Erik 2024-04-19

Friday, April 19, 2024 4:37 PM

Dnes:

bežné integrály
určitý integrál ako miera
Lebesgueov integrál
Mrkva a petržlen
Náhodné kráčanie na mriežke

$$\int_a^b f(x)\,dx = \sum_{(k)} h f(a+kh) \quad h \to 0$$

$$= F(b) - F(a)$$

$$F'(x) = f(x)$$

$$F(x) = \int_{-\infty}^{x} f(t)\,dt + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$F_a(x) = \int_a^x f(x)\,dx$$

$$= F(x) - F(a)$$

$$\int e^x\,dx = e^x + C$$

$$\int \sin x\,dx = -\cos x + C \qquad (\cos x)' = -\sin x$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int \ln x\,dx = x \ln x - x$$

$$(uv)' = u'v + v'u$$

$$uv = \int v\,du + \int u\,dv$$

$$uv = \int v \underbrace{du}_{u'dx} + \int u\,dv$$

$$\int \underline{u\,dv} = uv - \int v\,du$$

$u = \ln x \qquad v = x$

$u' = \frac{1}{x} \qquad v' = 1$ „Per partes"

$$\int \ln x\,dx = x\ln x - \int x\,\frac{1}{x}\,dx =$$
$$= x\ln x - x + C$$

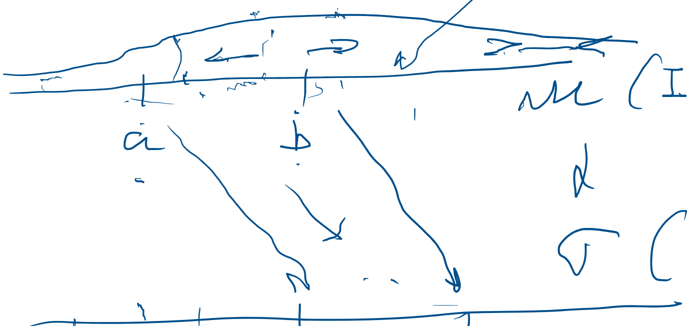
$$\int \frac{1}{1+x^2}\,dx = \operatorname{arctg} x + C$$

$$(\operatorname{tg})' = \left(\frac{\sin x}{\cos x}\right)' =$$
$$= \frac{\cos x}{\cos x} - \frac{\sin x(-\sin x)}{\cos^2 x}$$
$$= 1 + \frac{\sin^2 x}{\cos^2 x} = \begin{cases} \dfrac{1}{\cos^2 x} \\ 1 + \operatorname{tg}^2 x \end{cases}$$

$$(\operatorname{arctg})' = \frac{1}{(\operatorname{tg} y)'} = \frac{1}{1+\operatorname{tg}^2 y} = \frac{1}{1+x^2}$$

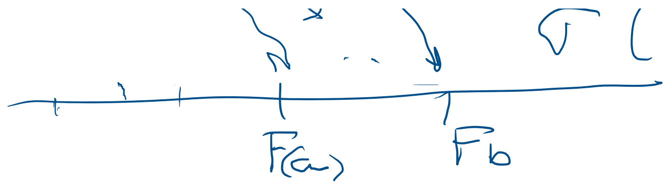
$$F(b) - F(a) = \int_a^b f(x)\,dx$$

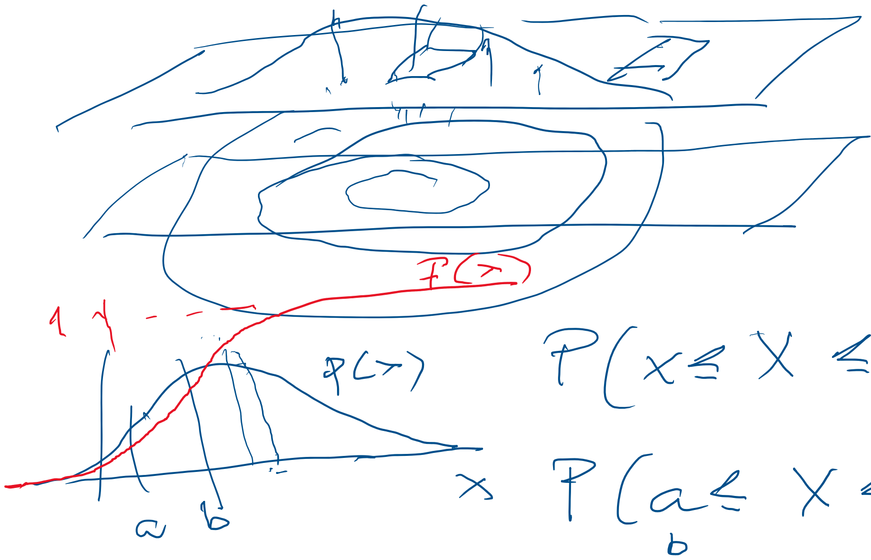
$$b - a \longrightarrow F(b) - F(a)$$



$$m(I_{ab}) = b - a$$
$$\wr$$
$$\sigma(I_{a,b}) = F(b) - F(a)$$

$\mu(\langle a, b \rangle) = F(b) - F(a)$





$P(x \leq X \leq x + dx) = p(x)$

hustota pravdepod.

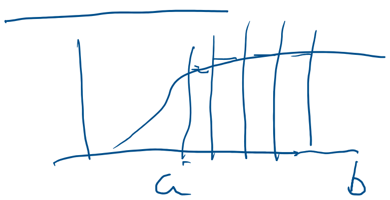
$P(a \leq X \leq b) = \int_a^b p(x)\,dx = F(b) - F(a)$

distribučná funkcia

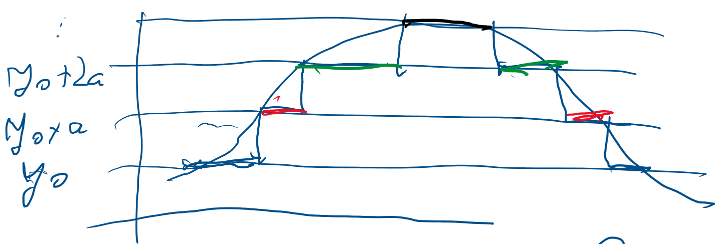
$$\int_{-\infty}^{\infty} p(x)\,dx = 1$$

$$F(x) \xrightarrow[x \to \infty]{} 1$$

$$F(x) \xrightarrow[x \to -\infty]{} 0$$

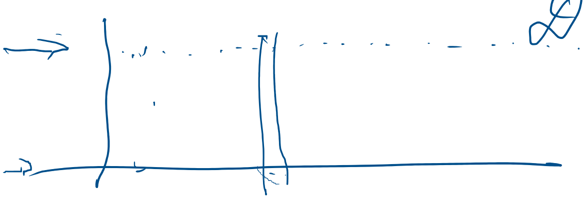


Lebesgue



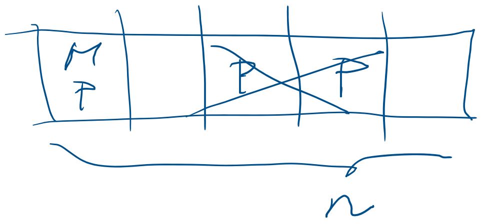
Dirichletova funkcia

$$\begin{cases} 1, & x \in \mathbb{Q} \\ 0 & \text{inak} \end{cases}$$



Mrkva a petržka

Mrkva a petržlen



$n=1$ $\quad$ $n=2$ $\quad$ $n=3$

M $\longrightarrow$ MM $\longrightarrow$ MMM

P $\longrightarrow$ MP

PM

MMP

MPM

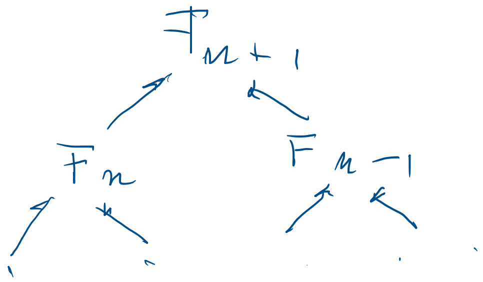
PMM

PMP

$2 \begin{matrix}1\\1\end{matrix}$ $\quad$ $3 \begin{matrix}2\\1\end{matrix}$ $\quad$ $5 \begin{matrix}3\\2\end{matrix}$ $\quad$ $8 \begin{matrix}5\\3\end{matrix}$ $\quad$ $13 \begin{matrix}8\\5\end{matrix}$

$$\begin{matrix}m\\p\end{matrix} \longrightarrow \begin{matrix}m+p\\m\end{matrix} \longrightarrow \underbrace{\begin{pmatrix}1 & 1\\ 1 & 0\end{pmatrix}}\begin{pmatrix}m\\p\end{pmatrix} = \begin{matrix}m+p\\m\end{matrix}$$

$$\begin{pmatrix}F_n\\F_{n-1}\end{pmatrix} \longrightarrow \underbrace{\begin{matrix}F_{n+1} = F_n + F_{n-1}\\ F_n = F_n\end{matrix}}$$



$F_0 \quad F_1 \quad \ldots$

$$\begin{pmatrix}m_n\\p_n\end{pmatrix} = \underbrace{\begin{pmatrix}1 & 1\\ 1 & 0\end{pmatrix}^{n-1}}\begin{pmatrix}1\\1\end{pmatrix} \begin{matrix}m_1\\p_1\end{matrix}$$

$$\underbrace{A\ \Sigma\ A^T}$$

vlastné vektory $\qquad$ diag. mat vl. hodnôt

$$\begin{pmatrix}1 & 1\\ 1 & 0\end{pmatrix}\vec{v} = \lambda\vec{v}$$

$$\left[\begin{pmatrix}1 & 1 \end{pmatrix} \quad \begin{pmatrix}\lambda & \end{pmatrix}\right]\vec{v} = 0$$

$$\left[\begin{pmatrix}1&1\\1&0\end{pmatrix} - \begin{pmatrix}\lambda & \\ & \lambda\end{pmatrix}\right]\vec v = 0$$

$$\begin{vmatrix}1-\lambda & 1\\ 1 & -\lambda\end{vmatrix} = 0$$

$$(1-\lambda)(-\lambda) - 1 = 0$$

$$-\lambda + \lambda^2 - 1 = 0$$

$$\lambda^2 - \lambda - 1 = 0$$

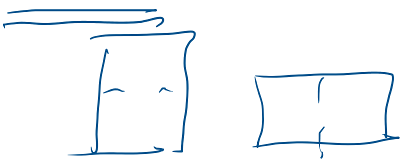
$$\lambda_{1,2} = \frac{1 \pm \sqrt{5}}{2}$$

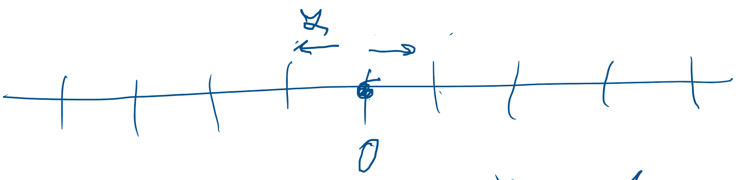
$$M^n = A \Sigma \cancel{A^T A \Sigma A^T} \ldots A \Sigma A^T$$

$$= A \Sigma^n A^T$$

$$\begin{pmatrix}\lambda_1 & \\ & \lambda_2\end{pmatrix}^n = \begin{pmatrix}\lambda_1^n & \\ & \lambda_2^n\end{pmatrix}$$

$$F_n \sim \left(\frac{1+\sqrt{5}}{2}\right)^n$$





$$x_{n+1} = \begin{cases} x_n - 1, & p = p_- \\ x_n, & p = p_0 \\ x_n + 1 & p = p_+ \end{cases}$$

NP NP

$$x_n = \sum_{i=1}^{n} \alpha_i$$

číslo číslo číslo

$X_n = \sum_{i=1}^{n} z_i$ číslo číslo

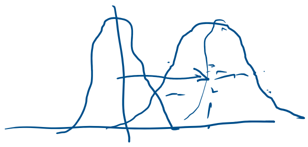
číslo

$$\bar{X}_n = \sum_{i=1}^{n} \bar{z}_i = n\bar{z}$$

$$\bar{z} = (-1)\,p_- + 0\,p_0 + 1\,p_+$$

$$= p_+ - p_- \quad \text{netto posun do} + x$$

$$\bar{X}_n = n(p_+ - p_-)$$



$$\langle (x_n - \bar{x}_n)^2 \rangle = \langle \left(\sum (z_n - \bar{z}) \right)^2 \rangle$$

$$= \langle \sum_{i,j} \underbrace{(z_i - \bar{z})}_{\langle\,\rangle = 0} (z_j - \bar{z}) \rangle$$

$$= \sum_{i,j} \langle z_i z_j \rangle - \bar{z} \langle z_i + z_j \rangle + \bar{z}^2$$

$$= \sum_{i,j} \langle z_i z_j \rangle - \bar{z}^2$$