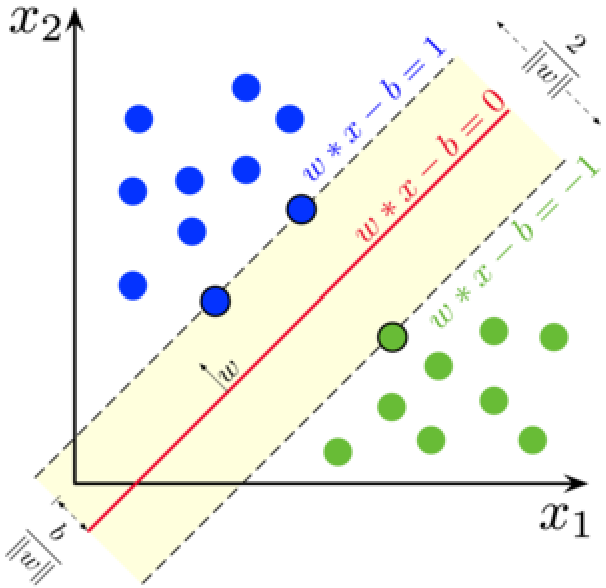
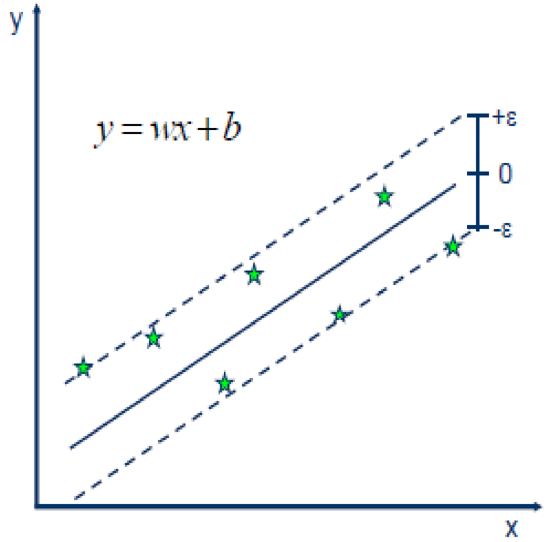
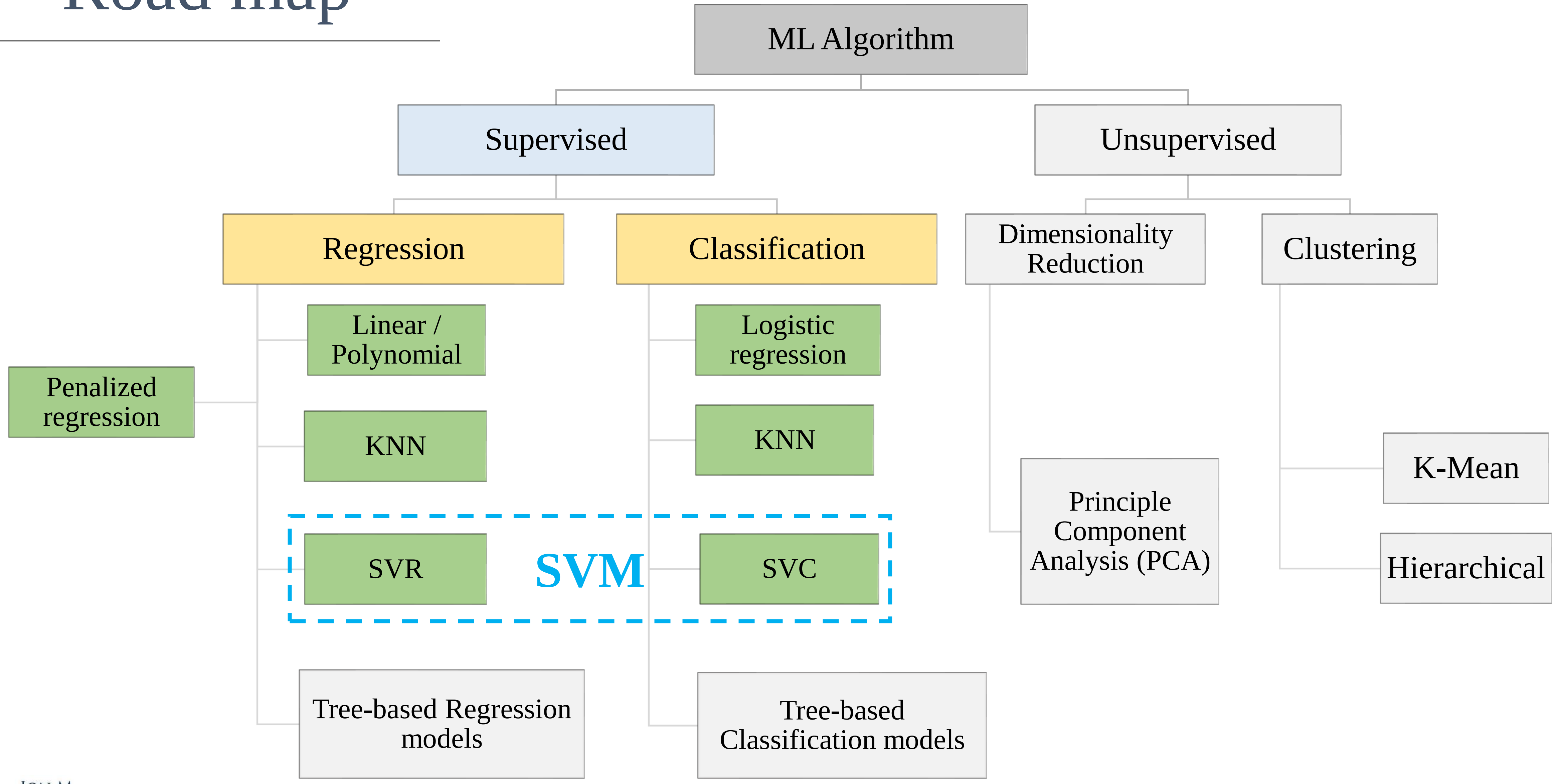
Class 14 –15 Support Vector Machines SVM

Prof. Pedram Jahangiry





Road map


ML Algorithm
Supervised
Unsupervised
Regression
Classification
Dimensionality Reduction
Clustering
Linear / Polynomial
Penalized regression
KNN
SVR
SVM
Tree-based Regression models
Logistic regression
KNN
SVC
Tree-based Classification models
Principle Component Analysis (PCA)
K-Mean
Hierarchical

Topics

Part I

1. SVM Geometry
2. SVM Motivation

Part II

1. Maximum Margin Classifier (MMC)
2. Support Vector Classifiers (SVC)
3. Support Vector Machines (SVM)

Part III

1. Support Vector Regressors (SVR)

Part IV

1. Tuning Hyperparameters
2. SVM pros and cons
3. SVM applications in Finance

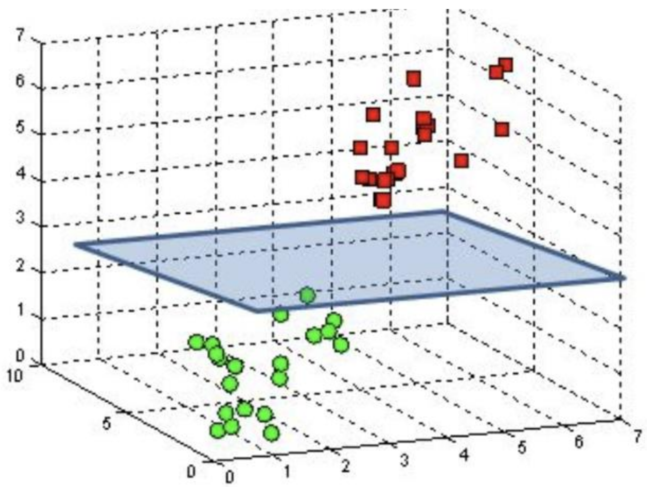
Part I

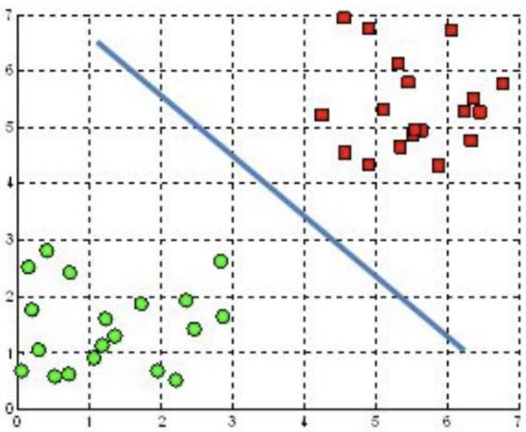
Geometry

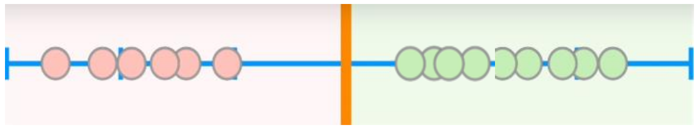
SVM Motivation

SVM Geometry

- In geometry, a hyperplane is a subspace whose dimension is one less than that of its ambient space. A hyperplane separates the space into two spaces.
- If a space is 3-dimensional then its hyperplanes are the 2-dimensional planes,
- If the space is 2-dimensional, its hyperplanes are the 1-dimensional lines.
- If the space is 1-dimensional, its hyperplanes are single points.





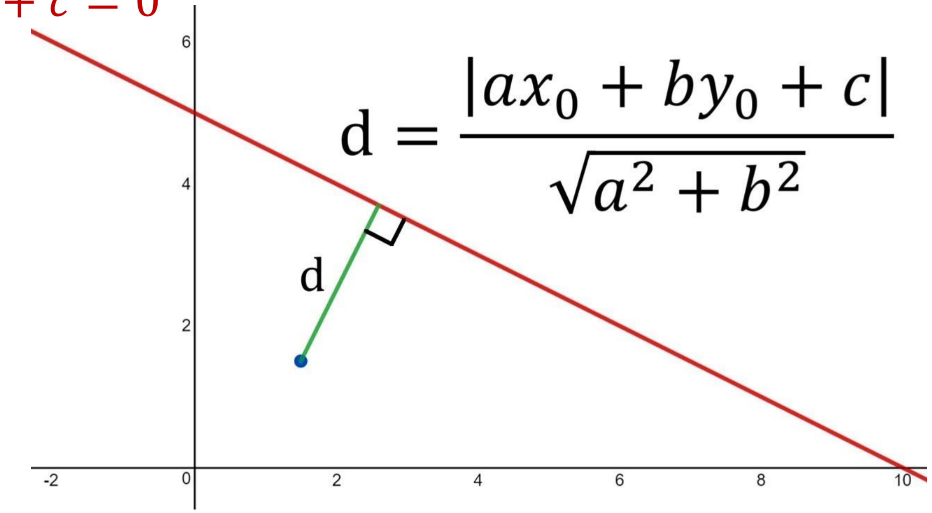


Geometry

- The perpendicular distance between two objects is the distance from one to the other, measured along a line that is perpendicular to one or both.
- The distance between a point (x_0, y_0) and a line parameterized by $ax + by + c = 0$ is equal to:

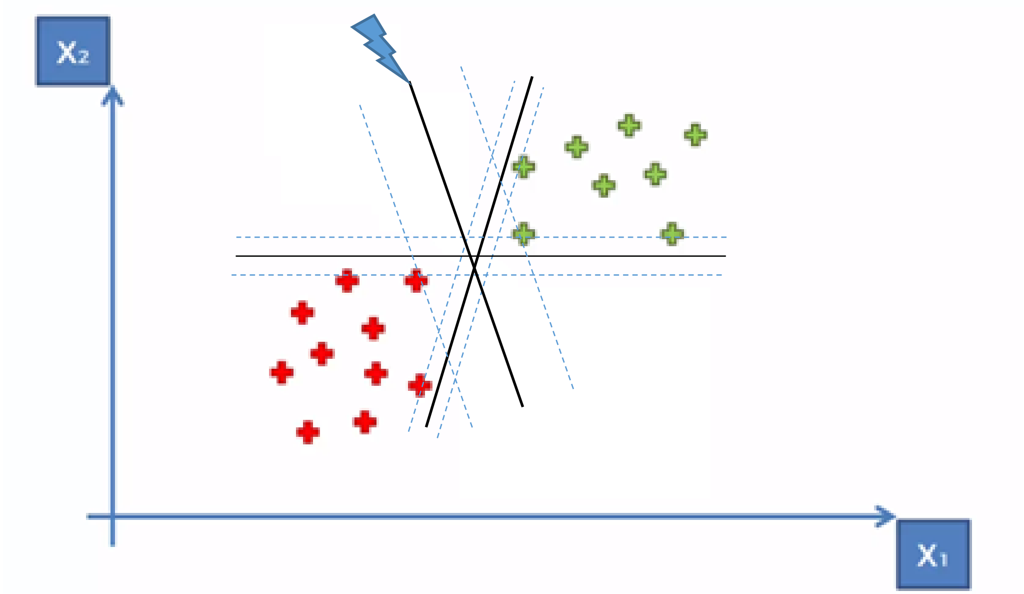
$ax + by + c = 0$

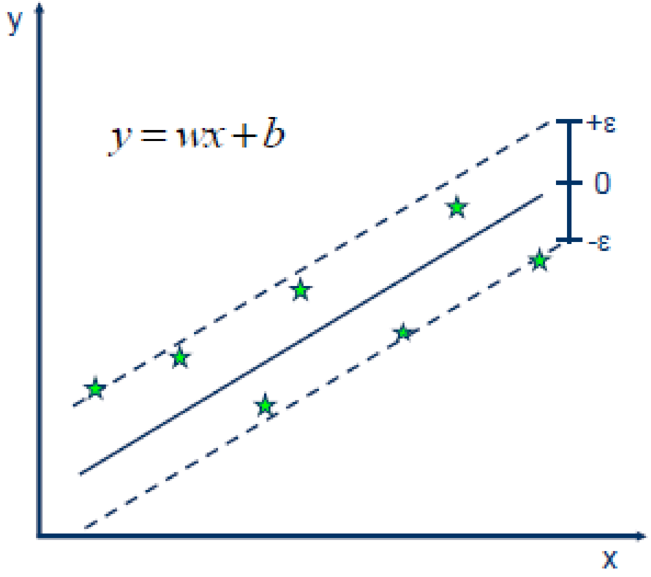
$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$



SVM Motivation

Support vector machine (SVM) is one of the most popular algorithms in machine learning. It is a powerful supervised algorithm used for classification and regression.





Part II

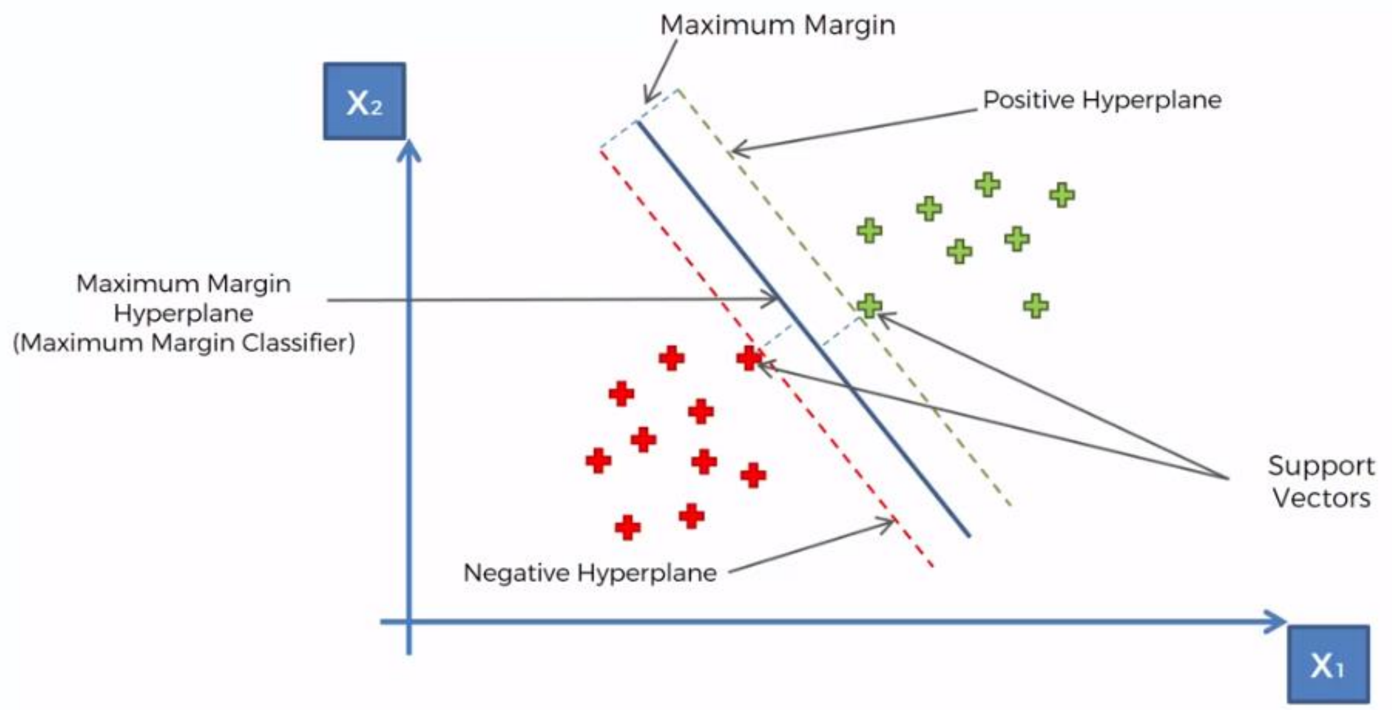
Maximum Margin Classifier (MMC)

Support Vector Classifiers (SVC)

Support Vector Machines (SVM)

Maximum Margin Classifier (MMC) – Hard Margin

MMC is the hyperplane that among all separating hyperplanes, find the one that makes the biggest gap (margin) between two classes.



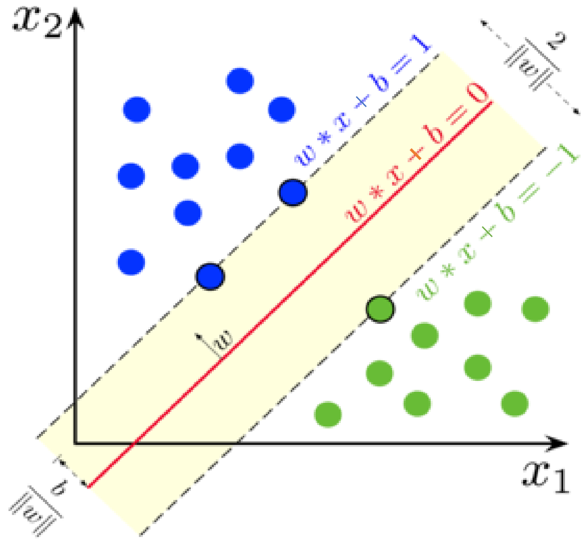
MMC optimization problem

- The core idea of hard margin is to maximize the margin, under the constraint that the classifier does not make any mistake.
- SVMs try to pick the most robust model (by finding the w^* and b^*) among all those that yield a correct classification. If we numerically define blue circles as +1 and green circles as -1, any **good** linear model is expected to satisfy:

$$\begin{cases} \sum_{k=1}^{K} w_k x_{i,k} + b \geq +1 & \text{when } y_i = +1 \\ \sum_{k=1}^{K} w_k x_{i,k} + b \leq -1 & \text{when } y_i = -1 \end{cases}$$

$$\underset{w,b}{\text{Min}} \ \frac{1}{2} \|w\|^2$$

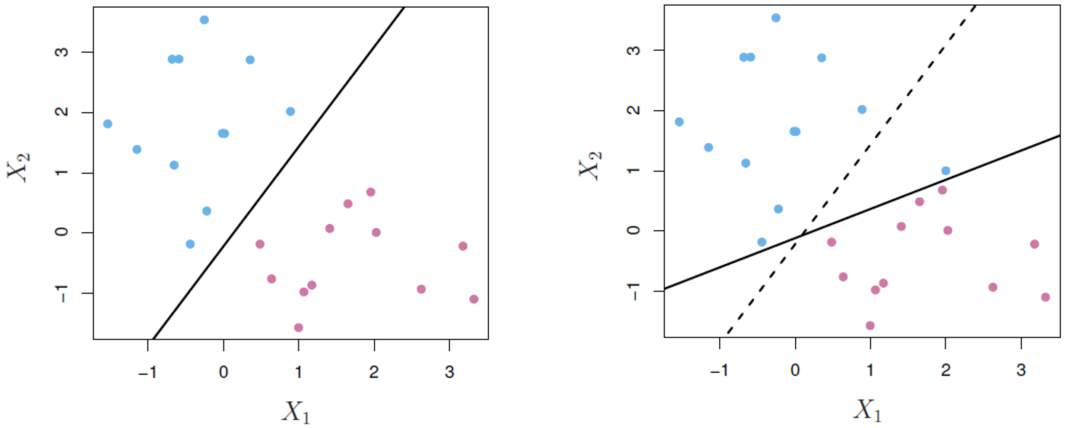
$$s.t.\ y_i \left(\sum_{k=1}^{K} w_k x_{i,k} + b \right) \geq 1$$



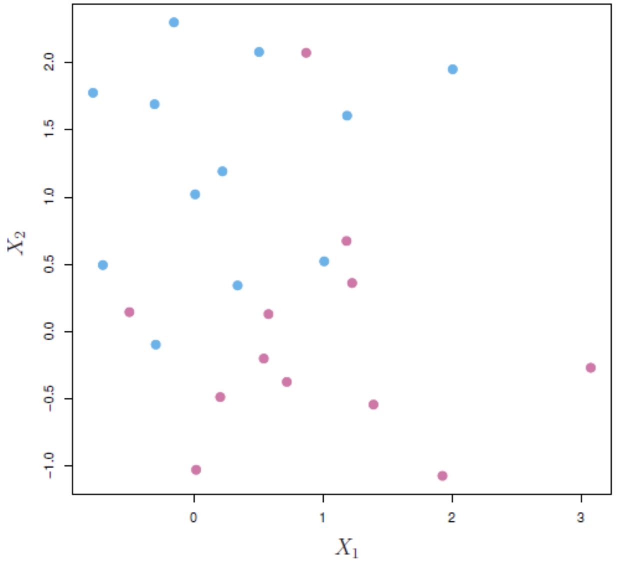
Support Vector Classifier (SVC) – Soft Margin

- The MMC optimization problem becomes infeasible whenever the condition cannot be satisfied, that is, when a simple line cannot perfectly separate the labels, no matter the choice of coefficients.
- This happens when:

1- The data is noisy
MMC is very sensitive to outliers

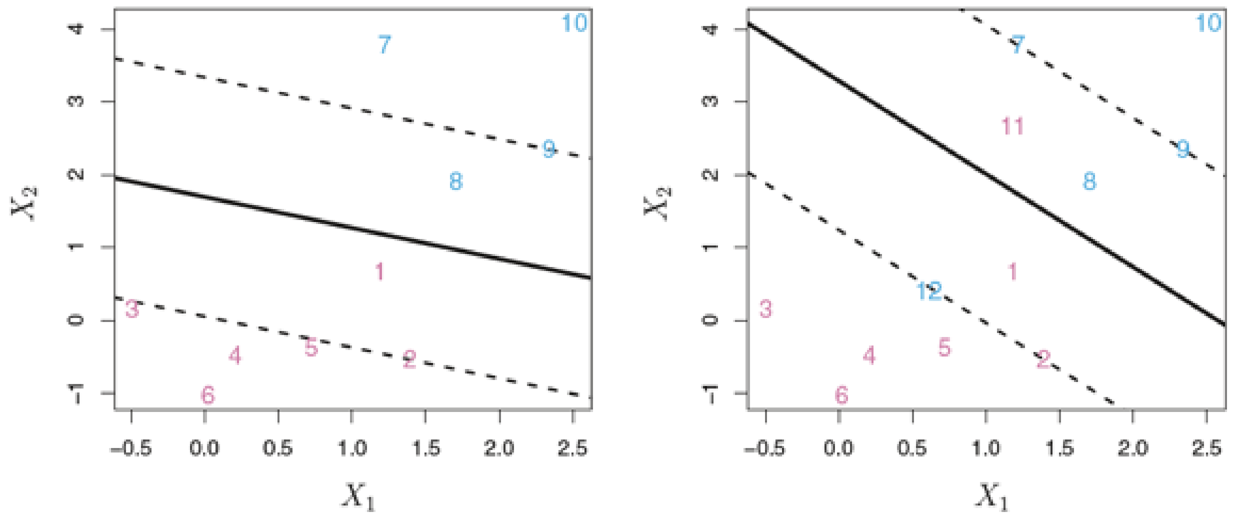


2- The data is non-separable (overlap)



Support Vector Classifier (SVC) – Soft Margin

- **Solution**: we can extend the concept of a separating hyperplane in order to develop a hyperplane that **almost** separates the classes, using a so-called **soft margin**.
- The generalization of the maximal margin classifier using soft margin is known as the support vector classifier (SVC).
- It could be worthwhile to misclassify a few training observations in order to do a better job in classifying the remaining observations.



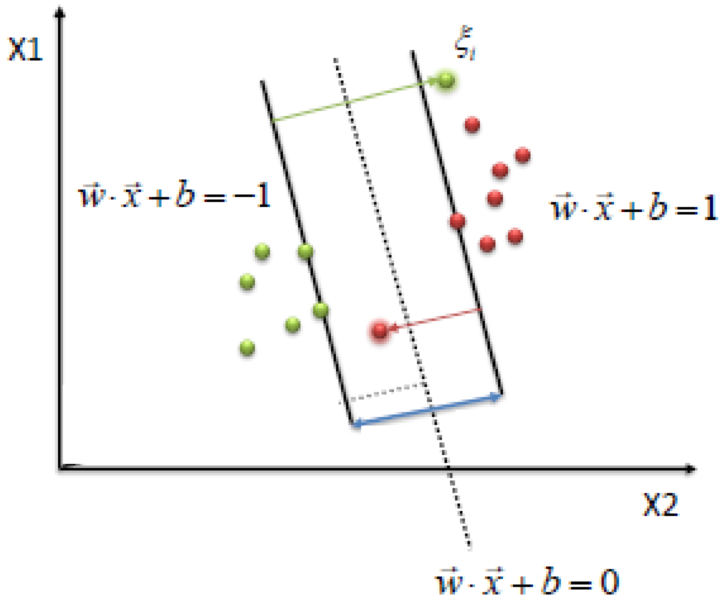
SVC optimization problem

- Soft margin classification adds a penalty (C) to the objective function for observations in the training set that are misclassified. In essence, the SVM algorithm will choose a decision boundary that optimizes the trade-off between a wider margin and a lower total error penalty.
- Slack variable ξ_i allow some observations to fall on the wrong side of the margin, but will penalized them by parameter C: Cost of misclassification

$$\begin{cases} \sum_{k=1}^{K} w_k x_{i,k} + b \geq +1 - \xi_i & \text{when } y_i = +1 \\ \sum_{k=1}^{K} w_k x_{i,k} + b \leq -1 + \xi_i & \text{when } y_i = -1 \end{cases}$$

$$\underset{w,b}{\text{Min}} \ \frac{1}{2} \|w\|^2 + C \sum_{i=1}^{I} \xi_i$$

$$s.t. \ \ y_i \left(\sum_{k=1}^{K} w_k x_{i,k} + b \right) \geq 1 - \xi_i, \qquad \xi_i \geq 0 \ \ \forall_i$$

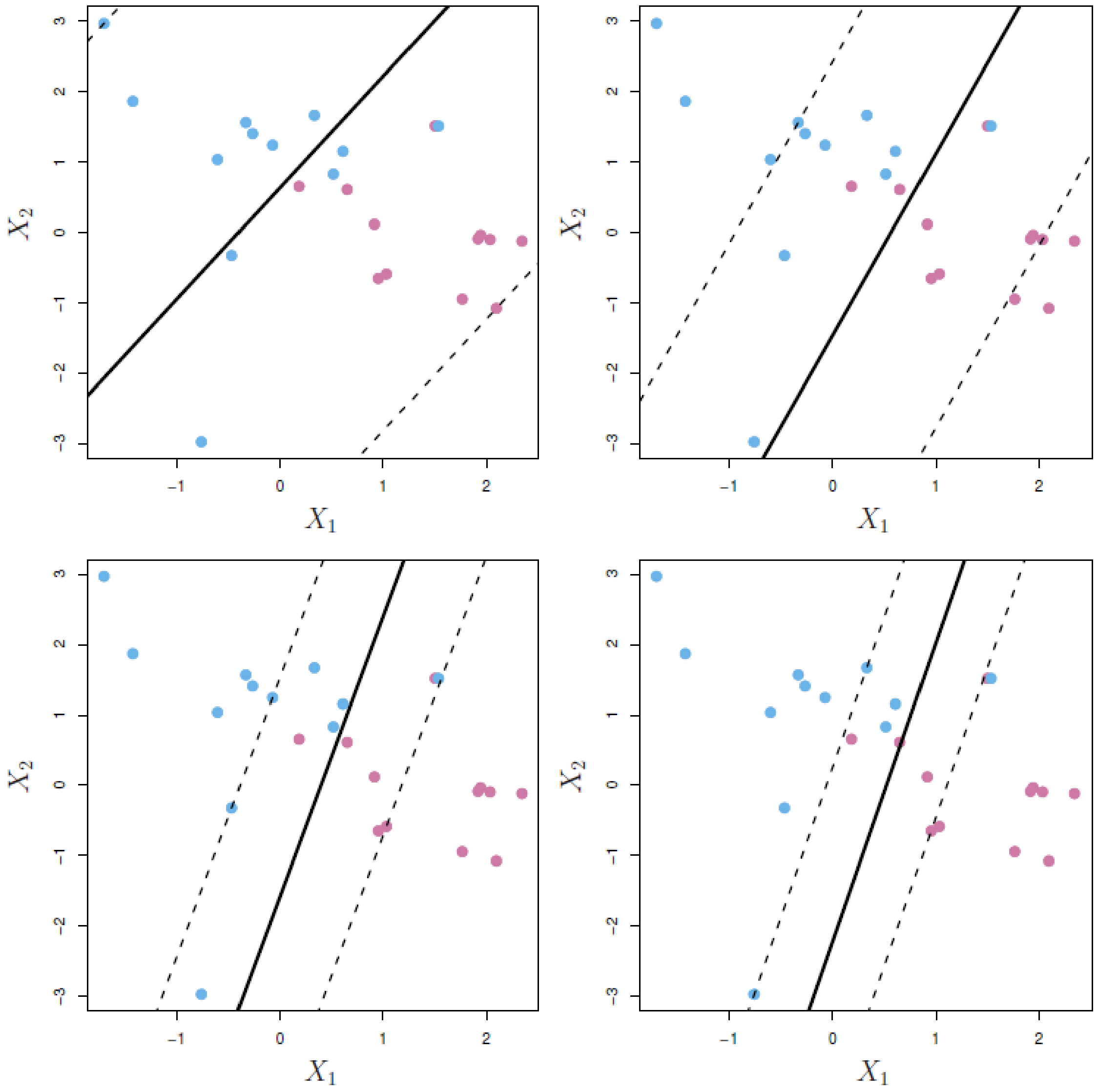


Regularization parameter

C: Cost of misclassification

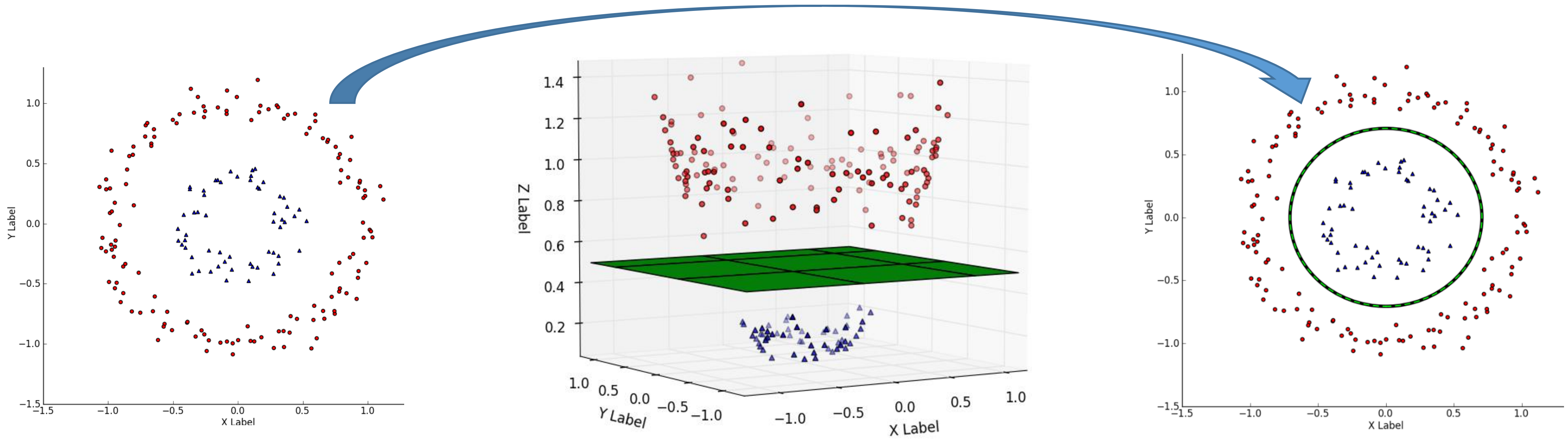
Small C :
wide margin : high bias : low variance

Large C:
narrow margin : low bias : High variance



Kernel Trick!

- Non-linearly separable data: sometime a linear boundary simply won't work, no matter what value of C.
- We need a non-linear decision boundary!
- Mapping to higher dimensional space, finding the hyper plane and projecting it back to low dimensional space can be computationally expensive.
- Solution: Kernel Trick!



Support Vector Machines (SVM)

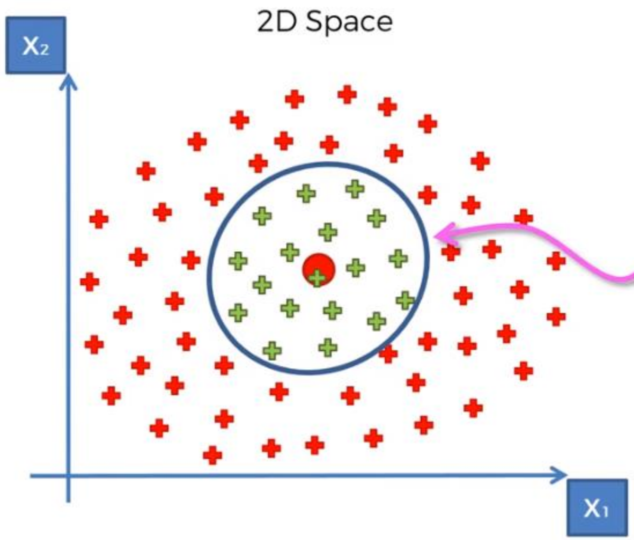
- SVM generalizes the SVC to a nonlinear model, via the kernel ϕ which is applied to the input points $x_{i,k}$.
- The Kernel $\phi\left(x_{i,k}\right)$ is a function that quantifies the similarities between observations by summarizes the relationship between every single pairs in the training set.

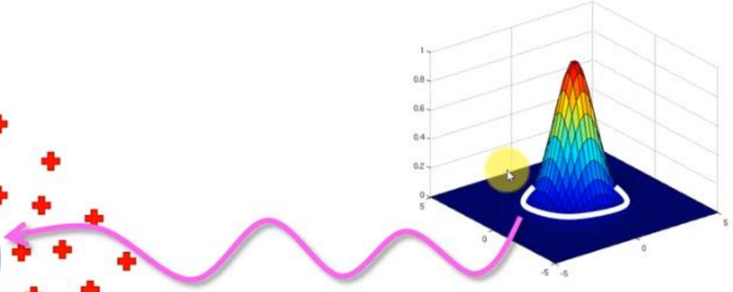
SVC + Non-linear Kernel = SVM

$$\underset{w,b}{\text{Min}} \ \frac{1}{2}\left|\left|w\right|\right|^2 + C\sum_{i=1}^{I}\xi_i$$

$$s.t. \ \ y_i\left(\sum_{k=1}^{K} w_k\phi(x_{i,k}) + b\right) \geq 1-\xi_i, \qquad \xi_i \geq 0 \ \ \forall_i$$

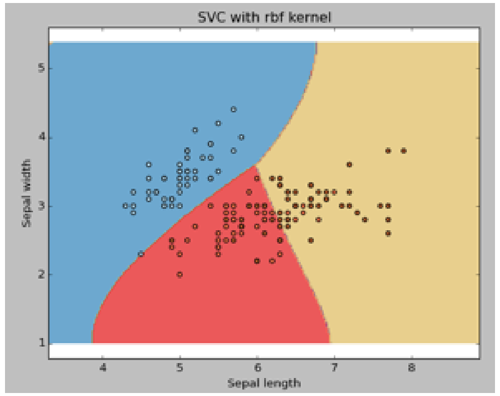
The Gaussian RBF Kernel (Radial Basis Function)



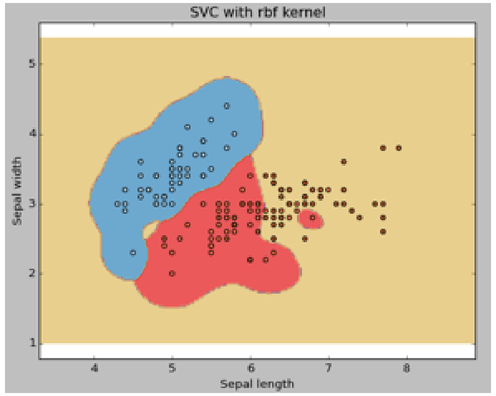


$$K\left(\vec{x}, \vec{l}^{i}\right) = e^{-\frac{\left\|\vec{x}-\vec{l}^{i}\right\|^{2}}{2\sigma^{2}}} \quad , \qquad \gamma = \frac{1}{2\sigma^{2}}$$

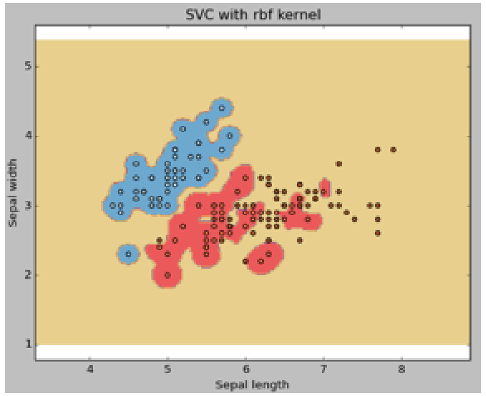
gamma =0



gamma =10

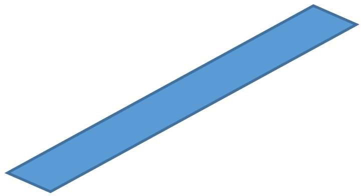


gamma =100

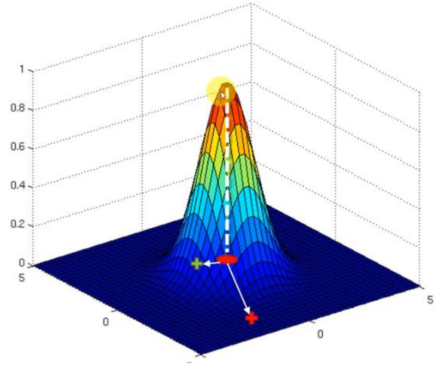


Most common types of Kernel

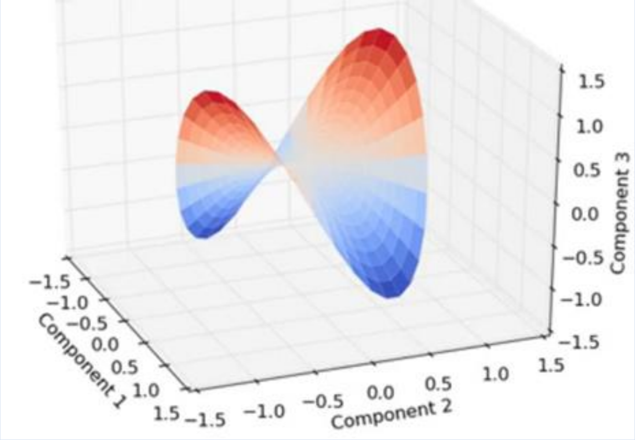
Linear Kernel



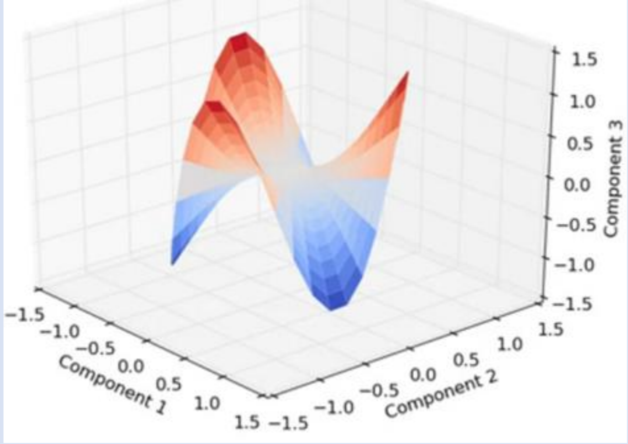
The Gaussian RBF Kernel



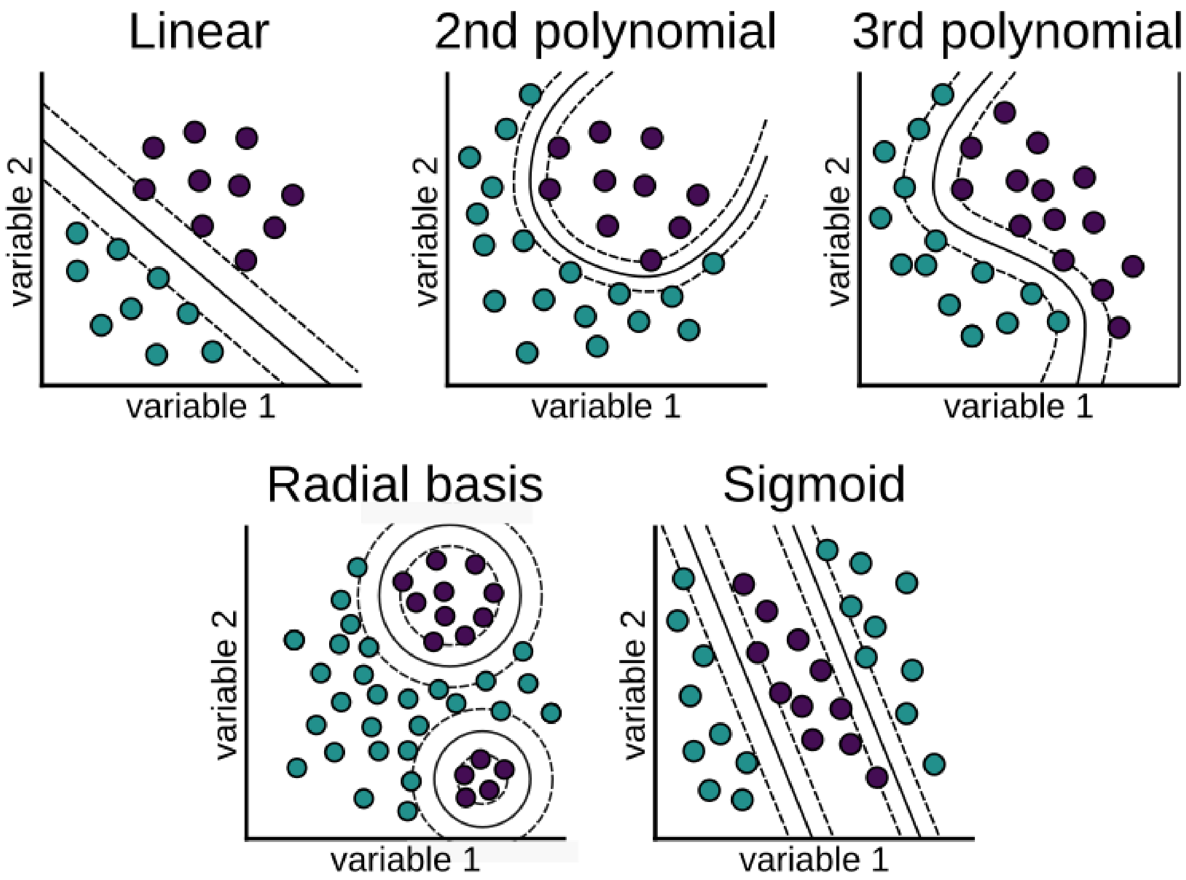
Polynomial Kernel



Sigmoid Kernel



Decision boundaries with different Kernels



Source: [Machine learning with R, tidyverse, and mlr]

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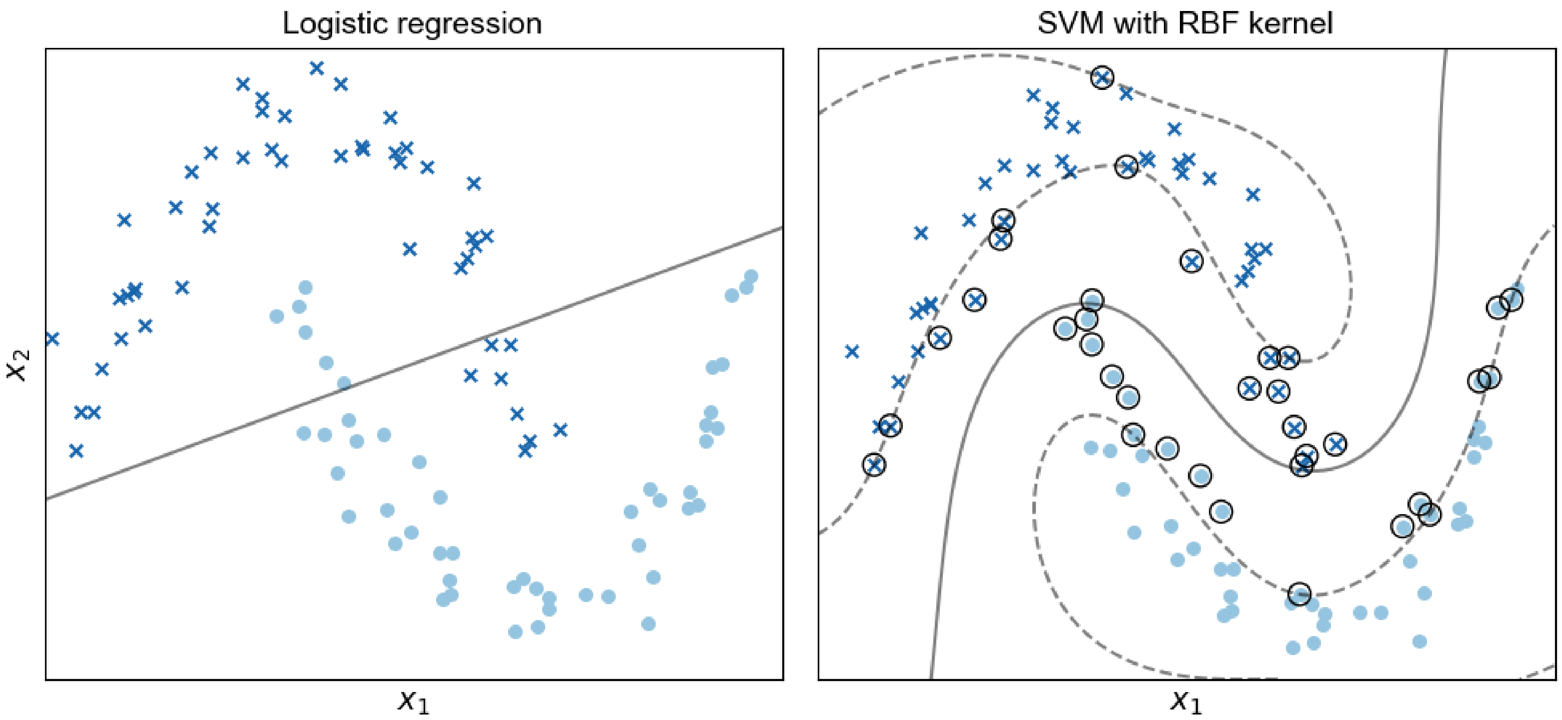
Comparing Logistic Regression and SVM



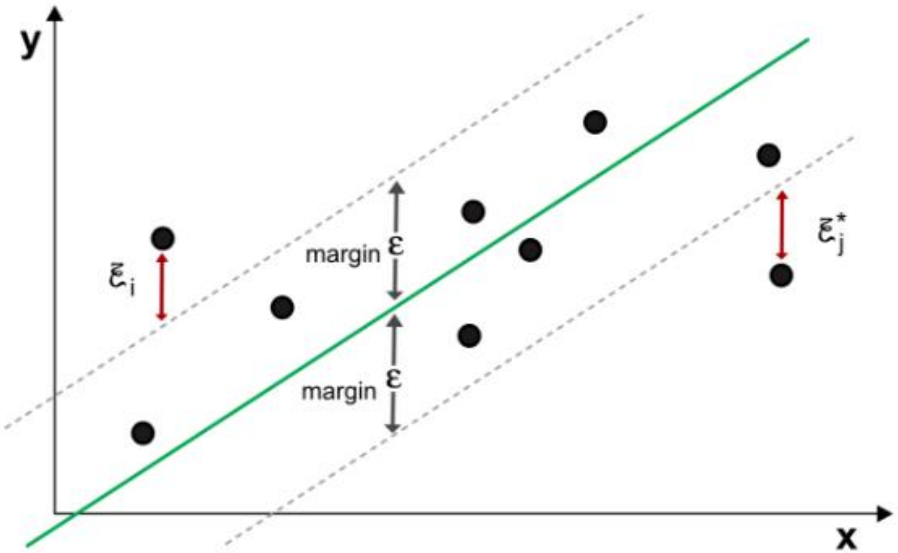
Source: http://gregorygundersen.com/blog/2019/12/23/random-fourier-features/

Part III

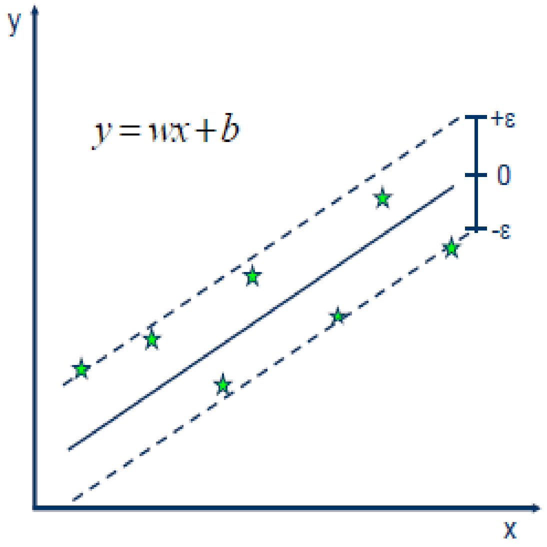
Support Vector Regressors (SVR)

SVM for regression (Support Vector Regressors)

- The idea of SVM classification can be transposed to regression problems.
- However, the role of the margin is different. Our objective, is to basically find the hyperplane that holds maximum training observations within the margin ϵ (tolerance level).



SVR optimization

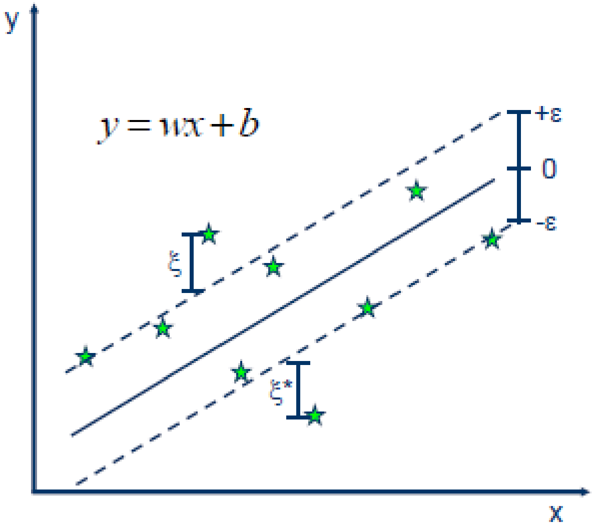


- Minimize:

$$\min \frac{1}{2}\|w\|^2$$

- Constraints:

$$y_i - wx_i - b \le \varepsilon$$
$$wx_i + b - y_i \le \varepsilon$$



- Minimize:

$$\frac{1}{2}\|w\|^2 + C\sum_{i=1}^{N}\left(\xi_i + \xi_i^*\right)$$

- Constraints:

$$y_i - wx_i - b \le \varepsilon + \xi_i$$
$$wx_i + b - y_i \le \varepsilon + \xi_i^*$$
$$\xi_i, \xi_i^* \ge 0$$

Source: https://www.saedsayad.com/support_vector_machine_reg.htm

Kernel SVR optimization

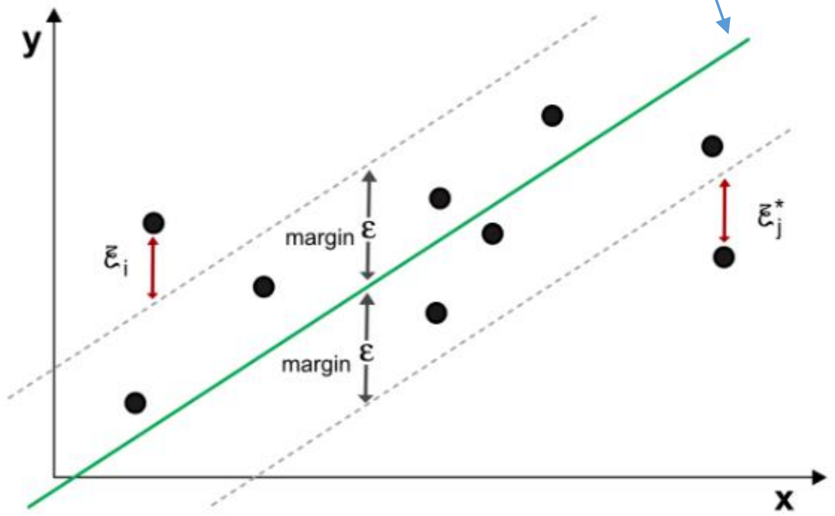
Predictions: $\sum_{k=1}^{K} w_k \phi(x_{i,k}) + b$

$$\underset{w,b}{\text{Min}} \;\; \frac{1}{2}\,||w||^2 \;+ C\sum_{i=1}^{I}(\xi_i + \xi_i^*)$$

$$\left(\sum_{k=1}^{K} w_k \phi(x_{i,k}) + b\right) - y_i \le \epsilon + \xi_i^*$$

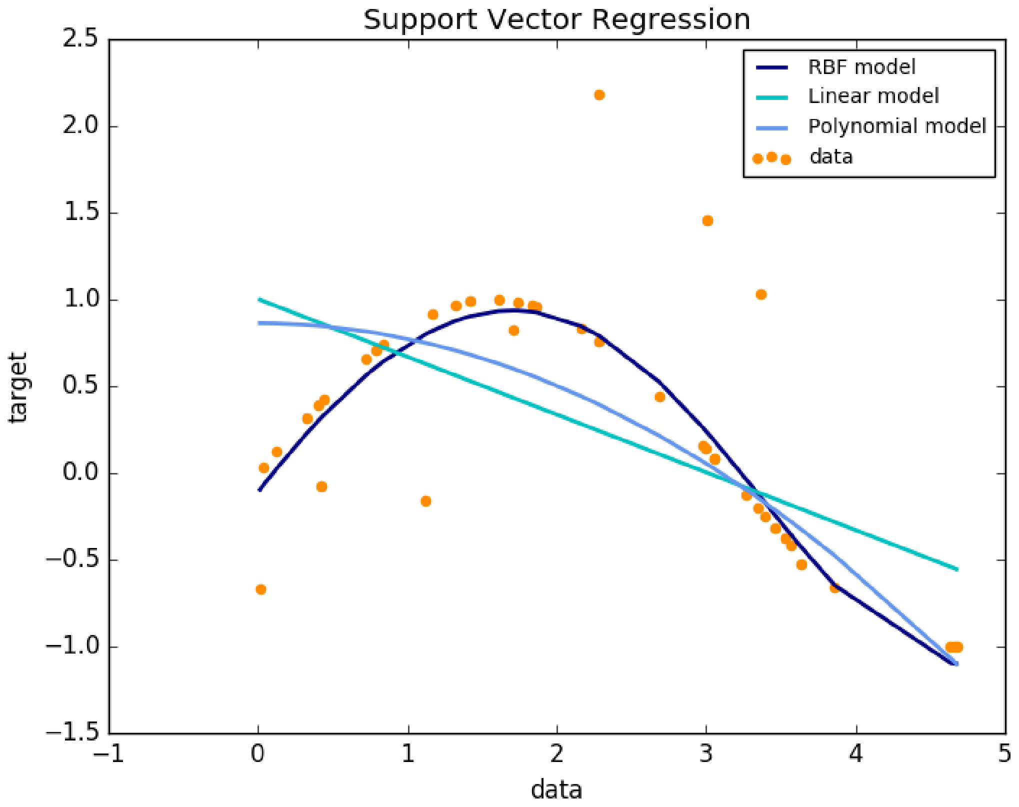
$$y_i - \left(\sum_{k=1}^{K} w_k \phi(x_{i,k}) + b\right) \le \epsilon + \xi_i$$

$$\xi_i, \xi_i^* \ge 0 \;\; \forall_i$$



- Goal: minimize the sum of squared weights subject to the error being small enough
- This is somewhat the opposite of the penalized linear regressions which seek to minimize the error, subject to the weights being small enough

SVR using Linear and Non-linear Kernels



Source: Scikit learn documentation

Part IV

Tuning hyperparameters

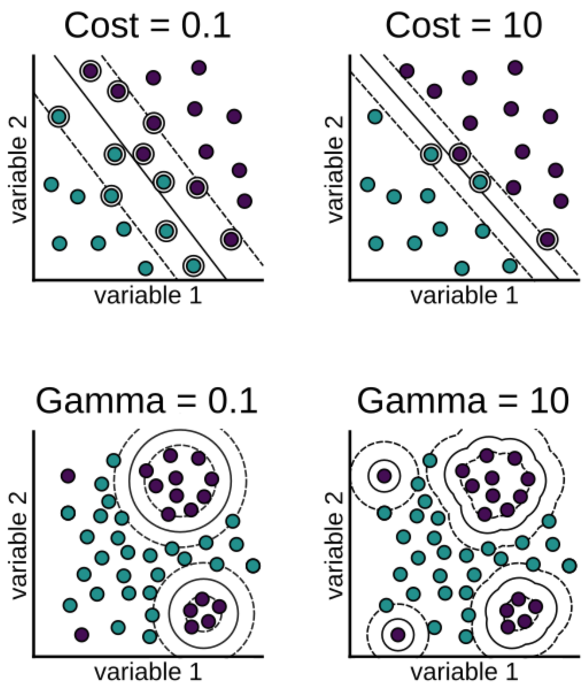
SVM pros and cons

SVM applications in Finance

Tuning hyperparameters

SVM hyperparameters:

1) C, Cost of misclassification: controls bias variance trade off
2) Kernel
3) Gamma, controls how far the influence of a single training set reaches

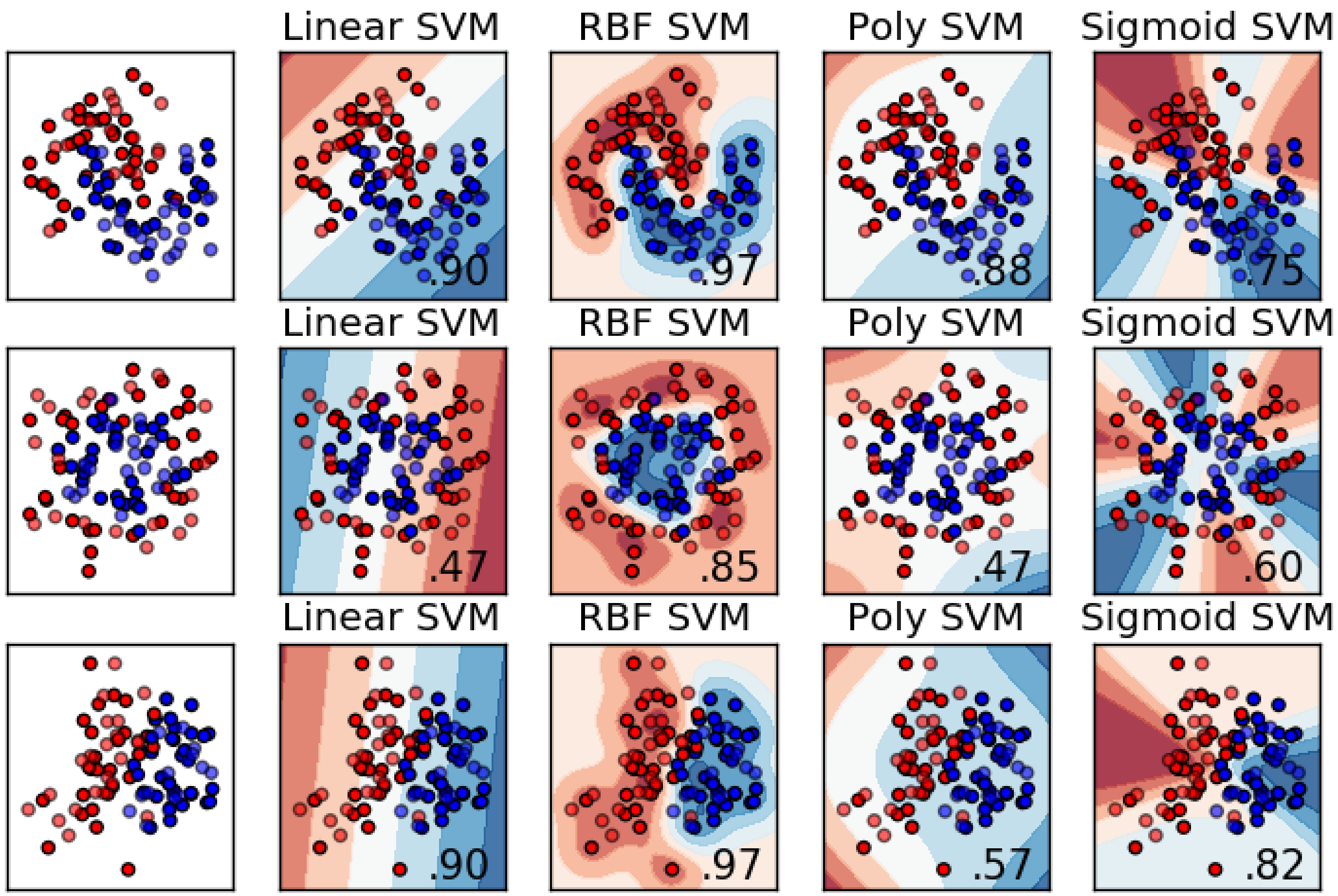


Source: Machine learning with R, tidyverse, and mlr

Grid search cross validation is used to tune the hyper parameters.

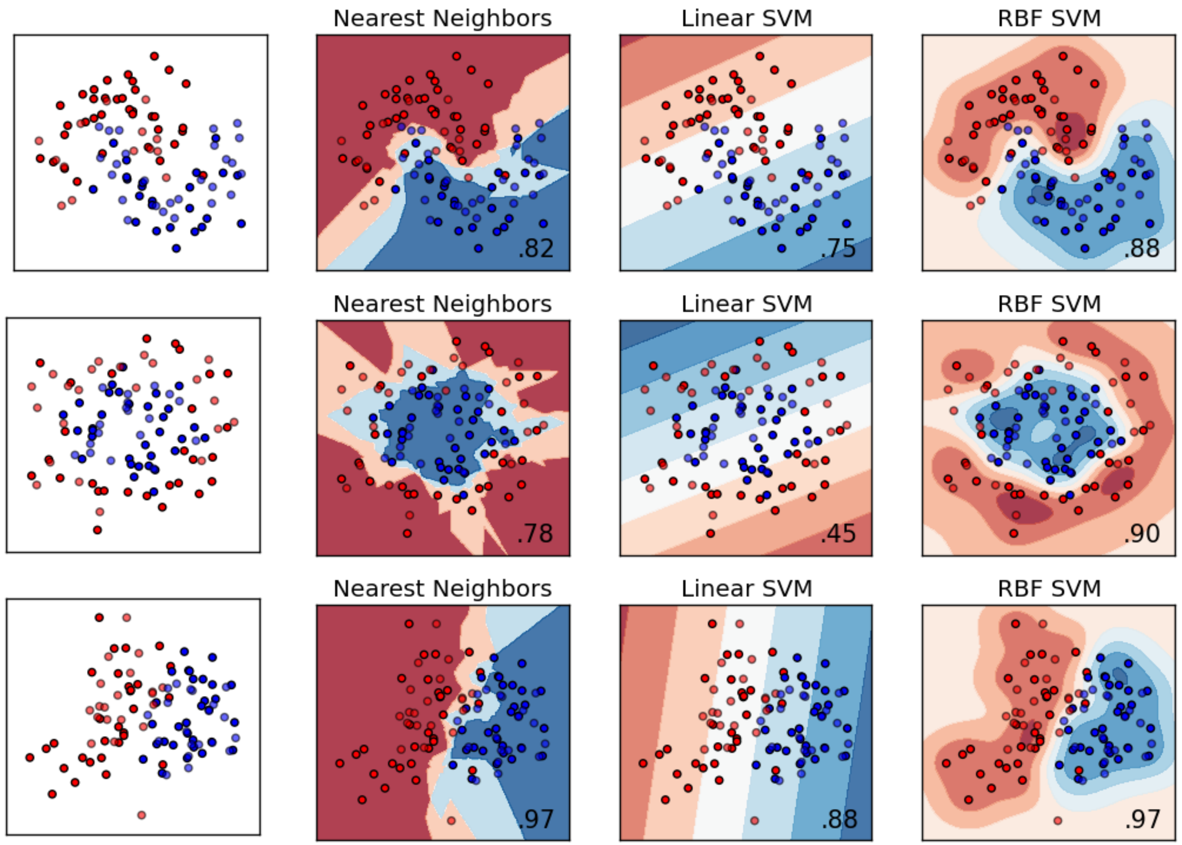
Kernel	C	Gamma	CV
Linear, rbf, poly, ….	0.1, 1, 10, 100, …	0.001, 0.01, 0.1, 1, …	5,10,…

Decision boundaries with different Kernels



Source https://www.kaggle.com/residentmario/kernels-and-support-vector-machine-regularization

Comparing classifiers (so far)

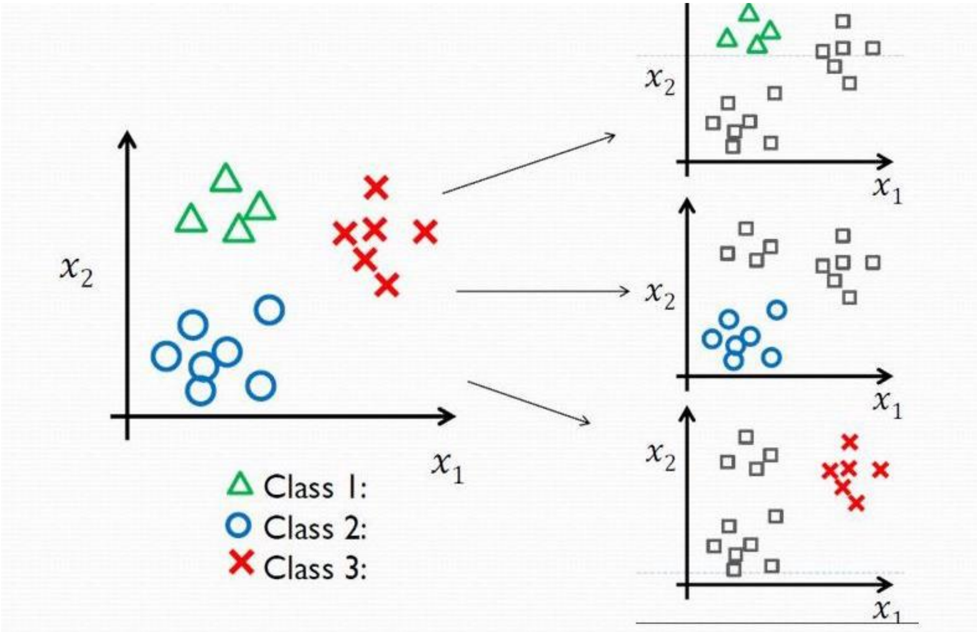


Source: https://scikit-learn.org/stable/auto_examples/classification/plot_classifier_comparison.html

K-Multiple class SVM

- One-VS-All (OVA)

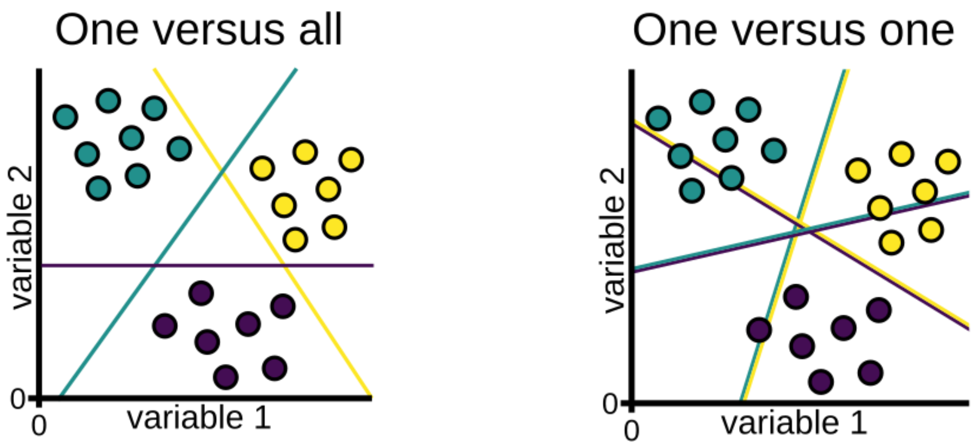
1. Fit K different 2-class SVM classifiers $\widehat{f_k}(x)$, each class versus the rest
2. Classify x_{te} to the class for which $\widehat{f_k}(x_{te})$ is largest.



K-Multiple class SVM

- One-VS-One (OVO)

1. Fit all $\binom{K}{2}$ pairwise classifiers $\widehat{f_{kl}}(x)$, each class versus the rest
2. Classify x_{te} to the class that wins the most pairwise competitions.



SVM’s Pros and Cons

Pros:

- SVM can be memory efficient! uses only a subset of the training data (support vectors)
- Can handle non-linear data sets
- Can handle high dimensional spaces (even when D>N)
- Used both for classification and regression
- SVM are not very sensitive to overfitting (soft margin; regularization)
- Can have high accuracy (even compared to NN)

Cons:

- No probability outcome!
- Long training time when we have large data sets.
- Limited interpretability (specially for Kernel SVM)
- Does not perform well with noisy data
- Suited for small to medium size data

SVM's Applications in finance

- Corporate financial statements and bankruptcy (high dimensional)
- Identifying stressed companies to short sell (using many fundamental and technical features)
- Sentiment analysis (classify text from documents e.g., news articles, company announcements, and company annual reports into useful categories for investors)
- Money laundering analysis and spam detection
- Loan management

Appendix

MMC solution

$$\underset{\mathbf{w},b}{\operatorname{argmin}} \frac{1}{2}||\mathbf{w}||^2 \text{ s.t. } y_i\left(\sum_{k=1}^{K} w_k x_{i,k} + b\right) \geq 1$$

$$L(\mathbf{w}, b, \boldsymbol{\lambda}) = \frac{1}{2}||\mathbf{w}||^2 + \sum_{i=1}^{I} \lambda_i \left(y_i \left(\sum_{k=1}^{K} w_k x_{i,k} + b \right) - 1 \right)$$

$$\frac{\partial L}{\partial \mathbf{w}} L(\mathbf{w}, b, \boldsymbol{\lambda}) = \mathbf{0}, \quad \frac{\partial L}{\partial b} L(\mathbf{w}, b, \boldsymbol{\lambda}) = 0,$$

$$\mathbf{w}^* = \sum_{i=1}^{I} \lambda_i u_i \mathbf{x}_i.$$


thank you all
CLASS DISMISSED