Stats & Probability

Experiment : Flipping a Coin twice & Count the # of heads

Discrete

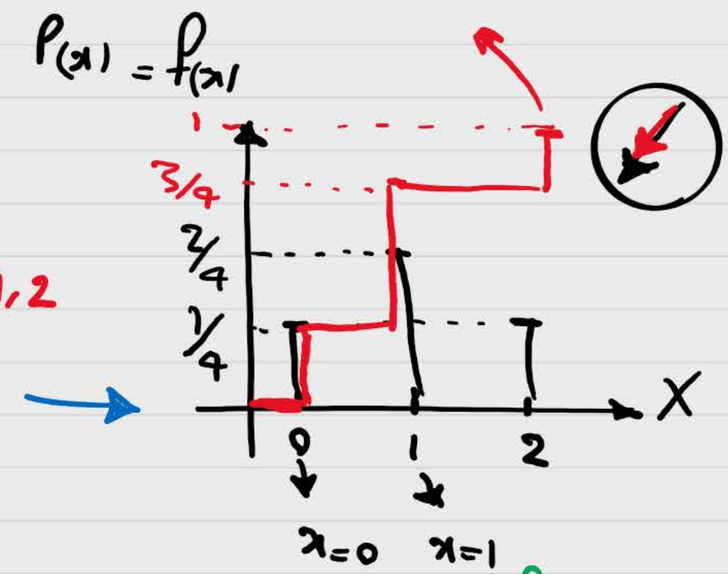
Outcomes : HH, HT, TH, TT

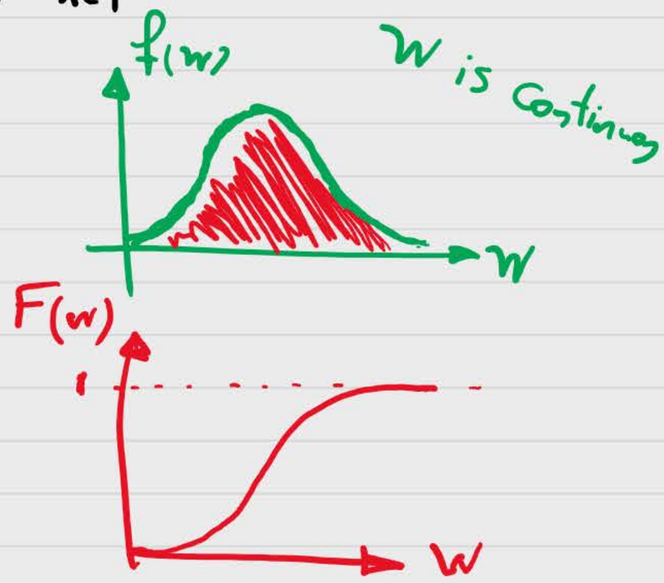
RV : X

	HH	HT	TH	TT	
RV : X	2	1	1	0	$x = 0, 1, 2$
PDF :	1/4	2/4		1/4	$\Rightarrow f(x)$
CDF :	1	3/4		1/4	$\Rightarrow F(x)$

$$CDF(1) = F(1) = f(x \leq 1) = \underbrace{P(x=0)}_{1/4} + \underbrace{P(x=1)}_{2/4}$$

CDF = Area under PDF





Example:

	Y = wage	X = gender
1	20	M
2	25	M
3	30	F
4	30	M
5	20	F

Conditional Prob

$P(Y=20 \mid X=M) = 1/3$

$P(Y=20 \mid X=M) = \frac{P(Y=20 \cap X=M)}{P(X=M)} = \frac{1/5}{3/5} = 1/3$

① is X and Y independent (strong): $P(Y \cap X) = P(Y)P(X) \Rightarrow P(Y|X) = P(Y)$

$P(Y|X=M) = P(Y) \to P(Y=20)$

$Y=20$

$1/3 \neq 2/5 \Rightarrow$ X & Y are Not indep

Conditional Exp

$E(Y|X=M) = 25$ → $f(Y|X)$

$E(Y|X=M) = \sum Y f(Y|X) = 20\, f(20|M) + 25\, f(25|M) + 30\, f(30|M) = \frac{20+25+30}{3} = 25$

($f(20|M) = 1/3$, $f(25|M) = 1/3$, $f(30|M) = 1/3$)

② is X and Y are Mean independent (weak indep): $E(Y|X) = E(Y)$

$E(Y|\text{male}) \overset{?}{=} E(Y) \to$? → X, Y are mean indep:

$25 \overset{\checkmark}{=} 25$

Not yet

$E(Y|\text{female}) \overset{?}{=} E(Y)$

$25 \overset{\checkmark}{=} 25$

Now can say

Yes! X & Y are Mean indep

③ $Corr(Y,X) = 0$

Yes

Corr = 0