Erik 2024-06-14

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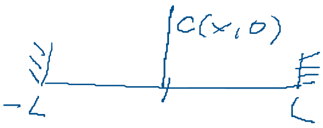


$\stackrel{?}{=} c(x,t)$

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$$

$$\left.\frac{\partial c}{\partial x}\right|_{\pm L} = 0$$

Dáme nejaké veci do poriadku.

$$c(x,t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{L}x + b_n \sin\frac{n\pi}{L}x$$

$$a_n = \frac{1}{L}\int_{-L}^{L} c(x,t)\cos\frac{n\pi}{L}x\,dx, \quad n = 0, 1, 2 \ldots$$

$$b_n = \frac{1}{L}\int_{-L}^{L} c(x,t)\sin\frac{n\pi}{L}x\,dx, \quad n = 1, 2, \ldots$$

Oblasť: $-L \ldots L$ dĺžka $2L$

Zákl. perióda: $n = 1$, od $-L$ do L prejdeme 2π

Hľadáme $a_n(t)$, $b_n(t)$:

$$\frac{1}{L}\int \frac{\partial c}{\partial t}\cos\frac{n\pi}{L}x\,dx = \frac{1}{L}\int D\frac{\partial^2 c}{\partial x^2}\cos\frac{n\pi}{L}x\,dx$$

$$\frac{\partial}{\partial t}\left(\frac{1}{L}\int_{-L}^{L} c(x,t)\cos\frac{n\pi}{L}x\,dx\right) = \frac{\partial a_n(t)}{\partial t} \qquad \frac{\partial}{\partial t}\left(\frac{1}{L}\int_{-L}^{L} c(x,t)\sin\frac{n\pi}{L}x\,dx\right) = \frac{\partial b_n(t)}{\partial t}$$

$$\frac{D}{L}\int \frac{\partial^2 c}{\partial x^2}\cos\frac{n\pi}{L}x\,dx =$$

$$(uv)' = u'v + u\cdot v'$$

$$\int_a^b (uv)'\,dx = \int_a^b u'v\,dx + \int_a^b uv'\,dx$$

$$[uv]_a^b = \int_a^b u'v\,dx + \int_a^b uv'\,dx$$

$$\int_a^b u'v\,dx = [uv]_a^b - \int uv'\,dx \quad \text{Per partes}$$

$$\int_{-L}^{L} c_{xx}\cos\frac{n\pi}{L}x\,dx = \Big[\underbrace{c_x}_{0}\cos\frac{n\pi}{L}x\Big]_{-L}^{L} + \frac{n\pi}{L}\int c_x \sin\frac{n\pi x}{L}\,dx =$$

$u' = c_{xx}$ $\quad v = \cos\frac{n\pi}{L}x$

$u = c_x$ $\quad v' = -\frac{n\pi}{L}\sin\frac{n\pi}{L}x$

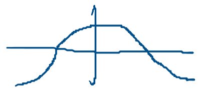
$$= \frac{n\pi}{L}\int c_x \sin\frac{n\pi x}{L}\,dx = \frac{n\pi}{L}\Big[c\underbrace{\sin\frac{n\pi x}{L}}_{0}\Big]_{-L}^{L} - \left(\frac{n\pi}{L}\right)^2\int c\cos\frac{n\pi x}{L}\,dx$$

$u' = c_x$ $\quad v = \sin\frac{n\pi x}{L}$

$u = c \qquad v' = \frac{n\pi}{L}\cos\frac{n\pi x}{L}$

$$= -\left(\frac{n\pi}{L}\right)^2 L a_n$$

$$\frac{1}{L} D \int \frac{\partial^2 c}{\partial x^2}\cos\frac{n\pi x}{L}\,dx = -D\left(\frac{n\pi}{L}\right)^2 a_n$$



$$\frac{\partial a_n}{\partial t} = -D\left(\frac{n\pi}{L}\right)^2 a_n \qquad \frac{\partial b_n}{\partial t} = -D\left(\frac{n\pi}{L}\right)^2 b_n$$

$$\frac{da_n}{a_n} = -D\left(\frac{n\pi}{L}\right)^2 dt \qquad \frac{dg}{dx} = \frac{df}{dx} \Downarrow g = f + C$$

$$\ln a_n(t) - \underbrace{\ln a_n(0)} = -D\left(\frac{n\pi}{L}\right)^2 t$$

$$a(t) = a(0) \cdot e^{-D\left(\frac{n\pi}{L}\right)^2 t}$$
$$b(t) = b(0) \cdot e^{-D\left(\frac{n\pi}{L}\right)^2 t}$$

$$c(x,0) = N\delta(x)$$

$$a_0 = \frac{1}{L}\int_{-L}^{L} N\delta(x)\,dx = \frac{N}{L}$$

$$a_n = \frac{1}{L}\int_{-L}^{L} N\delta(x)\cos\frac{n\pi x}{L}\,dx = \frac{N}{L}\cos\frac{n\pi x}{L}\Big|_{x=0} = \frac{N}{L}$$

$$b_n = \frac{N}{L}\sin\frac{n\pi x}{L}\Big|_{x=0} = 0$$

$$\frac{N}{2L} = c_0$$

$$c(x,0) = \frac{N}{2L} + \frac{N}{L}\sum_{n=1}^{\infty}\cos\frac{n\pi x}{L}$$

$$\boxed{c(x,t) = c_0 + 2c_0 \sum_1 e^{-D\left(\frac{n\pi}{L}\right)^2 t}\cos\frac{n\pi x}{L}}$$