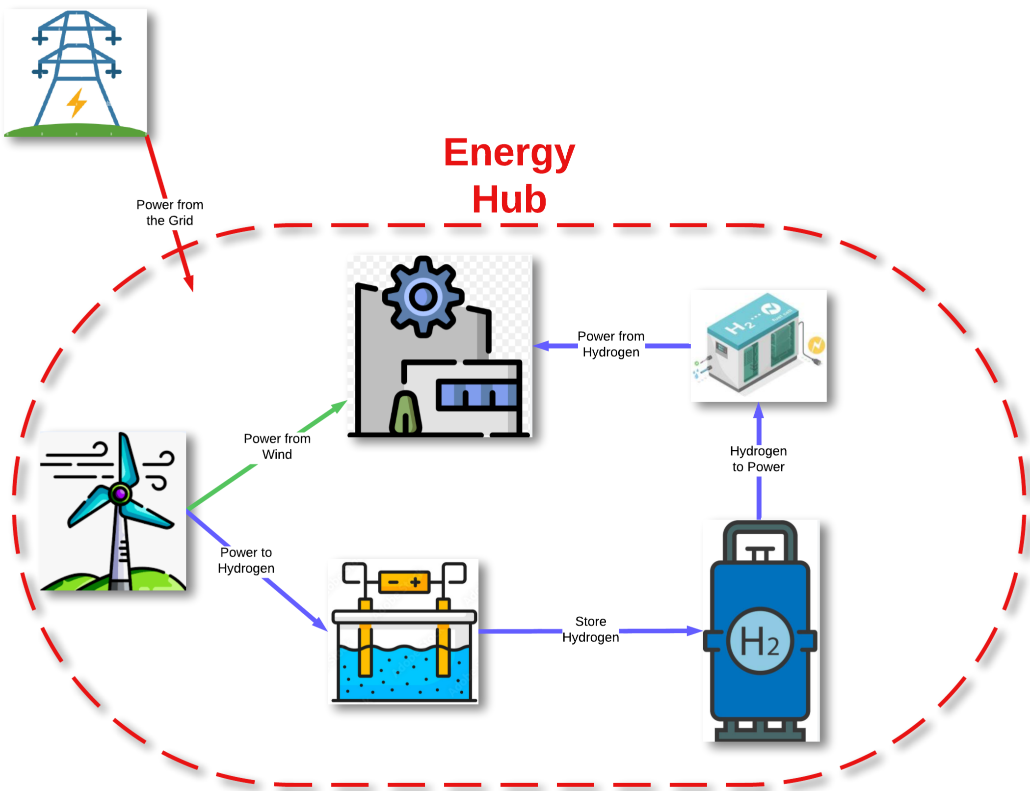
Decision Making under Uncertainty - Spring 2025
Assignment A

February 11, 2025



Problem Description

You are responsible for the operational decisions of an energy hub. The hub comprises:

- an industrial plant that has a known power demand D_t in each hour t.
- a wind turbine that generates electricity $p_t^{\texttt{wind}}$. The wind generation at a future timeslot t is uncertain and depends on wind generation at $t-1$ and $t-2$, based on a stochastic process that is known and given to you.
- a hydrogen plant that includes:
  1. An electrolyzer that turns electricity into hydrogen (up to an amount $\overline{\mathrm{P2H}}$ per hour). The conversion is linear with a conversion rate $\mathrm{R}^{\texttt{p2h}}$.
  2. A hydrogen storage tank of capacity C. When an amount of hydrogen is produced at t, it becomes stored and available in the tank from the next timeslot $t+1$.
  3. A Hydrogen-to-Power unit that can convert stored hydrogen back to electricity (up to an amount $\overline{\mathrm{H2P}}$ per hour). The conversion is linear with a conversion rate $\mathrm{R}^{\texttt{h2p}}$.

The hub can also draw power from the main grid at a per-unit price $\lambda_t^{\mathtt{grid}}$. The price at a future timeslot t is uncertain and depends on the price at $t-1$ and $t-2$ and it is also correlated to the hub's wind generation at t. This stochastic process is known and given to you. The hub cannot inject power into the grid; if there is excess power, it is just wasted.

The electrolyzer can be ON or OFF. When it is OFF, it cannot turn electricity into hydrogen. If it is activated (or deactivated) in timeslot t, it goes ON (respectively, OFF) from the **next** timeslot $t+1$. For every timeslot that the electrolyzer is ON, it bears a fixed operational cost of $\mathrm{C}^{\mathtt{elzr}}$.

Your objective is to serve the power demand of the industrial plant at the minimum possible cost. The relevant decisions you need to make include: whether to turn the electrolyzer ON or OFF, the amount of power to turn into hydrogen, and the amount of (previously stored) hydrogen to turn back into electricity.

Task 0: Optimal in Hindsight solution

Assume the wind power and the price is known and not uncertain. Formulate and implement the problem above as a deterministic mixed-integer linear program.

Deliverables:

1. pdf[1] describing the problem in math mode. Follow the following template: Begin by defining the notation (nomenclature). Use calligraphic fonts for sets, italics for variables and indices, and straight letters for known quantities or parameters.

   Then, define the constraints and the objective function. Explain each constraint in words, in a single sentence, just before the constraint. Be concise and avoid wordy explanations.

2. Python code that implements this MILP. Use the file `data` to define numerical values for the problem's parameters and the files `WindProcess` and `PriceProcess` to generate a time-series for wind and price. Plot the result for 2 example time-series of your choice, using the file `Plots` as a template.

3. pdf presenting the two figures (one for each example time-series containing the 8 subfigures of the `Plots` template). Briefly comment on whether the results make sense (around 3 sentences in total).

Task 1: Markov Decision Process

Formulate the problem as a Markov Decision Process and create a simulation environment in Python.

Deliverables:

1. pdf describing the MDP in math mode: Define the state variables, the decision variables, the transition dynamics, and the reward function. Use calligraphic fonts for sets, italics for states, decisions and indices, and straight letters for known quantities or parameters. Be concise and precise. Avoid overly wordy explanations.

2. Python code that takes a decision-making policy for this problem as input, and evaluates its average performance (output) in E independent Experiments (days). Use the file `data` to define numerical values for the problem's parameters and the files `WindProcess` and `PriceProcess` for simulating the wind and price stochastic processes and obtain the realized wind and price time-series for each experiment. To verify that the MDP code works, use it to evaluate the performance of a dummy policy that never uses the electrolyzer.

Task 2: Stochastic Programming

Formulate, implement, and evaluate a multi-stage stochastic programming policy for the above problem.

Deliverables:

1. pdf presenting the details of the policy: number of lookahead timeslots, number of scenarios (initial and reduced) and structure (number of branches at each stage etc), and the stochastic optimization problem (sets, variables, constraints, objective function).

2. python code with the policy: takes as input the current state and returns the here-and-now decisions.

[1] Include all pdf-deliverables (of all tasks) in a single pdf file.

Task 3: Approximate Dynamic Programming

Formulate, implement, and evaluate an approximate dynamic programming (ADP) policy for the problem.

Deliverables:

1. pdf presenting the policy, including the approximate value function along with its training/configuration process, as well as how exactly it is used to make a decision.
2. python code with the policy: takes as input the current state and returns the here-and-now decisions.

Task 4: Evaluation and Report

Report your findings.

Deliverable: pdf presenting the comparative evaluation of

1. the dummy policy
2. the optimal-in-hindsight solution of Task 0
3. the stochastic programming policy for different configurations (namely, longer lookahead horizon length but fewer branches/scenarios vs shorter lookahead horizon but more branches/scenarios). Try 4 configurations such that the total number of variables in each is not more than 1000.
4. the expected value policy i.e. a special case of the stochastic programming policy that considers a single (the expected) scenario.
5. the ADP policy.

Evaluate these policies on the MDP environment you implemented in Task 1. Use a set of 20 randomly drawn experiments (the same experiments for each policy) and plot a histogram with the policy's cost across the 20 different experiments. Report each policy's average cost. Briefly comment on the results (3-4 sentences).

Task 5: Credit Allocation

Report how each group member contributed to the Assignment.

Deliverable: pdf where

- Each group member writes their name and one paragraph about their own contributions to each Task (around 6-7 sentences in total). Be specific and comprehensive. The group reads, reviews, and approves each member's contributions. Take note that a false statement (for yourself or for others) is a breach of academic integrity.
- The group presents a credit assignment table, indicating the **percentage** of the group's work that should be allocated to each member (use names, not student numbers) **for each Task**. Nuclear option: If you cannot reach an agreement, you can ask for a meeting with the instructor (to be scheduled after your deliverables are evaluated) where each member's contributions and overall understanding of the concepts will be assessed. Use this option only as a last resort; note that it **can** result in reduced points for all members if the assessment reveals weaknesses not discovered through your deliverables alone.