Erik 2024-03-15

Friday, March 15, 2024 3:46 PM

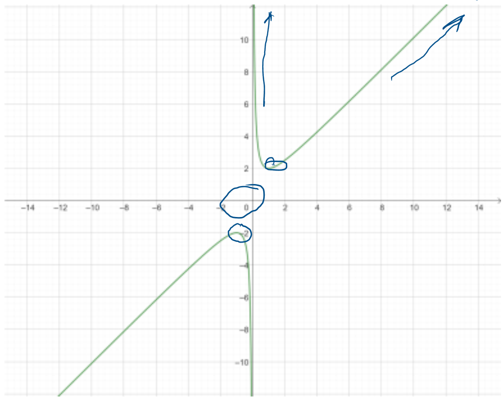
D.Ú.

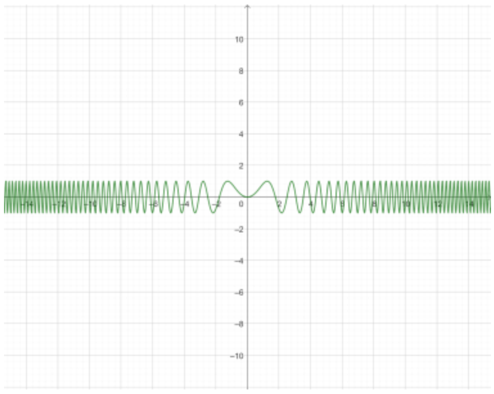
a) $y = x + \frac{1}{x}$

b) $y = \sin x^2$

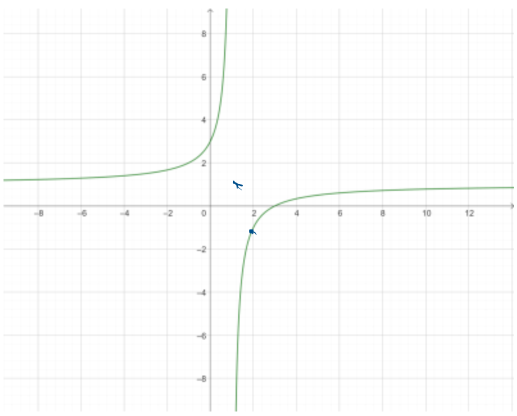
c) $y = \frac{x^2 - 5x + 6}{x^2 - 3x + 2}$

$x = 0$ $+\infty$





$-\infty$



Ďalšie funkcie na dnes:

$y = \ln(x - \sqrt{x^2 - 1})$

$y = \frac{3x^3 - 1}{2x^2 - 2}$

$y = \frac{x}{\sqrt{a^2 - x^2}}$

$y = \ln\left(\frac{1+x}{1-x}\right)$

Ešte iné veci:

- asymptoty horizontálne, vertikálne, šikmé
- inflexné body

Asymptota

$y = x + \frac{1}{x}$

$y = \boxed{p}\, x + q$

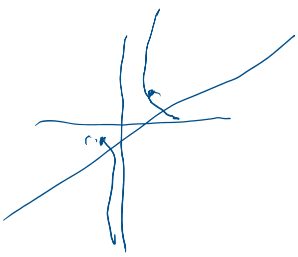
$$\lim_{x \to \pm\infty} y = \pm\infty$$

$y = 1 \cdot x$

$$\lim_{x \to \infty} \frac{y}{x} = \lim_{x \to \infty} \frac{x + \frac{1}{x}}{x} = 1$$

$$\lim_{x \to 0^+} x + \frac{1}{x} = +\infty$$

$$\lim_{x \to 0^-} x + \frac{1}{x} = -\infty$$



$$\lim_{x\to 0^-} x + \frac{1}{x} = -\infty$$

$y = \sin x^2$

$D: \mathbb{R}$

$\Omega: -1, 1$

$\lim\limits_{x\to\pm\infty} \sin x^2$ neexistuje

$\forall \varepsilon > 0 : \exists x_\varepsilon : x > x_\varepsilon : |a - \sin x^2| < \varepsilon$

Počnúc od x_0: $x \,|\, \sin x^2 = 1$ $\quad x \,|\, \sin x^2 = -1$

budú ľub. blízko.

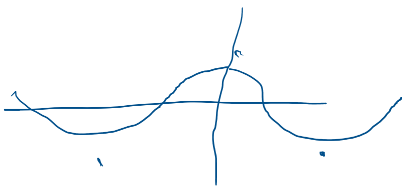
Extrémy:

$y' = \cos x^2 \cdot 2x = 0$

$x = 0$

$\cos x^2 = 0$

$x^2 = \frac{\pi}{2} + k\pi \quad k = 0, \pm 1, \pm 2 \dots$



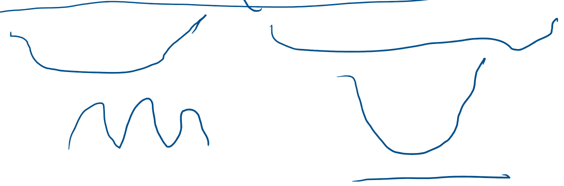
$y'' = (2x\cos x^2)' = 2\cos x^2 - (2x)^2 \sin x^2$

$= 2(\cos x^2 - x^2 \sin x^2)$

$= 2(1 - \sin x^2 - x^2 \sin x^2) =$

$= 2 - \sin x^2 (1 + x^2)$

$2 -$



$y = \sin x^2$

$x^2 = \frac{\pi}{2} + k\pi$

$$x^2 = \frac{\pi}{2} + k\pi$$

$$\sin x^2 = \sin\left(\frac{\pi}{2} + k\pi\right) =$$

$$= \underbrace{\sin\frac{\pi}{2}}_{1}\ \underbrace{\cos k\pi}_{(-1)^k} + \cos\frac{\pi}{2}\ \underbrace{\cancel{\sin k\pi}}_{0}$$

$$\sin x_S^2 = \sin\left(\frac{\pi}{2} + k\pi\right) = (-1)^k$$

$$y = \frac{x^2 - 5x + 6}{x^2 - 3x + 2}$$

$D: \{x : x^2 - 3x + 2 \neq 0\}$ $\quad R - \{1, 2\}$

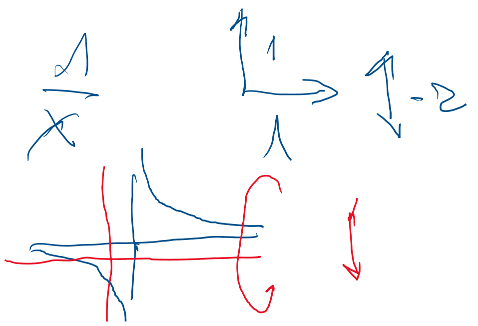
$$x^2 - 3x + 2 = (x-2)(x-1)$$

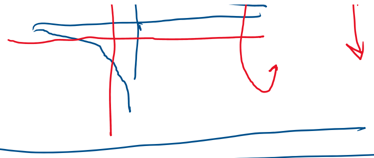
$$y = \frac{x^2 - 5x + 6}{(x-1)(x-2)} = 1 + \frac{-2x+4}{(x-1)(x-2)}$$

$$(x^2 - 5x + 6) : (x^2 - 3x + 2) = 1$$
$$\underline{-x^2 + 3x - 2}$$
$$-2x + 4$$

$$= 1 + \frac{(-2)(x-2)}{(x-1)(x-2)} = 1 - \frac{2}{x-1}$$

$\frac{1}{x}$





$$y = \frac{3x^3 - 1}{2x^2 - 2}$$

$$D: \{x : 2x^2 - 2 \neq 0\} = \mathbb{R} \setminus \{-1, 1\}$$

$$2x^2 - 2 = 2(x^2 - 1) = 2(x+1)(x-1)$$

$$(3x^3 - 1) : (2x^2 - 2) = \frac{3}{2}x$$

$$\underline{-3x^3 + 3x}$$

$$3x - 1$$

$$\longrightarrow 0 \text{ pre } x \to \pm\infty$$

$$y = \frac{3}{2}x + \boxed{\frac{3x - 1}{(x-1)(x+1)}}$$

Asymptota $y = \frac{3}{2}x$ pre $x \to +\infty$, $x \to -\infty$

$$\frac{3x - 1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

$$3x - 1 = A(x+1) + B(x-1)$$

$x = 1$

$$2 = A \cdot 2 + B \cdot 0$$

$$A = 1$$

$x = -1$

$$-4 = A \cdot 0 + B(-2)$$

$$B = 2$$

$$(3 - A - B)x + (-1 - A + B) = 0$$

$$3 = A + B$$

$$-1 = A - B$$

✓

$$A = \left((x-1)f\right)\Big|_{x=1}$$

$$= \frac{3x-1}{x+1}\Big|_{x=1} = \frac{2}{2} = 1$$

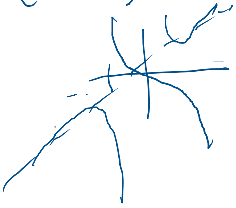
$$B = (x+1)f(x)\Big|_{x=-1} =$$

$$= \frac{3x-1}{x-1}\Big|_{x=-1} = \frac{-4}{-2} = 2$$

$$y = \frac{3}{2}x + \frac{1}{x-1} + \frac{2}{x+1}$$

$$\lim_{x\to -1^-} y = \frac{3}{2}(-1) + \frac{1}{-2} + (-\infty) = -\infty$$

$$\lim_{x\to -1^+} y = +\infty$$



Norman Wildberger

$$\lim_{y\to 1^-} \frac{1}{x-1} = -\infty$$

$x-1 \Rightarrow < 0$

$|x| < 1$

$$\lim_{y\to 1^+} \frac{1}{x-1} = +\infty$$

Extrémy:

$$\left(\frac{3}{2}x + \frac{1}{x-1} + \frac{2}{x+1}\right)' = \frac{3}{2} - \frac{1}{(x-1)^2} - \frac{2}{(x+1)^2} = 0$$

$$\left(\frac{3}{2}x + \frac{1}{x-1} + \frac{2}{x+1}\right)' = \frac{3}{2} - \frac{1}{(x-1)^2} - \frac{2}{(x+1)^2} = 0$$

D. Ú.

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