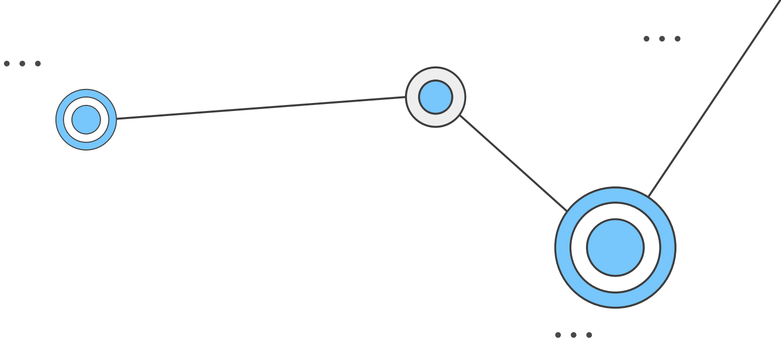

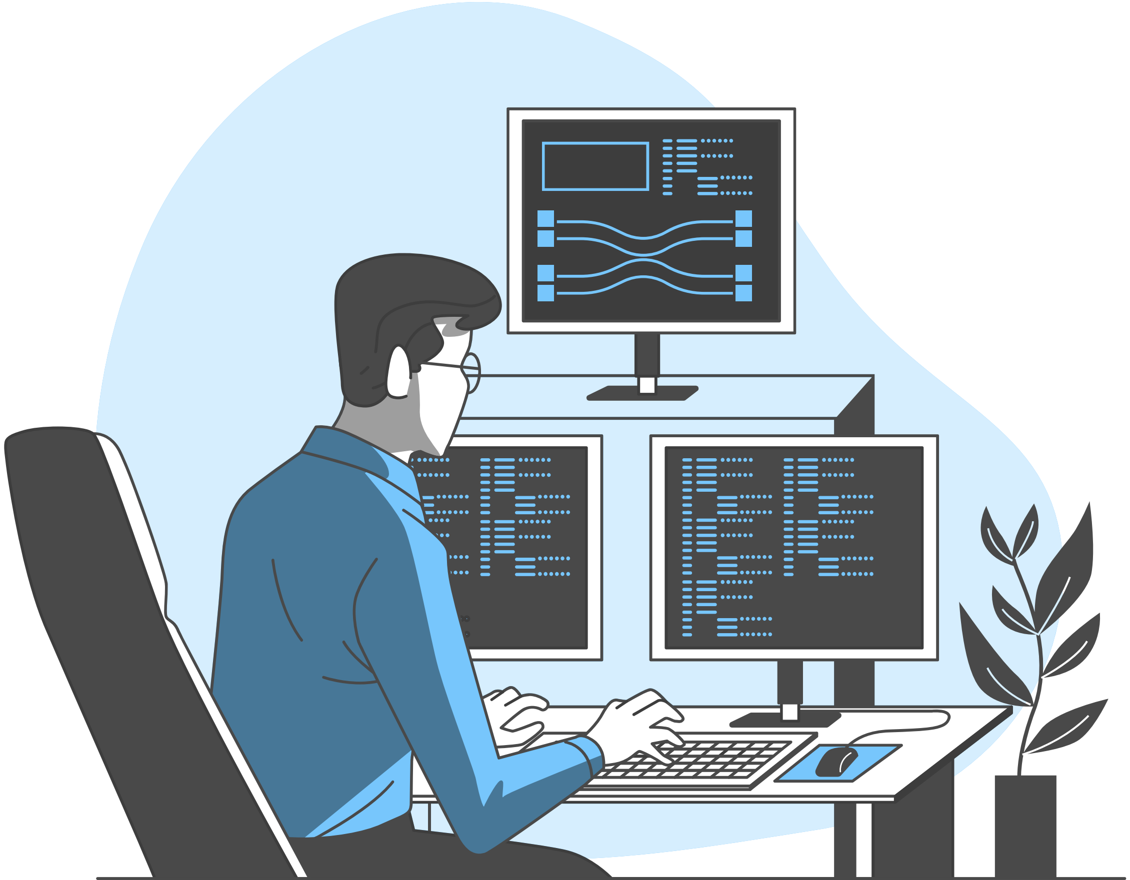

Optimization Under Uncertainty

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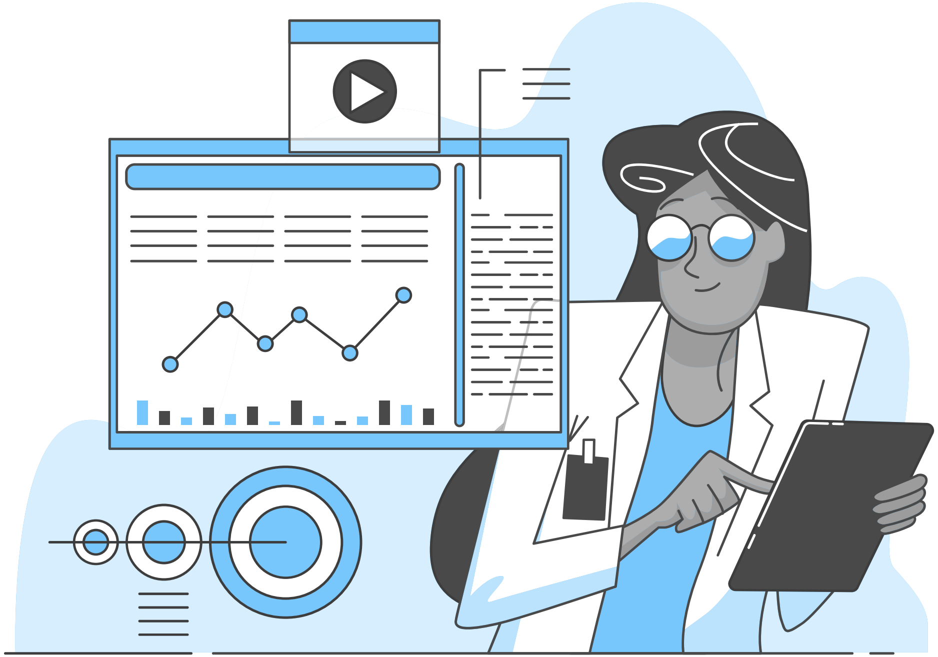
Contents

Problem Statement

To depict how introducing uncertainty causes deviation from classical mathematical techniques and discuss ways to mitigate or incorporate uncertain parameters within our model using Production Planning Problem.



Introduction

1. Uncertainty

UNCERTAINTY Decisions must often be taken in the face of the unknown.
Actions decided upon in the present will have consequences that can't fully be determined until a later stage. But there may be openings for corrective action later or even multiple opportunities for recourse as more and more becomes known.

2. Modelling of Uncertainty

Stochastic modelling: The uncertain elements in a problem can often be modelled as random variables to which the theory of probability can be applied. For this purpose, such elements must have a "known" probability distribution.

USE OF UNIFORM DISTRIBUTION

Uniform distributions are particularly useful for modelling uncertainties where all values within a range are considered equally likely. For example:

Process Variability: Uncertainties arising from inherent variations in a manufacturing or production process can be modelled using a uniform distribution. For example, the duration a certain task takes or the amount of resources consumed could follow a uniform distribution within a specified range.

Demand Variability: For uncertainties related to demand for a product or service, especially when the exact demand level is not known, a uniform distribution can be used. The range would capture the potential variability in customer demand.

USE OF NORMAL DISTRIBUTION

The normal distribution is widely used in various fields due to the central limit theorem and its applicability to many natural phenomena. For example :

Sampling Distributions: According to the central limit theorem, the distribution of sample means (or sums) from a sufficiently large number of samples tends to be normally distributed, even if the underlying population distribution is not normal.

Mathematical Formulation

$$PROFIT = \sum_j SP(j)*X(j) - \sum_j (PC_l(j)*L(j)+PC_m(j)*M(j)+PC_h(j)*H(j))$$

$$X(j)= cl(j) *L(j)+cm(j)*M(j)+ch(j)*H(j)$$

$$H(j) = 1 - Y(j)$$

$$L(j) \leq Y(j)$$

$$L(j)+M(j)+H(j) = Z(j)$$

$$X(j) \leq U * z(j)$$

$$\sum_j rm(k, j)*X(j)\leq R(k)$$

$$\sum_j (IC_l(j)*L(j)+IC_m(j)*M(j)+IC_h(j)*H(j)) \leq B$$

Naive Approach

We make following additions in the existing MILP formulation :

$$R_{lk} = (1 + \epsilon \cdot \xi) \cdot R_{lk}$$

To achieve Normal distribution in Gams we used,

$$R(r1) = ROrignal(r1) * (1 + 0.5 \cdot Normal(0, 0.2))$$

To achieve Uniform distribution in Gams we used,

$$R(r1) = ROrignal(r1) * (1 + 0.5 \cdot Uniform(-1, 1))$$

In this approach we,

1. **Execute the optimization model with different scenarios of raw material availability.**
2. **Observe variations in profits to understand the impact of uncertainty.**
3. **Analyse the results to identify patterns or trends in the relationship between raw material availability and profit.**

Robust Approach

Based on the proposed formulation of robust optimization we incorporate the formulation by relaxing inequality of availability of raw materials in existing formulation under K(Reliability level),δ(feasibility tolerance) and ε(max Error):

$$R_{lk} = (1 + \epsilon \cdot \xi) \cdot R_{lk}$$

$$\sum_j rm(k,j) * X(j) \le \widetilde{R(k)}$$

$$\sum_j rm(k,j) * X(j) \le R(k) - \varepsilon * (1 - 2 * k) * R(k) + \delta * \max(1, R(k))$$

This ensures that the results produced remain feasible under given uncertainties with a certain level of tolerance and reliability resulting in effective and reliable decision making. While also ensuring less computational load compared to naïve approach where each uncertainty we would have to reiterate resulting in greater computational loss. here: K(Reliability level)

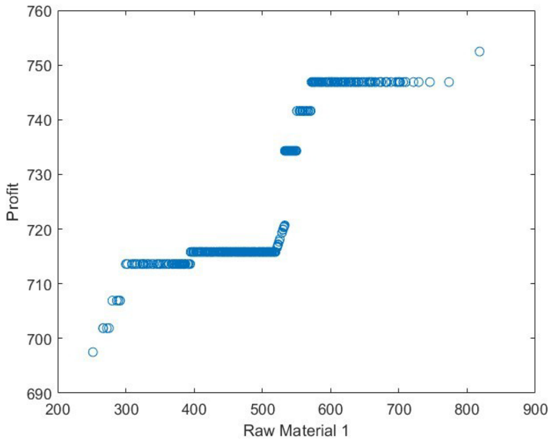
δ(feasibility tolerance)

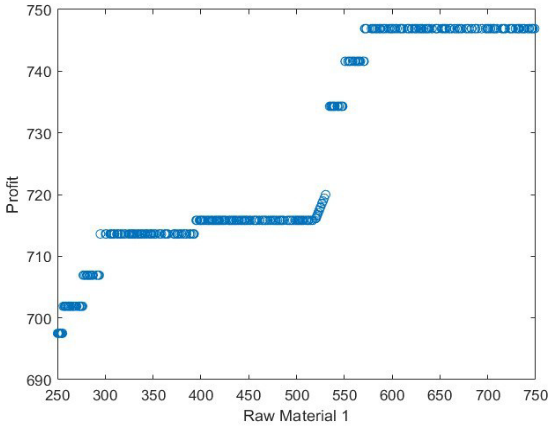
ε(max Error)

Observation

Naive Method

- Initially we were trying to compare and analyze the distribution of profit with different distribution of Raw material (R.V. - Normal/Uniform) .
- We expected somewhat linear dependency of raw material with profits .
- We observed bands of const. profit for wide range of Raw material .
- Profit distribution were similar to the RV used i.e. in Normally
- distributed Raw matrial we saw that Profit was concentrated in the middle distributed .





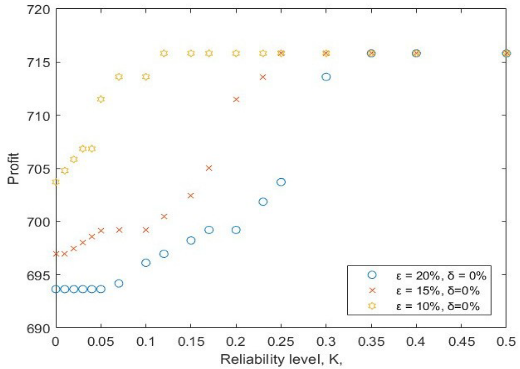
Observation

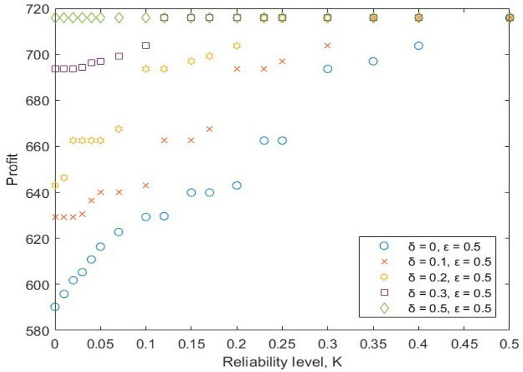
Robust Method

- A new constrain was added (prev. slide) where,
  K(Reliability level)
  δ(feasibility tolerance)
  ε(max Error)
- At the time of planning we don't have the actual value of the amount raw material available at time of production. So Robust Method says the probability of a plan to violate the Raw material constrain is k.

$$P(\textstyle\sum_j rm(k,j) * X(j) > R(k)) = K$$

- So we can make estimated decisions like what plan should we use with 95% chances of working withing bound i.e. (1-K=0.95)





Code Analysis

Naïve Method

- PPP code was modified .
- Loop was added .
- Value of Raw material 1 was changed using Uniform/Normal inbuilt functions .
- Max profit was calculated .
- File I/O was used in gams to save the data into .txt file .
- Data was taken into exel and then to Matlab for plots and analysis .

```
Parameter Solution1;

File outputTextFile / 'output.txt' /;

put outputTextFile;

put "Profit     Raw Matrial 1"   /;

outputTextFile.ap=1;

loop(i,

*        R('r1') = R_orignal('r1')*(1 + Normal(0,0.2));
       R('r1') = R_orignal('r1')*(1 + 0.5*uniform(-1,1));

    SOLVE petrochemical USING MIP MAXIMIZING OBJ;

    Solution1  = OBJ.l;

    put Solution1 , ';', R('r1')  /;

)
```

Code Analysis

Robust Method

- PPP code was modified and New Constrain was added .
- Value of k i.e. reliability level (d in the code) was iterated in loop .
- Values for epsilon and delta was manually set .
- Max profit was calculated .
- File I/O was used in gams to save the data into .txt file .
- Data was taken into excel and then to MATLAB for plots and analysis .

```
EqnRobust(k)..
    sum(j, rm(k, j) * X(j)) =l=
    R(k) - epsilon*(1 - 2*d)*R(k) + delta*max(1,R(k));

epsilon = 0.5;
delta = 0.3;

MODEL petrochemical /all/;

File outputTextFile / 'output.txt' /;

put outputTextFile;

put "Profit     K"  /;

outputTextFile.ap=1;

loop(o,
    d=rl(o);
    SOLVE petrochemical USING MIP MAXIMIZING OBJ;
    Solution1  = OBJ.l;
    put Solution1 , ';', d  /;
)
```

Conclusion

This project contributes to the field of optimization under uncertainty, specifically addressing challenges in production planning production, taking inspiration from available literature on scheduling problems. The study explores two distinct analytical approaches: a naive method involving multiple runs and a robust optimization approach. The naive approach provides valuable insights into the impact of uncertainty on profits, showcasing variations in different scenarios. On the other hand, the robust optimization approach introduces a systematic and computationally efficient methodology that ensures feasibility under specified uncertainty levels. Through the mathematical formulation of a Production Planning Problem (PPP), this research illuminates the complexities of decision-making in the presence of incomplete knowledge. The naive approach, relying on multiple runs with varied scenarios, sheds light on the variability of profits under different conditions. However, it lacks the systematic control over robustness that the robust optimization approach offers. The robust optimization approach, with its foundation in mathematical rigor, introduces a trade- off between optimality and robustness. Decision-makers can fine-tune the degree of conservatism, providing solutions that remain feasible even under uncertain conditions. This approach is computationally efficient, reducing the need for repetitive computations and offering a reliable framework for decision-making. Though modelling under this approach and probabilistic bounding is quite rigorous and time consuming.

Thank You