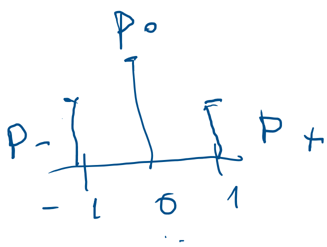
Erik 2024-04-26

Friday, April 26, 2024 5:02 PM



$$\vec{X}_n = \sum_i \underbrace{z_i}$$

$$|\vec{X}_n| = n(p_+ - p_-) \qquad \frac{|\vec{X}_n|}{n} = (p_+ - p_-)$$

$$\underline{\langle (\vec{X}_n - \langle \vec{X}_n \rangle)^2} \rangle = \langle \Big[\sum_i (z_i - \bar{z})\Big]^2 \rangle =$$

$$= \langle \sum_{i,j} (z_i - \bar{z})(z_j - \bar{z}) \rangle$$

$$n \to \infty \qquad P(z_i < \bar{z}) = P(z_i > \bar{z}) = \frac{1}{2}$$

$$z_i = \begin{cases} -1, & p_- \\ 0, & p_0 \\ 1, & p_+ \end{cases}$$

$$\bar{z} = p_+ - p_-$$

$$\bar{z} = (-1)p_- + 0 \cdot p_0 + 1 \cdot p_+ = p_+ - p_-$$

$$\underline{z - \bar{z}} = \begin{cases} -1 - p_+ + p_-, & p_- \\ -p_+ + p_-, & p_0 \\ 1 - p_+ + p_-, & p_+ \end{cases}$$

$$(-1 - \cancel{p_+} + p_-)p_- + (-p_+ + p_-)p_0 + (1 - p_+ + \cancel{p_-})p_+ =$$

$$= p_+ - p_- - p_+^2 + p_-^2 - p_+ \cdot p_0 + p_- \cdot p_0$$

$$= p_+(1 - p_0) - p_-(1 - p_0) - (p_+ + p_-)(p_+ - p_-)$$

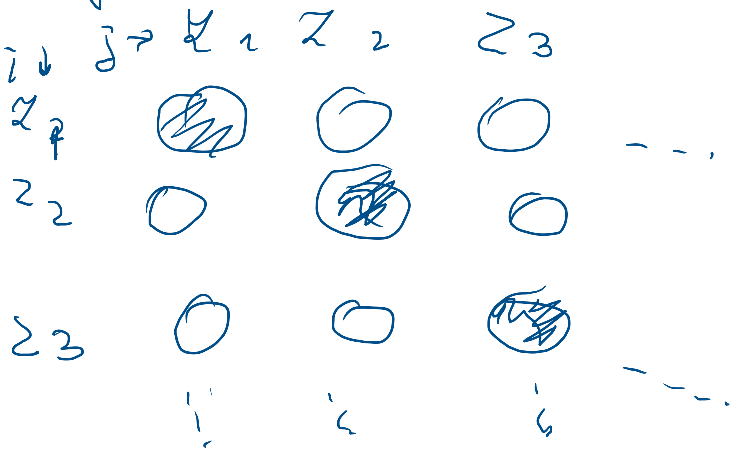
$$= (p_+ + p_-)(p_+ + p_- - p_+ + p_-) \qquad p_- + p_0 + p_+ = 1$$

$= 0$

$$\boxed{z - \bar{z} = 0}$$

$$\langle (x_n - \langle x_n \rangle)^2 \rangle =$$

$$\sum_{i,j} \langle (z_i - \bar{z})(z_j - \bar{z}) \rangle \xrightarrow[n \to \infty]{=} \sum_i \langle (z_i - \bar{z})^2 \rangle$$



$$= n \underbrace{\langle (z - \bar{z})^2 \rangle}_{\mathrm{var}\, z} \qquad \mathrm{var}\, X = \langle (X - \langle X \rangle)^2 \rangle$$

$$1 - p_+ + p_- = p_- + p_0 + p_+ - p_+ + p_- =$$

$$= 2p_- + p_0$$

$$-1 - p_+ + p_- = \cancel{-p_-} - p_0 - p_+ - p_+ + \cancel{p_-}$$

$$= -p_0 - 2p_+$$

$$z - \bar{z} = \begin{cases} -1 - p_+ + p_-, & p_- \\ -p_+ + p_-, & p_0 \\ 1 - p_+ + p_-, & p_+ \end{cases}$$

$$\langle (z - \bar{z})^2 \rangle = (-1 - p_+ + p_-)^2 p_- + (-p_+ + p_-)^2 p_0 + (+1 - p_+ + p_-)^2 p_+$$

$$+(-p_+ + p_-)p_0 + (+1 - p_+ + p_-)p_+$$

$$= \left[\boxed{(-1)^2} + 2(-1)(-p_+ + p_-) + (-p_+ + p_-)^2\right] p_-$$

$$+ (-p_+ + p_-)^2 \, p_0$$

$$+ \left[\boxed{(+1)^2} + 2(+1)(-p_+ + p_-) + (-p_+ + p_-)^2\right] p_+$$

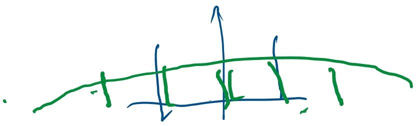
$$= p_- + p_+ + 2(-p_+ + p_-)(p_+ - p_-)$$

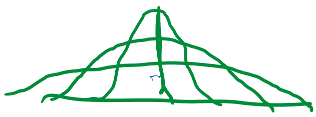
$$+ (-p_+ + p_-)^2 \underbrace{\left[p_- + p_0 + p_+\right]}_{1}$$

$$= p_- + p_+ - 2(-p_+ + p_-)^2 + (-p_+ + p_-)^2$$

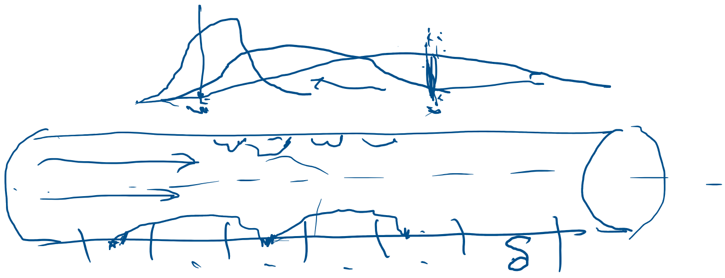
$$= \underbrace{p_- + p_+} - \underbrace{(-p_+ + p_-)^2}_{(p_+ - p_-)^2}$$

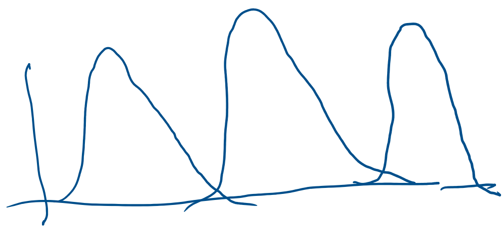
$$z^2$$





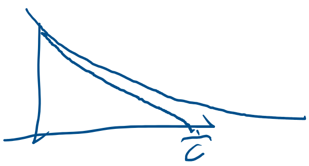
$$= \underbrace{2\,\frac{p_+ + p_-}{2}} - \underbrace{(p_+ - p_-)^2}$$





$$\bar{v} = (p_+ - p_-)\frac{\delta}{\tau} \qquad \langle z \rangle = (p_+ - p_-)\delta$$

$$\langle x_n \rangle = n \langle z \rangle$$



$$v = \frac{\langle x_n \rangle}{n\tau} = \frac{\cancel{n}(p_+ - p_-)\delta}{\cancel{n}\tau}$$

$$\langle x^2(t) \rangle = 2Dt \qquad \langle (x_n - \langle x_n \rangle)^2 \rangle =$$

$$\underline{n}\left(2\,\frac{p_+ + p_-}{2} - (p_+ - p_-)^2\right)\delta^2$$

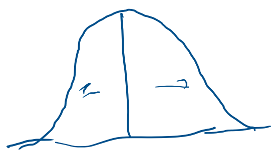
$$t = n\tau$$

$$n = \frac{t}{\tau}$$

$$= \langle l^2 \rangle = \left(2\,\frac{p_+ + p_-}{2} - (p_+ - p_-)^2\right)\frac{\delta^2 t}{\tau}$$

$$p_+ = p_-$$

$$\langle l^2 \rangle = 2\,\boxed{\frac{p_+ + p_-}{2}\,\frac{\delta^2}{\tau}}\,t$$



$$\boxed{D = \frac{p_+ + p_-}{2} \cdot \frac{\delta^2}{\tau}}$$