# AN EFFECTIVE FIELD THEORY FOR HEAVY QUARKS AT LOW ENERGIES ☆

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I construct a Lorentz invariant effective field theory description of QCD in the presence of heavy quarks at energies large compared to the QCD scale and small compared to the heavy quark mass, formalizing the ideas of Isgur and Wise and of Eichten and Hill. The theory is built by "integrating in" degrees of freedom to implement a superselection rule for the velocity of the heavy quark.

## 1. Introduction

This note represents the author's attempt to understand a series of brilliant papers [1–7] on the properties of heavy quark systems in QCD, where the energy scale for the QCD processes is small compared to the heavy quark mass. I will recast the results of these papers into the language of a low-energy effective field theory in a new way, which preserves the Lorentz invariance and which also explicitly exhibits the enormous set of new approximate symmetries that appear in the limit of large quark mass. I hope that the formalism set up here will make the results of refs. [1–7] accessible to a wider audience and will also make the underlying assumptions more clear and explicit.

## 2. The velocity superselection rule

The essential idea is a simple one. A heavy quark in a QCD bound state with light quarks carries most of the energy and momentum of the system. If the system is moving with 4-velocity $v^\mu$ ($v_\mu v^\mu = 1$, $v^0 < 0$), then the 4-momentum of the system has the form

$$P^\mu = m v^\mu , \tag{1}$$

where $m$ is the heavy quark mass, which for large $m$ is essentially the same as the mass of the bound state. For definiteness, consider the scattering of the heavy quark system by an external potential, into a new state of the same heavy quark, with momentum

$$P^\mu = m v'^\mu + k^\mu . \tag{2}$$

As $m \to \infty$ for fixed $k^\mu$, we must have $v^\mu = v'^\mu$. In the limit of infinite mass, the *velocity* of the heavy quark is unchanged by the scattering with fixed momentum transfer. We may hope that this is true not only for the final states, but also for *virtual* heavy quarks that appear in QCD radiative corrections to this process, because after appropriate renormalization (if such a thing exists) the typical momentum in virtual processes will be bounded (and of the same order as $k^\mu$). We can, of course, imagine a different process (or the same process in a different frame of reference) in which the heavy quark has a different velocity, but the two heavy quarks with different velocities do not talk to one another at all, because their momenta are infinitely different, and we are interested only in bounded momentum transfers.

In other words, when the momentum transfer is held fixed, as $m \to \infty$, there is a velocity superselection rule. We can keep track of the velocity of the heavy quark, independent of whatever else is going on (unless we explicitly annihilate the heavy quark and somehow absorb the infinite momentum through some external agency). Thus in the effective field theory description of the process, we will include *a different heavy quark field for each velocity*. That is,

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our heavy quark field will depend not only on space–time, but on $v^\mu$. We will denote this heavy quark field by $h_v$.

In the effective field theory description of processes like the scattering of a heavy system by an external potential, we want to eliminate the trivial, kinematic dependence of $P^\mu$ on the heavy quark mass. To implement this, we redefined the heavy quark fields so that the mass dependence is entirely in the commutator of the momentum operator with the field:

$$[P^\mu, h_v(x)] = (-m\not{v}v^\mu - \mathrm{i}\partial^\mu)\, h_v(x)\ . \tag{3}$$

The relation of $h_v$ with the usual heavy quark field $\psi$ is the following:

$$h_v(x) = \exp(\mathrm{i}m\not{v}v_\mu x^\mu)\ \psi(x)\ . \tag{4}$$

In the full QCD theory, it would be crazy to redefine the field in this way, because of the explicit dependence on the velocity, $v^\mu$. However, in the effective low energy theory, we have a separate heavy quark field for each $v^\mu$ because of the velocity superselection rule, so (4) is appropriate. With this redefinition, the mass dependence disappears from the heavy quark lagrangian, which becomes

$$\mathrm{i}\bar{h}_v\ \not{\partial} h_v\ . \tag{5}$$

We can simplify this still further if we divide up the field into two component subfields:

$$h_v = h_v^+ + h_v^-\ , \tag{6}$$

where

$$\not{v}h_v^\pm = \pm h_v^\pm\ . \tag{7}$$

Then if the residual momentum is small compared to the mass, $h_v^+$ annihilates heavy quarks with momentum $mv^\mu$, and $h_v^-$ creates heavy antiquarks with momentum $mv^\mu$. Now for the same reason that we are not interested in changes in the heavy quark velocity, we are not interested in terms in the lagrangian that produce a quark–antiquark pair. At low energies, such terms are irrelevant. If we ignore these terms, the heavy quark lagrangian becomes

$$\mathscr{L}_v = \mathrm{i}\overline{h_v^+}\ \not{\partial} h_v^+ + \mathrm{i}\overline{h_v^-}\ \not{\partial} h_v^- = \mathrm{i}\bar{h}_v\ \not{v}v_\mu\ \partial^\mu h_v\ . \tag{8}$$

This is the generalization of the Eichten–Hill lagrangian [6] to an arbitrary reference frame and to include heavy antiquarks as well as quarks.

To implement the mass superselection rule, we simply add the lagrangians for all values of $\boldsymbol{v}$ in a Lorentz-invariant way:

$$\mathscr{L}_h = \int \frac{\mathrm{d}^3 v}{2v^0}\, \mathscr{L}_v\ . \tag{9}$$

The lagrangian, (9), involves many more apparent degrees of freedom than the corresponding QCD lagrangian. It is equivalent, by the physical argument above, when the heavy quark mass goes to infinity for fixed distinct heavy quark velocities. This is very different from the usual procedure of integrating out heavy particle degrees of freedom. We have actually *"integrated" the extra degrees of freedom in!* For finite heavy quark mass we may get into trouble if we look at a process involving two or more heavy quarks or antiquarks with the same mass and very similar velocities, because then small residual momenta can cause confusion.

If we canonically quantize the lagrangian (9), we find

$$\{h_v, h_{v'}^\dagger\} = \not{v}\gamma^0 \delta_{v'v}\ . \tag{10}$$

Note that the $\delta$-function in (10) is a Kronecker $\delta$, not a Dirac $\delta$-function. The different values of $\boldsymbol{v}$ behave like independent particles, because of the velocity superselection rule.

Diagrammatically the velocity superselection rule is the statement that each heavy quark momentum flows through each Feynman diagram only on the corresponding heavy quark line, or into an external current or other source specifically designed to emit or absorb that heavy quark.

One advantage of this scheme is that after the mass superselection rule has been implemented, so that we have removed the kinematic dependence on the heavy quark mass, we can renormalize the effective theory with a mass independent procedure such as $\overline{\mathrm{MS}}$.

As in a conventional effective field theory, the matching is done at the boundary. In this instance, that means at a scale of the order of the heavy quark mass. Of course, at this scale, the effective field theory picture is totally useless for actually calculating anything, because the terms suppressed by powers of the heavy quark mass will be just as important as the leading terms. Nevertheless, the velocity superselection rule plus a mass independent renormalization scheme gives us an unambiguous prescription for finding the coefficients of the leading terms in the effective theory. As these terms are then evolved down to lower momentum scale, using the renormalization group, they become more important relative to the higher dimension terms. This is precisely analogous to the situation in a standard effective field theory in which the heavy quark degrees of freedom are integrated out rather than in.

## 3. Spin symmetry

The lagrangian, (9), has an enormous internal symmetry, including a symmetry under separate rotations in the spin space for each heavy quark and antiquark velocity. Define the "spin" operators for some fixed $v^\mu$:

$$S_j^\pm = \mathrm{i}\epsilon_{jkl}[\not{e}_k, \not{e}_l](1 \pm \not{v})/2\,, \tag{11}$$

where $e_j^\mu$ for $j=1, ..., 3$ is an orthonormal set of space-like vectors orthogonal to $v^\mu$,

$$e_{j\,\mu} e_k^\mu = -\delta_{jk}\,, \quad v_\mu e_j^\mu = 0\,. \tag{12}$$

The $S_j^\pm$ have the commutation relations of SU(2)×SU(2). Now the lagrangian is invariant under the transformation

$$\delta h_v^\pm = \mathrm{i}\boldsymbol{\epsilon}_\pm \cdot \boldsymbol{S}^\pm h_v^\pm\,. \tag{13}$$

It follows that

$$\delta \overline{h_v^\pm} = \mathrm{i}\overline{h_v^\pm}\, \boldsymbol{\epsilon}_\pm \cdot \boldsymbol{S}^\pm\,. \tag{14}$$

The transformation, (13) and (14), also preserves the form of the commutation relation (10). This transformation implements independent rotations in the spin space of the quark and antiquark of velocity $v$.

Note that there is a separate SU(2)×SU(2) spin symmetry for each $v$ because of the velocity superselection rule. The spin symmetries are the translation into the effective field theory of the fact that the spin-dependent interactions of the heavy quark are suppressed by powers of the heavy quark mass. The leading terms are spin independent. This symmetry is realized on the bound states of a heavy quark with light quarks in an obvious way. As discussed by Isgur and Wise, if there are $f$ types of heavy quark, then the spin symmetries can be extended to SU($2f$)×SU($2f$) symmetries in the tensor product space of spin and flavor. On the states of the system, these symmetries relate heavy quark bound states of different spins and flavors, but the same velocity.

If there are $f$ heavy quarks described by heavy quarks fields $h_v^j$ for $j=1, ..., f$, the leading term in the heavy quark effective lagrangian becomes

$$\mathcal{L}_h = \sum_{j=1}^{f} \int \frac{\mathrm{d}^3 v}{2v^0} \mathcal{L}_v^j\,, \tag{15}$$

where

$$\mathcal{L}_v^j = \mathrm{i}\overline{h_v^j}\, \not{v} v_\mu\, \partial^\mu h_v^j\,. \tag{16}$$

The lagrangian, (15), is invariant under the SU($2f$)×SU($2f$) symmetry generated by

$$\delta h_v^\pm = \mathrm{i}(\boldsymbol{\epsilon}_\pm \cdot \boldsymbol{S}^\pm + \epsilon'_\pm)\, h_v^\pm\,. \tag{17}$$

where $\boldsymbol{\epsilon}_\pm$ are three arbitrary hermitian matrices in flavor space and $\epsilon'_\pm$ is an arbitrary traceless hermitian matrix in flavor space. (17) contains the symmetries discussed by Isgur and Wise [5].

## 4. Conclusions

I have argued that the techniques of effective field theories at low energies can be extended to incorporate heavy quarks with masses much larger than the energies of interest, without sacrificing Lorentz invariance, or any of the other standard properties. The new feature of these low energy descriptions is the appearance of an infinite number of degrees of freedom, encoding the huge phase space of the very heavy quark. I hope that this description will be helpful in unleashing the full power of low energy effective theories for heavy quarks systems.

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