Erik 2024-08-23

Friday, August 23, 2024 4:56 PM

Σ minula :

$$P_k = pP_{k+1} + qP_{k-1} \qquad (R)$$

$$p+q=1$$

$$0 \le k \le N$$

$$P_0 = 0,\ P_N = 1$$

$$G(s) = \sum_{k=0}^{N} P_k s^k$$

(R) násobíme s^k a sčítáme

$$\sum_{k=0}^{N} P_k s^k = p\sum_{k=0}^{N} P_{k+1} s^k + q\sum_{k=0}^{N} P_{k-1} s^k$$

$\sum_{k=0}^{N} P_k s^k = G(s)$

$$G(s) = P_1 s^0 + P_2 s^1 + \dots P_N s^{N-1} + \cancel{P_{N+1} s^N}$$

$$\frac{1}{s}\left(P_1 s^1 + \dots + P_N s^N\right) = \frac{1}{s}\left(G(s) - P_0\right)$$

$$\sum_{k=0}^{N} P_{k-1} s^k = \cancel{P_{-1} s^0} + P_0 s + P_1 s^2 + \dots P_{N-1} s^N$$

$$= s\left(P_0 + P_1 s + \dots + P_{N-1} s^{N-1}\right) =$$

$$= s\left(G(s) - P_N s^N\right)$$

$$G(s) = \frac{p}{s}\left(G(s) - P_0\right) + qs\left(G(s) - P_N s^N\right)$$

$$G(s)\left[1 - \frac{p}{s} - qs\right] = -\frac{p}{s}P_0 - qs^{N+1}P_N$$

$$G(s) = -\frac{\frac{p}{s}P_0 + qs^{N+1}P_N}{1 - \frac{p}{s} - qs}$$

$$= +\frac{p\cancel{P_0} + qs^{N+2}\cancel{P_N}}{qs^2 - s + p}$$

$P_0 = 0,\ P_N = 1$

$$= \frac{qs^{N+2}}{qs^2 - s + p}$$

$$G(s) = \frac{q s^{N+2}}{q s^2 - s + p}$$

$$= q s^{N+2} \left(\frac{A}{s-\alpha} + \frac{B}{s-\beta} \right)$$

$\longrightarrow$ α, β sú korene $q s^2 - s + p = 0$

Ak máme iba a jeden koreň,

$$q s^2 - s + p = q (s-\alpha)^2$$

$$q s^2 - s + p = 0$$

$$s_{1,2} = \frac{1 \pm \sqrt{1 - 4pq}}{2q} = \frac{1 \pm (1-2p)}{2q} = \begin{cases} 1 \\ \frac{p}{q} \end{cases}$$

$$1 - 4pq = 1 - 4p(1-p) = 1 - 4p + 4p^2 = (1-2p)^2$$

$$q s^2 - s + p = q (s-1)\left(s - \frac{p}{q}\right)$$

$$G(s) = \frac{q s^{N+2}}{q (s-1)\left(s - \frac{p}{q}\right)} = \frac{s^{N+2}}{(s-1)\left(s - \frac{p}{q}\right)} =$$

$$= s^{N+2} \left(\frac{A}{s-1} + \frac{B}{s - \frac{p}{q}} \right)$$

$$A\left(s - \frac{p}{q}\right) + B(s-1) = 1$$

$s = 1$ $\qquad$ $s = \frac{p}{q}$

$$A = \frac{1}{1 - \frac{p}{q}} \qquad B = \frac{1}{\frac{p}{q} - 1} = -A$$

$$G(s) = s^{N+1} \frac{1}{1 - \frac{p}{q}} \left(\frac{1}{s-1} - \frac{1}{s - \frac{p}{q}} \right) =$$

$$= \frac{s^{N+1}}{1 - \frac{p}{q}} \left(-\frac{1}{1-s} + \frac{1}{\frac{p}{q}\left(1 - \frac{q}{p} s\right)} \right) =$$

$$s^{N} \left(1 - \frac{1}{\quad} - \right) -$$

$$= \frac{s^N}{1-\frac{p}{q}} \left(\frac{1}{1-\frac{1}{s}} - \frac{1}{1-\frac{p}{q}\frac{1}{s}} \right) =$$

$$= \frac{s^N}{1-\frac{p}{q}} \left(\sum_{k=0}^{\infty} s^{-k} - \left(\frac{p}{q} s\right)^{-k} \right) =$$

$$= \frac{1}{1-\frac{p}{q}} \sum_{k=0}^{\infty} \left(s^{N-k} - \left(\frac{p}{q} s\right)^{N-k} \right)$$

$P(0) = 0$

$P(N) = 1$

$P(0) = 0, N$

$$P(a)_{k=0} \quad \frac{1}{1-\frac{p}{q}} \left(1 - \left(\frac{p}{q}\right)^N\right)$$

$$G(s) = \sum_{k=0}^{\infty} \left(\frac{1 - \left(\frac{p}{q}\right)^{N-k}}{1-\frac{p}{q}} \right) s^{N-k}$$

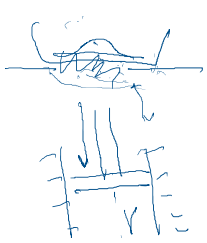
$n = N - k \qquad 0 \le k < \infty$

$-\infty < n \le N$

$$\overset{n \ge 0}{=} \sum_{n=0}^{N} \underbrace{\frac{1 - \left(\frac{p}{q}\right)^n}{1-\frac{p}{q}}}_{P_N} s^n$$

Termodynamika

Teplo a práca



$W = PdV$

Fds

$\int_S S ds$

Práca

Teplo

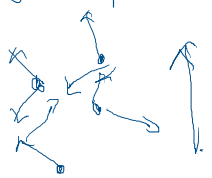
pelā d.f. ($\sim 10^{23}$)
spontánne

málo d.f. (~ 1)
zmeny ektorných parametrov

$\int F ds$ práce
$\int_S \sigma ds$
$\int_S p\, dV$

zmena vonkajších parametrov

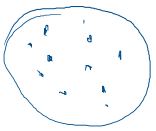
Externé parametre
- objem
- vonk. pole



Teplota

miera horúcosti telesa

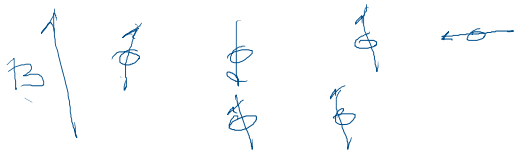
Plyn



$N \sim 10^{23}$ častíc

interagujú zrážkami

Feromagnet



$E = -B \sum_i s_i - K \sum_{i,j} s_i s_j$

Vzdialenie:
- Makr. rovnice
- $\sigma(\epsilon)$ do kelu
- Pokr. v TD