$P(Y|X) = P(Y) \Rightarrow E(Y|X) = E(Y) \Rightarrow Cov(Y,X) = 0$

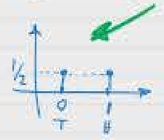
Ev_1 = flip a coin once

RV → X = T: 0, H: 1

PDF(x) = 1/2, 1/2

f(x)

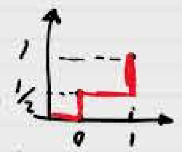
Discrete



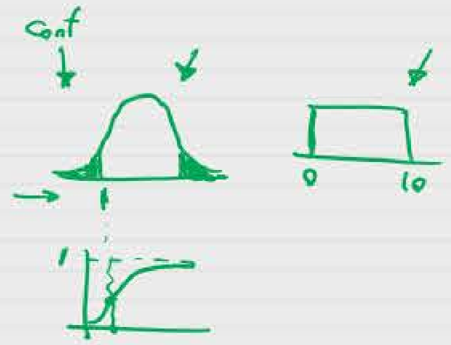
F(x) CDF(X=x)
= PDF(X ≤ x)

CDF(X=0) = PDF(X ≤ 0) → (1/2) (1)

CDF(X=1) = PDF(X ≤ 1)
= PDF(X=0) + PDF(X=1)
1/2 + 1/2 = 1



Cont

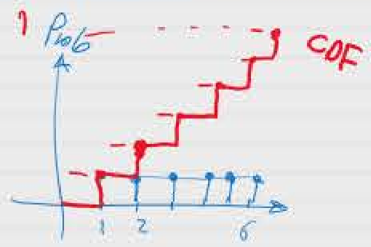


Ev_2 : Rolling a fair dice

Y = 1, 2, 3, 4, 5, 6

f(y) = 1/6, (1/6), ... 1/6

→ F(y) = 1/6, 2/6, 3/6, . . . 6/6 = 1



obs	Y_i	x_i
1	1	H: 1
2	2	1
:	3	1
.	4	1
.	5	1
.	6	1
.	1	0
.	2	0
.	3	0
.	4	0
.	5	0
12	6	0

① Strong indep ✓ $P(Y|X) = P(Y)$ ✓ $\forall$ (for all) $X = x_i$, $i = 0, 1$

(y=2, x=1) → $P(Y=2 | X=1) = \frac{1}{6} = \frac{P(Y \cap X)}{P(X)} = \frac{P(Y=2 \cap X=1)}{P(X=1)} = \frac{1/12}{1/2} = \frac{1}{6}$

$P(Y=2) = \frac{1}{6}$

② Mean indep

$E(Y|X) = E(Y)$

$E(Y|X=1) = \frac{1+2+3+\dots+6}{6} = 3.5$

$E(Y|X=0) = 3.5$

$E(Y) = 3.5 = \frac{1+2+3+4+5+6}{6}$ ← Average

$E(Y) = \sum y_i \cdot f(y_i) =$
$1(\frac{1}{6}) + 2(\frac{1}{6}) + 3(\frac{1}{6}) + \dots = 3.5$

③ Cov = Corr(Y, X) = 0 ; $Cov(X,Y) = E((X-\mu_X)(Y-\mu_Y))$

Y_i	X_i	μ_Y	μ_X	Ⓐ $(X-\mu_X)$	Ⓑ $(Y-\mu_Y)$	A×B
1	1	3.5	0.5	1−0.5	1−3.5	−1.25
2	1	3.5	'	0.5	−2.5	"
:	1	|	|	|	"	"
6	1	|	|	|	"	"
1	0	|	|	|	"	"
:	0	|		|	"	"
:	0	|		|	"	"
6	:	|		|	"	"
	0	3.5	0.5		"	"

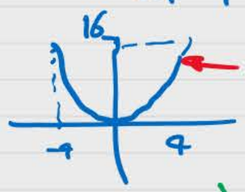
0

Ex (2) $P(y|x) = P(y)$ $\Leftarrow$ $E(y|x) = E(y)$ ~~$\Rightarrow$~~ $Corr(y,x) = 0$

$Corr(x,y) = ?$ Q

$X = [-4, 4]$

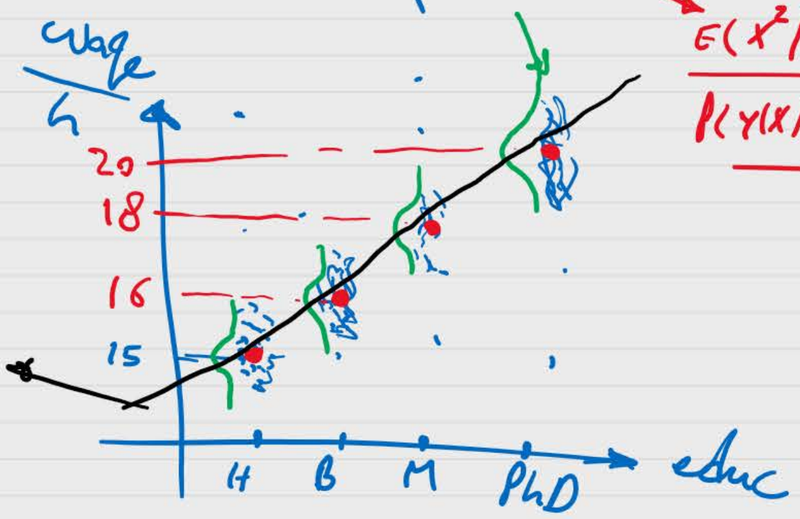
$Y = X^2$



$E(y|x) = ?$

$E(X^2|X) = X^2 = (Y) \neq E(Y)$

$P(y|x) = P(y|X)$



$E(wage \mid educ = H) = 15$

$E(w \mid educ = M) = 18$