Erik 2024-05-31

Friday, May 31, 2024 4:53 PM

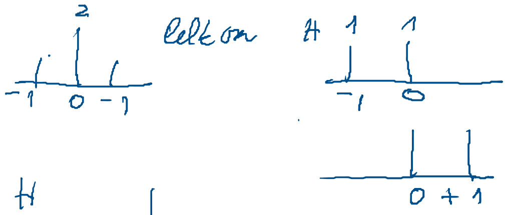
Head - tail

HT

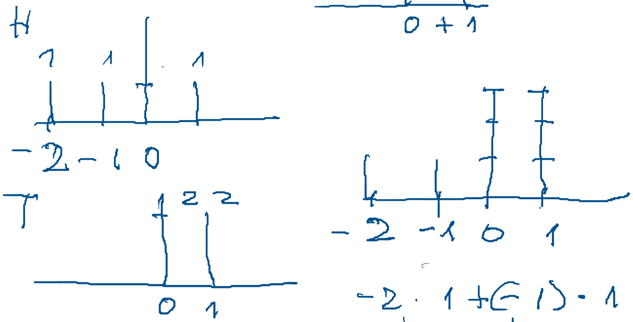
HH

HHTH ......

H	HH	-1
T	HT	$+1$
	TH	0
	TT	0

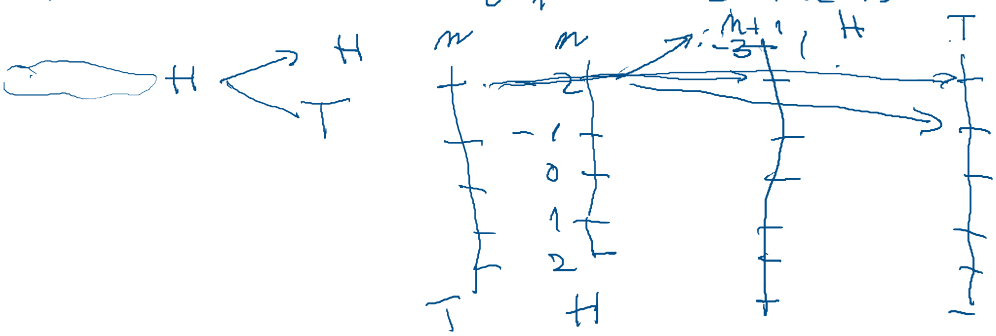


HHH	-2
HHT	0
HTH	1
HTT	1
THH	-1
THT	1
TTH	0
TTT	0



$$P_+ = \frac{3}{8} \qquad P_- = \frac{2}{8} \qquad P_0 = \frac{3}{8}$$

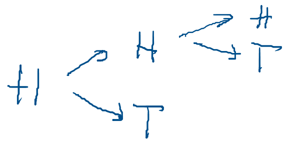
$$-2\cdot 1 + (-1)\cdot 1 + 3\cdot 0 + 3\cdot 1 = 0$$

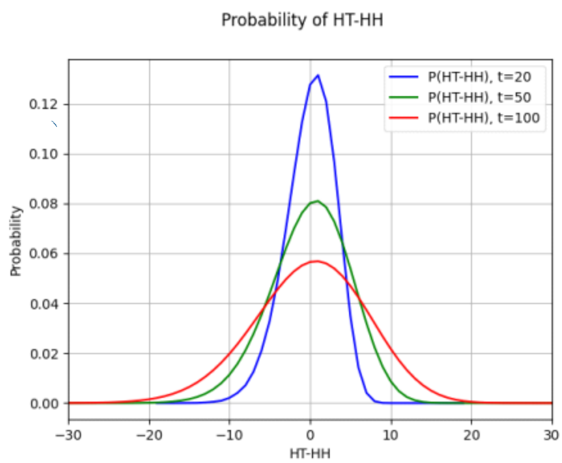


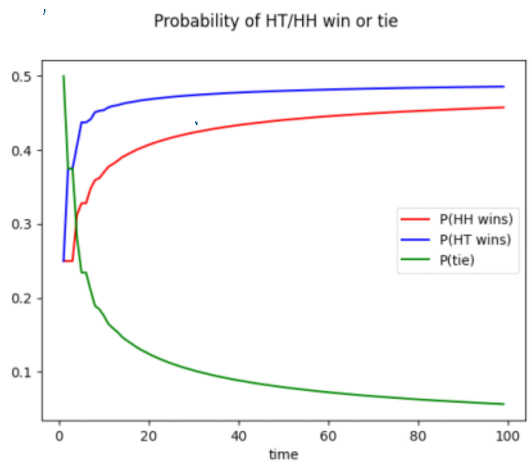
$$h(k, t+1) = h(k+1, t) + t(k, t)$$

$$t(k, t+1) = h(k-1, t) + t(k, t)$$

$$P(HT) - P(HH) \sim \frac{1}{2\sqrt{\pi n}}$$





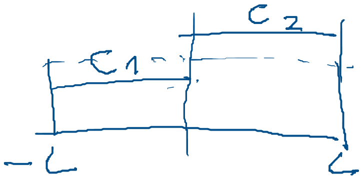


$$f(x) = \frac{a_0}{2} + \sum_{i=1}^{\infty} a_i \sin \frac{2\pi}{L} x + \sum_{i=1}^{\infty} b_i \sin \frac{2\pi}{L} x$$

$$x \in (-L, L)$$

$$a_k = \frac{1}{L} \int_{-\infty}^{\infty} f(x) \cos \frac{2\pi}{L} k x \, dx, \quad k = 0, 1 \ldots$$

$$b_k = \frac{1}{L} \int_{-\infty}^{\infty} f(x) \sin \frac{2\pi k}{L} x \, dx$$



$$\frac{\partial c}{\partial t} = D \Delta c$$

$$c = \frac{a_0}{2} + \sum a_k \cos \frac{2\pi k}{L} x + \sum b_k \sin \frac{2\pi k}{L} x$$

$$\frac{\partial c}{\partial t} \cos \frac{2\pi k}{L} x = D \frac{\partial^2}{\partial x^2} \cos \frac{2\pi k}{L} x$$

$$\frac{1}{L} \int \frac{\partial c}{\partial t} \cos \frac{2\pi k}{L} x \, dx = D \frac{1}{L} \int \frac{\partial^2}{\partial x^2} \cos \frac{2\pi k}{L} x \, dx$$

$$\frac{\partial}{\partial t} a_k(t) = -\underbrace{D \left(\frac{2\pi k}{L}\right)^2} a_k(t)$$

$$\frac{\partial^2}{\partial x^2} \cos \frac{2\pi k}{L} x = \left(\frac{2\pi k}{L}\right) \frac{\partial}{\partial x} \left(-\sin \frac{2\pi k}{L} x\right)$$

$$= -\left(\frac{2\pi k}{L}\right)^2 \cos \frac{2\pi k}{L} x$$

$$\boxed{a_k(t) = a_k(0) \, e^{-D \left(\frac{2\pi k}{L}\right)^2 t}}$$

$$\frac{dy}{dt} = -ky \qquad \frac{dy}{y} = -k \, dt \qquad \int_0^t$$

$$d \ln y = -k \, dt$$

$$\int_0^t d \ln y = -k \int_0^t dt$$

$$[\ln y]_{t=0}^{t=t}$$

$$\ln y(t) - \ln y(0) = -kt$$

$$\ln y(t) - \ln y(0) = -kt$$

$$\boxed{y(t) = y(0)\, e^{-kt}}$$

$$c(x,0) = \begin{cases} c_1, & -L \leq x < 0 \\ c_2, & 0 \leq x \leq L \end{cases}$$

$$a_k = \frac{1}{L}\int_{-L}^{L} c(x,0) \cos\frac{2\pi k}{L} x \, dx =$$

$$= \frac{1}{L} c_1 \int_{-L}^{0} \cos\frac{2\pi k}{L} x \, dx + \frac{1}{L} c_2 \int_{0}^{L} \cos\frac{2\pi k}{L} x \, dx$$

$$\frac{1}{L}\int_{-L}^{L} \cos\frac{2\pi k}{L} x \, dx = 0$$

$$\int_{-L}^{0} \cos\frac{2\pi k}{L} x \, dx = \frac{L}{2\pi k}\left[\sin\frac{2\pi k}{L} x\right]_{-L}^{0} =$$

$$= \frac{1}{2\pi k} \cdot 0$$

$$a_0 = \frac{1}{L}\int_{-L}^{L} c(x,0)\, dx = \frac{1}{L} c_1 \int_{-L}^{0} dx + \frac{1}{L} c_2 \int_{0}^{L} dx =$$

$$c_1 + c_2$$