Další funkcie na dnes:

$$y = \ln(x - \sqrt{x^2-1})$$

$$y = \frac{3x^3 - 1}{2x^2 - 2}$$

$$y = \frac{x}{\sqrt{a^2 - x^2}}$$

$$y = \ln\left(\frac{1+x}{1-x}\right)$$

$$y = \ln(x - \underbrace{\sqrt{x^2-1}}_{x^2 \geq 1})$$

D(f)

$$\boxed{x \geq 1}$$

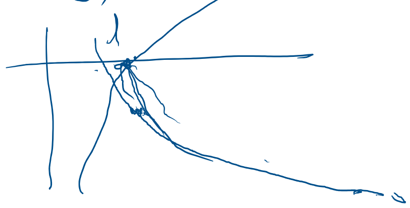
$$x - \underbrace{\sqrt{x^2-1}}_{\leq x} > 0 \qquad x \geq 1$$

x	$f(x)$
1	0
2	$\ln(2-\sqrt{3}) < 0$
10	$\ln(10-\sqrt{99}) \ll 0$

$$x - \sqrt{x^2-1} = x - x\sqrt{1 - \frac{1}{x^2}}$$

$$= x - x\left(1 - \frac{1}{2x^2}\right) = \frac{1}{2x}$$

$$\ln\left(\frac{1}{2x}\right) = -\ln(2x) = -\ln 2 - \ln x$$



$$f'(x) = \left[\ln\left(x - \sqrt{x^2-1}\right)\right]' =$$

$$\frac{1}{x - \sqrt{x^2-1}} \cdot \left(1 - \frac{1}{2\sqrt{x^2-1}} \cdot 2x\right) =$$

$$\frac{}{x-\sqrt{x^2-1}} \quad \left(1 - \frac{}{2\sqrt{x^2-1}}\right)$$

$$\bigcirc^{1/2} \to \bigcirc^{-\frac{1}{2}}$$

$$= \frac{1}{x-\cancel{\sqrt{x^2-1}}} \cdot \frac{\cancel{\sqrt{x^2-1}}-x}{\sqrt{x^2-1}} = -\frac{1}{\sqrt{x^2-1}}$$

$$f'(x) = -\frac{1}{\sqrt{x^2-1}}, \quad x \geq 1$$

$$x \to \quad f'(x) \sim \frac{1}{x}$$

$$f'(x) < 0 \quad x \geq 1$$

$$f: y = \frac{x}{\sqrt{a^2-x^2}} = \frac{\frac{x}{a}}{\sqrt{1-\left(\frac{x}{a}\right)^2}}$$

$$a \neq 0$$

$$D(f) = a^2 - x^2 > 0$$

$$-a < x < a$$

x	$f(x)$
0	0
$+a$	$+\infty$
$-a$	$-\infty$
$\frac{a}{2}$	$1/\sqrt{3}$
$-\frac{a}{2}$	$-\frac{1}{\sqrt{3}}$

$$\frac{\frac{a}{2}}{\sqrt{a^2-\left(\frac{a}{2}\right)^2}} = \frac{\frac{a}{2}}{a\sqrt{\frac{3}{4}}} = \frac{1}{\sqrt{3}}$$

$$\lim_{x\to a^-} \frac{x}{\sqrt{a^2-x^2}} = \lim_{x\to a^-} \frac{x}{\sqrt{a+x}\sqrt{a-x}} =$$

$$= \sqrt{\frac{a}{2}} \lim_{x\to a^-} \frac{1}{\sqrt{a-x}} = +\infty$$

Podobne

$$\lim \quad \frac{x}{} = \sqrt{\frac{a}{}} \quad \lim \quad \frac{1}{}$$

podobne

$$\lim_{x\to -a^+} \frac{x}{\sqrt{a^2-x^2}} = \sqrt{\frac{a}{2}} \lim_{x\to -a^+} \frac{1}{\sqrt{a+x}}$$

$$f'(x) = \frac{x'}{\sqrt{a^2-x^2}} + x\left(\frac{1}{\sqrt{a^2-x^2}}\right)' =$$

$$= \frac{1}{\sqrt{a^2-x^2}} + x\left(-\frac{1}{2}\right)\frac{1}{(a^2-x^2)}^{-3/2}(-2x)$$

$$= \frac{a^2-x^2+x^2}{(a^2-x^2)^{3/2}} = \underline{\underline{\frac{a^2}{(a^2-x^2)^{3/2}}}}$$

$$y = \ln\frac{1+x}{1-x}$$

$$\boxed{\frac{1+x}{1-x} > 0}$$

$$1-x \neq 0$$

$$1+x > 0 \quad\quad 1-x > 0 \quad \text{alebo} \quad 1+x < 0 \quad\quad 1-x < 0$$

$$x > -1 \quad\quad x < 1$$

$$\cancel{x < -1 \quad\quad x > 1}$$

$$x \to 1^-$$

$$\frac{1+x}{1-x} > 0, \quad \to +\infty$$

$$D(f) = (-1, 1)$$

$$x \to -1^+$$

$$\frac{1+x \to 0}{1-x \to 2}$$

x	$\ln\frac{1+x}{1-x}$
0	0
$+\frac{1}{2}$	$\ln\frac{3/2}{1/2} = \ln 3$

$+\frac{1}{2}$: $\ln \frac{3/2}{1/2} = \ln 3$

$-\frac{1}{2}$: $\ln \frac{1/2}{3/2} = \ln \frac{1}{3} = -\ln 3$

$$f(x) = \ln \frac{(1+x)}{1-x}$$

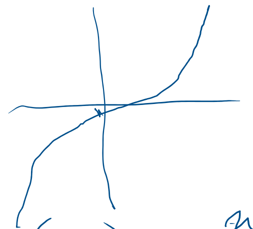
$$f(-x) = \ln \frac{(1-x)}{1+x} = -\ln \frac{(1+x)}{1-x} = -f(x)$$

f – nepárna funkcia

$$f'(x) = \frac{1}{\frac{1+x}{1-x}} \cdot \left(\frac{\overbrace{1}^{(1+x)'}}{1-x} + (1+x)\left(-\frac{1}{(1-x)^2}\right)(-1) \right)$$

$$= \frac{1-x}{1+x} \left(\frac{1}{1-x} + \frac{1+x}{(1-x)^2} \right) =$$

$$= \frac{1}{1+x} + \frac{1}{1-x}$$



$$f(a+x) = \sum_{n=0}^{\infty} \frac{1}{n!} f^{(n)}(a)\, x^n$$

D.Ú.

Napísať Taylorov rad pre

$\ln(1+x)$

$\ln(1-x)$

$\ln\left(\frac{1+x}{1-x}\right)$