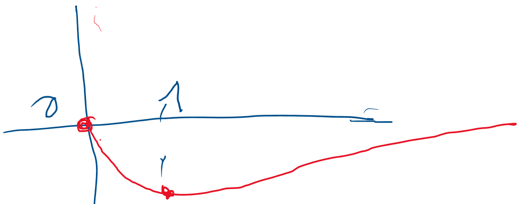
Derivácie 2024-03-08

Friday, March 8, 2024 5:03 PM

$$f(x) = \frac{x \ln x}{1 - x^2}$$



$\mathcal{D}(f) : \{ x : \exists f(x) \}$

- menovateľ: $x \neq \pm 1$
- ln: $x > 0$

$$\lim_{x \to 0^+} \frac{x \ln x}{1 - x^2}$$

$$\lim_{x \to 0^+} x \ln x = \lim_{x \to 0} \frac{\ln x}{\frac{1}{x}} = \lim_{x \to 0} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x \to 0} (-x)$$

$$\frac{\sin x}{x}, \ x \to 0$$

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

$$\lim_{x \to a} f(x) = 0$$

$$\lim_{x \to a} g(x) = 0$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{g(a+h) - g(a)} = \lim_{h \to 0} \frac{\frac{f(a+h) - f(a)}{h}}{\frac{g(a+h) - g(a)}{h}} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

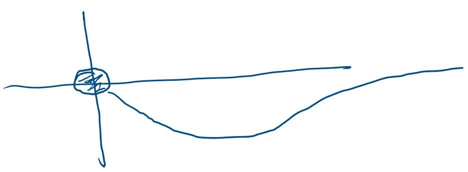
Príklad:

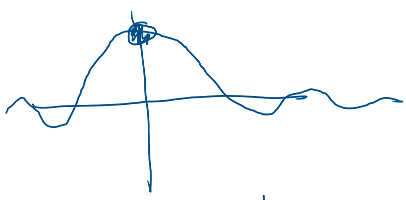
$$\lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \cos x = \cos 0 = 1$$

$$\lim_{x \to 0} x^{\varepsilon} \ln x = \lim_{x \to 0} \frac{\ln x}{x^{-\varepsilon}} \overset{\text{L'Hospital}}{=} \lim_{x \to 0} \frac{\frac{1}{x}}{-\varepsilon x^{-1-\varepsilon}} =$$

$$= \lim_{x \to 0} \left(-\frac{1}{\varepsilon}\right) x^{\varepsilon}$$

$$\lim_{x \to 0^+} \frac{x \ln x}{1 - x^2} = 0 \quad \text{môžeme si dodefinovať}$$





$$\lim \frac{x \ln x}{\ldots} = \lim \frac{\frac{1}{\varepsilon} \ln \frac{1}{\varepsilon}}{\ldots} =$$

$$\lim_{x\to\infty} \frac{x\ln x}{1-x^2} \underset{t=\frac{1}{x}}{=} \lim_{t\to 0} \frac{\frac{1}{t}\ln\frac{1}{t}}{1-\frac{1}{t^2}} =$$

$$= \lim_{t\to 0} \frac{\frac{1}{t}(-\ln t)}{\frac{t^2-1}{t^2}} = \lim_{t\to 0} \frac{t\ln t}{1-t^2} = 0$$

$$\lim_{x\to\infty} \frac{x\ln x}{1-x^2} = \lim_{x\to\infty} \frac{x\ln x}{-x^2} = 0$$

$\Rightarrow$

$$f(x) = \frac{x\ln x}{1-x^2} \qquad f'(x) = (x\ln x)'\frac{1}{1-x^2} +$$

$$+\, x\ln x\left(\frac{1}{1-x^2}\right)' = \frac{1}{1-x^2}\left(1\cdot\ln x + x\cdot\frac{1}{x}\right)$$

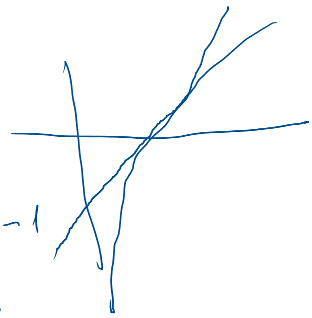
$$+\, x\ln x\,(-1)\frac{1}{(1-x^2)^2}(-2x) =$$

$$= \frac{1+\ln x}{1-x^2} + \frac{2x^2\ln x}{(1-x^2)^2} =$$

$$= \frac{1+\ln x - x^2 - x^2\ln x + 2x^2\ln x}{(1-x^2)^2}$$

$$= \frac{1-x^2+\ln x + x^2\ln x}{(1-x^2)^2} > 0$$

$$= \underbrace{(1-x^2)}_{>0} + \underbrace{(1+x^2)}_{>0}\ln x$$



$y = x-1$

$0 < x < 1$:

$x - 1 > \ln x$

$\boxed{0 < x < 1:}$

$$(1-x^2) + (1+x^2)\ln x$$

$$< (1-x^2) + (1+x^2)(x-1)$$

$$= (1-x)\left[1+x-1-x^2\right]$$

$$= (1-x)\left[1+x-1-x^2\right]$$

$$= \underset{>0}{(1-x)}\left[x-x^2\right] = (1-x)^2 x$$

$$f'(x) = \frac{1-x^2+(1+x^2)\ln x}{(1-x^2)^2}$$

$$\lim_{x\to 1} f'(x) = \lim_{x\to 1} \frac{-2x+\frac{1}{x}+2x\ln x+x^2\cdot\frac{1}{x}}{2(1-x^2)\cdot 2x}$$

$$= \lim_{x\to 1} \frac{-2x+\frac{1}{x}+2x\ln x+x}{4x\underset{0}{(1-x^2)}} =$$

$$= \lim_{x\to 1} \frac{-2-\frac{1}{x^2}+2\ln x+2+1}{4-12x^2}$$

$$= \frac{\boxed{-2-1+3}}{-8} = 0$$

D.Ú.

a) $y = x + \frac{1}{x}$

b) $y = \sin x^2$

c) $y = \frac{x^2-5x+6}{x^2-3x+2}$

$D = ?$

ako sa chová na hraniciach D?

minimá, maximá