Erik 2024-05-24

Friday, May 24, 2024 4:57 PM

D. Ú.

$$\vec{E} = \frac{1}{4\pi\varepsilon}\ \frac{q}{r^3}\vec{r} \qquad U = -\frac{1}{4\pi\varepsilon}\ \frac{q}{r}$$

$\mathrm{div}\, E$? $\qquad$ $\mathrm{grad}\, U$?

ΔU

$$\vec{\nabla} = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

$$\Delta = \vec{\nabla}\cdot\vec{\nabla} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\ \frac{q}{r^3}\vec{r} \qquad \vec{\nabla}\cdot\vec{E} = \mathrm{div}\,\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$E_x = \frac{1}{4\pi\varepsilon_0}\ \frac{q}{r^3}\, x$$

$$\frac{\partial E_x}{\partial x} = \frac{q}{4\pi\varepsilon_0}\ \frac{\partial}{\partial x}\left(\frac{x}{r^3}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{x}{r^3}\right) = \frac{1}{r^3} + x\cdot(-3)\frac{1}{r^4}\frac{\partial r}{\partial x} = \frac{1}{r^3} - \frac{3x}{r^4}\cdot\frac{x}{r} =$$

$$\frac{\partial r}{\partial x} = \frac{\partial}{\partial x}\left(\sqrt{x^2+y^2+z^2}\right) = \frac{1}{2}\ \frac{1}{r}\ 2x$$

$$\frac{1}{r^3} - 3\frac{x^2}{r^5} = \frac{\partial E_x}{\partial x}$$

$$\mathrm{div}\, E = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 3\frac{1}{r^3} - 3\frac{1}{r^5}\underbrace{(x^2+y^2+z^2)}_{r^2}$$

$$= 0$$

$\vec{E}$ 

$$U = -\frac{1}{4\pi\varepsilon_0}\ \frac{q}{r}$$

$$\vec{\nabla}U = i\frac{\partial U}{\partial x} + j\frac{\partial U}{\partial y} + k\frac{\partial U}{\partial z} = \mathrm{grad}\, U$$

$$\frac{\partial U}{\partial x} = -\frac{q}{4\pi\varepsilon}\cdot\frac{\partial}{\partial x}\left(\frac{1}{r}\right) =$$

$$\frac{\partial}{\partial x}\left(\frac{1}{r}\right) = (-1)\frac{1}{r^2}\frac{\partial r}{\partial x} = (-1)\frac{1}{r^2}\frac{x}{r}$$

$$\mathrm{grad}\, U = \frac{q}{\ldots}\ \frac{1}{r^3}\,(ix + jy + kz) = \frac{q}{\ldots}\ \frac{\vec{r}}{r^3} = \vec{E}(\vec{r})$$

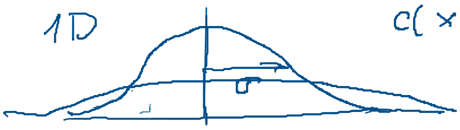
$$\mathrm{grad}\,U = \frac{q}{4\pi\varepsilon}\frac{1}{r^3}(\underbrace{ix+jy+kz}_{\vec r}) = \frac{q}{4\pi\varepsilon}\frac{\vec r}{r^3} = \vec E(\vec r)$$

tok sila

$$\frac{\partial c}{\partial t} = D\Delta c \qquad ① \quad \underline{\vec J_D = -D\,\mathrm{grad}\,c} \qquad F \to v$$

$$② \quad \frac{\partial c}{\partial t} + \mathrm{div}\,J_D = 0$$

1D

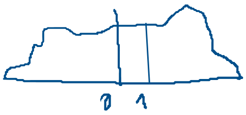


$$c(x, t=0) = N\delta(x)$$

$$c(x,t) = c_0\, e^{-\frac{x^2}{4Dt}} = \frac{N}{\sqrt{4\pi Dt}}\, e^{-\frac{x^2}{4Dt}}$$

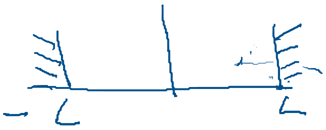
$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{x^2}{2\sigma^2}} \qquad \int P(x)\,dx = 1$$

$$\sigma^2 = 2Dt$$



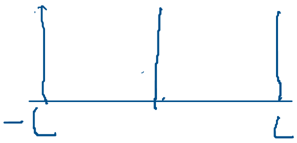
$$c(x,t) = c_0(x,t) \to c_1(x,t)$$

$$\boxed{c(x,t) = \int dy\, \frac{N(y)}{\sqrt{4\pi Dt}}\, e^{-\frac{(x-y)^2}{4Dt}}}$$

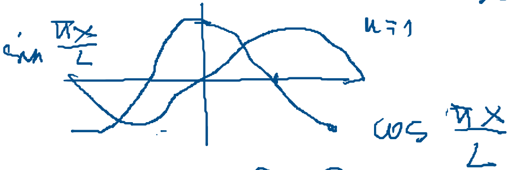


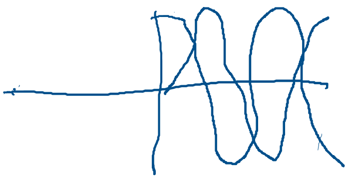
1. pohlcujúca hranica $c = 0$
2. odrážajúca hranica

$\vec J_L$ $J = 0$ odrážajúca hranica



$$1 \qquad \cos\left(\pi \frac{nx}{L}\right) \qquad \sin\left(\pi \frac{nx}{L}\right) \qquad n = 1, 2, \ldots$$





$$\int_{-L}^{L} 1 \cdot \cos\frac{n\pi x}{L}\,dx = 0$$

$$\int_{-L}^{L} 1 \sin\frac{n\pi x}{L}\,dx = 0$$

$$\int_{-L}^{L} \sin\frac{n\pi x}{L}\,\cos\frac{n\pi x}{L} = 0$$

$$\int \cos\frac{n\pi x}{L}\,dx = \frac{L}{n\pi}\left[\sin\frac{n\pi x}{L}\right]_{-L}^{L} = 0$$

$$\int \sin\frac{n\pi x}{L}\,dx = \frac{L}{}\left[-\cos\frac{n\pi x}{L}\right]^{L} =$$

$$\int \sin\frac{n\pi x}{L}\,dx = \frac{L}{n\pi}\left[-\cos\frac{n\pi x}{L}\right]_{-L}^{L} =$$

$$= -\cos n\pi + \underset{\cos n\pi}{\underset{\shortparallel}{\cos(-n\pi)}} = 0$$

$$\int \sin\frac{n\pi x}{L}\cos\frac{n\pi x}{L}\,dx = \frac{1}{2}\int \sin\frac{2n\pi x}{L}\,dx = 0$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin(\beta)\cos\alpha$$

$$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta$$

$$\alpha=\beta$$

na $(-L, L)$

$$f(x) = \frac{c_0}{2} + \sum a_n \sin\frac{n\pi x}{L} + \sum b_n \cos\frac{n\pi x}{L}$$

$$\int_{-L}^{L} f(x)\,dx = \frac{c_0}{2}\int_{-L}^{L} dx + \sum a_n \cancel{\int \sin\frac{n\pi x}{L}\,dx} +$$

$$+ \sum b_n \cancel{\int \cos\frac{n\pi x}{L}\,dx}$$

$$\int_{-L}^{L} f(x)\,dx = \frac{c_0}{2}\,2L = c_0$$

$$\boxed{c_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx}$$

$$\int_{-L}^{L} \sin\frac{n\pi x}{L}\sin\frac{m\pi x}{L}\,dx = 0 \qquad \int_{-L}^{L} \cos\frac{n\pi x}{L}\cos\frac{m\pi x}{L}\,dx = 0$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin\alpha\sin\beta = \frac{\cos(\alpha-\beta) - \cos(\alpha+\beta)}{2}$$

$$\alpha = \frac{n\pi x}{L} \quad \beta = \frac{m\pi x}{L}$$

$$\alpha-\beta = \frac{(n-m)\pi x}{L} \quad \alpha+\beta = \frac{(n+m)\pi x}{L}$$

$$\int_{-L}^{L} f(x)\sin\frac{k\pi x}{L}\,dx = a_k\int_{-L}^{L}\sin^2\frac{k\pi x}{L}\,dx$$

$$\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}} \qquad = a_k\int_{-L}^{L}\left(\frac{1-\underbrace{\cos\frac{2k\pi x}{L}}_{0}}{2}\right)dx$$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}} \qquad = a_k \cdot L$$

$$a_k = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{k\pi x}{L}\,dx$$

$$b_k = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{k\pi x}{L}\,dx$$

$$c_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

D.Ú.

Hanka bod za "HH"

Zdeno bod za "HZ"

100x hodiť HZZZHHZHHZHZ ---