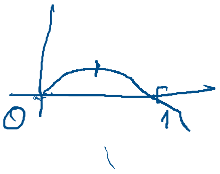
Erik 2024-09-27

Friday, September 27, 2024   5:01 PM

Entropia

$S = k_B \ln W$ termodynamika

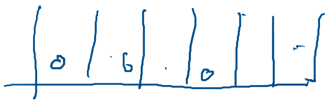
$H = -\sum_{(i)} p_i \ln p_i$ $\sum p_i = 1$ L'Hospital



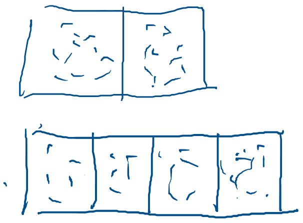
$$\lim_{x\to 0} x\ln x = \lim_{x\to 0} \frac{\ln x}{\frac{1}{x}} = \lim_{x\to 0} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0} (-x) = 0$$

$p = \frac{1}{N}$

$$H = -\sum_{(i)} \frac{1}{N} \ln \frac{1}{N} = -N \frac{1}{N} \ln \frac{1}{N} = \ln N$$



N guličiek m priehradok



Multiplicita

(n_1, n_2, n_3, n_4) mikrostav

$\sum n_i = N$

$$\binom{N}{k} = \frac{N!}{\underset{N_1}{k!}\,\underset{N_2}{(N-k)!}}$$

$$P(n_1, n_2, n_3, \dots n_m) = \binom{N}{n_1\, n_2\, n_3 \dots n_m} \underbrace{\left(\frac{1}{m}\right)^N}_{\text{konšt}}$$

$$\underbrace{\frac{N!}{n_1!\, n_2!\, n_3! \dots n_m!}}_{\text{Multiplicita}}$$

Maximalizujem

$\ln P(n_1 \dots n_m) = -N \ln m + \ln N! - \sum \ln N_i!$

Maximalizujem

$\ln P(n_1 \ldots n_m) = -N \ln m + \ln N! - \sum_i \ln N_i!$

$N \to \infty$

N veľké $\quad N! \sim \left(\frac{N}{e}\right)^N$ (stirling)

$\ln N! \sim N(\ln N - 1)$

$\ln P(n_1, \ldots n_m) = N \ln m + N \ln N - N -$

$- \sum_i N_i \ln N_i + \underbrace{\sum_i N_i}_{N}$

$= \underline{N \ln m} + \underline{N \ln N} - \sum_i N_i \ln N_i$

max multiplicita:

$(N_1^*, N_2^* \ldots N_m^*)$ $\ln P$ je max

a $\sum N_i^* = N$

$\sum_i N_i^* = N$

$\to \underbrace{\sum N_i^* - N}_{g(N_1 \ldots N_m)} = 0$

$\mathcal{L} = \ln P(n_1, n_2, \ldots n_m) - \lambda g(N_1, \ldots N_m)$

$\frac{\partial \mathcal{L}}{\partial N_i} = 0$

$\frac{\partial}{\partial N_i}\left(-\sum_k N_k \ln N_k - \lambda\left(\sum N_k - N\right)\right) = 0$

$(-N_i \ln N_i)' - \lambda = 0$

$-\ln N_i - 1 - \lambda = 0$

$N_i = \underbrace{e^{-1-\lambda}}_{konšt.}$

$\sum_i N_i = N \qquad mC = N \qquad C = \frac{N}{m}$ Rovnomerné rozdelenie

$\sum N_i = N$ $\quad mC = N$ $\quad C = \frac{N}{m}$ $\quad$ Rovnoměrné rozdělení

$1,4142\ldots - 1 = 0.414\ldots$

$(\sqrt{2} - 1)^{12} = ?\ A_n + B_n\sqrt{2}$ $\quad \boxed{a + b\sqrt{q}}$

$\sum_{k=0}^{12} \binom{12}{k} \sqrt{2}^k (-1)^{N-k}$ $\quad \frac{A}{B}$

$x = \sqrt{2} - 1$

$x + 1 = \sqrt{2}$

$(x+1)^2 = 2$

$x^2 + 2x + 1 = 2$

$\boxed{x^2 = 1 - 2x}$

$(x+1)^4 = 4$

$x^4 + 3x^3 + 3x^2 + 1 = 4$

$x^4 = 3 - 3x^2 - 3x \cdot x^2$

$= 3 - 3(1-2x) - 3x(1-2x)$

$= 6x - 3x + 6(1-2x) =$

$= 3x + 6 - 12x = 6 - 9x$

$x^8 = (x^4)^2 = a + bx$

$x^{12} = x^4 x^8 = A + Bx$