Erik 2024-10-11

Friday, October 11, 2024 11:11 AM

2. veta TD:

$S = k \ln W$; Multiplicita
Objem fázového priestoru

$\boxed{\Delta S = \frac{q}{T}}$ dodané teplo
teplota

$q = T \Delta S$

Rovnovážny (pomalý) proces.

$V_1 \to V_2$

Rýchly proces:

Rovnaký poč. a konečný stav

U_1 U_2

V_1 V_2

$W = 0$ $q = 0$

$\cancel{\Delta S = \frac{q}{T}}$

S_1 S_2

$\Delta S_{pom} = \Delta S_{rýchl}$.

$\boxed{\Delta S \geq \frac{q}{T}}$ v ad. systéme S neklesá.

rovnosť pre rovnovážny proces

Čierna diera

horizont

$v_{únik} = c$

Plocha povrchu nemôže klesať.

$S \to T \to$ žiarenie

Stefanov zákon σT^4

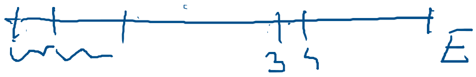
Princíp maximálnej entropie

$\sim (x-\mu)^2$

Princíp maximálnej entropie

$$\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

N častíc celková energia E
energ. hladiny $E_1, E_2 \dots$



Aké je rozdelenie?

$p_i = P$(častica má energiu E_i)

$$\sum p_i = 1$$
$$N \sum p_i E_i = E$$
$$\sum p_i E_i = \varepsilon \left(= \frac{E}{N}\right)$$

$$H = -\sum p_i \ln p_i$$

$$\mathcal{H} = -\sum p_i \ln p_i - \lambda\left(\sum p_i - 1\right) - \mu\left(\sum p_i E_i - \varepsilon\right)$$

$$\frac{\partial \mathcal{H}}{\partial p_i} = 0, \quad i = 1, 2 \dots$$

$$-\ln p_i - 1 - \lambda - \mu E_i = 0$$

$$p_i = e^{-1-\lambda-\mu E_i}$$

$$\sum p_i = 1$$

$$e^{-1-\lambda} \sum e^{-\mu E_i} = 1$$

$$e^{-1-\lambda} = \frac{1}{\sum e^{-\mu E_i}}$$

$$p_i = \frac{e^{-\mu E_i}}{\sum_k e^{-\mu E_k}} = \frac{1}{Z} e^{-\mu E_i}$$

partičná funkcia

Boltzmanovo rozdelenie

$$e^{-\frac{E}{kT}}$$

$$Z = \sum_k e^{-\mu E_k}$$

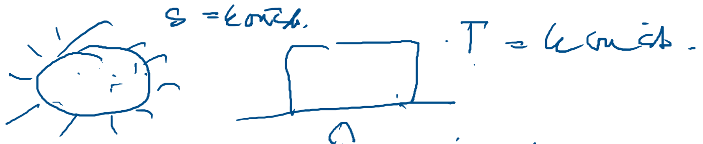
$$F = -kT \ln Z,$$

$$Z = \sum_k e^{-\beta E_k} \qquad \underline{F = -kT \ln Z}$$

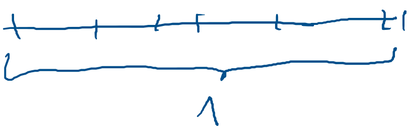
$$\frac{\partial Z}{\partial \beta} = -\sum E_k e^{-\beta E_k}$$

$$\varepsilon = \frac{\sum E_k e^{-\beta E_k}}{\sum e^{-\beta E_k}} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = -\frac{\partial \ln Z}{\partial \beta}$$

Poissonovo rozdelenie

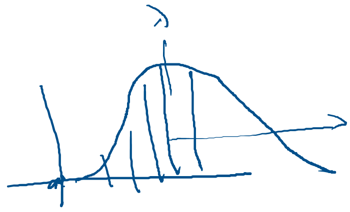


$E = konšt.$ $\qquad$ $\varepsilon = konšt.$



λ - intenzita procesu

Poissonovo rozdelenie



$$P(n) = \frac{\lambda^n}{n!} e^{-\lambda}$$

$$\sum_{n=0}^{\infty} P_n = e^{-\lambda} \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} = e^{-\lambda} e^{\lambda} = 1$$

$$\langle n \rangle = \sum_{n=0}^{\infty} n P_n = e^{-\lambda} \sum \frac{n \lambda^n}{n!} = e^{-\lambda} \sum_{n=1}^{\infty} \frac{\lambda^n}{(n-1)!}$$

$$\overset{n-1 \to n}{=} e^{-\lambda} \lambda \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} = \lambda e^{-\lambda} e^{\lambda} = \underline{\underline{\lambda}}$$

$$\sigma^2 = \langle (n - \langle n \rangle)^2 \rangle = \langle n^2 - 2\langle n \rangle n + \langle n \rangle^2 \rangle =$$

$$= \langle n(n-1) + n - 2\langle n \rangle n + \langle n \rangle^2 \rangle$$

$$= \underbrace{\langle n(n-1) \rangle}_{\lambda} - \langle n \rangle (\langle n \rangle - 1)$$

$$\langle n(n-1) \rangle = e^{-\lambda} \sum_{n=0}^{\infty} \frac{n(n-1)\lambda^n}{n!} = e^{-\lambda} \sum_{n=2}^{\infty} \frac{\lambda^n}{(n-2)!}$$

0 pre $n = 0, 1$

$$\overset{n-2 \to n}{=} e^{-\lambda} \lambda^2 \sum \frac{\lambda^n}{n!} = \underline{\underline{\lambda^2}}$$

$$\sigma^2 = \langle n(n-1)\rangle - \langle n\rangle(\langle n\rangle - 1) =$$
$$= \lambda^2 - \lambda(\lambda - 1) = \underline{\underline{\lambda}}$$

$$G(s) = \sum_n P_n s^n = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} e^{-\lambda} s^n = e^{-\lambda} \sum_n \frac{(\lambda s)^n}{n!}$$
$$= e^{-\lambda} e^{\lambda s} = e^{\lambda(s-1)}$$

$$\left.\frac{dG}{ds}\right|_{s=1} = \sum n P_n s^{n-1} \Big|_{s=1} = \sum n P_n = \langle n\rangle$$
$$\left.\frac{d^2G}{ds^2}\right|_{s=1} = \sum n(n-1) P_n = \langle n(n-1)\rangle$$

$$G = e^{\lambda s - \lambda}$$
$$\left.\frac{dG}{ds}\right|_{s=1} = \lambda \underline{e^{\lambda s - \lambda}} \Big|_{s=1} = \lambda$$
$$\frac{d^2G}{ds^2} = \lambda^2 e^{\lambda s - \lambda} \Big|_{s=1} = \lambda^2$$

t

$$P_n = \frac{\lambda^n}{n!} e^{-\lambda}$$
$$P_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}$$
$$\boxed{P_0(t) = e^{-\lambda t}}$$
$$\langle t\rangle = \int_0^{\infty} t\, e^{-\lambda t}\, dt = \frac{1}{\lambda^2} \int_0^{\infty} \underline{z e^{-z} dz}$$

$$=$$

$$\lambda t = z$$
$$t = \frac{z}{\lambda} \quad dt = \frac{dz}{\lambda}$$

$$0. \quad \int_0^{\infty} e^{-\lambda t} dt = \left(-\frac{1}{\lambda}\right)\left[e^{-\lambda t}\right]_0^{\infty} =$$
$$= \left(-\frac{1}{\lambda}\right)\left(e^{-\lambda\cdot\infty} - e^{-\lambda\cdot 0}\right) = \frac{1}{\lambda}$$

$$= \left(-\frac{1}{\lambda}\right)\left(\underbrace{e^{-\lambda\cdot\infty}}_{0} - \underbrace{e^{-\lambda\cdot 0}}_{1}\right) = \frac{1}{\lambda}$$

1. $$I = \int_0^\infty t\, e^{-\lambda t}\, dt = -\frac{d}{d\lambda} \underbrace{\int_0^\infty e^{-\lambda t}\, dt}_{\frac{1}{\lambda}}$$

$$= -\frac{d}{d\lambda}\left(\frac{1}{\lambda}\right) = -\left(-\frac{1}{\lambda^2}\right) = \frac{1}{\lambda^2}$$

2. $\lambda t = z$

$$\int_0^\infty t\, e^{-\lambda t}\, dt = \frac{1}{\lambda^2}\int_0^\infty z^{2-1} e^{-z}\, dz = \frac{1}{\lambda^2}\Gamma(2)$$

$$= \frac{1}{\lambda^2}\, 1! = \frac{1}{\lambda^2}$$