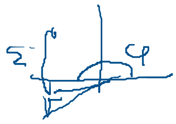
Erik 2024-07-05

Friday, July 5, 2024 4:52 PM

Δ v sferických súradniciach

$$\Delta f(r) = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial f}{\partial r}$$

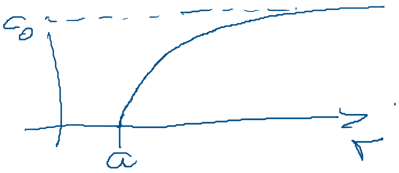
v cyl. súradniciach (r, φ, z)



$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial f}{\partial r}$$

$\Delta c = 0$ $\quad c_\infty = c_0$

$c(a) = 0$

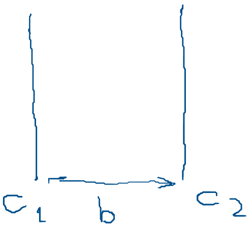


$$c = c_0\left(1 - \frac{a}{r}\right)$$

$$J = -D \operatorname{grad} c = -Dc_0\left(+\frac{a}{r^2}\right) = -Dc_0\frac{a}{r^2}$$

$$I = \left|4\pi a^2 J_{r=a}\right| = +4\pi a^2 D c_0 \frac{a}{a^2} = +4\pi D a\, c_0$$

$$I = \frac{1}{R}c_0 \qquad R = \frac{1}{4\pi D a}$$



$$\frac{\partial c}{\partial t} = 0$$

$$\Delta c = 0 \qquad \frac{\partial^2 c}{\partial c^2} = 0$$

$$c = Ax + B$$

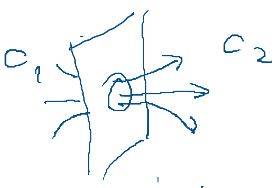
$c(0) = c_1 \qquad c_1 = A\cdot 0 + B \qquad B = c_1$

$c(b) = c_2 \qquad c_2 = Ab + c_1 \qquad A = \dfrac{c_2 - c_1}{b}$

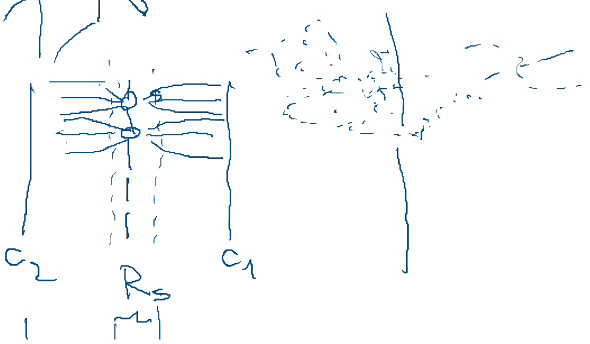
$$c = \frac{c_2 - c_1}{b}x + c_1$$

$$J = -D \operatorname{grad} c = -D\frac{c_2 - c_1}{b}$$

$$I = \left|AJ\right|_0\Big| = \underbrace{\boxed{\frac{AD}{b}}}_{1/R}(c_2 - c_1) \qquad R = \frac{b}{AD}$$

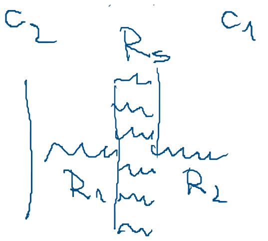


$$I = 2Ds(c_2 - c_1) \qquad R = \frac{1}{2Ds}$$



$$R = \underbrace{R_1 + R_2}_{\frac{b}{AD}} + \underbrace{\frac{R_s}{n}}_{\frac{1}{2Dsn}}$$

$$= \frac{b}{AD} + \frac{1}{2Dsn}$$



$$= \frac{b}{AD} + \frac{1}{2Dsn}$$

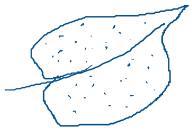
$$\frac{I}{I_0} = \frac{\dfrac{1}{b/AD + 1/2Dsn}\cancel{(c_2-c_1)}}{1/(b/AD)\cancel{(c_2-c_1)}}$$

$$= \frac{b/AD}{b/AD + 1/2Dsn} = \frac{1}{1 + \dfrac{A}{2bsn}}$$

$$\frac{I}{I_0} = \frac{1}{2} \qquad \frac{A}{2bsn} = 1$$

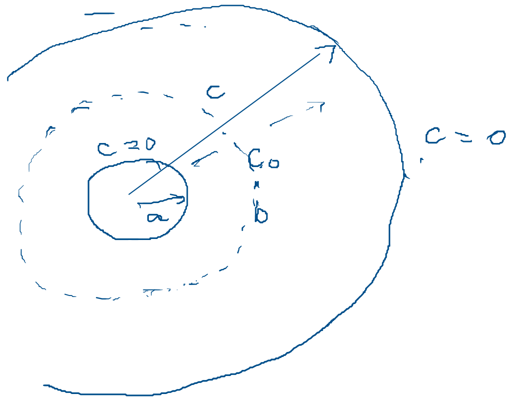
$A_s = 4\pi s^2$ plocha dierky

$$\frac{nA_s}{A} = \frac{4\pi s^2}{2bs} = 2\pi\frac{s}{b}$$



Difúzia so záchytom

- P(záchytu)
- čas do záchytu
-



r	a	b	c
c	0	c_0	0

$\Delta c = 0$

$\langle a, b \rangle$

$$\cancel{\frac{1}{r^2}}\frac{\partial}{\partial r} r^2 \frac{\partial c}{\partial r} = 0$$

$$r^2 \frac{\partial c}{\partial r} = K_1$$

$$\frac{\partial c}{\partial r} = \frac{K_1}{r^2}$$

$$c(r) = K_2 - \frac{K_1}{r}$$

$c(a) = 0$

$$K_2 - \frac{K_1}{a} = 0$$

$$K_2 = \frac{K_1}{a}$$

$c(b) = c_0$

$$\frac{K_1}{a} - \frac{K_1}{b} = c_0$$

$$K_1 = \frac{c_0}{\frac{1}{a} - \frac{1}{b}} = \frac{c_0 ab}{b-a}$$

$$c(r) = \frac{K_1}{a} - \frac{K_1}{r}$$

$$= \left(\frac{1}{a} - \frac{1}{r}\right)\frac{c_0 ab}{b-a}$$

$$= \frac{c_0 b}{b-a}\left(1 - \frac{a}{r}\right)$$

$$c(r) = \frac{c_0 b}{b-a}\left(1 - \frac{a}{r}\right), \quad a \le r \le b$$

$b - a$

$b \le r \le c$

$c(r) = K_2 - \frac{K_1}{r}$

$c(b) = c_0$

$c(c) = 0$

$0 = K_2 - \frac{K_1}{c} \quad K_2 = \frac{K_1}{c}$

$c(b) = c_0$

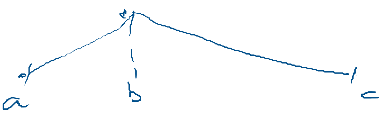
$c_0 = \frac{K_1}{c} - \frac{K_1}{b} \quad K_1 = \frac{c_0}{\frac{1}{c} - \frac{1}{b}} = \frac{-c_0 bc}{c-b}$

$c(r) = -\frac{c_0 bc}{c-b}\left(\frac{1}{c} - \frac{1}{r}\right)$

$$\boxed{\begin{array}{l} c(r) = \frac{c_0 b}{c-b}\left(\frac{c}{r} - 1\right) \\ b \le r \le c \end{array}}$$

$r = c$ ✓

$r = b$ ✓



$J_a = -D \operatorname{grad} c\big|_a = -D\frac{c_0 ba}{(b-a)a^2} = -D\frac{c_0 b}{(b-a)a}$

$I_a = 4\pi a^2 |J_a| = 4\pi D c_0 \frac{ab}{b-a}$

$I_c = 4\pi c^2 |J_c| = 4\pi D c_0 \frac{bc}{c-b}$

$P(a) = \frac{I_a}{I_a + I_c} = \frac{\frac{ab}{b-a}}{\frac{ab}{b-a} + \frac{bc}{c-b}} = \frac{1}{1 + \frac{c}{a}\frac{b-a}{c-b}} =$

$= \frac{a(c-b)}{\cancel{ac} - ab + cb - \cancel{ac}} = \frac{a(c-b)}{b(c-a)}$

$\lim_{c\to\infty} P(a) = \lim_{c\to\infty} \frac{a}{b}\cancel{\frac{c}{c}}\frac{\left(1 - \frac{b}{c}\right)}{\left(1 - \frac{a}{c}\right)} = \frac{a}{b}$