

Worksheet 1.2, Row Reduction and Echelon Forms

Worksheet Exercises

Recitations are meant to be active: students are encouraged to work with other students in recitation. As students are working through exercises, the TA should circulate around the room, helping students. Students may be asked to present their work using a document camera or write a solution on a whiteboard.

1. **Written Explanation Exercise** Answer each the following in one or two sentences.

- (a) What are some of the differences between echelon form and reduced echelon form? List at least three.
- (b) How can we use row reduction to determine whether an augmented matrix corresponds to a consistent system?

2. Which matrices are in RREF? In echelon form?

$$A = \begin{pmatrix} 0 & 1 & 2 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 5 & 5 \end{pmatrix}$$

3. List all 3×2 matrices in RREF. Use * for entries that can be arbitrary.
4. Indicate whether the statements are true or false.
- a) A linear system, whose 3×5 coefficient matrix has three pivotal columns, must be consistent.
 - b) The echelon form of a coefficient matrix is unique.
5. For any three distinct points in the plane, no two on a vertical line, there is a second degree polynomial $p(t) = a_0 + a_1t + a_2t^2$ that passes through those three points. Construct the polynomial that passes through $(1, 12)$, $(2, 15)$, and $(3, 16)$. That is, solve

$$p(1) = 12 = a_0 + a_1 + a_2$$

$$p(2) = 15 = a_0 + 2a_1 + 4a_2$$

$$p(3) = 16 = a_0 + 3a_1 + 9a_2$$

(The coefficient matrix is an instance of the *vandermonde matrix*.)