Homework 1

Friday, September 9, 2022 11:11 AM

Written Assignment

1. a, 2^n grows faster than n^3.

We have e^{2^n} and e^{n^3}

$\Rightarrow ne^2 \quad 3e^n$

$\Rightarrow n < e^n$

$\Rightarrow$ $\boxed{\text{$2^n$ grows slower than n^3 False}}$

b, $2^{2n} \quad O(2^n)$

$2^n \cdot 2^n \quad 2^n$

$2^n > 1$

$\Rightarrow \boxed{\text{False}}$

2. a, $O(2n^4 + 6n + 10000)$

Since constants are not important $\Rightarrow O(2n^4 + 6n + 10000) = O(2n^4 + 6n)$

Since only consider faster growth $\Rightarrow O(2n^4 + 6n) = \boxed{O(n^4)}$

b, $O(\log_2 n^2 + (\log_2 n)^2 + \log_2 n)$

Consider $\log_2 n^2$ and $(\log_2 n)^2 \Rightarrow 2\log_2 n$ and $\log_2^2 n$

$2\log_2 n$ and $(\log_2 n).(\log_2 n)$

$\log_2 n > 2 \Rightarrow (\log_2 n)^2 > \log_2 n^2$

Consider $\log_2 n$ and $(\log_2 n)^2 \Rightarrow \log_2 n$ and $(\log_2 n)(\log_2 n)$

$\Rightarrow \log_2 n > 1$

$\Rightarrow (\log_2 n)^2 > \log_2 n$

$\Rightarrow O(\log_2 n^2 + (\log_2 n)^2 + \log_2 n) = \boxed{O((\log_2 n)^2)}$

c, $O((n+3)^4 + (n+5)^2)$

Since $(n+3)^4 = n^4 \quad n^4 = n^2 \cdot n^2 > n^2 \cdot 1$

$(n+5)^2 = n^2$

$\Rightarrow O((n+3)^4 + (n+5)^2) = \boxed{O(n^4)}$

3.

```
public int calcSum1(int n) {
    int sum = 0;
    for ( int i = 1; i < n; i *= 2 ) {
        sum++;
    }
    return sum;
}
```

$1 \to 2 \to 4 \to \ldots \to 2^k = N$

$\Rightarrow k = \log N$

sum++ = const

$\Rightarrow \boxed{O(\log N)}$

```
public int calcSum2(int n) {
    int sum = 0;
    for (int i = 1; i < n; i++) {
        for (int j = 1; j < n/i; j++) {
            sum++;
        }
    }
    return sum;
}
```

Inner loop:

i	n
1	n
2	n/2
3	n/3
...	...
n-1	n/n-1

Outer loop: n

$\Rightarrow$ Inner loop $= n + \frac{n}{2} + \ldots + \frac{n}{n-1}$

$= n\left(1 + \frac{1}{2} + \ldots + \frac{1}{n-1}\right)$

$= \ln(n)$

$\Rightarrow N \times \ln(N) = \boxed{O(N\ln(N-1))}$

```
public int calcSum3(int n) {
    int sum = 0;
    for (int i = n; i > 1; i--) {
        for (int j = i; j > 1; j = j/3) {
            sum++;
        }
    }
    return sum;
}
```

Inner loop $n \to n/3 \to n/9 \to \ldots \to \frac{1}{3^k} = N \Rightarrow \left(\frac{1}{3}\right)^k = N \Rightarrow k = \log_{1/3} N = \log N$

Outer loop $n \to n-1 \to n-2 \to \ldots \to 1 = n$

$\Rightarrow N \times \ln\left(\frac{1}{N}\right) = \boxed{O(N\log N)}$

```
public int calcSum4 (int n) {
    int sum = 0;
    for (int i = 0; i < n; i++) {
        for (j = 1; j < i * i; j++) {
            if ( j % i == 0) {
                for (int k = 0; k < j; k++) {
                    sum++;
                }
            }
        }
    }
    return sum;
}
```

i-loop $= 0, 1, 2, \ldots, n-2, n-1 \Rightarrow kn = N$

j-loop contains if statement $\Rightarrow$ Runtime = if statement.

$j \% i = i^2 / i = i = N$ times

k-loop $0, 1, 2, \ldots, n^2 - 3 \Rightarrow kn^2 = N^2$

$\Rightarrow O(N \times N \times N^2) = \boxed{O(N^4)}$