Erik 2024-08-16

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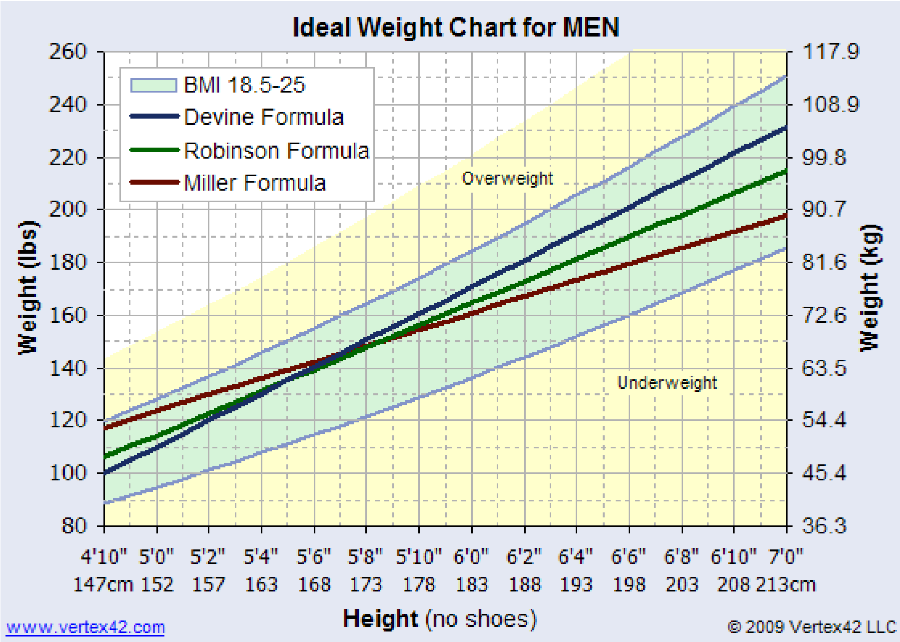

Pravdepodobnost záchytu

P_k

$0 \quad 1 \quad 2 \quad \ldots \quad a$

$x_i \to \begin{cases} p \to x_i + 1 \\ q \to x_i - 1 \end{cases} \qquad p + q = 1$

$$\boxed{P_k = p P_{k+1} + q P_{k-1}}$$

$$\boxed{P_k \sim C \lambda^k}$$

$$C\lambda^k = p C \lambda^{k+1} + q C \lambda^{k-1}$$

$$1 = p\lambda + q/\lambda$$

$$p\lambda^2 - \lambda + q = 0$$

$$\lambda_{1,2} = \frac{1 \pm \sqrt{1 - 4pq}}{2p}$$

$$1 - 4pq = 1 - 4p(1-p) = 1 - 4p + 4p^2 =$$

$$= (2p-1)^2 = (p-q)^2$$

$$\lambda_{1,2} = \frac{1 \pm (p-q)}{2p} = \begin{cases} \frac{1+p-q}{2p} = 1 \\ 1 - p + q \quad \ldots q \end{cases}$$

$$\lambda_{1,2} = \frac{1 \pm (p-q)}{2p} = \begin{cases} \frac{\quad + \quad}{2p} = 1 \\ \frac{1-p+q}{2p} = \frac{q}{p} \end{cases}$$

$\lambda_1 = 1$

$\lambda_2 = \frac{q}{p}$

$$P_k = C_1 + C_2 \left(\frac{q}{p}\right)^k$$

$P_0 = 0 \quad P_0 = C_1 + C_2 = 0 \quad C_1 = -C_2$

$P_a = 1 \quad P_a = 1 = C_1 + C_2 \left(\frac{q}{p}\right)^a$

$= -C_2 + C_2 \left(\frac{q}{p}\right)^a$

$$C_1 = -\frac{1}{-1 + \left(\frac{q}{p}\right)^a} \qquad C_2 = \frac{1}{-1 + \left(\frac{q}{p}\right)^a}$$

$$P_k = \frac{-1 + \left(\frac{q}{p}\right)^k}{-1 + \left(\frac{q}{p}\right)^a}$$

$q = p \quad \frac{q}{p} = 1 \quad P_k = \frac{-1+1}{-1+1}$

$\{P_k\}_0^\infty \qquad G(s) = \sum P_k s^k$

Vytvárajúca funkcia

$G(s) \xrightarrow{?} P_k$?

$G(s) = P_0 + P_1 s + P_2 s^2 + \dots$

$P_0 = G(s)\Big|_{s=0}$

$G'(s) = P_1 + 2P_2 s + 3P_3 s^2 \dots$

$P_1 = G'(s)\Big|_{s=0}$

$G'' = 2P_2 + 6P_3 s + \dots$

$$G'' = 2P_2 + 6P_3 s + \dots$$

$$P_2 = \frac{1}{2} G''(s) \Big|_{s=0}$$

$$P_k = \frac{1}{k!} G^{(k)}(s) \Big|_{s=0}$$

$$P_k = pP_{k+1} + qP_{k-1} \qquad / \times s^k$$

$$G(s) = \sum P_k s^k$$

$$\sum_{k=0}^{\infty} P_k s^k = p \sum_{k=0}^{\infty} P_{k+1} s^k + q \sum_{k=0}^{\infty} P_{k-1} s^k$$

$$G(s) =$$

$$\sum_{k=0}^{\infty} P_{k+1} s^k = \frac{1}{s} \sum_{k=0}^{\infty} P_{k+1} s^{k+1}$$

$$= \frac{1}{s} \sum_{k=1}^{\infty} P_k s^k = \frac{1}{s}\left(G(s) - P_0\right)$$

$$k+1 \to k$$

$$\sum_{k=0}^{\infty} P_{k-1} s^k = s \sum_{k=0}^{\infty} P_{k-1} s^{k-1} = s\,G(s)$$

$$G(s) = \frac{p}{s}\left(G(s) - P_0\right) + q\,s\,G(s)$$

$$G(s) = \frac{p}{s} G(s) - \frac{p}{s} P_0 + q\,s\,G(s)$$

$$\left(1 - \frac{p}{s} - qs\right) G(s) = -\frac{p}{s} P_0$$

$$G(s) = \frac{-\frac{p}{s} P_0}{1 - \frac{p}{s} - qs} = \frac{-pP_0}{(s-\alpha)(s-\beta)}$$

α, β sú korene

α, β sú korene

$s - p - q s^2 = 0$

$$= \frac{B}{s-\alpha} + \frac{C}{s-\beta}$$

$$\frac{1}{s-\alpha} = -\frac{1}{\alpha\left(1-\frac{s}{\alpha}\right)} = -\frac{1}{\alpha}\sum\left(\frac{s}{\alpha}\right)^k$$

$$\frac{1}{s-\beta} = \frac{1}{\beta\left(1-\frac{s}{\beta}\right)} = -\frac{1}{\beta}\sum\left(\frac{s}{\beta}\right)^k$$

$$G(s) = -\sum_{k=0}^{\infty} \underbrace{\left(\frac{A}{\alpha}\alpha^{-k} + \frac{B}{\beta}\beta^{-k}\right)}_{P_k} s^k$$

$$s - p - q s^2 = 0$$

$$q s^2 - s + p = 0$$

$$s_{1,2} = \frac{1 \pm \sqrt{1-4pq}}{2q}$$

$$= \frac{1 \pm (p-q)}{2q} = \begin{cases} \frac{p}{q} & \beta \\ 1 & \alpha \end{cases}$$

$$G(s) = pP_0 \sum_{k=0}^{\infty} \underbrace{\left(A + B\left(\frac{p}{q}\right)^{-k-1}\right)}_{P_k} s^k$$

$$P_k = pP_0\left(A + B\left(\frac{q}{p}\right)^{k+1}\right)$$

$$\frac{1}{1-x} = 1 + x + x^2 + \dots$$

$$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + \dots$$

$n\, x^{n-1}$

$$\frac{}{(1-x)^2} = 1 + 2x + 3x^2 + \dots$$

$$nx^{n-1}$$

$$\left(\frac{1}{1-x}\right)' = (-1)\frac{1}{(1-x)^2}\cdot(-1) = \frac{1}{(1-x)^2}$$

$$\left((1-x)^{-1}\right)' = (-1)(1-x)^{-2}(-1) = \frac{1}{(1-x)^2}$$