STATS551 Homework2

Junjie Xu, Minxuan Chen

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Question1 (Junjie Xu)

Bayesian Coverage:

Scenario:

We employ Bayesian methodology, incorporating information about age (X), community engagement (Z), and screen time (V), to construct a credible interval for the influence of coffee consumption on residents from different countries.

Explanation:

In Bayesian coverage, considering all available information, including age, community engagement, and screen time, we build a probability distribution for the impact of coffee consumption. From this, we derive a credible interval. This interval signifies the uncertainty surrounding the influence of coffee consumption on specific demographic groups within the different countries considered. Decision-makers can utilize this interval to comprehend the variability in coffee consumption impact across different national groups and develop corresponding strategies.

Frequentist Coverage:

Scenario:

We employ Frequentist methodology, based on observed data (age X, community engagement Z, coffee consumption U, screen time V), to construct a confidence interval for the average coffee consumption among residents from different countries.

Explanation:

In Frequentist coverage, relying on sample data (i.e., observed age, community engagement, coffee consumption, and screen time), we estimate the average coffee consumption within various national groups. We construct a confidence interval, providing an estimate of the average coffee consumption for residents in different countries while accounting for the uncertainty in the estimation. Decision-makers can utilize this interval to compare coffee consumption patterns across different countries, gaining insights into these estimates' reliability.

Problem 2 (Minxuan Chen)

(a)

Prior

$$\theta_A \sim \text{gamma}(120,10), \theta_B \sim \text{gamma}(12,1), p\left(\theta_A,\theta_B\right) = p\left(\theta_A\right) \times p\left(\theta_B\right)$$

Likelihood

$$\begin{aligned}
&y_A|\theta_A \sim \mathcal{P}(\theta_A), y_A \text{ i.i.d.} \\
&y_B|\theta_B \sim \mathcal{P}(\theta_B), y_B \text{ i.i.d.} \\
&y_A \perp\!\!\!\perp y_B
\end{aligned}$$

Posterior

$$\theta_A|\mathbf{y}_A, \mathbf{y}_B \sim \text{gamma}(120 + 10\bar{y}_A, 10 + 10) = \text{gamma}(237, 20)$$
$$\theta_B|\mathbf{y}_A, \mathbf{y}_B \sim \text{gamma}(12 + 13\bar{y}_B, 1 + 13) = \text{gamma}(125, 14)$$

```r
ya <- c(12,9,12,14,13,13,15,8,15,6)
yb <- c(11,11,10,9,9,8,7,10,6,8,8,9,7)
sum(ya)
```

```
## [1] 117
```

```r
sum(yb)
```

```
## [1] 113
```

Posterior mean

$$E(\theta_A|\mathbf{y}_A, \mathbf{y}_B) = \frac{237}{20} = 11.85, \quad E(\theta_B|\mathbf{y}_A, \mathbf{y}_B) = \frac{125}{14} = 8.929$$

Posterior variance

$$Var(\theta_A|\mathbf{y}_A, \mathbf{y}_B) = \frac{237}{400}, \quad Var(\theta_B|\mathbf{y}_A, \mathbf{y}_B) = \frac{125}{196}$$

Posterior confidence interval

```r
#theta_A
cat("theta_A: ", qgamma(c(.025, .975), 237, 20), "\n")
```

```
## theta_A:  10.38924 13.40545
```

```r
#theta_B
cat("theta_B: ", qgamma(c(.025, .975), 125, 14))
```

```
## theta_B:  7.432064 10.56031
```

(b)

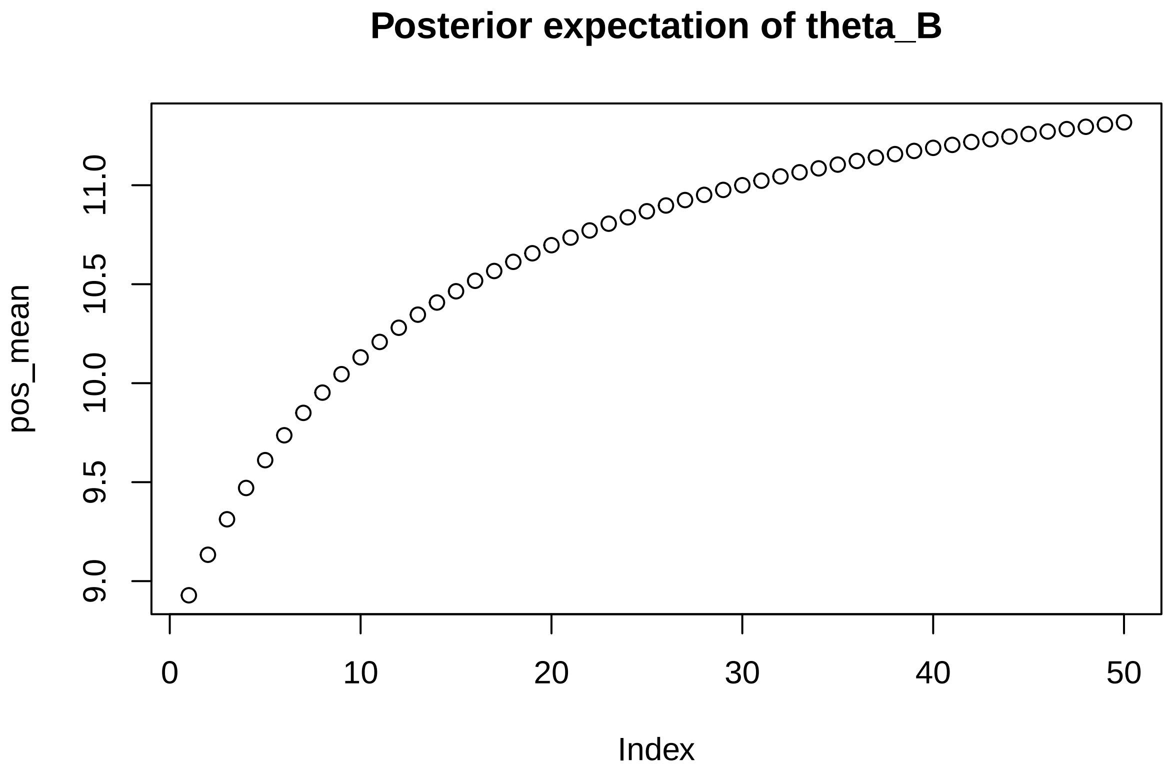
Posterior

$$\theta_B|\mathbf{y}_A, \mathbf{y}_B \sim \text{gamma}(12n_0 + 13\bar{y}_B, n_0 + 13) = \text{gamma}(12n_0 + 113, 13 + n_0)$$

```r
pos_mean <- (12*1:50+113)/(13+1:50)
print(pos_mean)
```

```
##  [1]  8.928571  9.133333  9.312500  9.470588  9.611111  9.736842  9.850000
##  [8]  9.952381 10.045455 10.130435 10.208333 10.280000 10.346154 10.407407
## [15] 10.464286 10.517241 10.566667 10.612903 10.656250 10.696970 10.735294
## [22] 10.771429 10.805556 10.837838 10.868421 10.897436 10.925000 10.951220
## [29] 10.976190 11.000000 11.022727 11.044444 11.065217 11.085106 11.104167
## [36] 11.122449 11.140000 11.156863 11.173077 11.188679 11.203704 11.218182
## [43] 11.232143 11.245614 11.258621 11.271186 11.283333 11.295082 11.306452
## [50] 11.317460
```

```r
plot(pos_mean, main="Posterior expectation of theta_B")
```




```r
#posterior expectation is (12n_0+185)/(13+n_0)
(12*1:50+113)/(13+1:50)
```

```
##  [1]  8.928571  9.133333  9.312500  9.470588  9.611111  9.736842  9.850000
##  [8]  9.952381 10.045455 10.130435 10.208333 10.280000 10.346154 10.407407
## [15] 10.464286 10.517241 10.566667 10.612903 10.656250 10.696970 10.735294
## [22] 10.771429 10.805556 10.837838 10.868421 10.897436 10.925000 10.951220
## [29] 10.976190 11.000000 11.022727 11.044444 11.065217 11.085106 11.104167
## [36] 11.122449 11.140000 11.156863 11.173077 11.188679 11.203704 11.218182
## [43] 11.232143 11.245614 11.258621 11.271186 11.283333 11.295082 11.306452
## [50] 11.317460
```

For arbitrary gamma prior, $\theta_B \sim \text{gamma}(\alpha, \beta)$, to make the posterior expectation of θ_B to be close to that of θ_A, just make them equal and solve

$$\frac{\alpha + 113}{\beta + 13} = \frac{237}{20}$$

we get

$$\alpha = 41.05 + 11.83\alpha, \quad \alpha, \beta > 0$$

Therefore, if the parameters of θ_B's prior locate at the line $\alpha = 41.05 + 11.83\beta$, we can get exactly the same posterior expectation. To get a close result, you can add small perturbation to this line.

(c)

No. Since the prior $p(\theta_A, \theta_B) = p(\theta_A) \times p(\theta_B)$ implies the independence between type A and type B. In other words

$$p(\theta_B | \mathbf{y}_A, \mathbf{y}_B) = p(\theta_B | \mathbf{y}_B)$$

We can prove it.
Proof

$$p(\theta_A, \theta_B | \mathbf{y}_A, \mathbf{y}_B) \propto p(\mathbf{y}_A, \mathbf{y}_B | \theta_A, \theta_B) p(\theta_A, \theta_B)$$

Note that type A and type B are independent given θ_A, θ_B (due to different type, i.e. two populations), so

$$\begin{aligned}
p(\theta_A, \theta_B|\mathbf{y}_A, \mathbf{y}_B) &\propto p(\mathbf{y}_A|\theta_A, \theta_B)p(\mathbf{y}_B|\theta_A, \theta_B)p(\theta_A)p(\theta_B) \\
&\propto p(\mathbf{y}_A|\theta_A)p(\theta_A) \cdot p(\mathbf{y}_B|\theta_B)p(\theta_B) \\
&\propto p(\theta_A|\mathbf{y}_A)p(\theta_B|\mathbf{y}_B)
\end{aligned}$$

The second line is because in the sampling model we assume type A or B is only influenced by its own parameter.
We integrate over θ_A,

$$p(\theta_B|\mathbf{y}_A, \mathbf{y}_B) = C \cdot p(\theta_B|\mathbf{y}_B)$$

Note that both sides are pdf of θ_B, so if we integrate over θ_B again, we must have $C = 1$, i.e.

$$p(\theta_B|\mathbf{y}_A, \mathbf{y}_B) = p(\theta_B|\mathbf{y}_B)$$

It doesn't make sense to have $p(\theta_A, \theta_B) = p(\theta_A)p(\theta_B)$ because it's known that type B mice are related to type A mice. The previous prior implies there's no connection between type A and B.

(d)

The test quantity is

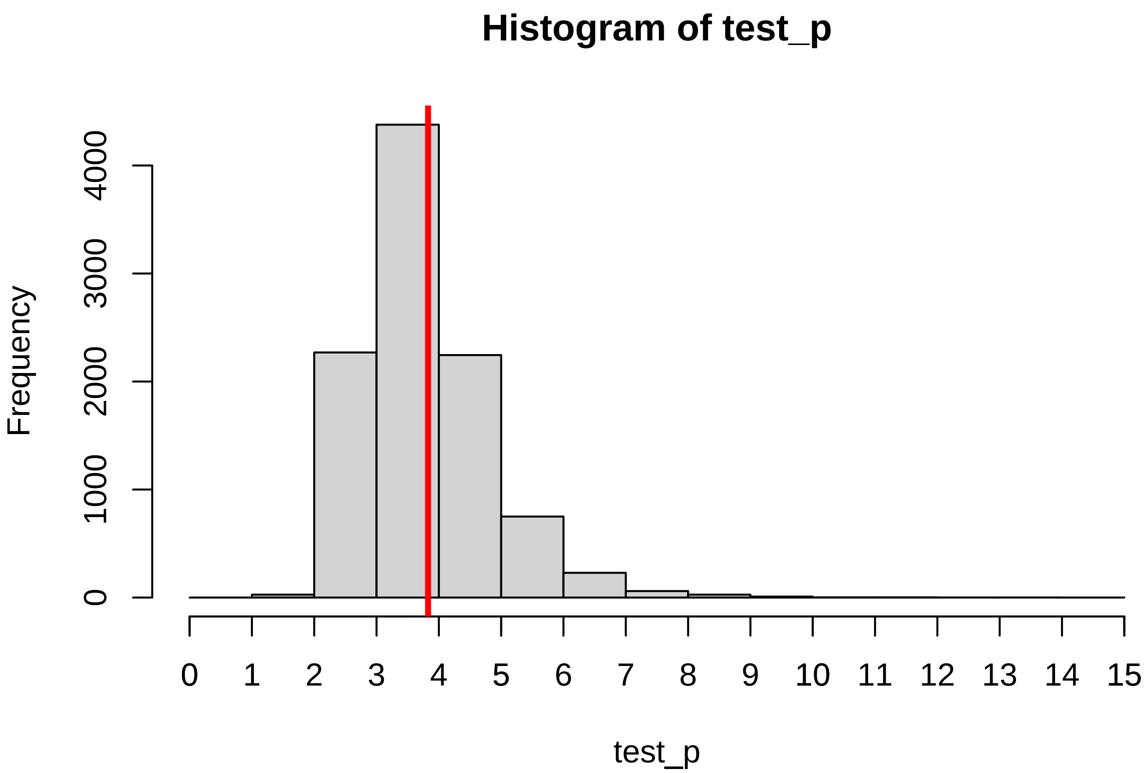
$$T(y, \theta) = \frac{\bar{\mathbf{y}}_A}{sd(\bar{\mathbf{y}}_A)}$$

We need to calculate this value in the replicated data.

```r
set.seed(551)
S <- 10000

post_test <- function(a,b,n){
  #sample posterior theta
  theta_p <- rgamma(S,a,b)

  #sample posterior y, one row is one replicated data
  y_p <- matrix(rpois(n*S, rep(theta_p, n)), ncol=n)

  #calculate posterior test quantity
  test_p <- rowMeans(y_p)/apply(y_p, 1, sd)

  return (test_p)
}

test_p <- post_test(237, 20, 10)

#observed test quantity
test_obs <- mean(ya)/sd(ya)
#calculate bayesian p-value
hat_p_value <- mean(test_p>=test_obs)
print(hat_p_value)
```

```
## [1] 0.3937
```

```r
hist(test_p, breaks=seq(0, 15), cex=1, xlim=c(0,15))
axis(side = 1, at = seq(0,15))
abline(v=test_obs, col='red', lwd=3)
```



From the Bayesian p-value (0.3937, not extreme) and the distribution of test quantity, we conclude that using Poisson sampling model is appropriate and adequate.

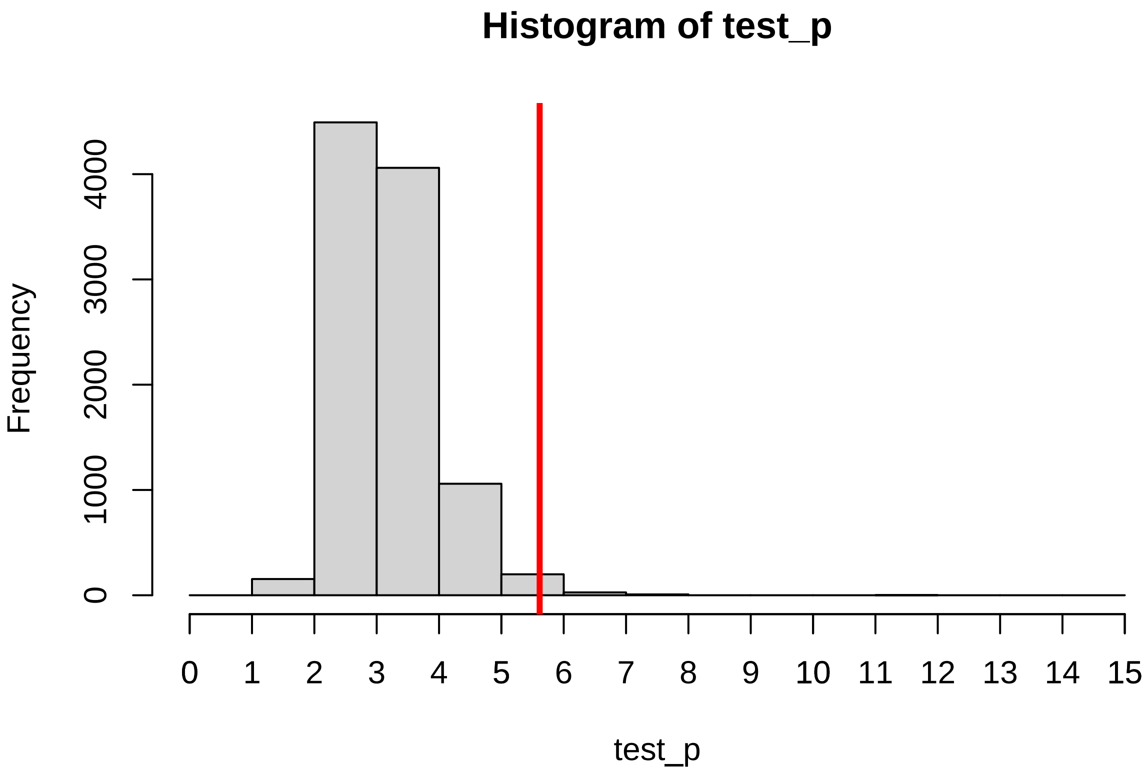
(e)

Similarly

```r
test_p <- post_test(125, 14, 13)

#observed test quantity
test_obs <- mean(yb)/sd(yb)
#calculate bayesian p-value
hat_p_value <- mean(test_p>=test_obs)
print(hat_p_value)
```

```
## [1] 0.0083
```

```r
hist(test_p, breaks=seq(0, 15), cex=1, xlim=c(0,15))
axis(side = 1, at = seq(0,15))
abline(v=test_obs, col='red', lwd=3)
```



Note that the Bayesian p-value is extreme (0.0083), which shows that the Poisson sampling model and the prior of type B is not adequate.

Problem 3 (Minxuan Chen)

(a)

Prior

$$\theta \sim \Gamma(\alpha, \beta) \propto \theta^{\alpha-1} e^{-\beta\theta}$$

Likelihood

$$y_i|\theta \sim \mathcal{E}(\theta) \propto \theta e^{-\theta y_i}, y_i \text{ i.i.d.}$$
$$\rightarrow p(\vec{y}|\theta) \propto \prod_i \theta e^{-\theta y_i} = \theta^n e^{-\theta \sum_i y_i} = \theta^n e^{-\theta \cdot n\bar{y}}$$

Posterior

$$p(\theta|\vec{y}) \propto p(\vec{y}|\theta)p(\theta) \propto \theta^n e^{-\theta \cdot n\bar{y}} \theta^{\alpha-1} e^{-\beta\theta} \propto \theta^{\alpha+n-1} e^{-(\beta+n\bar{y})\theta}$$
$$\sim \Gamma(\alpha + n, \beta + n\bar{y})$$

Since the posterior distribution of θ is still gamma distribution, which is in the same probability distribution family as the prior $p(\theta)$, by definition, the gamma prior distribution is conjugate for exponential distribution likelihood.

(b)

For $\Gamma(\alpha, \beta)$, the coefficient of variation is

$$\frac{\sqrt{\alpha/\beta^2}}{\alpha/\beta} = \frac{1}{\sqrt{\alpha}}$$

It's known that this value is 0.5, so for the prior distribution

$$\frac{1}{\sqrt{\alpha}} = 0.5 \rightarrow \alpha = 4$$

In the posterior distribution of θ, the coefficient of variation is

$$\frac{\sqrt{\alpha+n/(\beta+n\bar{y})^2}}{\alpha+n/(\beta+n\bar{y})}=\frac{1}{\sqrt{\alpha+n}}=0.1$$

Therefore

$$n=96$$

So, 96 light bulbs need to be tested.

(c)

We need to consider the prior and posterior distribution of $\phi=1/\theta$. Since the prior of θ is $\Gamma(\alpha,\beta)$, the prior of ϕ should be

$$\phi\sim\text{Inv-}\Gamma(\alpha,\beta)$$

the coefficient of variation is

$$\frac{\sqrt{\beta^2/((\alpha-1)^2(\alpha-2))}}{\beta/(\alpha-1)}=0.5\rightarrow\alpha=6$$

Similarly, the posterior of ϕ should be

$$\phi|\vec{y}\sim\text{Inv-}\Gamma(\alpha+n,\beta+n\bar{y})$$

so

$$\frac{\sqrt{(\beta+n\bar{y})^2/((\alpha+n-1)^2(\alpha+n-2))}}{(\beta+n\bar{y})/(\alpha+n-1)}=0.1\rightarrow n=96$$

The answer doesn't change.

(d)

Prior

$$\theta\sim\Gamma(\alpha,\beta)\propto\theta^{\alpha-1}e^{-\beta\theta}$$

Data

$$y|\theta\sim\mathcal{E}(\theta),y\geq 100,\text{ and no exact value}$$

Since we don't know the exact value of y, so now the condition of posterior becomes $y\geq 100$,i.e.

$$\begin{aligned}p(\theta|y\geq 100)&\propto p(y\geq 100|\theta)p(\theta)\propto\left(\int_{100}^{+\infty}\theta e^{-\theta y}dy\right)\cdot\theta^{\alpha-1}e^{-\beta\theta}\\&\propto e^{-100\theta}\theta^{\alpha-1}e^{-\beta\theta}\\&\propto\theta^{\alpha-1}e^{-(\beta+100)\theta}\\&\sim\Gamma(\alpha,\beta+100)\end{aligned}$$

Therefore, posterior mean and variance is

$$E(\theta|y\geq 100)=\frac{\alpha}{\beta+100},\quad Var(\theta|y\geq 100)=\frac{\alpha}{(\beta+100)^2}$$

(e)

By part (a), when $y=100$, posterior distribution of θ is

$$\theta|y=100\sim\Gamma(\alpha+1,\beta+100)$$

Posterior mean and variance is

$$E(\theta|y=100)=\frac{\alpha+1}{\beta+100},\quad Var(\theta|y=100)=\frac{\alpha+1}{(\beta+100)^2}$$

We find that this posterior variance is higher than that in part (d).

As for the reason, by the law of total variance

$$Var(\theta) = Var(E(\theta|Y)) + E(Var(\theta|Y))$$

thus

$$Var(\theta) \geq Var(E(\theta|Y)), Var(\theta) \geq E(Var(\theta|Y))$$

Note that we can apply these inequalities to conditional variance, i.e.

$$Var(\theta|y \geq 100) \geq E(Var(\theta|y)|y \geq 100)$$

This inequality implies that, **on average**, the variance of θ decreases as we acquire more information.

However, it's important to note that simply taking $y = 100$ on the right-hand side is different from averaging over the distribution $y|y \geq 100$.

In this particular case, the increase in variance may be attributed to a numerical coincidence rather than a natural phenomenon. For instance, in (e), if we are told that $y = 100000$, it's highly probable that the variance would decrease, as the term $(\beta + 100000)^2$ significantly influences the comparison between $\alpha/(\beta + 100)^2$ and $(\alpha + 1)/(\beta + 100000)^2$.

Question4 (Junjie Xu)

The joint density of nine observations is

$$\begin{aligned} p(Y_1, ..., Y_9|\theta, \sigma^2) &= \prod_{i=1}^{9} p(Y_i|\theta, \sigma^2) \\ &= (2\pi\sigma^2)^{-\frac{9}{2}} \exp\left\{ \frac{1}{2\sigma^2} \sum_{i=1}^{9} (y_i - \theta)^2 \right\} \end{aligned}$$

Assume that the prior distribution of θ follows normal distribution with

$$\theta \sim \mathcal{N}(\mu_0, \tau_0^2)$$

From the lecture notes, we know the posterior distribution of $\theta \sim \mathcal{N}(\mu_n, \tau_n^2)$, with

$$\mu_n = \frac{\frac{1}{\tau_0^2}\mu_0 + \frac{n}{\sigma^2}\bar{y}}{\frac{1}{\tau_0^2} + \frac{n}{\sigma^2}}, \tau_n^2 = \frac{1}{\frac{1}{\tau_0^2} + \frac{n}{\sigma^2}}$$

From the instruction, we know that $\bar{y} = 17.653$, $\sigma = 0.12$, $n = 9$, and $\tau_0 \to \infty$. Then,

$$\begin{aligned} \mu_n &= \frac{\frac{9}{0.12^2} \times 17.653}{\frac{9}{0.12^2}} = 17.653 \\ \tau_n^2 &= \frac{1}{\frac{9}{0.12^2}} = 0.0016 \end{aligned}$$

Thus, the posterior distribution follows $\theta|y_1, ..., y_9, \sigma^2 \sim \mathcal{N}(17.653, 0.0016)$.

Then, for the predictive distribution, we know $Y_{10}|\sigma^2, y_1, ..., y_9 \sim \mathcal{N}(\mu_n, \tau_n^2 + \sigma^2)$.

Here, we can compute $\tau_n^2 + \sigma^2 = 0.016$. So,

$$Y_{10}|\sigma^2, y_1, ..., y_9 \sim \mathcal{N}(17.653, 0.016)$$

```r
z_lower <- qnorm(0.005)
z_upper <- qnorm(0.995)

mean <- 17.653
std_dev <- sqrt(0.016)

lower_limit <- mean + z_lower * std_dev
upper_limit <- mean + z_upper * std_dev

cat("lower bound of 99% confidence interval:", lower_limit, "\n")
```

```
## lower bound of 99% confidence interval: 17.32718
```

```r
cat("upper bound of 99% confidence interval:", upper_limit, "\n")
```

```
## upper bound of 99% confidence interval: 17.97882
```

Use the formula calculation, the lower bound is $17.653 + \sqrt{0.016} \times Z_{0.005} = 17.32718$, and the upper bound is $17.653 + \sqrt{0.016} \times Z_{0.995} = 17.97882$.

Thus, the 99% confidence interval is $(17.32718, 17.97882)$.

Question5 (Junjie Xu)

###(a)

```r
# Loading Data

y1 = scan('./dataset/school1.dat')
y2 = scan('./dataset/school2.dat')
y3 = scan('./dataset/school3.dat')
```

```r
# Preparation

#Prior Parameters
mu0=5; s20=4; k0=1; nu0=2
# prior
n_1 = length(y1); ybar_1 = mean(y1); s2_1 = var(y1)
n_2 = length(y2); ybar_2 = mean(y2); s2_2 = var(y2)
n_3 = length(y3); ybar_3 = mean(y3); s2_3 = var(y3)
```

Here, the posterior parameters are

$$
\begin{aligned}
\kappa_n &= \kappa_0 + n \\
\mu_n &= \frac{\frac{\kappa_0}{\sigma^2}\mu_0 + \frac{n}{\sigma^2}\bar{y}}{\frac{\kappa_0}{\sigma^2} + \frac{n}{\sigma^2}} = \frac{\kappa_0\mu_0 + n\bar{y}}{\kappa_n} \\
\nu_n &= \nu_0 + n \\
\sigma_n^2 &= \frac{1}{\nu_n}\left[\nu_0\sigma_0^2 + (n-1)s^2 + \frac{\kappa_0 n}{\kappa_n}(\bar{y}-\mu_0)^2\right]
\end{aligned}
$$

```r
#Posterior parameters
nun_1 = nu0 + n_1 ; kn_1 = k0 + n_1
nun_2 = nu0 + n_2 ; kn_2 = k0 + n_2
nun_3 = nu0 + n_3 ; kn_3 = k0 + n_3

# Posterior inference
```

```r
mun_1 = (k0*mu0 + n_1*ybar_1)/kn_1
s2n_1 = (nu0*s20 + (n_1-1)*s2_1 + k0*n_1*(ybar_1-mu0)^2/(kn_1))/nun_1
mun_2 = (k0*mu0 + n_2*ybar_2)/kn_2
s2n_2 = (nu0*s20 + (n_2-1)*s2_2 + k0*n_2*(ybar_2-mu0)^2/(kn_2))/nun_2
mun_3 = (k0*mu0 + n_3*ybar_3)/kn_3
s2n_3 = (nu0*s20 + (n_3-1)*s2_3 + k0*n_3*(ybar_3-mu0)^2/(kn_3))/nun_3
```

```r
# monte carlo sampling
set.seed(551)

s2sample_1 = 1/rgamma(n=10000, shape=nun_1/2, rate=s2n_1*(nun_1/2))
s2sample_2 = 1/rgamma(n=10000, shape=nun_2 / 2, rate=s2n_2*(nun_2/2))
s2sample_3 = 1/rgamma(n=10000, shape=nun_3 /2, rate=s2n_3*(nun_3/2))

thetasample_1 = rnorm(n=10000, mean=mun_1, sd=sqrt(s2sample_1/kn_1))
thetasample_2 = rnorm(n=10000, mean=mun_2, sd=sqrt(s2sample_2/kn_2))
thetasample_3 = rnorm(n=10000, mean=mun_3, sd=sqrt(s2sample_3/kn_3))

postpred_y1 = rnorm(n=10000, mean=thetasample_1 ,sd=sqrt(s2sample_1))
postpred_y2 = rnorm(n=10000, mean=thetasample_2 ,sd=sqrt(s2sample_2))
postpred_y3 = rnorm(n=10000, mean=thetasample_3 ,sd=sqrt(s2sample_3))
```

```r
result.a = rbind(
c(mean(thetasample_1), quantile(thetasample_1 , c(.025,.975)),
mean(sqrt(s2sample_1)), quantile(sqrt(s2sample_1), c(.025,.975))), c(mean(thetasample_2),
        quantile(thetasample_2, c(.025,.975)),
mean(sqrt(s2sample_2)), quantile(sqrt(s2sample_2), c(.025,.975))), c(mean(thetasample_3),
        quantile(thetasample_3, c(.025,.975)),
mean(sqrt(s2sample_3)), quantile(sqrt(s2sample_3), c(.025,.975))))

result.a
```

```
##                    2.5%    97.5%             2.5%    97.5%
## [1,] 9.293830 7.771127 10.77869 3.902452 2.993188 5.150213
## [2,] 6.953888 5.163655  8.73679 4.393086 3.346825 5.859199
## [3,] 7.810491 6.178661  9.44799 3.753616 2.790842 5.148818
```

The output is solved above, with mean values in the first and fourth columns.

(b)

```r
t123 <- mean(thetasample_1 < thetasample_2 & thetasample_2 < thetasample_3)
t132 <- mean(thetasample_1 < thetasample_3 & thetasample_3 < thetasample_2)
t213 <- mean(thetasample_2 < thetasample_1 & thetasample_1 < thetasample_3)
t231 <- mean(thetasample_2 < thetasample_3 & thetasample_3 < thetasample_1)
t312 <- mean(thetasample_3 < thetasample_1 & thetasample_1 < thetasample_2)
t321 <- mean(thetasample_3 < thetasample_2 & thetasample_2 < thetasample_1)

cbind(t123, t132, t213, t231, t312, t321)
```

```
##        t123  t132   t213   t231   t312   t321
## [1,] 0.0052 0.004 0.0811 0.6766 0.0154 0.2177
```

$$P(\theta_1 < \theta_2 < \theta_3) = 0.0052$$
$$P(\theta_1 < \theta_3 < \theta_2) = 0.004$$
$$P(\theta_2 < \theta_1 < \theta_3) = 0.0811$$
$$P(\theta_2 < \theta_3 < \theta_1) = 0.6766$$
$$P(\theta_3 < \theta_1 < \theta_2) = 0.0154$$
$$P(\theta_3 < \theta_2 < \theta_1) = 0.2177$$

(c)

```
y123 <- mean(postpred_y1 < postpred_y2 & postpred_y2 < postpred_y3)
y132 <- mean(postpred_y1 < postpred_y3 & postpred_y3 < postpred_y2)
y213 <- mean(postpred_y2 < postpred_y1 & postpred_y1 < postpred_y3)
y231 <- mean(postpred_y2 < postpred_y3 & postpred_y3 < postpred_y1)
y312 <- mean(postpred_y3 < postpred_y1 & postpred_y1 < postpred_y2)
y321 <- mean(postpred_y3 < postpred_y2 & postpred_y2 < postpred_y1)

cbind(y123, y132, y213, y231, y312, y321)
```

```
##        y123   y132   y213  y231   y312   y321
## [1,] 0.1028 0.1055 0.1843 0.276 0.1365 0.1949
```

$$P(\tilde{Y}_1 < \tilde{Y}_2 < \tilde{Y}_3) = 0.1028$$
$$P(\tilde{Y}_1 < \tilde{Y}_3 < \tilde{Y}_2) = 0.1055$$
$$P(\tilde{Y}_2 < \tilde{Y}_1 < \tilde{Y}_3) = 0.1843$$
$$P(\tilde{Y}_2 < \tilde{Y}_3 < \tilde{Y}_1) = 0.276$$
$$P(\tilde{Y}_3 < \tilde{Y}_1 < \tilde{Y}_2) = 0.1365$$
$$P(\tilde{Y}_3 < \tilde{Y}_2 < \tilde{Y}_1) = 0.1949$$

(d)

```
t1max <- mean(thetasample_1 > thetasample_2 & thetasample_1 > thetasample_3)
y1max <- mean(postpred_y1 > postpred_y2 & postpred_y1 > postpred_y3)
cbind(t1max, y1max)
```

```
##       t1max  y1max
## [1,] 0.8943 0.4709
```

$$P(\theta_1 > \max\{\theta_2, \theta_3\} = 0.8943)$$
$$P(\tilde{Y}_1 > \max\{\tilde{Y}_2, \tilde{Y}_3\} = 0.4709)$$