STATS 500 HW1

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Github repo: https://github.com/PKUniiiiice/STATS_500

```r
library(faraway)
library(ggplot2)
library(GGally)
library(reshape)
```

1 Problem 1

The dataset `teengamb` concerns a study of teenage gambling in Britain.

Glimpse

```r
data(teengamb)
head(teengamb)
```

```
  sex status income verbal gamble
1   1     51   2.00      8    0.0
2   1     28   2.50      8    0.0
3   1     37   2.00      6    0.0
4   1     28   7.00      4    7.3
5   1     65   2.00      8   19.6
6   1     61   3.47      6    0.1
```

Descriptive Statistics

Note that `sex` is a categorical variable so here we only provide descriptions for the other variables.

```r
summary(teengamb[,2:5])
```

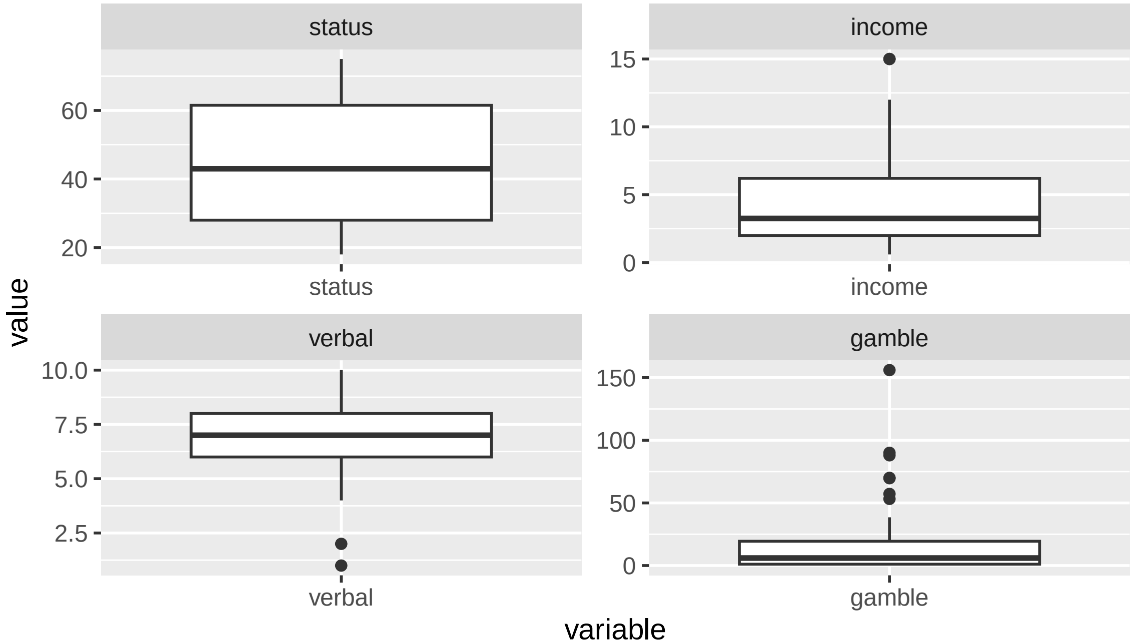
```
     status          income           verbal          gamble     
 Min.   :18.00   Min.   : 0.600   Min.   : 1.00   Min.   :  0.0  
 1st Qu.:28.00   1st Qu.: 2.000   1st Qu.: 6.00   1st Qu.:  1.1  
 Median :43.00   Median : 3.250   Median : 7.00   Median :  6.0  
 Mean   :45.23   Mean   : 4.642   Mean   : 6.66   Mean   : 19.3  
 3rd Qu.:61.50   3rd Qu.: 6.210   3rd Qu.: 8.00   3rd Qu.: 19.4  
 Max.   :75.00   Max.   :15.000   Max.   :10.00   Max.   :156.0  
```

It seems that the variable `gamble` is heavily right-skewed.

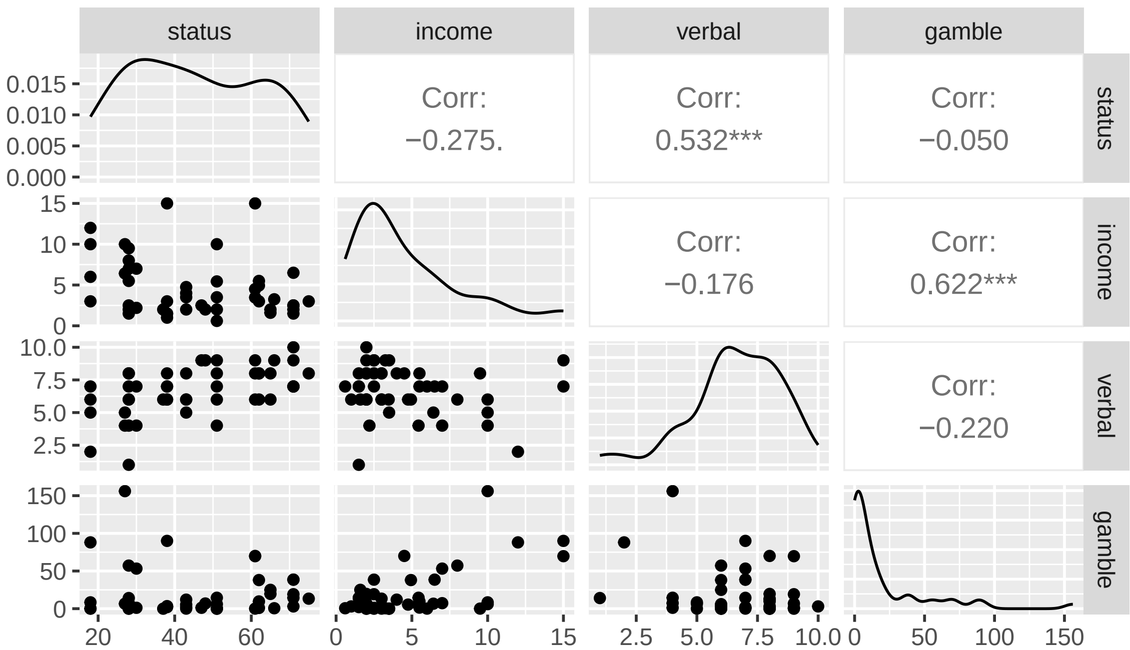
Distribution and Correlation

```r
md <- melt(teengamb[2:5], id.vars=NULL)
ggplot(md, aes(variable, value)) +
  geom_boxplot() +
```

```
    facet_wrap(~variable, scales="free")
```




```
ggpairs(teengamb[2:5])
```

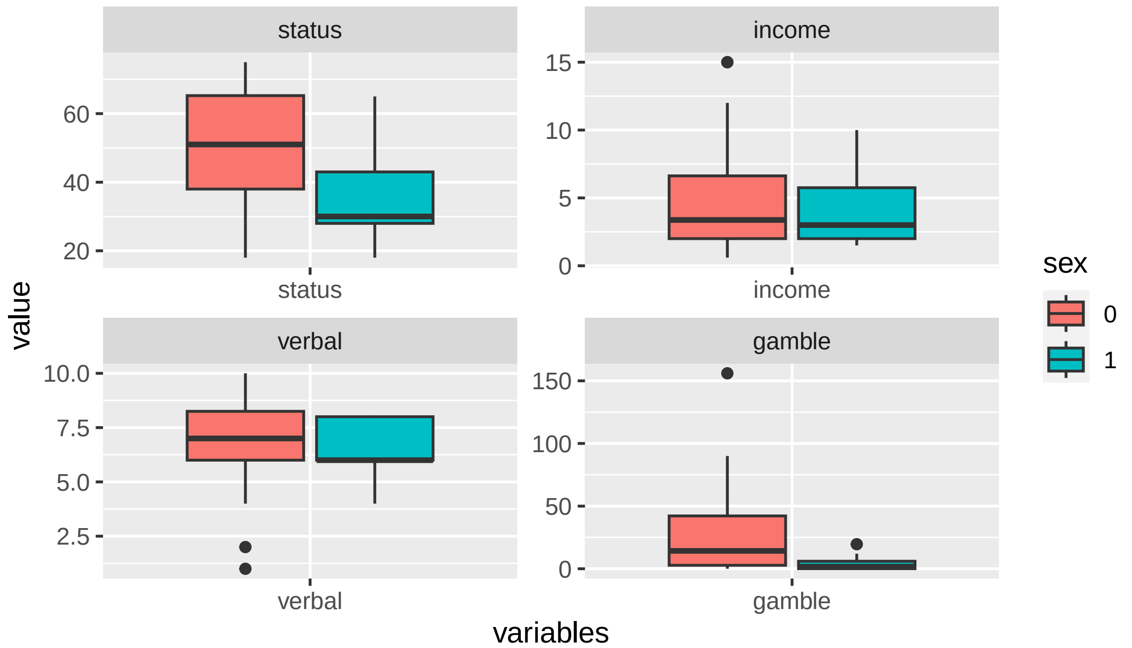


In the boxplot, we can observe outliers in all variables except for `status`, with `gamble` having the highest number of outliers.

Regarding correlation, significant linear relationships are apparent between `status` and `verbal`, as well as between `income` and `gamble`. These relationships are consistent with the meanings of these variables.

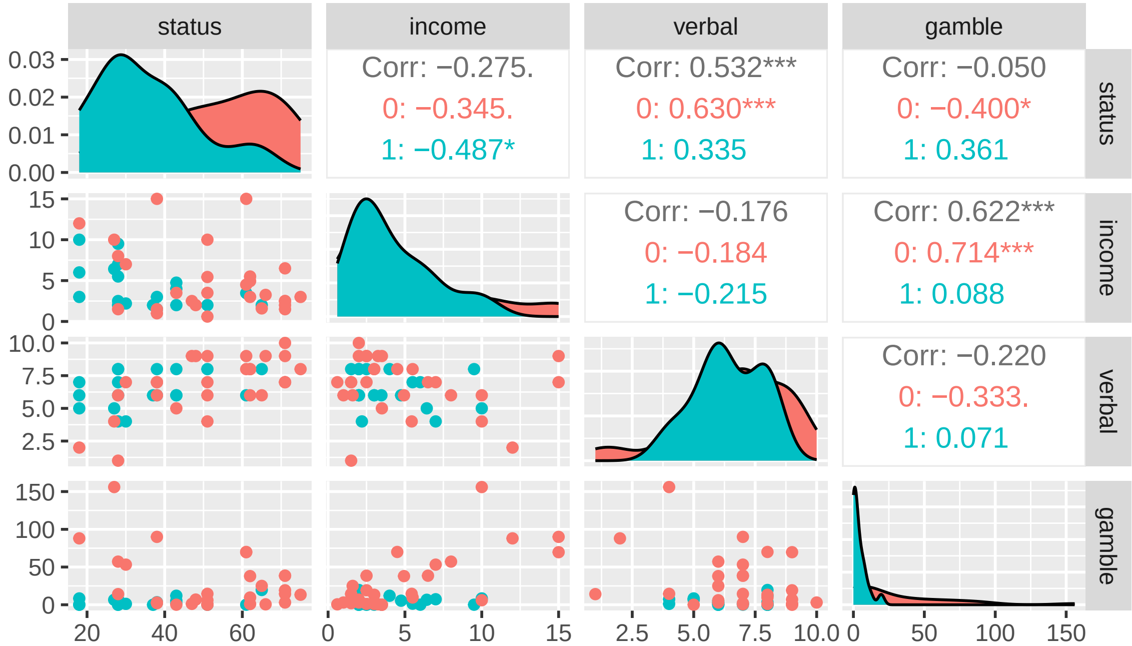
Gender Differences

```
teengamb$sex <- as.factor(teengamb$sex)
md <- melt(teengamb, id.var=c("sex"), var="variables")
p <- ggplot(data=md, aes(x=variables, y=value, fill=sex))+
      geom_boxplot()+
      facet_wrap(~variables, scale="free")
show(p)
```



In the variables `status` and `gamble`, there appears to be noticeable differences between males and females. Males tend to have higher socioeconomic status scores as well as higher expenditures on gambling.

```
ggpairs(teengamb, columns=2:5, ggplot2::aes(color=sex))
```



Note that in the relationships `status-verbal` and `income-gamble`, there are noticeable differences between males and females (i.e. different corr coef). For instance, in the case of `income-gamble`, in males, teenagers with higher income tend to have higher expenditures on gambling. However, in females, gambling expenditure remains low, regardless of their income level.

2 Problem 2

2.1 (a)

By definition, for $E(AZ) = AE(Z)$, we need to verify

$$[E(AZ)]_i = [AE(Z)]_i$$

Note that

$$\begin{aligned}
[E(AZ)]_i &= E([AZ]_i) \quad \text{(By def.)} \\
&= E\left(\sum_{j=1}^{m} A_{ij} Z_j\right) \\
&= \sum_{j=1}^{m} A_{ij} E(Z_j) \quad \text{(Linearity)} \\
&= \sum_{j=1}^{m} A_{ij} [E(Z)]_j \quad \text{(By def.)} \\
&= [AE(Z)]_i
\end{aligned}$$

Similarly, for $\operatorname{Cov}(AZ) = A\operatorname{Cov}(Z)A^\top$, we need to verify

$$\operatorname{Cov}(AZ)_{ij} = [A\operatorname{Cov}(Z)A^\top]_{ij}$$

Note that

$$\begin{aligned}
LHS &= \operatorname{Cov}([AZ]_i, [AZ]_j) \\
&= \operatorname{Cov}\left(\sum_{s=1}^m A_{is}Z_s, \sum_{t=1}^m A_{jt}Z_t\right) \\
&= \sum_{t=1}^m \sum_{s=1}^m A_{is}A_{jt}\operatorname{Cov}(Z_s, Z_t) \\
&= \sum_{t=1}^m \sum_{s=1}^m A_{is}A_{jt}\operatorname{Cov}(Z)_{st}
\end{aligned}$$

and

$$\begin{aligned}
RHS &= [A\operatorname{Cov}(Z)A^\top]_{ij} \\
&= \sum_{t=1}^m [A\operatorname{Cov}(Z)]_{it}\,[A^\top]_{tj} \\
&= \sum_{t=1}^m \left(\sum_{s=1}^m A_{is}\operatorname{Cov}(Z)_{st}\right)[A^\top]_{tj} \\
&= \sum_{t=1}^m \left(\sum_{s=1}^m A_{is}\operatorname{Cov}(Z)_{st}\right)A_{jt} \quad (A_{jt} = [A^\top]_{tj}) \\
&= \sum_{t=1}^m \sum_{s=1}^m A_{is}A_{jt}\operatorname{Cov}(Z)_{st} \\
&= LHS
\end{aligned}$$

2.2 (b)

We have

$$Y = \begin{bmatrix} Z_1 + 2Z_2 \\ Z_1 - 2Z_2 \\ -Z_1 + Z_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = AZ$$

and

$$\operatorname{Cov}(Z) = \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}$$

Therefore,

$$\operatorname{Cov}(Y) = A\operatorname{Cov}(Z)A^\top$$

```r
A <- matrix(c(1,1,-1,2,-2,1),ncol=2)
cov.z <- matrix(c(3,-1,-1,1),ncol=2)
cov.Y <- A%*%cov.z%*%t(A)
cov.Y
```

```
     [,1] [,2] [,3]
[1,]    3   -1    0
[2,]   -1   11   -8
[3,]    0   -8    6
```

and

$$\mathrm{Corr}(Y) = [\mathrm{diag}(\mathrm{Cov}(Y))]^{-\frac{1}{2}} \mathrm{Cov}(Y)[\mathrm{diag}(\mathrm{Cov}(Y))]^{-\frac{1}{2}}$$

```
diag(diag(cov.Y)^(-1/2))%*%cov.Y%*%diag(diag(cov.Y)^(-1/2))
```

```
           [,1]       [,2]       [,3]
[1,]  1.0000000 -0.1740777  0.0000000
[2,] -0.1740777  1.0000000 -0.9847319
[3,]  0.0000000 -0.9847319  1.0000000
```

3 Problem 3

It's known that

$$s_y = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2}, s_x = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$r = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{s_x \cdot s_y}$$

We go from $y = \hat{\beta}_0 + \hat{\beta}_1 x$

$$\begin{aligned}
y &= \bar{y} - \hat{\beta}_1 \bar{x} + \hat{\beta}_1 x \\
&= \bar{y} + \hat{\beta}_1 (x - \bar{x}) \\
&= \bar{y} + \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}(x - \bar{x}) \\
&= \bar{y} + \frac{n \cdot r \cdot s_x \cdot s_y}{\sum_{i=1}^{n}(x_i - \bar{x})^2}(x - \bar{x}) \\
&= \bar{y} + \frac{r \cdot s_x \cdot s_y}{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}(x - \bar{x}) \\
&= \bar{y} + \frac{r \cdot s_x \cdot s_y}{s_x^2}(x - \bar{x})
\end{aligned}$$

Therefore, we have

$$\frac{y - \bar{y}}{s_y} = r \cdot \frac{x - \bar{x}}{s_x}$$