Quantitative Macro Final Project
Checking An Importance Of A Technology Spillovers Using Bayesian Estimation.

Piotr Królak

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Abstract

The main aim of my final project is to examine the importance and existence of a technological spillover across the countries for a model based in the DSGE framework. To verify this, I use Bayesian estimation with a prior distribution, that make it insignificant. I start with an explanation of a technology spillover concept, followed by a brief literature review on it. Then analysis of a real-world data is performed, trying to show the existence of the spillovers. After that, I briefly summarize the the theoretical basics of Bayesian estimation. Finally, I write a full model, describe solution method and state the results(the spillovers have slightly negative impact on the production function).

Contents

1 Technology spillover - description and literature

The aim of this section is to present the notion of technology spillover between the countries and briefly describe the literature about it.

Technology spillover is (according to the IGI Global dictionary) a beneficial impact of a new technological knowledge on the productivity of the other firms or countries. The main goal of my research is to check, whether is it possible that a country that does not invest much in the R&D could still be technologically advanced due to the closeness of other regions, states that spend more on technology. What could be the reason for this? Nowadays, in a globalized world the stream of technology may flow easier through borders, because of existence of multinational companies that operate in many countries, direct investments of foreign companies or workers that return from job emigration.

Briefly describing the literature, there is a plethora of articles concerning a technological spillover between firms (see Jaffe,1986) or within a multinational company (Keller, Yeaple, 2003). There exist another branch of articles presenting effects of the foreign direct investments on the developing countries economy (Harrison, Aitken, 1999), in that process the spillover plays an important role. In similar research (Kolasa 2007), the author states that particular economy sectors or even the whole could benefit from the flow of technology from abroad. The main focus point in my research is on cross-country technological spillover, similar to a paper (Acemoglu et all, 2009) where authors bring it into a model to asses technical change in a growth model with environmental constraints and limited resources, which is a bit closer to my point of interest.

2 Do technology spillovers matter - empirical evidence

The aim of this section is to provide the analysis of the real-world data to state whether technology spillovers matter, and if they do what is their effect. The data concerning technology usually covers last 20-25 years. It is the reason why I limit the analysis to the members of the European Communities from 1986, but without British Isles. The second reason, is that the previously mentioned countries could be characterised with a high level of economical integration and cooperation, especially those that signed Treaties of Rome in 1957 (Luxembourg is deliberately omitted here, due to completely different economic characteristics).

The analysis starts with comparison of TFP shocks cross-correlations between the countries, without any distinction of their origin (foreign or domestic). It is based on the data provided by Penn World Table concerning TFP from 1954 to 2017. For each country AR(1) model was estimated and the TFP shock was computed as a model residual. The results are following:

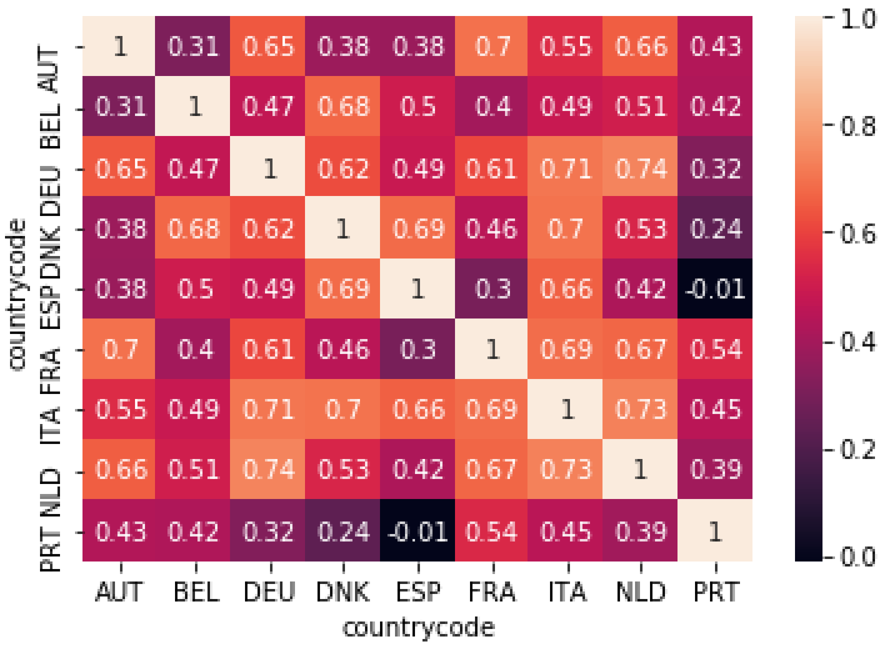

Figure 1: Cross-correlation of TFP shocks

One may notice that apart from one exception (Spain vs. Portugal) the correlation is moderate reaching values from 0.24 even to 0.74. There are some countries with a lower shocks correlation (Portugal) and a group with relatively high levels (Netherlands, France, Italy). To improve the analysis let us look also at the graph of the TFP shocks.

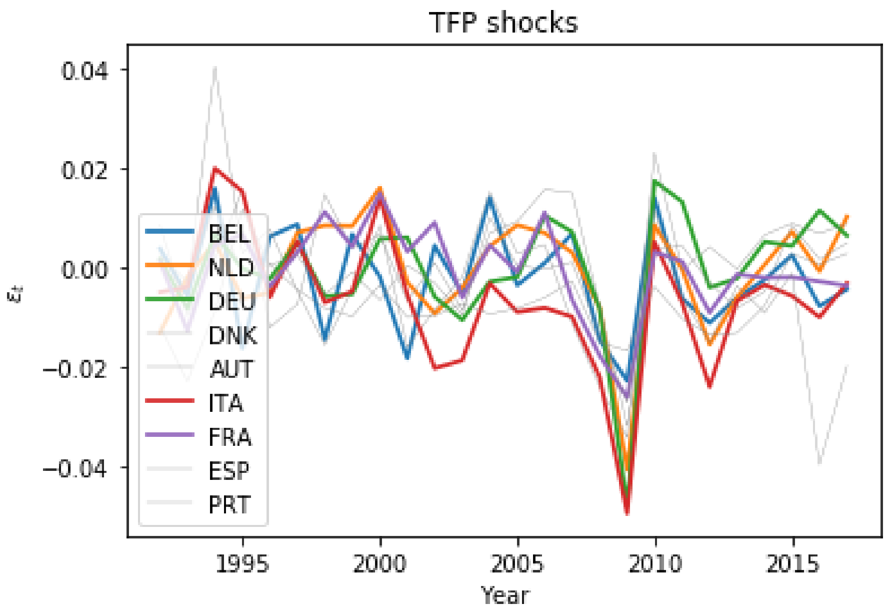

Figure 2: TFP shocks

The graph presents TFP shocks since 1992, with bold countours for the signatories of Treaties of Rome. It is noticeable that, the time series generally follow the same patterns with some exceptions. Also, one may notice that when the shock is major, than the differences between countries are relatively small. In addition, outliers on the graph are provided by countries that are not in bold.

From this part of the section we may conclude that there is a correlation between TFP shocks across the countries, it may be that it is resulted from the technology flows between them.

Now, I want to try to present the ways of distinction between effects of foreign and domestic technology shock that drives the general TFP shock.

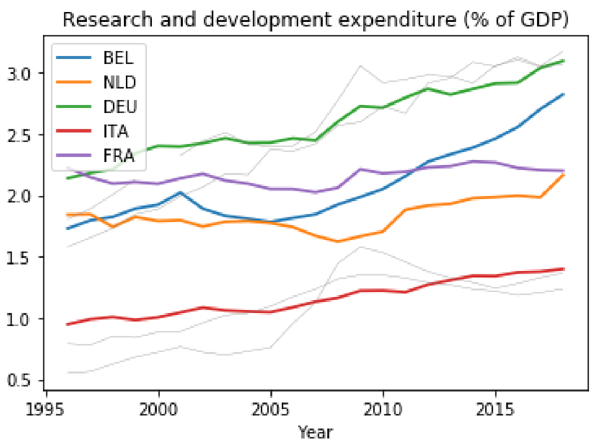

Figure 3: R&D expenditures

Let us start with the domestic one. We need to think of a proxy variable(s) of the domestic technology changes. One can think of the Research and Development expenditures (% of GDP, figure above). However, correlation of its changes with TFP shocks is moderately negative (-0.266) and effects of for example 2008 crisis not visible. I would argue, that the better proxy for the domestic shocks is a variable Charges for the use of intellectual property, payments (BoP, current US$) from the World Bank Data. It could measure the level of domestic technology, countries with better technology should receive more payments for it. In addition, changes of this variable is correlated with the TFP shocks (correlation coefficient of 0.1566).
Now, let us scan for the proxy variable for the foreign shock. Here the Foreign direct investment, net inflows (% of GDP) seems to be the best candidate. Its role for the technology spillovers is widely described in the literature and has a meaningful explanation, with the direct investments bring know-how, technology, management structure or other labour practices.

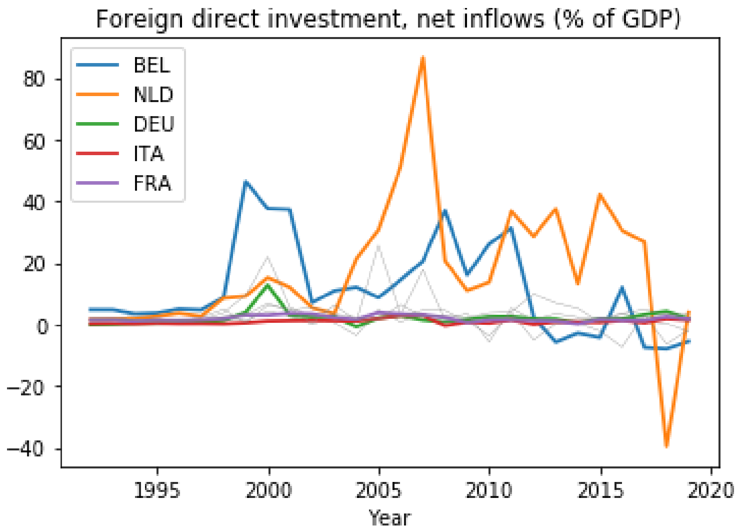

Figure 4: R&D expenditures

The correlation of changes is here bit lower (0.077). In addition, one may notice that there are some countries that base on the domestic technology and investments (usually bigger ones, France, Italy, Germany from the picture), they are characterised with very low level of FDI. Whereas, some countries like Belgium or Netherlands take the advantage of the foreign investments.

3 Bayesian estimation - short overview

To solve the model the Bayesian approach was chosen. It has a number of interesting features. Contrary to the frequentist approach, each parameter of the model is treated as a random variable with some distribution. In addition, the primary beliefs are provided by **a prior distribution** - $\Pr(\zeta)$ (usually, parameters are denoted by θ, but in the model I use this symbol for parameter of risk aversion). Then, after observing the data, the primary belief is updated to the **posterior distribution** - $\Pr(\zeta|Y)$. All is based on the Bayes' Theorem:

$$\Pr(\zeta|Y) = \frac{\Pr(Y|\zeta)\Pr(\zeta)}{\Pr(Y)}$$

The total probability of Y in the denominator only normalizes the distribution such that the cumulative density function sums up to one, hence very often it is written that it is proportional:

$$\Pr(\zeta|Y) \propto \Pr(Y|\zeta)\Pr(\zeta)$$

The term $\Pr(Y|\zeta)$ will be called **likelihood function**. As it is discussed in the next chapter, setting a prior distributions is relatively easy, whereas constructing likelihood function or drawing from the posterior distribution may require more attention. Applying Bayesian approach for the DSGE framework enables using out-of-sample data, information. It could come from different and independent data sources or researches that analyse phenomena not explained by the model, for example microeconomics studies. Here, in my project this approach has an additional advantage, namely setting a prior that makes the effect of technology spillovers insignificant.

4 Model setup

Let us set up the following model (it is a neoclassical RBC model with representative agent and endogenous labour). I will provide an extension, such that it enables capturing the technology spillover effect. All variables introduced below are stationary variables (as if they were divided by a level of effective technology and population).

4.1 Households

The households choose consumption, capital next period, savings and supplied labour such that it maximizes their expected discounted lifetime utility for each period t:

$$\max_{c_t, i_t, l_t, k_{t+1}} E_t \left[\sum_{s=0}^{\infty} \beta^{t+s} \left(\frac{c_{t+s}^{1-\theta}}{1-\theta} - \kappa \frac{l_{t+s}^{1+\frac{1}{v}}}{1+\frac{1}{v}} \right) \right]$$

such that the following budget constraint holds:

$$c_t + i_t = \omega_t l_t + (R_t) k_t$$

The capital law of motion is given by:

$$k_{t+1} = (1-\delta) k_t + i_t \tag{1}$$

The supplied labour must be within the $[0,1]$ interval. We construct the following Lagrangean substituting the capital low of motion into the constraint.

$$\mathcal{L}(c_t, k_{t+1}, l_t) = E_t \left[\sum_{s=0}^{\infty} \beta^{t+s} \left(\frac{c_{t+s}^{1-\theta}}{1-\theta} - \kappa \frac{l_{t+s}^{1+\frac{1}{v}}}{1+\frac{1}{v}} - \lambda_{t+s} \left(c_{t+s} + k_{t+1+s} - \omega_{t+s} l_{t+s} - (\delta + 1 + R_{t+s}) k_{t+s} \right) \right) \right]$$

Taking he First Order Conditions we obtain:

$$\frac{\partial \mathcal{L}}{\partial c_t} : \lambda_t = c_t^{-\theta}$$

$$\frac{\partial \mathcal{L}}{\partial l_t} : \omega_t = \frac{l_t^{\frac{1}{v}}}{\lambda_t}$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} : \lambda_t = \beta E_t \left[\lambda_{t+1} (\delta + 1 + R_{t+1}) \right]$$

Substituting for lambdas we obtain:

$$c_t^{-\theta} = \beta E_t \left[c_{t+1}^{-\theta} (-\delta + 1 + R_{t+1}) \right] \tag{2}$$

$$1 = \beta E_t \left[\frac{\omega_t}{\omega_{t+1}} \left(\frac{l_{t+1}}{l_t} \right)^{\frac{1}{v}} (-\delta + 1 + R_{t+1}) \right] \tag{3}$$

$$\omega_t = l_t^{\frac{1}{v}} c_t^{\theta} \tag{4}$$

4.2 Firms' sector and market clearing

In this sections I present firms' sector of a standard RBC model with some extensions that should cover adding technology spillovers.

For each period t, firms operating on the perfectly competitive market choose production volume, demand for labour and capital to maximize its profits:

$$\max_{y_t, k_t, l_t} y_t - R_t k_t - \omega_t l_t$$

such that the production function is given by:

$$y_t = (1 + \eta_t) A_t k_t^{\alpha} l_t^{1-\alpha} \tag{5}$$

Here arrives the effect of a technology spillovers from abroad countries. I assume that in period t, after capital, technology and labour is formed, the abroad technology flows and enters in a multiplicative manner production function. It is represented by a random variable η, that follows a normal distribution with mean μ_f and variance σ_f.
This results in the following FOC's:

$$w_t = (1 + \eta_t) A_t k_t^{\alpha} l_t^{-\alpha} (1 - \alpha) \tag{6}$$

$$R_t = (1 + \eta_t) A_t k_t^{\alpha - 1} l_t^{1-\alpha} \alpha \tag{7}$$

The market clearing for the product market is given by the following equation:

$$y_t = c_t + i_t \tag{8}$$

4.3 Technology

In the DSGE models the technology level is a stochastic AR(1) process. In my project, I assume that the domestic shocks to technology are incorporated here in an additive way. Suppose, that it is given by the following equation:

$$\ln(A_t) = \rho \ln(A_{t-1}) + \epsilon_t \tag{9}$$

Where the random variable ϵ follows a normal distribution with mean μ_d and variance σ_d.

5 Solving the model

5.1 Log-linearization

Due to nonlinearities and presence of expectations, the model does not have a closed-form solution, to solve it I will use the log-linear approximation for the equations 1-8 that fully describe the model. By hat variables I denote log-deviations from the steady state for any variable. $\hat{x_t} = \ln(\frac{x_t}{\tilde{x}})$, where the tilde variable denotes the steady state value. To do the log linearization we approximate the x_t with the first order Taylor approximation around the steady state.
For the equation 1 we obtain:

$$\hat{k}_{t+1} = (1 - \delta)\hat{k}_t + \delta \hat{i}_t \tag{10}$$

Equation 2:

$$(1-\beta(1-\delta))E_t[\hat{R}_{t+1}] + E_t[\hat{c}_t - \hat{c}_{t+1}] = 0 \tag{11}$$

Equation 3:

$$(1-\beta(1-\delta))E_t[\hat{R}_{t+1}] + E_t[\hat{w}_t - \hat{w}_{t+1}] + \frac{1}{v}E_t[\hat{l}_{t+1} - \hat{l}_t] = 0 \tag{12}$$

Equation 4:

$$\hat{w}_t = \frac{1}{v}\hat{l}_t + \theta\hat{c}_t \tag{13}$$

Equation 5:

$$\hat{y}_t = \ln(1+\eta_t) + \hat{A}_t + \alpha\hat{k}_t + (1-\alpha)\hat{l}_t \tag{14}$$

Equation 6:

$$\hat{\omega}_t = \ln(1+\eta_t) + \hat{A}_t + \alpha\hat{k}_t - \alpha\hat{l}_t \tag{15}$$

Equation 7:

$$\hat{R}_t = \ln(1+\eta_t) + \hat{A}_t + (\alpha-1)\hat{k}_t - (\alpha-1)\hat{l}_t \tag{16}$$

Equation 8 after the approximation looks following:

$$\hat{y}_t = \frac{\tilde{c}}{\tilde{y}}\hat{c}_t + \frac{\tilde{i}}{\tilde{y}}\hat{i}_t$$

After some algebra and substituting r with its steady state value it may be rewritten as:

$$\hat{y}_t = \left(1 - \frac{\alpha\delta}{\frac{1}{\beta}-1+\delta}\right)\hat{c}_t + \left(\frac{\alpha\delta}{\frac{1}{\beta}-1+\delta}\right)\hat{i}_t \tag{17}$$

Equation 9:

$$\hat{A}_t = \rho\hat{A}_{t-1} + \epsilon_t + \upsilon_t \tag{18}$$

After some reshuffling the we may reduce model to 3 equations. Firstly, to equation 10 equations 13, 14, 15 and 17 may be plugged in, such that we get next period capital as a function of current capital, technology and consumption:

$$\begin{aligned}&\hat{k}_{t+1} = \hat{k}_t\left[1-\delta+\frac{\delta\alpha}{m}\left(1+\frac{1-\alpha}{\alpha+\frac{1}{v}}\right)\right] - \\ &\frac{\delta}{m}\left(1+m+(1-\alpha)\frac{\theta}{\alpha+\frac{1}{v}}\right)\hat{c}_t + \\ &\frac{\delta}{m}\left(1+(1-\alpha)\frac{1}{\alpha+\frac{1}{v}}\right)\left[\hat{A}_t+\ln(1+\eta_t)\right]\end{aligned}$$

where $m = \frac{\alpha\delta}{\frac{1}{\beta}-1+\delta}$.

The second equation is equation 11 combined with equations 13, 15, 16, in that manner a consumption in the next period, as a function of current technology, capital and consumption in the current period, is obtained:

$$E_t\hat{c}_{t+1} = \frac{1}{d}\hat{c}_t + \frac{(1-\beta(1-\delta))(\alpha-1)\left(1-\frac{\alpha}{\alpha+\frac{1}{v}}\right)}{d}\hat{k}_{t+1}+$$
$$\frac{(1-\beta(1-\delta))\left(1-\frac{\alpha-1}{\alpha+\frac{1}{v}}\right)}{d}\hat{A}_t$$

where, $d = 1-(1-\beta(1-\delta))\theta\frac{\alpha}{\alpha+\frac{1}{v}}$ and k_{t+1} is determined by the previous equation. The third equation, which describes changes in technology level, is equation number 18. However, we want the solution to take the form:

$$s_t = \Phi_1(\zeta)s_{t-1} + \Phi_\epsilon(\zeta)\Theta_t$$

where s_t is a vector of states or exogenous processes - $[k_{t+1}, A_t]$, ζ is a vector of parameters, $\beta, \theta, \delta, v, \alpha, \rho, \mu_d, \sigma_d, \mu_d, \sigma_f$ in the model and Θ vector of stochastic components. It is called state transition equation. The Φ matrices of coefficients are a functions of parameters. To obtain this model representation, I implement Blanchard - Kahn method, for more information see Blanchard, Kahn (1980).

We may write write also so-called measurement equation for the observable variables (denote them with B_t). Because the previously mentioned equations fully describe the model, I solve the model in terms of $B_t = [\tilde{c}_t]$. From which, I can obtain the other variables of interest. I use the

$$B_t = \Psi_1(\zeta)s_t + v_t$$

With v_t being a measurement error with normal distribution with mean zero and variance 0.2. Measurement equation together with state transition equation establish so-called state-space representation.

5.2 Likelihood function for the model

In order to perform the Bayesian estimation a likelihood function for a model is necessary - $p(Y|\zeta)$. One may notice that states are partially unobservable (f.e A_t). To mitigate this concern the Kalman fiter is used to construct the likelihood function (see Herbst, Schorfheide 2014). The Kalman filter is feeded with with the log-deviations of consumption, that were calculated using a Hodrick-Prescott filter. As a datasource, yearly data for consumption at constant prices from Belgium since 1992 are used.

5.3 Priors

The following prior distributions were chosen for the model. Following (Del Negro, Schorfheide 2010) approach for capital share, depreciation rate and discount factor are dogmatic choices with very low variances. An important choice is the prior for labour elasticity, which can vary significantly. Here, the gamma distribution with mean 2 and variance of one is used (same as in Del Negro, Schorfheide 2010). For the autocorrelation coefficient of

Variable	Distribution	Mean	Variance
β	Beta	0.96	0.0036
θ	Gamma	2	1.5
δ	Beta	0.1	0.0001
v	Gamma	2	1
α	Beta	0.33	0.0004
ρ	Beta	0.96	0.000225
μ_d	Normal	0	0.04
σ_d	Inverse Gamma	0.01	0.0032
μ_f	Normal	0	0.04
σ_f	Inverse Gamma	0.05	0.01

Table 1: Prior distributions

technology (ρ), I use the same distribution (Beta), but with different moments that come from results of equation 9 estimation for different EU countries from 1960 to 2018. For random variables that correspond to technology shocks I use normal distributions with means zero, such that the effects of technology spillover is irrelevant. Speaking of variance of the shocks it is noticeable in the data, that the foreign shocks proxy has significantly higher variance.

5.4 The posterior distribution

This DSGE model is strongly nonlinear, that makes the Ψ and Φ matrices a nonlinear functions of parameters. Because of this nonlinear relationship the posterior distribution, does not resemble any familiar distribution. To obtain it we must simulate (generate) it. In my project I perform Random-Walk Metropolis algorithm, that belongs to the Monte Carlo Markov Chain (MCMC) methods. The algorithm consists of following steps:

Step 1 Maximize numerically the log posterior: $\ln \Pr(Y|\zeta) + \ln \Pr(\zeta)$, denote the maximand as $\tilde{\zeta}$.

Step 2 Compute the inverse of the (negative) Hessian at $\tilde{\zeta}$, denote it $\tilde{\Sigma}$.

Step 3 Draw a starting values of parameters from the normal distribution with mean $\tilde{\zeta}$ and variance $\tilde{\Sigma}$ or specify it.

Step 4.1 For $s = 1, 2, ..., N$ draw ξ from distribution $N(\zeta^{s-1}, c^2\tilde{\Sigma})$. Where c is coefficient that controls acceptance rate, which shoould be around 50 %.

Step 4.2 Compute:

$$r(\zeta^{s-1}, \xi|Y) = \frac{\Pr(Y|\xi)\Pr(\xi)}{\Pr(Y|\zeta^{s-1})\Pr(\zeta^{s-1})} \tag{19}$$

Step 4.3 Accept the movement (jump) with probability $\min(1, r(\zeta^{s-1}, \xi|Y))$, accepting means that $\zeta^s = \xi$, whereas rejecting $\zeta^s = \zeta^{s-1}$.

The first part of results is not taken into consideration (burning-in), usually it is about 20% of the generated sample.

6 Results

Main goal of the project is to assess importance of the technology spillovers across the countries. But before doing it, let us look at the distribution of μ_d that captures the mean of the domestic shocks. I provide here analysis of the distributions of both shocks and a table that summarises all the posterior distributions.
The previously described Random Walk Metropolis-Hastings algorithm was used with 1 million of draws, acceptance rate of 31.19% was obtained, the posterior distribution of a mean of domestic shock is following:

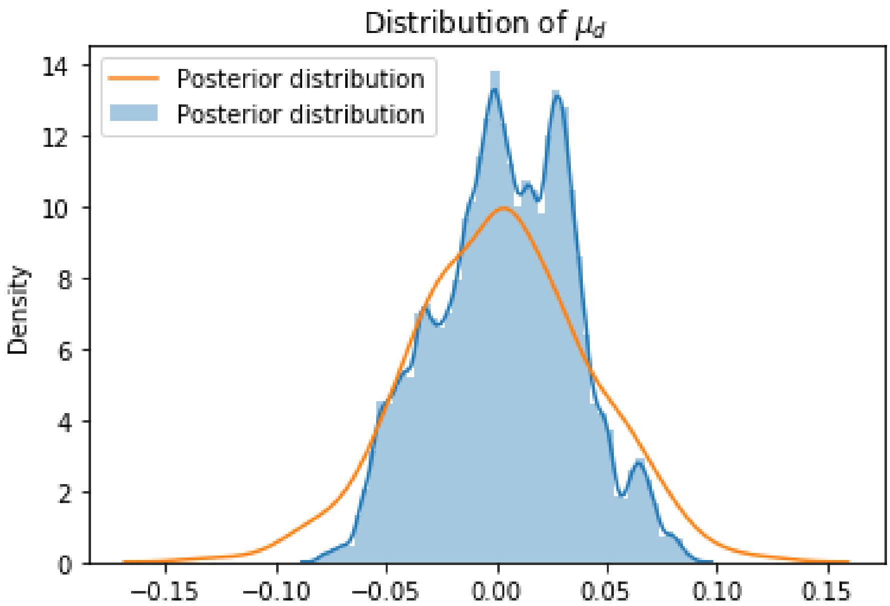

Figure 5: Distributions of domestic mean

The posterior distribution is different than the prior. The variance is much lower (0.00098 compared to 0.04). It is shifted rightwards, with two modes of different signs. The mean is equal to 0.0042748, which is not a big number, but slightly above zero.
Now, let us verify if the technology spillover matter in the examined model. We may expect that the mean of the shock should be significantly different from zero and positive. Let us see the distribution.

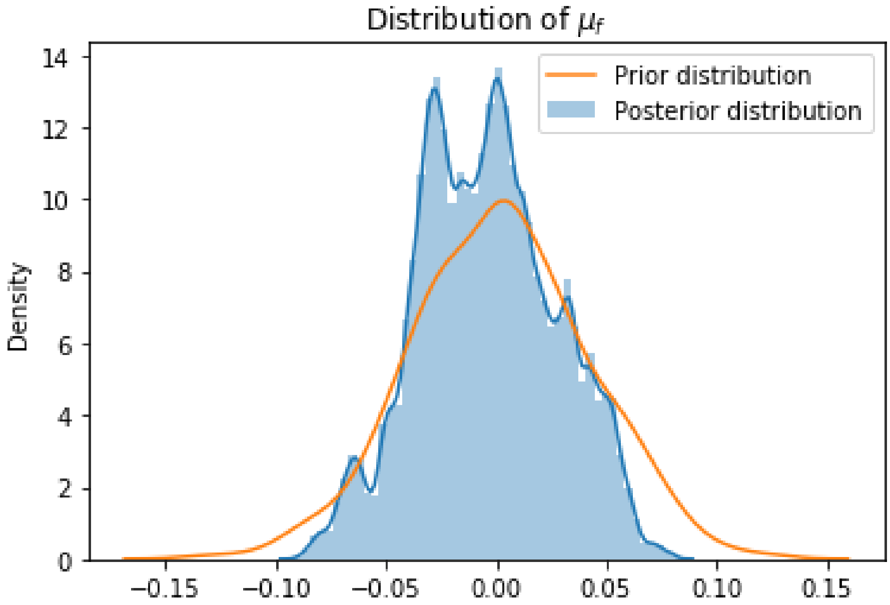

Figure 6: Distributions of foreign mean

Without any doubts, the distribution here is also moderately different than the prior. It resembles much the previous distribution, however it is shifted leftwards. The mean is equal to -0.004152, while the variance is also very low (0.000978). Hence, we may conclude that the technology spillovers are a statistically significant feature of the model, with moderate importance and negative impact on the technology.

Looking at the posterior distributions, one may notice that generally variances of all

Variable	Mean	Variance
β	0.99458	0.000067
θ	2.33834	0.00308
δ	0.09755	0.000092
υ	1.72511	0.063668
α	0.329905	0.00037
ρ	0.94045	0.00039
μ_d	0.0042748	0.00098
σ_d	0.002951	0.000008
μ_f	-0.004152	0.000978
σ_f	0.0252938	0.000277

Table 2: Posterior distributions

random variable are much lower than for a priors, it could be that the c coefficient that controls the acceptance rate was too low, or computed hessian was flawed (step 2 of the algorithm). For some parameters, obtained results are similar to the priors (discount rate, depreciation rate or capital share of output). Whereas, for other they differ slightly (risk aversion or elasticity of labour supply).

7 Concluding remarks

In my research I have tried to state whether the technology spillovers have a significant effect on the economy in a standard RBC framework. I have provided analysis stating that productivity shocks are correlated between countries that was followed by a Bayesian estimation of the model with some extensions should also capture the potential effect of the technology spiilovers between the countries. Starting with a prior distribution that made the effect irrelevant the results were obtained, that the foreign shocks have slightly negative impact on the production function.

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