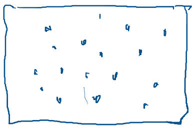
Erik 2024-09-20

Friday, September 20, 2024 5:04 PM



μ-stav: $\{[x_1, r_1], [x_2, r_2] \dots\}$

M-stav: $\{N, P, V, T\}$ $P = \frac{N}{V} k_B T$, $E = $ konšt.

$H = H(\mu\text{-stav}) = H(x_1, p_1, x_2, p_2 \dots)$

Evolúcia μ-stavu

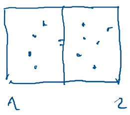
$$\dot{p}_i = -\frac{\partial H}{\partial x_i} \qquad \dot{x}_i = \frac{\partial H}{\partial p_i}$$

$\Omega(x_1, p_1, x_2, p_2 \dots) = $ konšt.

μ-stav $\longleftrightarrow$ M-stav

multiplicita

Rovnovážny stav: max(multiplicita)



$$P(N_1) = \frac{1}{2^N}\binom{N}{N_1}$$

$$P(N_1) = \binom{N}{N_1}\underbrace{\left(\frac{1}{2}\right)^{N_1}\left(\frac{1}{2}\right)^{N-N_1}}$$

$$k = \frac{N}{2} - N_1 = 0, \pm 1, \pm 2 \dots$$

$N, N_1 \gg 1$

$$\binom{N}{\frac{N}{2}-k} = \frac{N!}{\left(\frac{N}{2}-k\right)!\left(\frac{N}{2}+k\right)!} \qquad N \to \infty$$

$$\boxed{\text{Stirling: } n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n}$$

$$\binom{N}{\frac{N}{2}-k} \sim \frac{\cancel{\sqrt{2\pi N}}\left(\frac{N}{\cancel{e}}\right)^N}{\cancel{\sqrt{2\pi\left(\frac{N}{2}-k\right)}}\,\cancel{\sqrt{2\pi\left(\frac{N}{2}+k\right)}}\left(\frac{\frac{N}{2}-k}{\cancel{e}}\right)^{\frac{N}{2}-k}\left(\frac{\frac{N}{2}+k}{\cancel{e}}\right)^{\frac{N}{2}+k}}$$

$$= \frac{N^N}{\left(\frac{N}{2}-k\right)^{\frac{N}{2}-k}\left(\frac{N}{2}+k\right)^{\frac{N}{2}+k}} = \frac{N^N}{\left(\frac{N}{2}\right)^N}\left[\left(1-\frac{2k}{N}\right)^{\frac{N}{2}-k}\left(1+\frac{2k}{N}\right)^{\frac{N}{2}+k}\right]^{-1}$$

$= 2^N$

$$\left(1+\frac{2k}{N}\right)^{\frac{N}{2}-k} \qquad \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$

$$N \to \infty \quad \frac{2k}{N} \to 0 \qquad \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$$

$$\Bigg(\underbrace{1+\frac{2k}{N}}_{\frac{1}{x}}\Bigg)^{\frac{N}{2k}\cdot\underbrace{\frac{2k}{N}\left(\frac{N}{2}+k\right)}_{\sim 1}} = e^{k+\frac{2k^2}{N}}$$

$$\underbrace{\left(1+\frac{\ldots}{N}\right)}_{\frac{1}{x}} \qquad \underbrace{\qquad}_{\sim 1} \qquad e$$

$$\left(1-\frac{2k}{N}\right)^{\frac{N}{2}} = \underbrace{\left(1-\frac{2k}{N}\right)^{-\frac{N}{2k}}}_{e}{}^{\left(-\frac{2k}{N}\right)\left(\frac{N}{2}-k\right)} = e^{-k+\frac{2k^2}{N}}$$

$\frac{1}{x}$

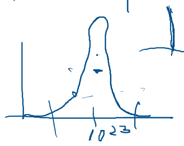
$$x = -\frac{N}{2k}$$

$$\binom{N}{\frac{N}{2}-k} \sim 2^N \frac{1}{e^{k+\frac{2k^2}{N}}\, e^{-k+\frac{2k^2}{N}}} = \boxed{2^N e^{-\frac{4k^2}{N}}}$$

Normálne (Gaussovo) rozdelenie

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}} = \frac{1}{\sqrt{\frac{N 2\pi}{4}}}\, e^{-\frac{4k^2}{N}}$$

$$\mu = 0, \quad 2\sigma^2 = \frac{N}{4} \qquad \sigma = \sqrt{\frac{N}{8}} = \frac{1}{2}\sqrt{\frac{N}{2}}$$



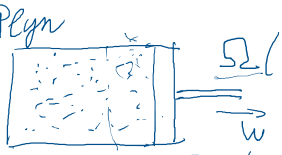
$$(\mu - 2\sigma, \ \mu + 2\sigma) : 0{,}95$$
$$\mu - \sigma, \ \mu + \sigma \quad : 0.68$$

$$\sigma \sim \sqrt{N}$$
$$N = 10^{23}$$
$$\sqrt{N} \sim 10^{12}$$

$$\frac{\delta N}{N} = \frac{k}{N} \sim \frac{10^{12}}{10^{23}} = 10^{-11}$$

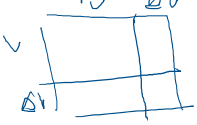
$$\boxed{n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n}$$

Plyn



Ω(rom. skr.) $\sim V$ pre 1 časticu

V^N pre N častíc



$q \qquad T = \text{konšt.}$

$\Downarrow$

$E = \text{konšt.}$

$$q = \underbrace{\Delta E}_{0} + W_{P\Delta V}$$

$$q = P\Delta V$$

Ω môžeme priamo spočítať

$$\frac{\Omega_{koniec}}{\Omega_{poč}} = \left(\frac{V+\Delta V}{V}\right)^N = \left(1+\frac{\Delta V}{V}\right)^N$$

tu dosadíme dostatí q

$$q = P\Delta V = \frac{N}{V} k_B T \Delta V$$

$$\frac{\Delta V}{V} = \frac{q}{N k_B T}$$

$$\frac{\Omega_k}{\Omega_p} = \left(1+\frac{q}{N k_B T}\right)^N \underset{N\to\infty}{\approx} \left(1+\frac{1}{\left(\frac{N k_B T}{q}\right)}\right)^{\frac{N k_B T}{q}\cdot\frac{q}{k_B T}} \to e^{\frac{q}{k_B T}}$$

$$\ln\frac{\Omega_k}{\Omega_p} = \frac{q}{k_B T}$$

stavové veličina

$\ln \frac{\Omega_k}{\Omega_p} = \frac{q}{k_B T} \rightarrow e^{\frac{q}{k_B T}}$

stavové veličina

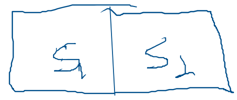
$q = T k_B (\ln \Omega_k - \ln \Omega_p)$

$= S_k - S_p \Rightarrow \underline{T \Delta S}$

$\boxed{S = k_B \ln \Omega}$ pre rovnovážny (pomalý)

$q = \Delta E + w \rightarrow$ ext. par. stav. veličiny

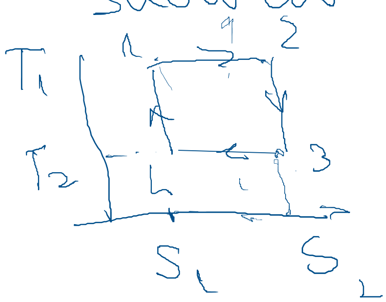
utorok 18^{00}

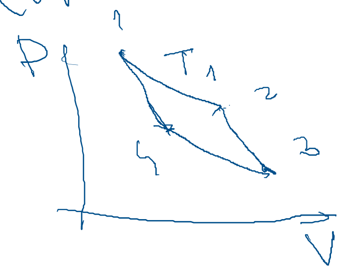


$S = S_1 + S_2$

Intenzívne veličiny (sily)
P, T, B, ...
Extenzívne veličiny
V, N, S ...
aditívne

Sadi Carnot





$Q_{12} = T_1 (S_2 - S_1)$ (ohrie)

$Q_{34} = T_2 (S_1 - S_2)$ (chladič)

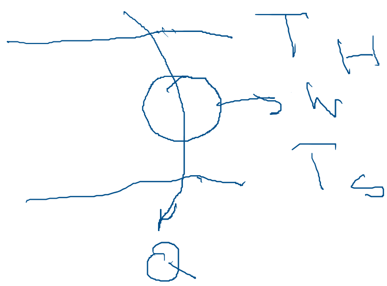
$Q_{1234} = (T_1 - T_2)(S_2 - S_1)$

$\hookrightarrow W$

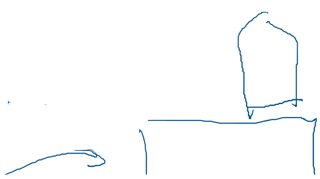
$\eta = \frac{W}{Q_{12}} = \frac{(T_1 - T_2)(S_2 - S_1)}{T_1 (S_2 - S_1)} = \frac{T_1 - T_2}{T_1}$

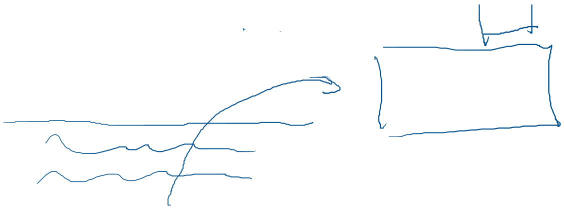
$T_1 > T_2$

$\eta = 1$ iba ak $T_2 = 0\,K$



2. veta v klasickom znení.





Obrátený Carnotov cyklus:
- ako definovať účinnosť
- aká je účinnosť?

W: Q_{34}
vstup