# Class-18 MRM Asymptotic

Pedram Jahangiry

JON M. HUNTSMAN SCHOOL OF BUSINESS
UtahStateUniversity

# CHAPTER 5

## Multiple Regression Analysis:

## OLS Asymptotics

# OLS Asymptotics

- So far we focused on properties of OLS that hold for any sample
- Properties of OLS that hold for **any sample size**
  - Expected values/unbiasedness under MLR.1 – MLR.4
  - Variance formulas under MLR.1 – MLR.5
  - **Gauss-Markov** Theorem under MLR.1 – MLR.5
  - Exact **sampling distributions** under MLR.1 – MLR.6

- Properties of OLS that hold in large samples
  - Consistency under MLR.1 – MLR.4
  - Asymptotic normality under MLR.1 – MLR.5

Without assuming nomality of the error term!

# Consistency of OLS

**THEOREM 5.1** **CONSISTENCY OF OLS**

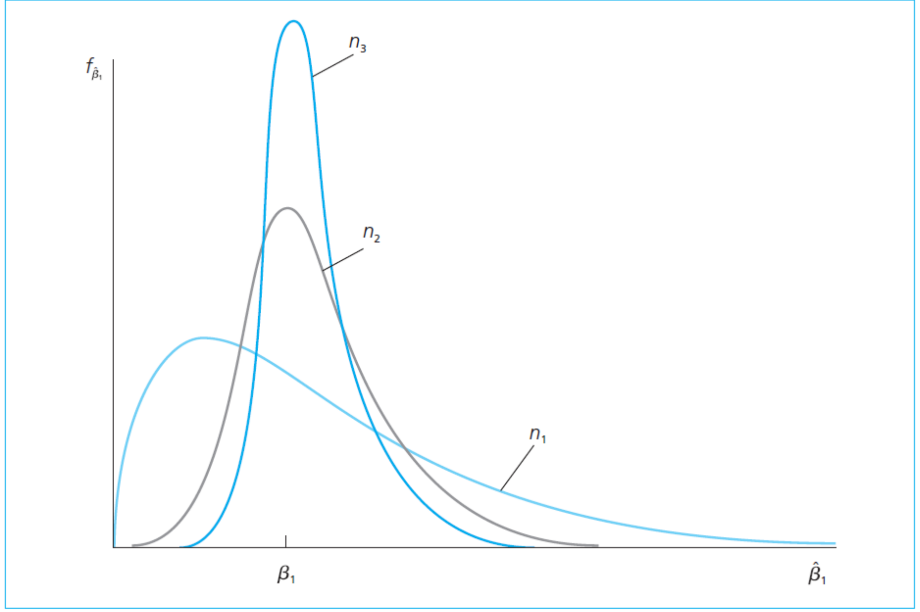
Under Assumptions MLR.1 through MLR.4, the OLS estimator $\hat{\beta}_j$ is consistent for $\beta_j$, for all $j = 0, 1, \ldots, k$.

Consistency: How far the estimator is likely to be from the parameter as sample size increase indefinitely.

An estimator is consistent if: $plim(\widehat{\theta_n}) = \theta$

- ❑ **Unbiasedness** is a feature of an estimator for a given sample size
- ❑ **Consistency** involves the behavior of the sampling distribution of the estimator as the sample size gets large.

**FIGURE 5.1** Sampling distributions of $\hat{\beta}_1$ for sample sizes $n_1 < n_2 < n_3$.



# Consistency of OLS (cont'd)

**Assumption MLR.4'** **Zero Mean and Zero Correlation**

$\mathrm{E}(u) = 0$ and $\mathrm{Cov}(x_j, u) = 0$, for $j = 1, 2, \ldots, k$.

All explanatory variables must be uncorrelated with the error term. This assumption is <u>weaker</u> than the zero conditional mean assumption MLR.4.

Independence $\Rightarrow$ Mean independence $\Longrightarrow$ Zero correlation

$$P(Y|X) = P(Y) \Rightarrow E(Y|X) = E(Y) \Rightarrow Corr(Y, X) = 0$$

BUT

Independence $\not\Leftarrow$ Mean independence $\not\Leftarrow$ Zero correlation

# Asymptotic analog of omitted variable bias

**TABLE 3.2 Summary of Bias in $\tilde{\beta}_1$ When $x_2$ Is Omitted in Estimating Equation**

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u$$

| | $\text{Corr}(x_1, x_2) > 0$ | $\text{Corr}(x_1, x_2) < 0$ |
|---|---|---|
| $\beta_2 > 0$ | Positive bias | Negative bias |
| $\beta_2 < 0$ | Negative bias | Positive bias |

- There is no omitted variable bias $\beta_2\delta_1$ if the omitted variable is irrelevant or uncorrelated with the included variable

- Deriving the **inconsistency**: $\text{plim}\,\tilde{\beta}_1 = \beta_1 + \beta_2\delta_1,$ where, $\delta_1 = \text{Cov}(x_1, x_2)/\text{Var}(x_1)$

  - For practical purposes, we can view the inconsistency as being the same as the bias.

  For example if $\beta_2 > 0$ and $x_1$ *and* $x_2$ are positively correlated, then the inconsistency in $\widetilde{\beta_1}$ is positive **(asymptotic positive bias)**

  - The difference is that the inconsistency is expressed in terms of the population variance of $x_1$ and the population covariance between $x_1$ and $x_2$, while the bias is based on their sample counterparts

# Asymptotic normality and large sample inference

- In practice, the normality assumption MLR.6 is often **questionable**
- If MLR.6 does not hold, the results of t- or F-tests may be wrong
- Fortunately, F- and t-tests still work if the sample size is large enough
- Also, OLS estimates are normal in large samples even without MLR.6 (this is due to the [central limit theorem](central limit theorem))

# Theorem 5.2 (Asymptotic normality of OLS)

Under assumptions MLR.1 – MLR.5:

$$\frac{(\hat{\beta}_j - \beta_j)}{se(\hat{\beta}_j)} \overset{a}{\sim} \text{Normal}(0, 1)$$

In large samples, the standardized estimates are normally distributed

also $$plim\ \hat{\sigma}^2 = \sigma^2$$

# Practical consequences of Asymptotic normality

- In large samples, **the t-distribution** is close to the **Normal(0,1)** distribution
- As a consequence, t-tests are valid in **large samples without MLR.6**
- The same is true for **confidence intervals** and **F-tests**
- Important: MLR.1 – MLR.5 are still necessary, esp. Homoskedasticity

## Asymptotic analysis of the OLS sampling errors

This is why large samples are better!

$$\widehat{Var}(\hat{\beta}_j) = \frac{\hat{\sigma}^2}{SST_j(1 - R_j^2)}$$

Converges to $\sigma^2$

$n \cdot Var(x_j)$

$\widehat{Var}(\hat{\beta}_j)$ shrinks with the rate $1/n$

$se(\hat{\beta}_j)$ shrinks with the rate $\sqrt{1/n}$