Plainsound Harmonic Space Calculator | basic
User Guide

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Description

The Plainsound Harmonic Space Calculator | basic (abbreviated as HS CALC | basic) is a tool for composers and musicians who are interested in discovering and working with properties of intervals tuned in just intonation.

One of HS CALC | basic's key objectives is to supplement and encourage use of the Extended Helmholtz-Ellis JI Pitch Notation (HEJI) developed by Marc Sabat and Wolfgang von Schweinitz. HEJI is based on the Pythagorean series of untempered perfect fifths, notated on the traditional five-line staff using flats, naturals, and sharps. It explicitly notates the raising and lowering of these pitches by specific microtonal ratios, defined for each prime, providing visually distinctive *logos* distinguishing *families* of justly tuned intervals that relate to the harmonic series. These take the form of strings of additional accidental symbols based on historical precedents.

HS CALC | basic allows users to compute and compare relative pitch heights of any frequencies based on a chosen reference. Pitches may be inputted in HEJI notation and/or as ratios; outputs include HEJI, ratios, absolute cents, tuning meter read-out, software pitch bends, and steps in an arbitrary equal division of the octave, etc.

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Contents

1   Basic Workflow

In general, movement of information through the calculator flows from left to right across the screen on desktop and top to bottom on mobile.

After setting the **REFERENCE** (1/1), just intonation relationships—whether as notation or ratios—may be entered into the calculator using the **HEJI notation palette** or the **JI ratio** input fields (the desired input method must be selected).

The calculator computes various types of information about the **INPUT**, including notation, ratio, tuning meter cent deviation, frequency, software pitch bend values, etc. The results are displayed in the **OUTPUT** area. Precision (i.e. number of decimal places) may be specified with the **calculator precision** drop-down menu.

Additionally, two pitches may be compared to determine their **MELODIC DISTANCE** (i.e. the interval between them). Ratios may be directly entered in the input fields or "loaded" by re-routing previously calculated pitches from the **OUTPUT** area.

1.1   System Requirements

HS CALC | basic is a simple JavaScript application able to run in any HTML5-compatible web browser on a wide variety of system configurations.

On desktop, the app has been optimised for full HD and higher resolution displays (i.e. 1080p or greater). On displays with lower resolutions (e.g. 720p HD), "areas" may reflow to form new rows. On mobile, the areas form a single vertical column.

The calculator requires an active connection to the internet in order to guarantee proper functionality. For information about offline compatibility, please contact Thomas Nicholson at `thomas@thomasnicholson.ca`.

2   The REFERENCE Area

By default, **HS CALC | basic**'s harmonic reference (1/1) is the pitch written as ♮A in SPN octave 4 and tuned to 440 Hz. This may be considered a *fundamental* or a *generator* for all the other tones in the harmonic space, similar to Harry Partch's concept of *Monophony*. In *Genesis of a Music*, he describes it as "an organization of musical materials based upon the faculty of the human ear to perceive all intervals and to deduce all principles of musical relationship as an expansion from unity, as 1 is to 1."

Changing the **REFERENCE** pitch's octave, diatonic pitch, and/or accidental automatically updates its frequency—*1/1 frequency (Hz)*—as a 12-EDO tempered pitch with respect to A4 440 Hz. This frequency may subsequently be adjusted to any desired value.

> **Example.** Selecting octave 4, tempered diatonic pitch G, and tempered accidental ♮ automatically calculates a frequency of 391.9954 Hz, a 12-EDO wholetone below A4 440 Hz ($440 \div 2^{1/6}$). To work in the harmonic space of Harry Partch, the user may adjust this value to 392.0000 Hz.

2.1   Tuning meter setting

The options under **tuning meter setting** affect how outputted pitches are read on a tuning meter by controlling the frequency of the tuning meter's own reference or "Kammerton" (typically tempered A4 set to 440 Hz). The default setting, *1/1 = 0 cents*, automatically computes the required frequency of tempered A4 in the meter's settings such that the selected reference pitch 1/1 will read ±0 cents.

> **Example.** If 1/1 is defined to be ♮G4 at 392 Hz, then ♮A4 (9/8) is at 441 Hz (a step of 203.91 cents from ♮G4); the expected tuning meter indication would read A+3.91 cents. However, if the frequency of tempered ♮A4 on the tuning meter (Kammerton) were left at 440 Hz, then the required cent deviation of ♮A4 is slightly larger than the expected display value (A+3.93 cents in order to account for the redefinition of tempered ♮G4 as 392 Hz). The calculator *automatically* updates the required frequency of tempered ♮A4 (i.e. ♮A4 on meter is 440.0051 Hz).

Toggling *1/1 cents relative to meter setting* allows for independent control over the desired frequency of 1/1 as well as the relative cent values indicated by a tuning meter. Adjusting the frequency of tempered A4 in the meter's settings allows *any pitch* to read ±0 cents.

> **Example.** Sometimes it may be useful to redefine 1/1—e.g. the note ♮G4 at 392 Hz—but allow some other pitch to remain at 0 cents—e.g. ♮A4 (9/8). With *1/1 cents relative to meter setting* selected, calculated cent deviation output may be arbitrarily redefined by directly adjusting the *frequency of tempered A4 on meter*. If the frequency is changed to 441 Hz, then the frequency of Pythagorean ♮A4 (9/8) will be equal to the frequency of tempered ♮A4, reading A±0 cents, while 1/1 will read G−3.91 cents.

> **Example.** Catherine Lamb has written a series of pieces based on a 10 Hz fundamental, choosing to notate this 1/1 as ♭E−1. Since each partial is a multiple of 10, the Kammerton frequency on the tuning meter may be made equal to the 44th partial (written ↓♭A4 sounding with frequency 440 Hz) in order to provide practical cent indications for performers. This allows for a potential simplification of the notation, where all of the pitches may be written a quartertone lower than sounding (with respect to the tuning meter)—i.e. the fundamental is notated as the pitch-class ♭E but sounds as ♭E+48.7 cents.

2.2   EDO cross-comparison

A reference equal temperament (EDO, "equal divisions of the octave") may be specified as a cross-comparison to untempered just intonation. Outputs are mapped to steps in the specified EDO (default 53-EDO). For more information, reference section 4 ("ratio in EDO steps + error").

3   The INPUT Areas

Before inputting pitches into HS CALC | basic, the desired input method must be selected.

- HEJI notation palette (default)

- JI ratio (input × offset)

3.1   HEJI notation palette

The HEJI notation palette is straightforward: an input pitch may be defined according to its SPN octave, Pythagorean diatonic pitch, and HEJI accidental(s). Palette input accidentals are limited to allow up to three Pythagorean apotomes ($\pm$) and three steps ($\pm$) in each of the prime dimension 5 through 31 (see below for the ratios used in the HEJI accidentals).

Prime dimension	HEJI symbol	Symbol alters by
5	♭♮♯ | ♭♮♯	81/80
7	˻ | ˻	64/63
11	◁ | ≁	33/32
13	◁ | ≁	27/26
17	≋ | ≋	256/255*
19	` | ´	513/512
23	↓ | ↑	736/729
29	≋ | ≋	145/144*
31	◁ | ≁	32/31

* both 17 and 29 in the current HEJI standard require further modification involving the prime dimension 5 to represent 17/16 and 29/16 respectively. A revised Pythagorean-only modification of HEJI is forthcoming.

3.2   JI ratio

Ratio input comprises two components.

- the *offset ratio* – a local 1/1 (with respect to the global 1/1 defined in REFERENCE)

- the *input ratio* – a ratio which is automatically multiplied by the *offset ratio*

> **Example.** If the *offset ratio* is 1/1 and the *input ratio* is 10/9, then the OUTPUT ratio is calculated as 10/9 with respect to the REFERENCE. However, if the *offset ratio* were set to 9/8, then the OUTPUT ratio would be calculated as 10/9 with respect to the REFERENCE *offset* by 9/8 (the *offset ratio* 9/8 functions as a local 1/1). The result is a compounding of the two input ratios, outputting 5/4 (10/9 $\times$ 9/8) with respect to the REFERENCE.

Whole numbers or decimal values may be used (the calculator will automatically convert decimals into whole number fractions when possible).

3.3   Moving between input methods

Making changes in an inactive input method will have no effect on the output until that method is made active (i.e. selected). However, calculated outputs (irrespective of the active input method) may be loaded at any time into the *offset ratio* and the *input ratio* fields by means of the **load output** buttons. This allows pitches inputted with the **HEJI notation palette** to be easily stored as offset or input values for later use in **JI ratio** input.

> **Example.** With default **REFERENCE** values, the pitch ⌐B4 may be selected using **HEJI notation palette** and loaded into the *offset ratio* (8/7) within **JI ratio** input. Selecting input by **JI ratio** initially produces the same output, but 8/7 may now function as a new local 1/1. Inputting 5/4 in *input ratio* compounds the input and offset, resulting in a calculated output of 10/7 (⌐♯D) with respect to the **REFERENCE** (5/4 × 8/7).

Additionally, while input by **JI ratio** is active, an input interval may easily be stacked multiple times by repeatedly loading the calculated output into the *offset ratio*.

> **Example.** With default **REFERENCE** values, the ratio 8/7 may be loaded or directly inputted into *input ratio*. With input by **JI ratio** active, repeatedly clicking **load output** under *offset ratio* generates a stack of 8/7s by updating the offset ratio with the current output and then recalculating the result.

4   The OUTPUT Area

HS CALC | basic computes information based on the **INPUT** and the **REFERENCE** values. Toggling **normalise** optionally reduces output results to pitch classes by reducing them to the span between 1/1 and the octave 2/1 (i.e. 3/1 ⇒ 3/2; 1/3 ⇒ 4/3; etc.).

- *HEJI notation* – The corresponding HEJI notation of a ratio in terms of the spelling of 1/1. If the output ratio goes beyond the 31-limit or if its prime powers extend beyond the scope of the palette, then the message *undefined* is displayed. The 5-dimension is a slight exception: an extra step ± is possible in the **OUTPUT** (i.e. four arrows up/down).

  > **Example.** After inputting the pitch ⥯C4 with the HEJI palette, it may be loaded into the *offset ratio*. Toggling **JI ratio** input and entering 5/4 in the *input ratio* outputs the pitch ⥯E.

- *ratio* – The calculated output expressed as a ratio of whole numbers with respect to the **REFERENCE**. If values become very large, the ratio is automatically converted to a decimal value.

- *ratio in EDO steps + error* – The output ratio quantised in steps of the EDO-temperament specified in **EDO cross-comparison**; indicated using the standard backslash notation (number-of-steps\steps-per-octave). Deviation from the exact ratio is given in cents.

- *tuning meter read-out* – The output ratio as a microtonal deviation falling within 50 cents of the nearest tempered chromatic note (MIDI pitch class).

- *frequency* – The frequency of the output ratio.

- *cents from reference* – The size of the output ratio in cents.

- *Tenney harmonic distance (HD)* – A measure of harmonic complexity of the output ratio b/a, defined as $\log_2(b \times a)$. This value represents the distance in octaves between the ratio's implicit fundamental and common partial ($b \times a$), or alternately the shortest distance from the reference along the prime axes of harmonic space.

- *harmonic space coordinates (monzo)* – The output ratio factorised and displayed as an array of powers with prime bases 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, and 31. Each exponent represents the number of steps along the corresponding prime axis of harmonic space. If the ratio goes beyond the 31-limit, the monzo is displayed as a 31-limit array multiplied by a remainder ratio.

  > **Example.** The ratio 53/52 has coordinates **-2,0,0,0,0,-1,0,0,0,0,0 × (53/1)**. The monzo represents the value $1/(4 \times 13)$—i.e. the 31-limit component.

- *software pitch bends* – Depending on the preferred workflow when faking microtonal accidentals in notation software that does not natively support playback of them, bends may be calculated

  - with respect to the written 12-EDO **diatonic** notes (i.e. A–G without any chromatic alterations), or
  - with respect to the nearest 12-EDO **chromatic** pitch (i.e. any sharps/flats as information for the software have been hidden, then overlayed with HEJI symbols).

  > **Example.** If the pitch bend value for ↯C is calculated from the diatonic note, it would be a modification of tempered C. If calculated from the chromatic, it would be a modification of tempered D. The choice depends on whether the underlying tempered accidentals are in fact present (but hidden) in the score.

The *pb range* may be specified between 1–12 semitones (i.e. 100–1200 cents). With default software settings, **Sibelius** has a *pb range* of ±200 cents, **Finale** (with human playback enabled) has a *pb range* of ±1200 cents, and **Musescore** has a currently fixed *pb range* of ±200 cents.

　　Dorico natively supports playback of user-defined microtonal accidentals. To set up HEJI, the first step is to create a new *Tonality System* with 12000 divisions per octave, achieved by setting the wholetones A–B, C–D, D–E, F–G, and G–A to 2040 divisions and the semitones B–C and E–F to 900 divisions. In principle, this allows for an accuracy of up to $\frac{1}{10}$th of a cent while simultaneously defining the diatonic wholetones in terms of the Pythagorean wholetone 9 : 8 and the diatonic semitones in terms of the Pythagorean

limma 256 : 243 (the backbones of HEJI). Accidental symbols may then be created using font glyphs[1], which the user associates with *pitch deltas.* Like HEJI accidentals themselves, the software makes use of these deltas to modify the previously defined Pythagorean diatonic notes.

Pitch delta values may be easily computed using information computed by the calculator. With default **REFERENCE** (A4) and **calculator precision** (1 decimal place), the desired accidentals modifying an A4 input may be selected using the HEJI palette. A4 is chosen becaues, in this case, it has no intrinsic deviaxtion and may, therefore, be use to construct pitch delta values. The calculated value under *cents from reference* may be converted into a Dorico pitch delta by removing the decimal (i.e. by multiplying by 10).

> **Example.** The accidental ♯ (2187/2048) represents a step of +113.7 cents and would require a Dorico pitch delta of 1137. Likewise, the accidental ♯♭ (63/65) represents a step of −54.1 cents and would require a Dorico pitch delta of −541.

As a staff notation, one of HEJI's highlights is the ability to transpose pitches while maintaining spelling of microtonal modifications. At its current stage of development, Dorico unfortunately does not support this type of transposition.

5   The MELODIC DISTANCE Area

To find the interval between two pitches, computed **OUTPUT**s may be saved by clicking the **load output** buttons or ratios may be typed directly into the **MELODIC DISTANCE** input fields. The calculation is sensitive to the order of input, as *ratio II ÷ ratio I*. The output ratio—in the form *denominator : numerator*—expresses melodic distance as a melody between partials in the harmonic/subharmonic series shared by the two input ratios.

> **Example.** Setting *ratio I* to 81/64 and *ratio II* to 5/4, the calculator computes a melodic distance of 81 : 80 (a downward step). Reversing the ratios results in a melodic distance of 80 : 81 (an upward step).

Output information under **MELODIC DISTANCE** is similar to that found under **OUTPUT**:

- *melodic ratio* – The interval from *ratio I* to *ratio II*, expressed as a "melodic" ratio with the form $a : b$ ("the step from partial a to b").

- *melodic ratio in EDO steps + error* – The melodic distance quantised in steps of the EDO-temperament specified in **EDO cross-comparison**; indicated using the standard backslash notation (number-of-steps\steps-per-octave). Deviation from the exact ratio is given in cents.

- *melodic step in cents* – The size of the melodic distance in cents.

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[1]Font files may be found under JI/MICROTONAL RESOURCES → SOFTWARE on the Plainsound Music Edition homepage – `www.plainsound.org`.

- *frequency difference* – The difference of frequency between *ratio I* and *ratio II* (with respect to the REFERENCE).

- *Tenney harmonic distance (HD)* – A measure of harmonic complexity of the melodic step $a : b$. Refer to page 6 for more information about HD.

- *harmonic space coordinates (monzo)* – The melodic distance factorised and displayed as an array of powers with prime bases 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, and 31. Refer to page 6 for more information about harmonic space coordinates.