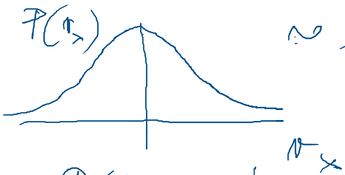
Erik 2024-09-13

Friday, September 13, 2024 4:59 PM

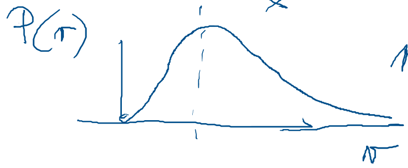
$$P = \frac{2}{3}\frac{N}{V}\left\langle \frac{1}{2} m v^2 \right\rangle$$



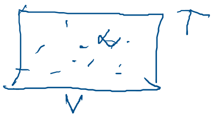
$$\sim e^{-\frac{mv^2}{2k_BT}}$$

Maxwellovo rozdelenie

$$v^2 = v_x^2 + v_y^2 + v_z^2$$



$$pV = Nk_BT$$



$$\left\langle \frac{1}{2} m v^2 \right\rangle = \frac{3}{2} k_B T$$

hmotnosť molekuly

$$\langle E_{molekuly} \rangle_{1P} = \frac{df}{2} k_B T$$

$$\int_{-\infty}^{\infty} e^{-x^2} dx$$

$$\cancel{u = x^2}$$

$$\cancel{x = \sqrt{u}}$$

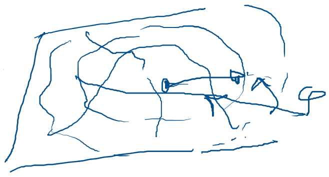
$$\cancel{dx = \frac{1}{2}\frac{1}{\sqrt{u}} du}$$

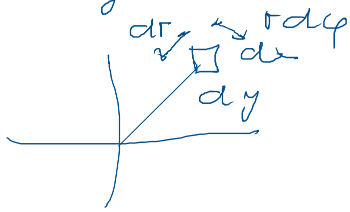
$$\frac{1}{2}\int \frac{1}{\sqrt{u}} e^{-u} du$$

$$I = \int_{-\infty}^{\infty} e^{-x^2} dx$$

$$I^2 = \int_{-\infty}^{\infty} e^{-x^2} dx \cdot \int_{-\infty}^{\infty} e^{-y^2} dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \underbrace{dx\,dy}\; e^{-\underbrace{(x^2+y^2)}_{r^2 = x^2+y^2}}$$





$$r^2 = x^2 + y^2$$

$$\varphi = \operatorname{arctg} \frac{y}{x}$$

$$dx\,dy \Rightarrow r\,d\varphi\,dr$$

$$= \int_0^{\infty} r\,dr \int_0^{2\pi} d\varphi\; e^{-r^2} = 2\pi \int^{\infty} r e^{-r^2} dr$$

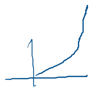
0 0 ∞

$u = r^2$
$du = 2r\,dr$

$$= \pi \int_0^\infty e^{-u}\,du$$
$$= \pi \left[-e^{-u}\right]_0^\infty =$$
$$= \pi(0-(-1)) = \pi$$

$I^2 = \pi$

$I = \sqrt{\pi}$



"Profil"

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \int_{-\infty}^{0} e^{-x^2}\,dx + \int_0^\infty e^{-x^2}\,dx$$

$u = x^2$

$x = \sqrt{u}$

$$= 2\int_0^\infty e^{-x^2}\,dx$$

$dx = \frac{1}{2}\frac{1}{\sqrt{u}}\,du$

$$I = 2\,\frac{1}{2}\int_0^\infty u^{-\frac{1}{2}} e^{-u}\,du = \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$\Gamma(z) = \int_0^\infty x^{z-1} e^{-x}\,dx$$

$\Gamma(z) = (z-1)!$ pre $z \in N$ $\quad \underbrace{\Gamma(z+1) = z\,\Gamma(z)}$

$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

$\Gamma\left(\frac{3}{2}\right) = \frac{1}{2}\cdot\Gamma\left(\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2}$ . . .

$\langle \frac{1}{2} m v^2 \rangle = \frac{3}{2} kT$

$\langle E_{mol} \rangle = \frac{d.f.}{2} kT$

$\downarrow$

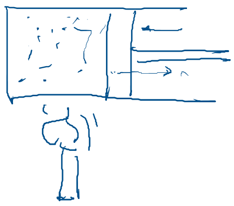
$\frac{E}{N} \longrightarrow U$ celková energia molekúl plynu
vnútorná energia

$dQ = dU + dW$

(mn. tepla) (zmena vn.) (práca)

$$dQ = dU + dW$$

(mn. tepla priv. do systému) (zmena vn. energie) (práca vykoná telesom)



Nemáme prácu:

$$dQ = dU$$

$$dQ = \frac{3}{2} N k_B \, dT$$

$$\frac{dQ}{dT} = \frac{3}{2} N k_B = C_V$$ tepelná kapacita

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V$$

Máme prácu:

$$F = p \cdot S$$

$$dW = p \cdot \underbrace{S \cdot dl}_{dV} = p\, dV$$

$$dQ = dU + p\, dV$$
$$= \frac{3}{2} NkT + NkT = \frac{5}{2} NkT$$

$$C_p = \left(\frac{\partial U}{\partial T}\right)_p = \frac{5}{2} NkT = C_V + Nk$$

1 mól:

$$C_p = C_V + N_A k_B = C_V + R$$

$$C_X = \left(\frac{\partial U}{\partial T}\right)_X$$

Adiabatický systém

= izolovaný

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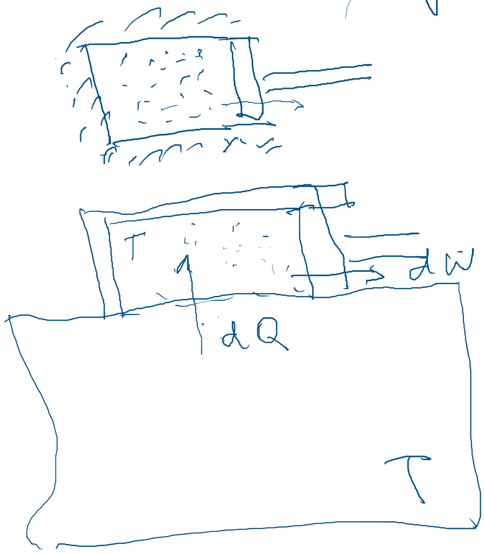
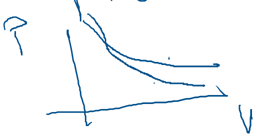


$$p = \frac{N}{V} k_B T$$

$p \cdot V =$ konst

izotermický proces



$$dQ = 0$$

$$0 = dU + p\,dV$$

$$0 = C_V dT + \frac{N}{V} k_B T\, dV$$

$$0 = C_V \frac{dT}{T} + N k_B \frac{dV}{V}$$

$$0 = \frac{dT}{T} + \frac{N k_B}{C_V} \frac{dV}{V}$$

$$0 = d\left(\ln T + \frac{N k_B}{C_V} \ln V \right)$$

$$\ln T + \frac{N k_B}{C_V} \ln V = K$$

$$T V^{\frac{N k_B}{C_V}} = K'$$

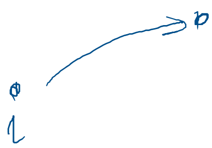
$$T V^{\varkappa - 1} = K$$

$$N k_B = C_p - C_V$$

$$\frac{N k_B}{C_V} = \underbrace{\frac{C_p}{C_V}}_{\varkappa} - 1$$

$$pV = N k_B T$$

$$T = pV \cdot \frac{1}{N k_B}$$



$$p V V^{\varkappa - 1} = K''$$

$$p V^{\varkappa} = K''$$

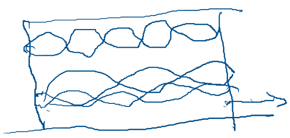
$$p_1 V_1^{\varkappa} = p_2 V_2^{\varkappa}$$

C_p $\qquad \underline{5}\, Nk$

$$PV^{\varkappa} = k$$

$$\varkappa = \frac{C_p}{C_V}$$

$$\frac{\frac{5}{2}Nk}{\frac{3}{2}Nk} = \frac{5}{3}$$



$$\psi(x)$$

$$P(x) \sim |\psi|^2$$

nabudúce Ut 18^{30}:

- nevratné procesy
- entropia