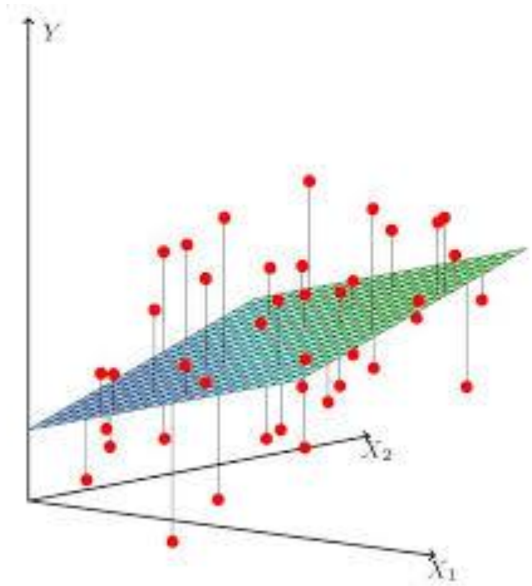
Module 3 – Linear Regression Models

Econometrics Approach



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Class Modules

- Module 1- Introduction to Deep Learning
- Module 2- Setting up Machine Learning Environment
- **Module 3- Linear Regression (Econometrics approach)**
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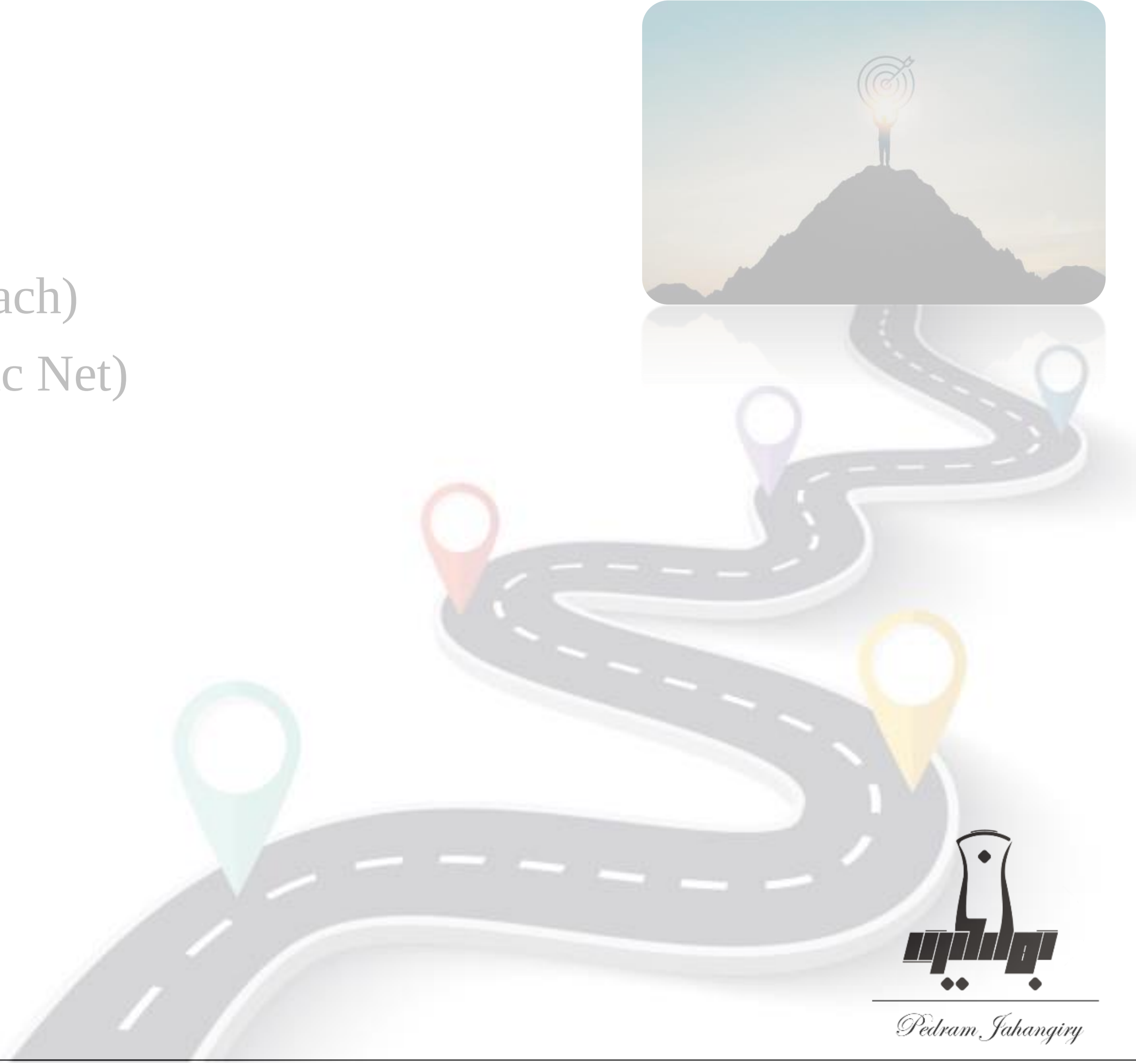

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What is Econometrics?

- Econometrics is the branch of economics that develops and uses statistical methods for estimating economic relationships

- Typical goas of econometrics analysis are:
  - Estimating relationships between random variables
  - Testing hypothesis
  - Predicting / Forecasting random variables

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Steps in Econometrics analysis

1) Specifying the regression model
2) Collecting data
3) Quantify the model

Example:

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The Multiple Regression Model (MRM)

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ \ldots + \beta_k x_k + u$$

- In econometrics language, β_s are the coefficients and u is the error term.

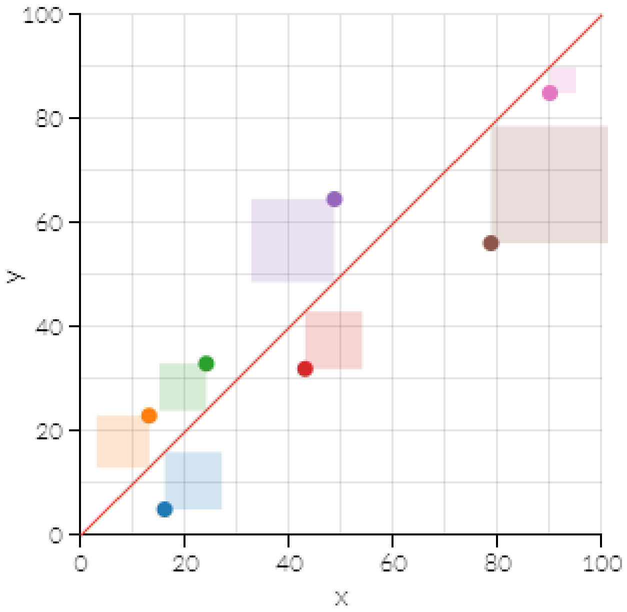
Y	*X*
Dependent variable	Independent variable
Explained variable	Explanatory variable
Response variable	Control variable
Predicted variable	Predictor variable
Regressand	Regressor

- How to estimate the coefficients? It's all about the error term u

Estimating the model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ \ldots + \beta_k x_k + u$$

- OLS (Ordinary Least Squared) is one way to estimate the coefficients.
- In order to estimate this model (finding out the β_s) , we need to make some assumptions.



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Interpreting the model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ \dots + \beta_k x_k + u$$

- Interpreting β_j

  if x_j increases by 1 unit, holding everything else constant, on average, y will increase by β_j units.

- In order to interpret this model (interpreting the β_s) , we need to make some assumptions.

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Correlation vs Causation

- Correlation refers to the linear relationship between two variables, and how they change together.
- Causation refers to the cause and effect, where the one event is a result of another event .
- Regression analysis cannot prove causality!
- Given some assumptions, we hope to get causality with statistical significancy.
- What are these assumptions?

The Gauss-Markov assumptions

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The Gauss-Markov assumptions (cont'd)

- Assumption 1: Linearity in parameters
- Assumption 2: Random Sampling
- Examples:

The Gauss-Markov assumptions (cont'd)

- Assumption 3: No perfect collinearity and $var(x) \neq 0$
- Examples:

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The Gauss-Markov assumptions (cont'd)

- Assumption 4: Zero Conditional Mean. Given any values of X, the errors are on average zero (conditional expectation). $E(u|X) \neq 0$
- Endogeneity violates this assumption! $Corr(X,u) \neq 0$
- Examples:

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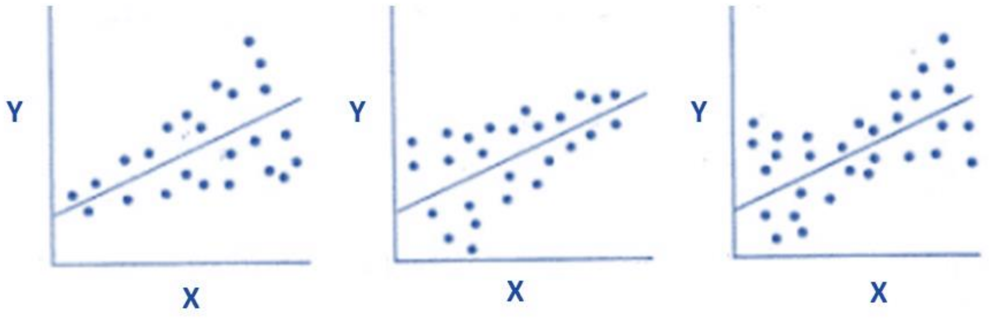
The Gauss-Markov assumptions (cont'd)

- Assumption 5: Homoskedasticity (same conditional variance) : $var(u|X) \neq \sigma^2$
- Examples:



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The Gauss-Markov assumptions (summary)

OLS estimators are unbiased

1) Assumption 1: Linearity in parameters
2) Assumption 2: Random Sampling
3) Assumption 3: No perfect collinearity and $var(x) \neq 0$
4) Assumption 4: Zero Conditional Mean
5) Assumption 5: Homoskedasticity

There is a formula for variance of OLS estimators

The scope of our course!

- If any of the Gauss-Markov assumptions are not met, it is important to exercise caution when interpreting the results of the econometric model, as the model's predictions and parameter estimates may be unreliable.
- Statistical tests and tools can be used to verify any of these assumptions, but it's beyond the scope of this course.

Statistical Inference
Hypothesis Testing

(quick review)

Statistical inference in the regression model

- So far, given the GMA, we know something about the expected value and the variance of OLS estimators. What about its distribution?
- We need one more assumption! Oh no!
- Assumption 6: the error terms are normally distributed, $u \sim N(0\,, \sigma^2)$
- Assumptions 1 through 6 is called, Classical Linear Model (CLM) assumptions.
- Good news: if the sample size is large enough, we can relax the normality assumption (because of central limit theorem)

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Theorem: t distribution for the estimators

- Under the CLM assumptions,

$$\frac{\hat{\beta}_j - \beta_j}{se(\hat{\beta}_j)} \sim t_{n-k-1}$$

- Now we can do hypothesis testing! Yay!
- Review hypothesis testing if needed.

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Evaluation Metrics

- Now let's focus on the performance aspect of linear regression models:

$$R^2 = 1 - \frac{\sum(y_i - \hat{y})^2}{\sum(y_i - \bar{y})^2} = 1 - \frac{SS_{residuals}}{SS_{total}}$$

$$Adjusted\ R^2 = 1 - (1 - R^2) * \frac{n - 1}{n - k - 1}$$

$$MAE = \frac{1}{N}\sum_{i=1}^{N} |y_i - \hat{y}|$$

$$MSE = \frac{1}{N}\sum_{i=1}^{N} (y_i - \hat{y})^2$$

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} (y_i - \hat{y})^2}$$

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