计算物理 HW3 Problem2

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2021 年 11 月 30 日

题目 2: 方程求根

解答: (a) 二分法

Initial condition: [a,b]=[1.5,2]

No.1 iteration: [a,b]=[1.750000,2.000000]

No.2 iteration: [a,b]=[1.875000,2.000000]

No.3 iteration: [a,b]=[1.875000,1.937500]

No.4 iteration: [a,b]=[1.875000,1.906250]

No.5 iteration: [a,b]=[1.890625,1.906250]

No.6 iteration: [a,b]=[1.890625,1.898438]

No.7 iteration: [a,b]=[1.894531,1.898438]

No.8 iteration: [a,b]=[1.894531,1.896484]

No.9 iteration: [a,b]=[1.894531,1.895508]

No.10 iteration: [a,b]=[1.895020,1.895508]

No.11 iteration: [a,b]=[1.895264,1.895508]

No.12 iteration: [a,b]=[1.895386,1.895508]

No.13 iteration: [a,b]=[1.895447,1.895508]

No.14 iteration: [a,b]=[1.895477,1.895508]

No.15 iteration: [a,b]=[1.895493,1.895508]

No.16 iteration: [a,b]=[1.895493,1.895500]

The root of x - 2sin(x)=0 is 1.895496

显然是第 16 次迭代后误差达到精度要求

牛顿法

Initial condition: x0=1.5

No.1 iteration: x1=2.076558

No.2 iteration: x2=1.910507

No.3 iteration: x3=1.895622

No.4 iteration: x4=1.895494

No.5 iteration: x5=1.895494

The root of x - 2sin(x)=0 is 1.895494

显然是第 5 次迭代后误差达到精度要求

割线法

Initial condition: x0=1.5, x1=2.076558

No.1 iteration: x1=2.076558

No.2 iteration: x2=1.847217

No.3 iteration: x3=1.890869

No.4 iteration: x4=1.895628

No.5 iteration: x5=1.895494

No.6 iteration: x6=1.895494

The root of x - 2sin(x)=0 is 1.895494

显然是第 6 次迭代后误差达到精度要求

(b) 二分法

Initial condition: [a,b]=[1.5,2]

No.1 iteration: [a,b]=[1.750000,2.000000]

No.2 iteration: [a,b]=[1.875000,2.000000]

No.3 iteration: [a,b]=[1.937500,2.000000]

No.4 iteration: [a,b]=[1.968750,2.000000]

No.5 iteration: [a,b]=[1.984375,2.000000]

No.6 iteration: [a,b]=[1.992188,2.000000]

No.7 iteration: [a,b]=[1.996094,2.000000]

No.8 iteration: [a,b]=[1.998047,2.000000]

No.9 iteration: [a,b]=[1.999023,2.000000]

No.10 iteration: [a,b]=[1.999512,2.000000]

No.11 iteration: [a,b]=[1.999756,2.000000]

No.12 iteration: [a,b]=[1.999878,2.000000]

No.13 iteration: [a,b]=[1.999939,2.000000]

No.14 iteration: [a,b]=[1.999969,2.000000]

No.15 iteration: [a,b]=[1.999985,2.000000]

No.16 iteration: [a,b]=[1.999992,2.000000]

The root of x - 2sin(x)=0 is 1.999996

显然这里二分法求解已经不适用了, 因为初始条件输入的 [a,b]=[1.5,2] 不满足 f(a)f(b)<0 条件，而 f(x) 关于 x 很敏感，f(x)=0 是病态问题

牛顿法

Initial condition: x0=1.5

No.1 iteration: x1=1.716413

No.2 iteration: x2=2.385759

No.3 iteration: x3=2.238376

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No.243 iteration: x243=1.897433

No.244 iteration: x244=1.897425

No.245 iteration: x245=1.897416

The root of x-2sin(x)=0 is 1.897416

数值结果和真实解的误差在 10^{-3} 量级，显然这里牛顿法求解已经不适用了，因为分母里含 $f'(x) = 2x - 4\sin x - 4x\cos x + 4\sin x\cos x$，在 $x = 1.897416$ 附近误差较大，对迭代求零点造成了病态影响

割线法

Initial condition: x0=1.5, x1=1.716413

No.1 iteration: x1=1.716413

No.2 iteration: x2=1.801018

No.3 iteration: x3=1.839107

No.4 iteration: x4=1.861795

No.5 iteration: x5=1.874951

No.6 iteration: x6=1.882927

No.7 iteration: x7=1.887769

No.8 iteration: x8=1.890737

No.9 iteration: x9=1.892560

No.10 iteration: x10=1.893683

No.11 iteration: x11=1.894376

No.12 iteration: x12=1.894803

No.13 iteration: x13=1.895067

No.14 iteration: x14=1.895231

No.15 iteration: x15=1.895331

No.16 iteration: x16=1.895394

No.17 iteration: x17=1.895432

No.18 iteration: x18=1.895456

No.19 iteration: x19=1.895470

No.20 iteration: x20=1.895480

The root of x - 2sin(x)=0 is 1.895480

显然是第 20 次迭代后误差达到精度要求