6. As a corollary to both Eq.s(1,2), prove the expression for the sum ("or"):

$$p(X+Y|I) = p(X|I) + p(Y|I) - p(XY|I) \tag{14}$$

Hint – Think how to relate "X+Y" to products of the X, Y propositions and their negations, which can be then manipulated using the rules of logic.

From a logical perspective, the phrase "X or Y" is equivalent to the phrace "X and Y or X and not Y or not X and Y". Or expressed mathematically: $X + Y = XY + X\bar{Y} + \bar{X}Y$

The following set is exhaustive: $\{xy, \bar{x}y, x\bar{y}, \bar{x}\bar{y}\}$ Therefore

$p(xy|I) + p(\bar{x}y|I) + p(\bar{y}x|I) + p(\bar{x}\bar{y}|I) = 1$. On the other hand

$p(x+y|I) + p(\overline{x+y}|I) = 1$ Noting that $\overline{x+y} = \bar{x}\bar{y}$

we have $p(x+y|I) = 1 - p(\overline{x+y}|I) = 1 - p(\bar{x}\bar{y})$

$= p(xy|I) + p(\bar{x}y|I) + p(yx|I) + p(\cancel{\bar{x}\bar{y}|I}) - p(\cancel{\bar{x}\bar{y}|I}) =$

$= p(xy|I) + p(\bar{x}y|I) + p(\bar{y}x|I)$

$\therefore P(X+Y|I) = P(XY|I) + P(\bar{X}Y|I) + P(X\bar{Y}|I) =$

$= P(X|Y,I)\,P(Y|I) + P(\bar{X}|Y,I)\,P(Y|I) + P(X|\bar{Y},I)\,P(\bar{Y}|I)$

$= P(Y|I)\underbrace{\left[P(X|Y,I) + P(\bar{X}|Y,I)\right]}_{=1} + P(X|\bar{Y},I)\,P(\bar{Y}|I)$

$= P(Y|I) + P(X|\bar{Y},I)\,P(\bar{Y}|I)$ Commutativity

$= P(Y|I) + P(\bar{Y}|X,I)\,P(X|I)$

$$= P(Y|I) + \underbrace{\left(1 - P(Y|X, I)\right)}_{P(Y|X,I) + P(\bar{Y}|X,I) = 1} p(x|I)$$

$$= P(Y|I) + P(X|I) - \underbrace{P(Y|X,I)\,P(X|I)}_{P(X,Y|I)}$$

$$= \boxed{P(Y|I) + P(X|I) - P(X,Y|I)}$$