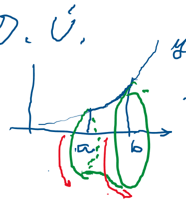
Erik 2024-04-12

Friday, April 12, 2024 4:55 PM

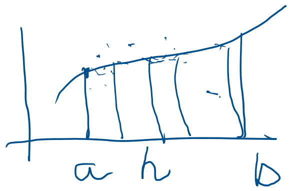
Program na dnes:
- integrály
- náhodné kráčanie a náhodné premenné

D. Ú.



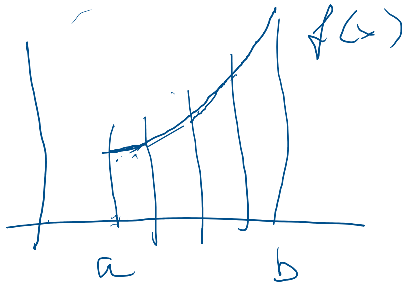
- povrch
- objem rotačného telesa

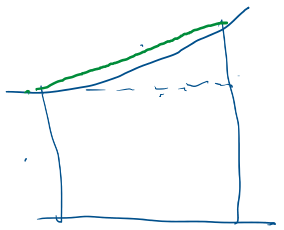
$$D_x: a < x_1 < x_2 \dots < b$$
$$D_\xi: a \le \xi_1 \le x_1 \le \xi_2 \dots \le b$$



$y = f(x)$

$$h = \frac{b-a}{n} \qquad S_{a,b} = \sum h \cdot f(a+ih)$$





$$[f(x+h)-f(x)]^2 + h^2 = l^2$$

$$l = \sqrt{h^2 + (f(x+h)-f(x))^2} = h\sqrt{1+\left(\frac{f(x+h)-f(x)}{h}\right)^2}$$

$$L = \sum_{i=0}^{n-1} h\sqrt{1+f'(a+ih)^2}, \quad n = \frac{b-a}{h}$$

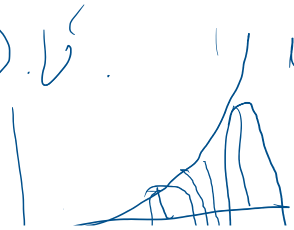
$$f(x) = x \quad a = 0 \quad b = 1$$

$$f'(a+ih) = 1, \quad i = 0, 1 \dots$$

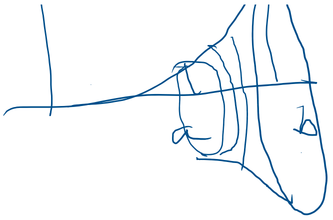
$$L = \sum_{i=1}^{n} h \cdot \sqrt{2} = \sqrt{2}\, h n = \sqrt{2}\,(b-a) = \sqrt{2}$$

$$f(x) = x^2$$

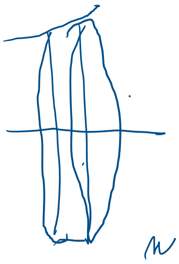
D.Ú.



$y = x^2$



$b - a = nh$



$V_i = \pi f(x_i)^2 \cdot h$

$$V = \sum_{i=1}^{n} \pi f(a+ih)^2 \cdot h = \pi h \sum_1^n (a+ih)^2$$

$\boxed{f(x) = x}$

$$= \pi h \sum_1 (a^2 + 2aih + i^2h^2) =$$

$$= \pi h a^2 n + \pi h^2 \cdot 2a \sum_1^n i + \pi h^3 \sum_1^n i^2$$

$$= \pi a^2(b-a) + \cancel{2}\pi h^2 a \frac{n(n+1)}{\cancel{2}} +$$

$$+ \pi h^3 \frac{n(n+1)(2n+1)}{6} =$$

$$= \pi a^2(b-a) + \pi a h^2 n^2 \left(1+\frac{1}{n}\right) +$$

$$+ \frac{1}{6} \pi h^3 n^3 \left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$$

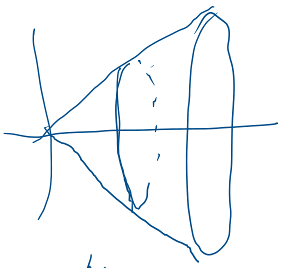
$$\xrightarrow[n\to\infty]{} \pi a^2(b-a) + \pi a(b-a)^2 +$$

$$+ \frac{1}{6} \pi (b-a)^3 \cdot 2 =$$

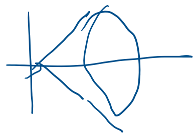
$$= \frac{\pi}{3}\left[(b-a)^3 + 3(b-a)^2 a + 3(b-a)a^2 + a^3\right] - \pi\frac{a^3}{3} =$$

$$= \frac{\pi}{3}(b-a+a)^3 - \frac{\pi}{2}a^3 = \frac{\pi}{2}(b^3 - a^3)$$

$$= \frac{\pi}{3}(b-a+a)^3 - \frac{\pi}{3}a^3 = \underline{\frac{\pi}{3}(b^3-a^3)}$$

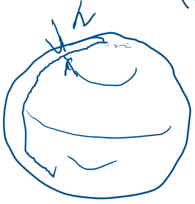


$a=0$



$$V = \frac{1}{3}\pi b^2 \cdot b = \frac{1}{3}\pi b^3$$

$=$



$$V = \frac{4}{3}\pi r^3$$

$$V^+ = \frac{4}{3}\pi(r+h)^3$$

$$S \approx \frac{V^+ - V}{h}\Big|_{h\to 0} = \frac{\frac{4}{3}\pi(r+h)^3 - \frac{4}{3}\pi r^3}{h}\Big|_{h\to 0}$$

$$= \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi 3r^2 = \underline{\underline{4\pi r^2}}$$

$$\int_a^b f(x)\,dx = \lim_{h\to 0}\sum_{i=1}^{\left[\frac{b-a}{h}\right]} h f(a+ih)$$

$$\int_a^b x^2\,dx = \frac{b^3}{3} - \frac{a^3}{3}$$

$$F(x) = \int_{-1}^{x} t^2\,dt = \frac{x^3}{3} - \frac{(-1)^3}{3}$$

$$\int_a^b t^2\,dt = F(b) - F(a)$$

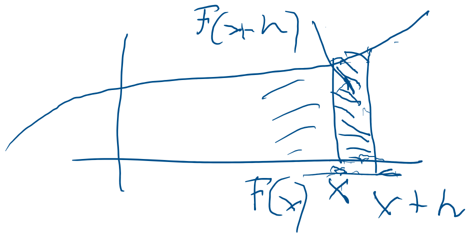
$$F(b) - F(a) = \frac{b^3}{3} - \cancel{\frac{(-1)^3}{3}} - \frac{a^3}{3} + \cancel{\frac{(-1)^3}{3}}$$

$$f(b) - f(a) = \frac{-}{3} - \cancel{\frac{\ }{3}} - \frac{\ }{3} + \cancel{\frac{\ }{3}}$$

$$F(x) = \int_{-\infty}^{x} f(t)\, dt$$

$$\Downarrow$$

$$F'(x) = f(x)$$



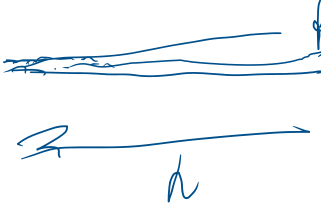
$$F'(x) = \lim_{h \to 0} \frac{F(x+h) - F(x)}{h}$$

$$= \lim_{h \to 0} f(x + \varepsilon h) \qquad 0 \leq \varepsilon \leq 1$$

$$= f(x)$$

$$x \leq \xi \leq x+h$$

$$h \to 0 \qquad f(\xi) \to f(x)$$



$$f(x) = x^2 \qquad F(x) = ?$$

$$(x^3)' = 3x^2$$

$$\left(\frac{x^3}{3}\right)' = \frac{1}{3} 3x^2 = x^2$$

$$f(x) = \cos x \qquad F(x) = \sin x + C$$

$$f(x) = \sin x \qquad F(x) = -\cos x + C$$

$$f(x) = \frac{1}{x} \qquad F(x) = \ln|x| + C$$

$$f(x) = e^x \qquad F(x) = e^x + C$$

→ primitivna f.

$f(x) = e^x$ ↘ primitívna f.

$$f(x) = \cos(ax)$$

$$\frac{d}{dx}\left(\frac{1}{a}\sin ax\right) = \cos ax$$

$$f(ax) \rightarrow \frac{1}{a}F(ax)$$

$$x^n \rightarrow \frac{x^{n+1}}{n+1}$$

$\frac{d}{dx}$

=

Domáca úloha

Basic Integratio...

Basic Integration Problems

I. Find the following integrals.

1. $\int(5x^2 - 8x + 5)dx$
2. $\int(-6x^3 + 9x^2 + 4x - 3)dx$
3. $\int(x^{\frac{1}{2}} + 2x + 3)dx$
4. $\int\left(\frac{8}{x} - \frac{5}{x^2} + \frac{6}{x^3}\right)dx$
5. $\int(\sqrt{x} + \frac{1}{3\sqrt{x}})dx$
6. $\int(12x^{\frac{3}{4}} - 9x^{\frac{5}{3}})dx$
7. $\int\frac{x^2 + 4}{x^2}dx$
8. $\int\frac{1}{x\sqrt{x}}dx$
9. $\int(1 + 3t)t^2\,dt$
10. $\int(2t^2 - 1)^2\,dt$
11. $\int y^2\sqrt[3]{y}\,dy$
12. $\int d\theta$
13. $\int 7\sin(x)dx$
14. $\int 5\cos(\theta)d\theta$
15. $\int 9\sin(3x)dx$
16. $\int 12\cos(4\theta)d\theta$
17. $\int 7\cos(5x)dx$
18. $\int 4\sin\left(\frac{x}{3}\right)dx$
19. $\int 4e^{-7x}dx$
20. $\int 9e^{\frac{x}{4}}dx$
21. $\int -5\cos\ \pi x\ \ dx$
22. $\int -13e^{6t}dt$

II. Evaluate the following definite integrals.

1. $\int_{1}^{4}(5x^2 - 8x + 5)dx$

2. $\int_{1}^{9}(x^{\frac{3}{2}} + 2x + 3)dx$

3. $\int_{4}^{9}(\sqrt{x} + \frac{1}{3\sqrt{x}})dx$

4. $\int_{1}^{4}\frac{5}{x^3}dx$

5. $\int_{-1}^{2}(1 + 3t)t^2\, dt$

6. $\int_{-2}^{1}(2t^2 - 1)^2\, dt$

Solutions

I. Find the following integrals.

1. $\int(5x^2 - 8x + 5)dx = \boxed{\frac{5x^3}{3} - 4x^2 + 5x + C}$

2. $\int(-6x^3 + 9x^2 + 4x - 3)dx = \boxed{\frac{-3x^4}{2} + 3x^3 + 2x^2 - 3x + C}$

3. $\int(x^{\frac{3}{2}} + 2x + 3)dx = \boxed{\frac{2x^{\frac{5}{2}}}{5} + x^2 + 3x + C}$

4. $\int\left(\frac{8}{x} - \frac{5}{x^2} + \frac{6}{x^3}\right)dx = \int\left(\frac{8}{x} - 5x^{-2} + 6x^{-3}\right)dx$

$= 8Ln(x) - \frac{5x^{-1}}{-1} + \frac{6x^{-2}}{-2} = \boxed{8Ln(x) + \frac{5}{x} - \frac{3}{x^2} + C}$

5. $\int(\sqrt{x} + \frac{1}{3\sqrt{x}})dx = \int\left(x^{\frac{1}{2}} + \frac{1}{3}x^{-\frac{1}{2}}\right)dx$

$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1}{3}\frac{x^{\frac{1}{2}}}{\frac{1}{2}} = \boxed{\frac{2}{3}x^{\frac{3}{2}} + \frac{2}{3}x^{\frac{1}{2}} + C}$

6. $\int(12x^{\frac{3}{4}} - 9x^{\frac{5}{3}})dx = \boxed{\frac{48x^{\frac{7}{4}}}{7} - \frac{27x^{\frac{8}{3}}}{8} + c}$

7. $\int\frac{x^2 + 4}{x^2}dx = \int 1 + 4x^{-2}\ \ dx = \boxed{x - \frac{4}{x} + C}$

8. $\int\frac{1}{x\sqrt{x}}dx = \int x^{-\frac{3}{2}}dx = \boxed{-\frac{2}{\sqrt{x}} + C}$

9. $\int(1 + 3t)t^2\, dt = \int t^2 + 3t^3\ \ dt = \boxed{\frac{t^3}{3} + \frac{3t^4}{4} + C}$

10. $\int(2t^2 - 1)^2\, dt = \int 4t^4 - 4t^2 + 1\ \ dt = \boxed{\frac{4t^5}{5} - \frac{4t^3}{3} + t + C}$

11. $\int y^2 \sqrt[3]{y}\, dy = \int y^{\frac{7}{3}} dy = \boxed{\frac{3y^{\frac{10}{3}}}{10} + C}$

12. $\int d\theta = \boxed{\theta + C}$

13. $\int 7\sin(x)dx = \boxed{-7\cos(x) + C}$

14. $\int 5\cos(\theta)d\theta = \boxed{5\sin(\theta) + C}$

15. $\int 9\sin(3x)dx = \boxed{-3\cos(3x) + C}$

16. $\int 12\cos(4\theta)d\theta = \boxed{3\sin\ 4\theta\ + C}$

17. $\int 7\cos(5x)dx = \boxed{\frac{7\sin(5x)}{5} + C}$

18. $\int 4\sin\left(\frac{x}{3}\right)dx = \boxed{-12\cos\left(\frac{x}{3}\right) + C}$

19. $\int 4e^{-7x}dx = \boxed{-\frac{4e^{-7x}}{7} + C}$

20. $\int 9e^{\frac{x}{4}}dx = \boxed{36e^{\frac{x}{4}} + C}$

21. $\int -5\cos\ \pi x\ dx = \boxed{-\frac{5\sin(\pi x)}{\pi} + C}$

22. $\int -13e^{6t}dt = \boxed{-\frac{13e^{6t}}{6} + C}$

II. Evaluate the following definite integrals.

1. $\int_1^4 (5x^2 - 8x + 5)dx = \left(\frac{5x^3}{3} - 4x^2 + 5x\right)_1^4 \Bigg| = \frac{188}{3} - \frac{8}{3} = \boxed{60}$

2. $\int_1^9 (x^{\frac{1}{2}} + 2x + 3)dx = \left(\frac{2x^{\frac{5}{2}}}{5} + x^2 + 3x\right)\Bigg|_1^9 = \frac{1026}{5} - \frac{22}{5} = \boxed{\frac{1001}{5} = 200.2}$

3. $\int_4^9 (\sqrt{x} + \frac{1}{3\sqrt{x}})dx = \left(\frac{2}{3}x^{\frac{3}{2}} + \frac{2}{3}x^{\frac{1}{2}}\right)\Bigg|_4^9 = 20 - \frac{20}{3} = \boxed{\frac{40}{3} = 13.333}$

4. $\int_1^4 \frac{5}{x^3}dx = -\frac{5}{2x^2}\Bigg|_1^4 = -\frac{5}{32} + \frac{5}{2} = \boxed{\frac{75}{32} = 2.344}$

5. $\int_{-1}^2 (1+3t)t^2\, dt = \left(\frac{t^3}{3} + \frac{3t^4}{4}\right)\Bigg|_{-1}^2 = \frac{44}{3} - \frac{5}{12} = \boxed{\frac{57}{4} = 14.25}$

6. $\int_{-2}^1 (2t^2 - 1)^2\, dt = \left(\frac{4t^5}{5} - \frac{4t^3}{3} + t\right)\Bigg|_{-2}^1 = \frac{7}{15} + \frac{254}{15} = \boxed{\frac{87}{5} = 17.4}$