.

$$
\begin{aligned}
X_k &= \sum_{n=0}^{N-1} x_n \cdot e^{-i\,2\pi\,k\,n\,/\,N} \\
&= \sum_{m=0}^{N/2-1} x_{2m} \cdot e^{-i\,2\pi\,k\,(2m)\,/\,N} + \sum_{m=0}^{N/2-1} x_{2m+1} \cdot e^{-i\,2\pi\,k\,(2m+1)\,/\,N} \\
&= \sum_{m=0}^{N/2-1} x_{2m} \cdot e^{-i\,2\pi\,k\,m\,/\,(N/2)} + e^{-i\,2\pi\,k\,/\,N} \sum_{m=0}^{N/2-1} x_{2m+1} \cdot e^{-i\,2\pi\,k\,m\,/\,(N/2)}
\end{aligned}
$$



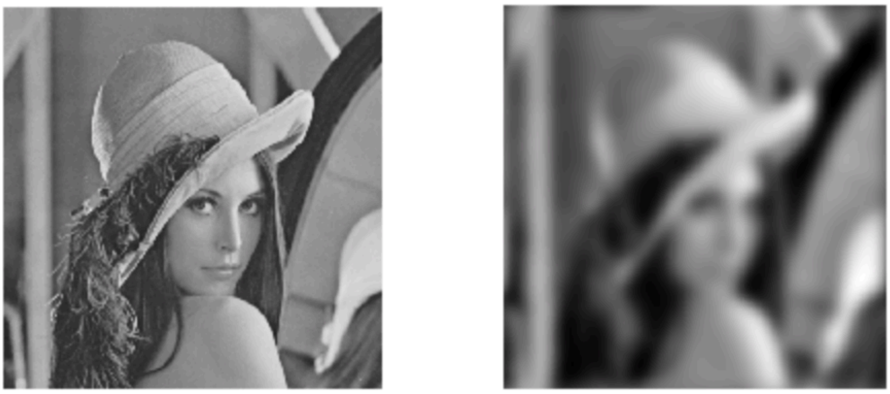




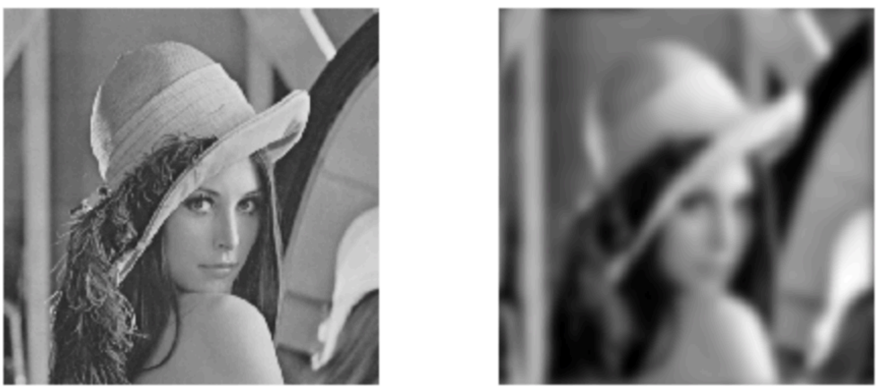
Q2.

Ideal Low Pass Filter-

Butterworth Low Pass Filter-

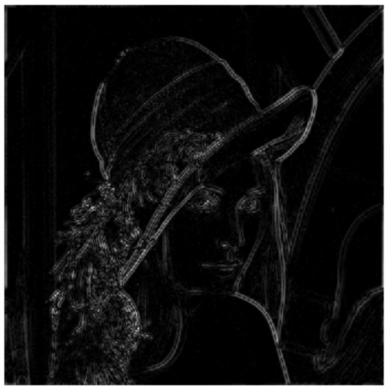


Gaussian Low Pass Filter-



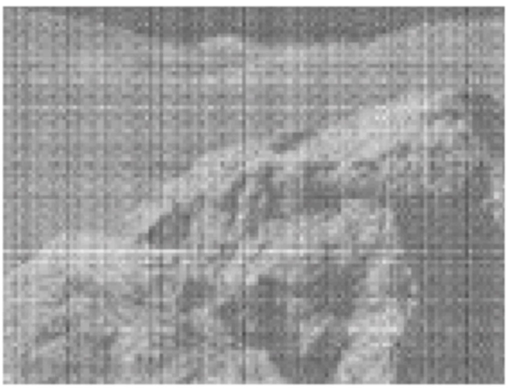
a = 10, b = 20

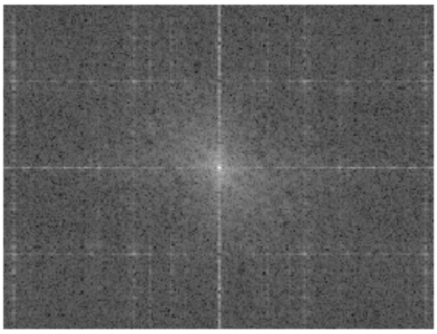
a = 20, b = 50

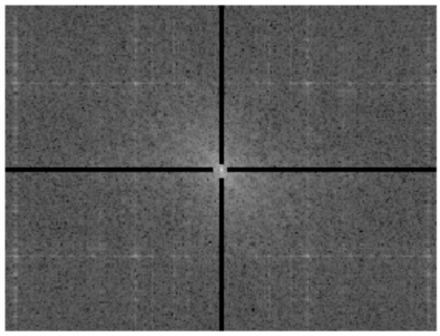


a = 50, b = 80

Q4.







Q6.
If f is [246, 300] and h is [250,189]-

COMMAND WINDOW

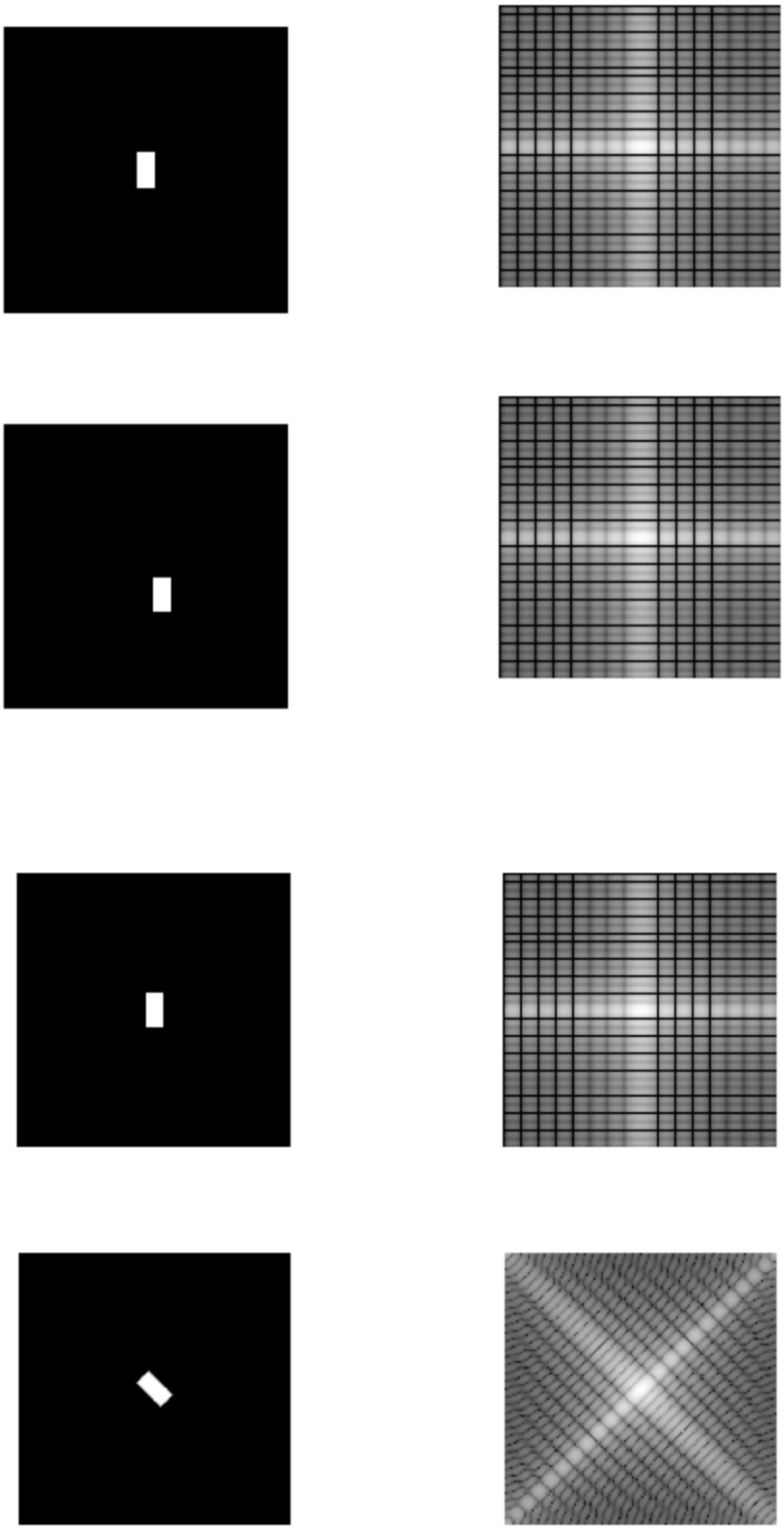
```
Elapsed time is 0.504984 seconds.
Elapsed time is 0.012282 seconds.
>>
```

If f is [530, 670] and h is [230, 860]-

COMMAND WINDOW

```
Elapsed time is 8.856954 seconds.
Elapsed time is 0.048376 seconds.
>>
```

Q8.



classmate
Date
Page

$g(x,y) \xrightarrow{FFT} G(f_x, f_y)$

8.1 2D Rotation

$g(x,y) \rightarrow$ original

(x', y') be the new coordinate system

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$g(x', y')$ $\rightarrow$ new

or

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$x = x'\cos\theta - y'\sin\theta$

$y = x'\sin\theta + y'\cos\theta$

$$F(g(x',y')) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x',y')\, e^{-2\pi j(xf_x + yf_y)}\, dx\, dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x',y')\, e^{-2\pi j\left(x'(f_x\cos\theta + f_y\sin\theta) + y'(-f_x\sin\theta + f_y\cos\theta)\right)}\, dx\, dy$$

$$\& \quad dx'\,dy' = \begin{vmatrix} \frac{\partial x'}{\partial x} & \frac{\partial y'}{\partial x} \\ \frac{\partial x'}{\partial y} & \frac{\partial y'}{\partial y} \end{vmatrix} dx\,dy$$

$$= \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} dx\,dy = dx\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x',y')\, e^{-2\pi j\left(x'(f_x\cos\theta + f_y\sin\theta) + y'(-f_x\sin\theta + f_y\cos\theta)\right)}\, dx'\,dy'$$

$$= G(f_x\cos\theta + f_y\sin\theta,\ -f_x\sin\theta + f_y\cos\theta)$$

$\Downarrow$

Rotation of fft G

classmate

Date ______
Page ______

$\xrightarrow{FFT} G(F_x, f_y)$

8.2 Translation $g(x,y) \rightarrow$ original

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x+a \\ y+b \end{bmatrix} \quad ; \quad \begin{aligned} x' &= x+a \\ y' &= y+b \end{aligned}$$

$$dx'dy' = dx\,dy$$

$$F(g(x',y')) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x',y')\, e^{-2\pi j(xf_x + yf_y)}\, dx\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x',y')\, e^{-2\pi j((x'-a)f_x + (y'-b)f_y)}\, dx'dy'$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x',y')\, e^{-2\pi j(x'f_x + y'f_y - (af_x+bf_y))}\, dx'dy'$$

$$= e^{-2\pi j(-af_x+bf_y)} \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x',y')\, e^{-2\pi j(x'f_x + y'f_y)}\, dx'dy'}$$

$$= K\, G(f_x, f_y)$$

Q3.

3. $out1 = f1 + h2 * f2$

$out2 = f2 + h1 * f1$

$F(out1) = F(f1) + F(h2 * f2)$

$= F(f1) + F(h2) F(f2)$ ——— (i)

$F(out2) = F(f2) + F(h1) F(f1)$ ——— (ii)

solving for $F(f1)$

$$F(out2) = \frac{F(out1) - F(f1)}{F(h2)} + F(h1) F(f1)$$

$$F(out2) F(h2) = F(out1) - F(f1) + F(h1) F(h2) F(f1)$$

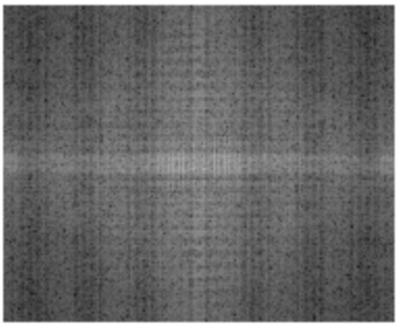
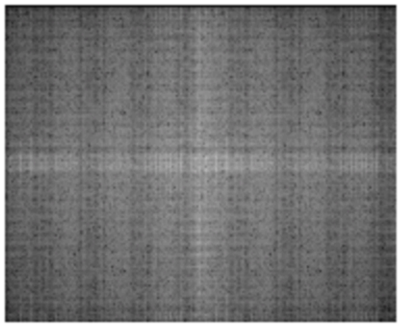
$$F(f1) = \frac{F(out2) F(h2) - F(out1)}{F(h1) F(h2) - [1]}$$ ⟶ matrix with all 1's

$||^{ly}$, $$F(f2) = \frac{F(out1) F(h1) - F(out2)}{F(h1) F(h2) - [1]}$$

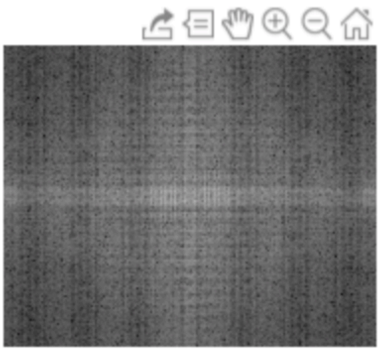
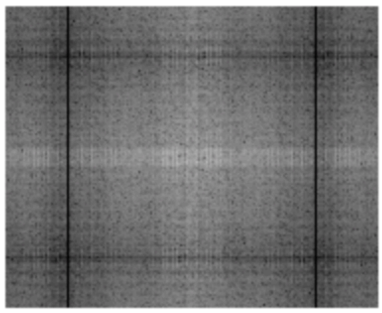
but if $F(h1) F(h2) = [1]$

then this is wrong.

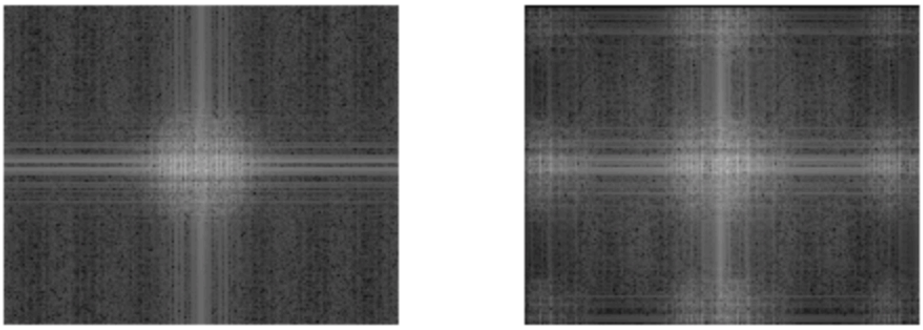
Q7.

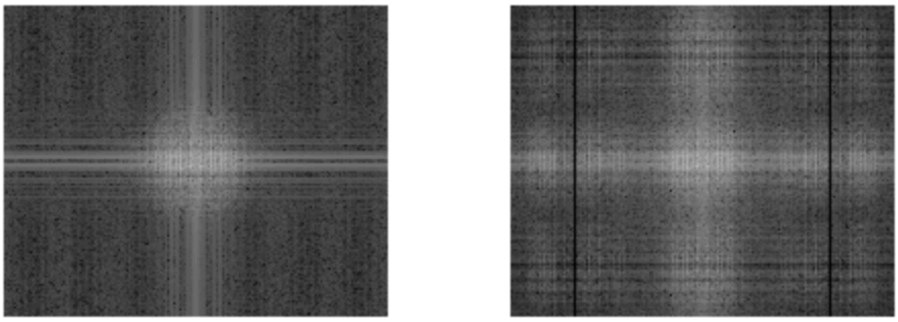
nx = 2, ny = 2

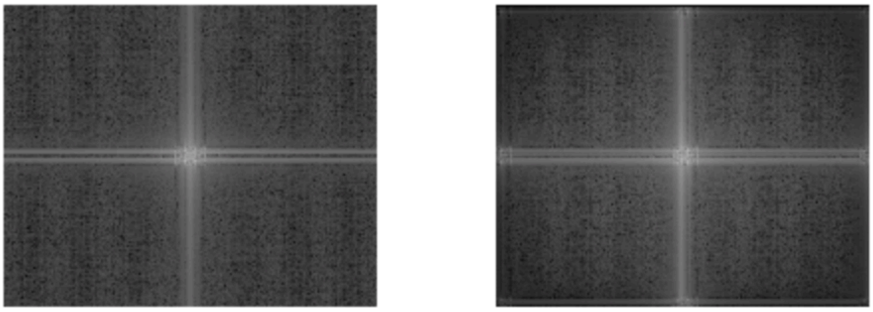
nx = 3, ny = 3



nx = 2, ny = 2 and sigma = 2



nx = 3, ny = 3 and sigma = 2



nx = 2, ny = 2 and sigma = 8

Q5.

COMMAND WINDOW

```
>> H = [[0,1,0];[1,2,1];[0,1,0]];
>> m = dftmtx(3)

m =

   1.0000 + 0.0000i   1.0000 + 0.0000i   1.0000 + 0.0000i
   1.0000 + 0.0000i  -0.5000 - 0.8660i  -0.5000 + 0.8660i
   1.0000 + 0.0000i  -0.5000 + 0.8660i  -0.5000 - 0.8660i

>> H_fft = m*H*m

H_fft =

   6.0000 + 0.0000i  -1.5000 - 2.5981i  -1.5000 + 2.5981i
  -1.5000 - 2.5981i   0.0000 + 0.0000i   0.0000 + 0.0000i
  -1.5000 + 2.5981i   0.0000 + 0.0000i   0.0000 + 0.0000i

>> H_fft = fftshift(H_fft)

H_fft =

   0.0000 + 0.0000i  -1.5000 + 2.5981i   0.0000 + 0.0000i
  -1.5000 + 2.5981i   6.0000 + 0.0000i  -1.5000 - 2.5981i
   0.0000 + 0.0000i  -1.5000 - 2.5981i   0.0000 + 0.0000i

>> H_fft = abs(H_fft)

H_fft =

     0     3     0
     3     6     3
     0     3     0
```

Which is a low pass filter.