Erik 2024-04-05

Friday, April 5, 2024 5:02 PM

D.Ú.

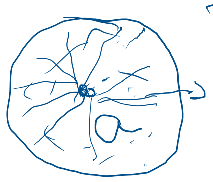
Napísať Taylorov rad pre

$\ln(1+x)$

$\ln(1-x)$

$\ln\left(\frac{1+x}{1-x}\right)$

$$f(x) = \underbrace{\sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!} (x-a)^k} + \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-a)^{n+1}$$



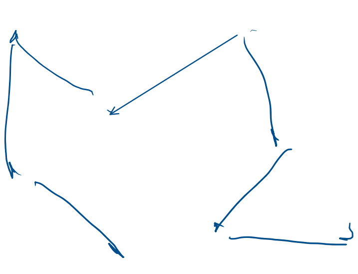
$$T_n(x|a) \xrightarrow[n\to\infty]{} f(x)$$

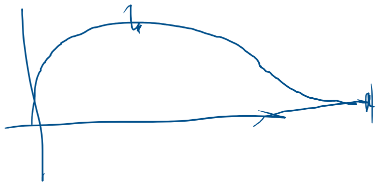
$\vec{v}_i \quad |v_i| = \ell$ náhodný smer

$$\vec{R} = \sum_{i=1}^{n} \vec{v}_i$$

$\langle \vec{v}_i \rangle = 0 \qquad \overline{\vec{v}}^{(1)} = \frac{\sum \vec{v}_i^{(1)}}{N} \xrightarrow[N\to\infty]{} 0$

$\langle \vec{R} \rangle = 0$





$$\langle R^2 \rangle = \langle \underbrace{\sum_i \vec{v}_i}_{R} \underbrace{\sum_j \vec{v}_j}_{R} \rangle = N\ell^2 + 2\sum_i \sum_{j<i} \langle v_i v_j \rangle$$

$= \langle \vec{v} \, \vec{v} \rangle \qquad \langle v_i^2 \rangle = \langle \vec{v}_i \cdot \vec{v}_i \rangle$

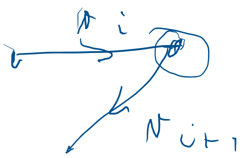
$$= \sum_{i,j} \langle \vec{r}_i \vec{r}_j \rangle$$

$$\langle r_i^2 \rangle = \langle \vec{r}_i \vec{r}_i \rangle = \langle l^2 \rangle = l^2$$

$$\begin{array}{c|cccc}
 & 1 & 2 & 3 & \cdots \\
\hline
1 & \langle r_1^2 \rangle & \langle r_1 r_2 \rangle & \langle r_1 r_3 \rangle & \\
2 & \langle r_2 r_1 \rangle & \langle r_2^2 \rangle & \langle r_2 r_3 \rangle & \\
3 & \langle r_3 r_1 \rangle & \langle r_3 r_2 \rangle & \langle r_3^2 \rangle & \\
\vdots & & & & \ddots \\
N & & & & \langle r_N^2 \rangle
\end{array}$$

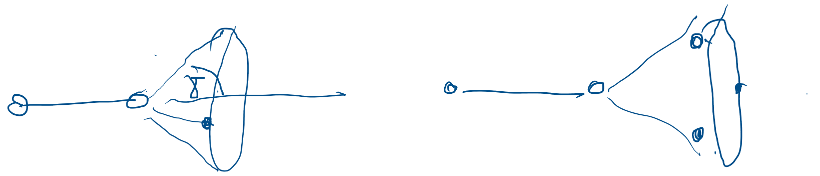
$$= N l^2 + 2 \sum_i \sum_{j<i} \langle \vec{r}_i \vec{r}_j \rangle$$

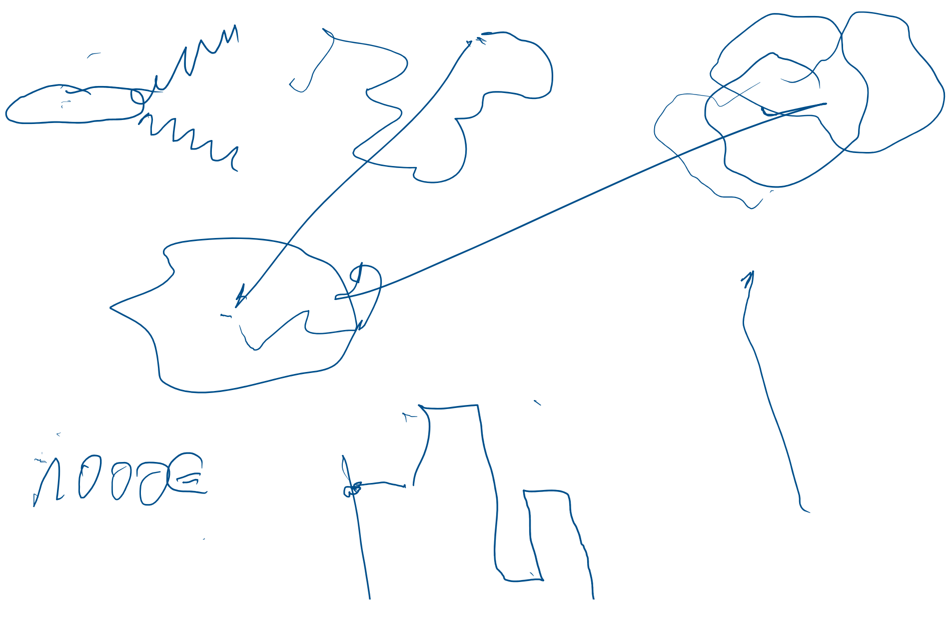
$$\langle r_i r_j \rangle = \delta_{ij} l^2$$

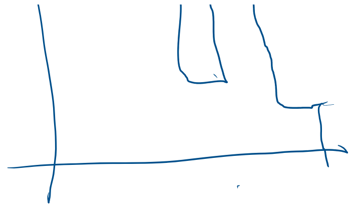


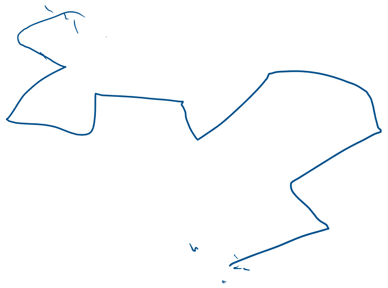
$$\langle \vec{R}^2 \rangle = N l^2$$

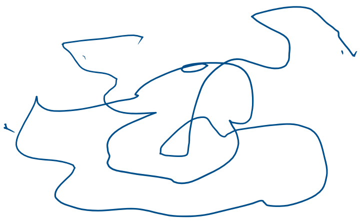
$$\sqrt{\langle R^2 \rangle} \sim \sqrt{N}$$



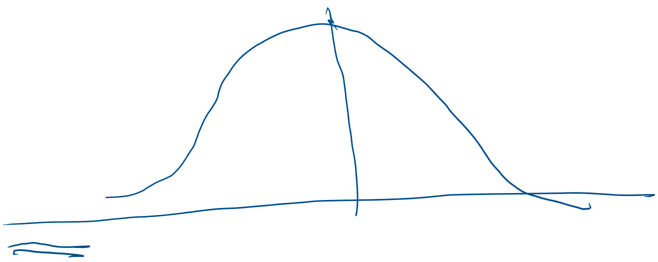


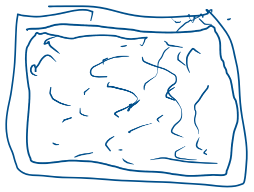


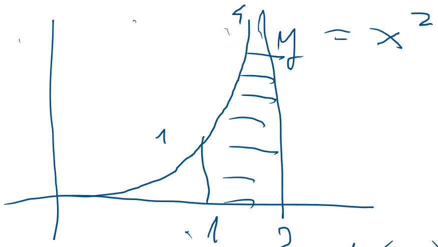


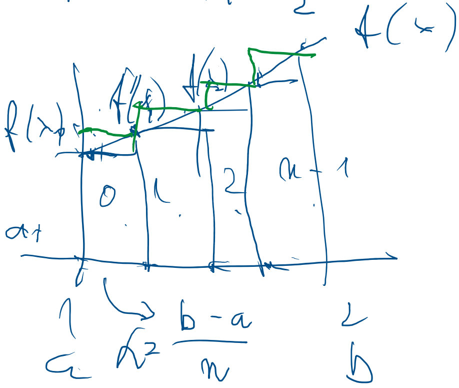


$\sqrt[n]{a}$









$h = \frac{b-a}{n}$ šírka stĺpca

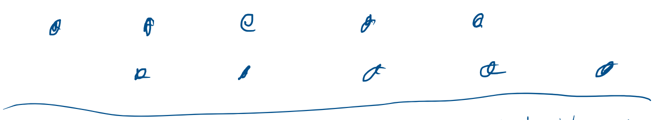
$x_i = a + ih,\ i = 0, \dots, n-1$

$$S_n = \sum_{i=0}^{n-1} h \cdot f(x_i)$$

$$= \frac{b-a}{n} \sum_{i=0}^{n-1} f(x_i)$$

$$S_n^- = \frac{b-a}{n} \sum_{i=0}^{n-1} f(a+ih)$$

$$S_n^+ = \frac{b-a}{n} \sum_{i=1}^{n} f(a+ih)$$



$a+(n-1)h \quad a+nh=b$

$a \quad a+h \quad - \quad - \quad a+(n-1)h \quad a+nh=b$

$\frac{1}{2} \quad 1 \quad 1 \quad - \quad 1 \quad - \quad 1 \ldots \frac{1}{2}$

$f(x)=x^2$

$$S_n^- = \frac{b-a}{n} \sum f(a+ih) =$$

$$= h\sum_{i=0}^{n-1}(a+ih)^2 = nha^2 + 2ah^2\sum_{i=0}^{n-1} i$$

$$+ h^3 \sum_{i=0}^{n-1} i^2 =$$

$$nh = n\frac{b-a}{n} = b-a$$

$$nha^2 = a^2(b-a)$$

$$\sum_{i=0}^{n-1} i = 0+1 - + n-1 = \frac{n(n-1)}{2}$$

$$2ah^2\frac{n(n-1)}{2} = a(b-a)^2 \frac{n-1}{n}$$

$$\sum_{i=0}^{n-1} i^2 = 0+1^2 + \ldots + (n-1)^2 =$$

$$= \frac{(n-1)(n)(2n-1)}{6}$$

$$h^3\sum i^2 = \frac{(b-a)^3}{6} \frac{(n-1)\cancel{(n)}(2n-1)}{n^2}$$

$$a^2(b-a) + a(b-a)^2 \frac{n-1}{n} + (b-a)^3 \frac{(n-1)(2n-1)}{6n^2}$$

S_n^-

$$a(b-a)[a+b-a]$$

$$\lim_{n\to\infty} S_n^- = a^2(b-a) + a(b-a)^2$$

$$+(b-a)^3 \frac{1}{2}$$

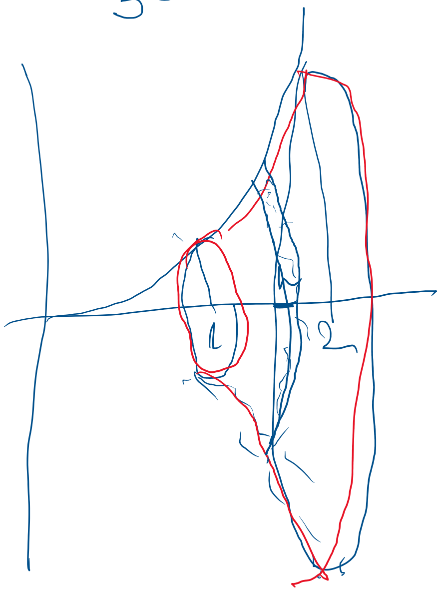
$n \to \infty$

$$+ (b-a)^3 \frac{1}{3}$$

$$= ab(b-a) + \underbrace{(b-a)^3}_{} \frac{1}{3}$$

$$= \frac{1}{3}\left(b^3 - \cancel{3b^2a} + \cancel{3ba^2} - a^3 + \cancel{3ab^2} - \cancel{3a^2b}\right)$$

$$= \frac{1}{3}\left(b^3 - a^3\right)$$



D. ú.

rot. okolo x

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x - plocha