Homework 6

Wednesday, November 30, 2022 9:17 PM

Problem 1

a, Adjacency List

A ⟶ B(4) ⟶ C(3) ⟶ D(2) ⟶ Z(30).null
B ⟶ A(4) ⟶ E(5).null
C ⟶ A(3) ⟶ E(5) ⟶ Y(6) null
D ⟶ A(2) ⟶ Y(3) null
E ⟶ B(5) ⟶ C(5) ⟶ W(2) ⟶ X(1) null
W ⟶ E(2) ⟶ Z(9) null
X ⟶ E(1) ⟶ Y(5) null
Y ⟶ C(6) ⟶ D(3) ⟶ X(5) null
Z ⟶ A(30) ⟶ W(9) null

b. Adjacency Matrix.

	A	B	C	D	E	W	X	Y	Z
A	0	4	3	2	0	0	0	0	30
B	4	0	0	0	5	0	0	0	0
C	3	0	0	0	5	0	0	6	0
D	2	0	0	0	0	0	0	3	0
E	0	5	5	0	0	2	1	0	0
W	0	0	0	0	2	0	0	0	9
X	0	0	0	0	1	0	0	5	0
Y	0	0	6	3	0	0	5	0	0
Z	30	0	0	0	0	9	0	0	0

c.

Step	Vertex	N	D(B), P(B)	D(C), P(C)	D(D), P(D)	D(E), P(E)	D(W), P(W)	D(X), P(X)	D(Y), P(Y)	D(Z), P(Z)
0	A	A	4, A	3, A	2, A (min)	∞	∞	∞	∞	30, A
1	D	A, D	4, A	3, A (min)	2, A	∞	∞	∞	5, D	30, A
2	C	A, D, C	4, A (min)	3, A	2, A	8, C	∞	∞	5, D	30, A
3	B	A, D, C, B	4, A	3, A	2, A	8, C	∞	∞	5, D (min)	30, A
4	Y	A, D, C, B, Y	4, A	3, A	2, A	8, C (min)	∞	10, Y	5, D	30, A
5	E	A, D, C, B, Y, E	4, A	3, A	2, A	8, C	10, E	9, E (min)	5, D	30, A
6	X	A, D, C, B, Y, E, X	4, A	3, A	2, A	8, C	10, E (min)	9, E	5, D	30, A
7	W	A, D, C, B, Y, E, X, W	4, A	3, A	2, A	8, C	10, E	9, E	5, D	19, W
8	Z	A, D, C, B, Y, E, X, W, Z.	4, A	3, A	2, A	8, C	10, E	9, E	5, D	$\boxed{19, W}$

Step 0 : A is start point → minimum cost
Step 1 : D (2) has smallest cost other than A
Step 2 : C(3) has smallest cost other than A, D
Step 3 : B(4) has smallest cost other than A, C, D
Step 4 : Y(5) has smallest cost other than A, B, C, D
Step 5 : E(8) has smallest cost other than A, B, C, D, Y
Step 6 X (10) has smallest cost other than A, B, C, D, E, Y
Step 7 W (10) has smallest cost other than A, B, C, D, E, X, Y
Step 8 Z (19) has smallest cost other than A, B, C, D, E, W, X, Y

Shortest Path A → C(3) → E(8) → W(10) → Z(19)

d.

State

0 Vertices that link A — D(Z), A: 30 ↳ Choose; D(B), A: 4; D(C), A: 3; D(D), A: 2

1 Vertices that link Z — D(W): 9 (max) ↳ Choose; D(B): 4; D(C): 3; D(D): 2

2 Vertices that link AZW — D(B), A: 4 (max) ↳ Choose; D(C), A: 3; D(D), A: 2; D(E), W: 2

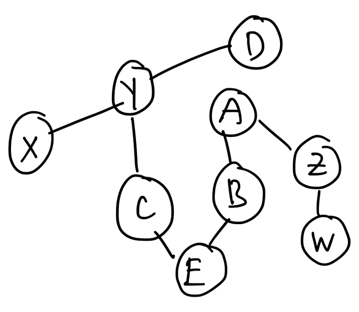
3 AZWB — D(E), B: 5 (max); D(C), A: 3; D(D), A: 2; D(E), W: 2

4 AZWBE — D(C), E: 5 (max); D(C), A: 3; D(D), A: 2; D(E), W: 2; D(X), E: 1

5 AZWBEC — D(Y), C: 6 (Max); D(C), A: 3; D(D), A: 2; D(E), W: 2; D(X), E: 1

AZWBECY — D(X), Y: 5 (max); D(C), A: 3; D(D), A: 2; D(E), W: 2; D(X), E: 1

AZWBECYX — D(C), A: 3 (Max); D(D), A: 2; D(E), W: 2; D(X), E: 1



Problem 2

a, The smallest number of edges in G is N-1 edges. In this case, the graph's is acylic because the graph has linear list, meaning it cannot make cycles

b If G is a complete graph, the number of spanning trees are n^{n-2}.
- Since there's n vertices, each vertex has n-1 edges since we're finding spanning tree
- From Cayley's formula & Kirchhoff's matrix theorem ⇒ Number of spanning tree $= \frac{1}{n} M_0 M_1 N_2 \dots N_{n-1} = \frac{n^{n-1}}{n} = n^{n-2}$

Problem 3.

a, For every vertex isolated, every edge is isolated ⇒ no 2 edges have the same vertices
Each edge need 2 vertices
⇒ Number of non-isolated vertices = $2m$
⇒ Number of isolated vertices = $n - 2m$

Therefore, the minimum number of isolated vertices is $\boxed{n - 2m}$ when $n < 2m$, meaning number of vertices smaller or equal than twice number of edges.

⇒ Algorithm : have input of number of vertices (n) and number of edges (m)
If $n \le 2m$ then return minimum = 0.
$n > 2m$ then return minimum = $n - 2m$

b. The maximum number of isolated vertices occured when all edges connect with minimum number of vertices. This only occurred in polygon that all vertex connect and has diagnal to other vertex.

The number of diagnal edges is $\frac{n(n-3)}{2}$
The number of non-diagnal edges is n
⇒ Number of edges $m = \frac{n(n-1)}{2}$

⇒ Maximum number of isolated vertices $\frac{n(n-1)}{2}$

Problem 4

a,

From \ To	0	1	2	3	4
0	∞	8	∞	9	4
1	∞	∞	1	∞	∞
2	∞	2	∞	3	∞
3	∞	∞	2	∞	7
4	∞	∞	1	∞	∞

b,

Step	V	Vertex Set	[0]	[1]	[2] ✓	[3]	[4] ✓
0	0	0	∞	8	∞	9	4 (min)
1	4	04	∞	8	5 (min)	9	4
2	2	042	∞	7 (min)	5	8	4
3	1	0421	∞	7	5	8 (min)	4
4	3	04213	∞	7	5	8	4