DRAG COEFFICIENT OF A FALLING CUPCAKE LINER

What is the drag coefficient of a cupcake liner by measuring its terminal velocity and varying its mass?

Commented [A1]: Research design: The research question is clear. We assume the coefficient is the same for different masses. Details should follow.

Background & Theory

I will explore the essence of one of the most important forces in existence - the drag force, more commonly known as air resistance. When an object is falling through the atmosphere, it experiences two forces on it, the force of gravity and the drag force. The drag force is a force that opposes the object's motion and slows it down. Since this force is negligible on heavy objects with a small area, I decided to use cupcake liners as they are incredibly light and have a large surface area, hence the drag force will have a noticeable effect on it.

Commented [A2]: Research design: Perhaps the drag force reduces the acceleration rate, but the resulting speed keeps increasing at a decreasing rate until terminal speed is reached.

I will not be calculating the drag force as it could simply be calculated using the equation $F_{drag} = mg$, at terminal velocity. Instead, I will be finding the drag coefficient of my object as it is a unique quantity. The drag coefficient is a dimensionless quantity that is used to quantify the drag of an object in a fluid environment. It is used in the drag equation in which a lower drag coefficient indicates that the object will have less aerodynamic drag. (Nancy Hall). It is always associated with a particular surface area and depends on the shape of the object.

Commented [A3]: Research design: The candidate states “a lower drag coefficient indicates that the object will have less aerodynamic drag”—this definition seems circular, but it does tell us how less and lower relate.

It is known that the drag force is proportional to velocity, but that velocity is raised to an exponent. For most objects, it is a squared relationship between the two (Nancy Hall). This relationship can be mathematically represented as $F_{drag} \propto v^2$.

Commented [A4]: Research design: The exponential power requires calculus to determine but as a general rule for high-speed motion the resisting force is approximately proportional to the speed squared. The key issues here are “high-speed” and “approximately”. The candidate does not appreciate this formal context. Are we dealing with high speeds?

Considering other factors that affect the drag, we arrive at the equation (Nancy Hall)

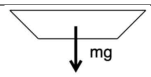
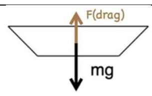
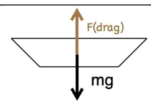
$$F_{drag} = \frac{1}{2}\rho A C_d v^2$$

where the quantities are: ρ = fluid density, for air this is about 1.2 kg m^{-3}, v = velocity of the object, C_d = drag coefficient, and A = reference area of the object (usually the cross sectional area). For the sake of simplicity, all the constants can be written as $b = \frac{1}{2}\rho A C_d$ so now we can write the drag force as $F_{drag} = b\,v^2$.

Commented [A5]: Research design: An interesting and helpful move.

Diagrams of the relationship between drag force and the force of gravity.

Commented [A6]: Research design: Nicely reproduced textbook or online images. Basic material does not need a reference, but the same images are available online.

$u_1 = 0$ Where the initial velocity is zero, the only force acting on the object is the force of gravity The weight) denoted as *mg*.	
$u_2 > u_1 > 0$ Here, the object falls and accelerates. The velocity is downwards and the force of air resistance acts upwards.	
$u_3 > u_2$ As the object's speed increases the air resistance increases and at some point the force is equal to the force of gravity. At this point the object travels at constant speed. This is the terminal velocity.	

Terminal velocity can be represented using forces: $F_{net} = m\,g - F_{drag} = 0$ which is to say that the drag force is equal to the weight: $F_{drag} = m\,g$. We can calculate the terminal velocity from $F_{drag} = b\,v^2 = m\,g$.

Commented [A7]: Research design. An interesting twist.

To calculate the terminal velocity (v_T), I need to first collect experimental data to construct a position vs time graph. This is important because the gradient of a position vs time graph gives the velocity of the object. Although the graph will initially be curved due to the acceleration the object faces ($m\,g > F_{drag}$), there will also be a linear section that represents constant intervals in the change of the object's position with time. The gradient of that linear section would give me the object's terminal velocity.

Commented [A8]: Research design: Correct interpretation—we understand the candidate's approach here with graphs and motion.

The equation above can be manipulated by dividing both sides by *m* and *b,* so we get:

$$\frac{v_T^2}{m} = \frac{g}{b}$$

This equation implies that the gradient of the graph of terminal velocity squared vs mass would allow me to calculate the drag constant *(b)*. Therefore, first, I would need to create a position vs time graph for 5 different masses, so I can also calculate the terminal velocities for the 5 different masses. Furthermore, I would need to square all my terminal velocity to construct a terminal speed squared against mass graph which should have a linear correlation.

Commented [A9]: Research design: But only if speed squared is the correct function. Interestingly, this assumption will work for the investigation.

Commented [A10]: Research design: We have a clear understanding of the approach to this investigation. The details of equations and forces provide a relevant context.

Methodology

Commented [A11]: Research design: Details of the method are clearly stated. This is a textbook investigation, however, and related details are easily available online; the candidate did reference key content.

Variables Table: Independent, dependent, and control variables.

Commented [A12]: Research design: Variables are identified. Details of the dependent variable are found in the procedure.

Variables	Description
Independent	Mass of the cupcake liners (The mass can be changed by stacking the cupcake liners). Denoted by m and measured in kg. The masses are $0.00023, 0.00046, 0.00069, 0.00092,$ and $0.00115\mathrm{kg}$. The masses are in increments of $0.00023\mathrm{kg}$ because a single cupcake liner has a mass of $0.00023\mathrm{kg}$. Conducting the experiment at 5 different masses increases the reliability of the results.
Dependent	Terminal velocity of the falling cupcake liner. Denoted by v_T and measured in ms^{-1}.
Control	• Height of drop (Approximately $2.5\mathrm{m}$, but it does not matter significantly since the object reaches terminal velocity extremely quickly). • Air conditions (The air conditioner in the classroom will be turned off). • Cross sectional area of the object (Stacking the cupcake liners ensures that the mass can be changed while the cross-sectional area remains the same). • Initial speed of zero.

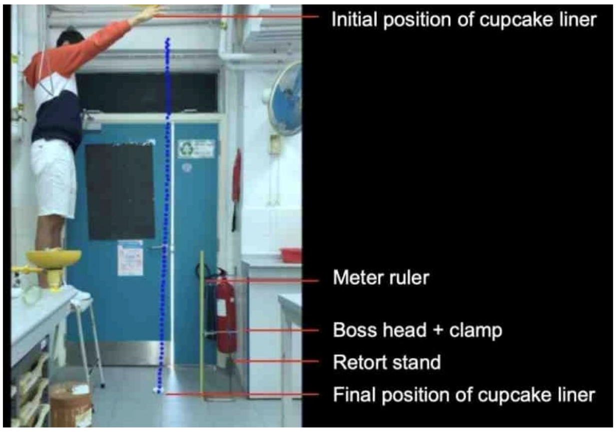
Pre-testing

To record the object' s position over a time period, a motion sensor could be used, however, from pre-testing I determined that my cupcake liner is to small and light to follow a perfectly straight path, hence it does not fall directly on top of the motion sensor and so measurements from this device were not accurate. Instead, I decided to record the drop using my phone at 4K 60 frames per second so I could conduct a frame by frame motion analysis on LoggerPro (A data collection and analysis software). The apparatus is shown below.

Commented [A13]: Research design: The preliminary work is done well and in a thoughtful way.

Photograph

Commented [A14]: Research design: A sketch would have been more useful in understanding the setup.



Experimental Procedures

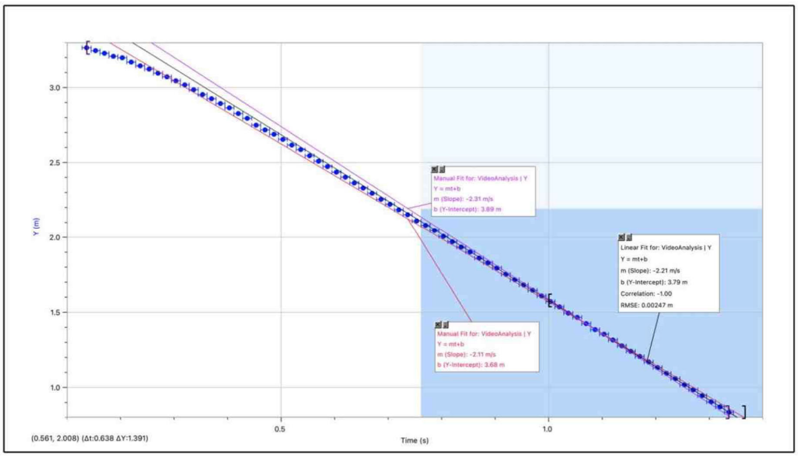
1. Place a meter ruler vertically, parallel to the position of drop using a retort stand as shown above to provide a scale and frame of reference for the motion analysis.
2. Check mass of cupcake liner on a weighing scale and record. (e.g. 0.00023 kg)
3. Place the phone/camera a few metres away from the position of drop to allow a clearview of the initial and final position of the cupcake liner. The distance away from the drop is to ensure the release point is in the field of view and ensure the recording device is straight to reduce parallax error while analysing the footage.
4. Start recording on phone/camera.
5. Release cupcake liner from a height of 2.5 m.
6. Stop recording after the cupcake liner touches the floor.
7. Repeat the procedure 4 more times for the mass currently experimenting on.
8. Repeat steps 4 to 6, 5 times for the 4 other masses. The cupcake liners should bestacked to increase the mass.
9. Import videos into LoggerPro to conduct frame by frame motion analysis.

No safety precautions were required for this experiment.

Commented [A15]: Research design: Repeated measurements are mentioned as well as a range of masses.

Commented [A16]: Research design: How does the candidate calibrate the video images? There is a scale function in LoggerPro. Presumably a metre ruler is used but there are error and uncertainty issues to be addressed, such as perspective, parallax, etc.

Sample frame by frame analysis of a drop for liner mass 0.00069 kg



From the line of best fit, it is evidence that initially the object experiences acceleration (represented by the curve) as there is minimal force of air resistance acting on it. However, as it quickly gains velocity, the force of air resistance increases thereby allowing the object to approach a constant speed (around 0.5 s from drop). The terminal velocity is represented by the linear section of the graph. From the line of best fit (black color) it is evident that the terminal velocity is reached around 0.8s after the drop since the line of best fit perfectly intersects most of the points after that time.

Furthermore, the correlation coefficient for the section highlighted is –1, which means the data has a perfect negative correlation and strongly supports the idea that the velocity is constant during that period. The slope of that highlighted portion is –2.21 m s^{-1}, which is the object's terminal velocity. The negative sign refers to the 'downward' direction hence theabsolute value will be used for calculations.

Commented [A17]: Research design: The image is hard to see. We must accept the candidate's data analysis.

Commented [A18]: Data analysis: A sound analysis of terminal speed.

Commented [A19]: Data analysis: The candidate clearly understands the meaning of this analysis.

Commented [A20]: Research design: The readings of the graph are correct and appropriate, but a correlation of –1 described as "perfect" fails to appreciate the limit of data precision and the fact that all experimental data has some degree of uncertainty. Still, this approach is valid.

The maximum and minimum slope lines (see above) represented by the colors purple and red respectively, will be used to determine the uncertainty in the terminal velocity. The maximum and minimum slope lines are only drawn using the uncertainty boxes of the points in the highlighted section.

Commented [A21]: Data analysis: The graph image is too difficult to see. Did the candidate follow the appropriate process to determine the range, or did they just use the first and last data points?

Raw Data

Commented [A22]: Data analysis: The software determined the gradient of the linear section of the graph, and hence the speed. But we only see one example of the determination of gradient uncertainty. The uncertainties listed here would not be for individual or raw data.

Terminal velocity of the cupcake liner at different masses

Commented [A23]: Data analysis: The data tables are clear, precise and we assume accurate.

Mass, m /kg $\Delta m \pm 0.00001$kg	Terminal velocity, v_T /ms^{-1} Δv_T /ms^{-1}				
	Trial 1	2	3	4	5
0.00023	1.12 ± 0.08	1.07 ± 0.06	1.11 ± 0.08	1.10 ± 0.05	1.11 ± 0.06
0.00046	1.46 ± 0.06	1.57 ± 0.05	1.47 ± 0.07	1.49 ± 0.06	1.52 ± 0.08
0.00069	1.94 ± 0.09	1.98 ± 0.08	1.99 ± 0.08	1.91 ± 0.07	1.93 ± 0.08
0.00092	2.19 ± 0.09	2.17 ± 0.12	2.23 ± 0.07	2.26 ± 0.10	2.21 ± 0.10
0.00115	2.42 ± 0.11	2.49 ± 0.10	2.50 ± 0.12	2.43 ± 0.09	2.53 ± 0.11

Uncertainty of the variables and the reasons for the uncertainty

Commented [A24]: Data analysis: The uncertainty in terminal velocity needs explication. However, we know what the candidate did here.

Commented [A25]: Data analysis: The uncertainty of a frame is not half the frame period. There is also a judgement call on defining the position in an image. More detail is wanted.

Uncertainty of variables	Reasons
Mass: ± 0.00001kg	The weighing scale was precise to 0.01g.
Position: Insignificant	The uncertainty in position comes from the uncertainty of the meter ruler and the random error of me not clicking the exact same point on the cupcake liner at each frame, but this was insignificant.
Time: ± 0.0083s	The video was taken at 60 frames per second, hence the single frame where an event happened gives the time to within $\frac{1}{60}$s, however, the actual time of the event could be anytime in that window, hence I took the uncertainty as $\frac{1}{2} \times \frac{1}{60} = 0.0083$s.
Terminal velocity: Varies	The uncertainty in the terminal velocity is derived from the maximum and minimum slope lines that were created from the uncertainty bars in time only, because the uncertainty in position was insignificant.

Processed Data

Average terminal velocity and average velocity squared

Mass m /kg $\Delta m \pm 0.00001$kg	Average terminal velocity $\overline{v_T}$ /ms^{-1}	Uncertainty in average terminal velocity $\Delta\overline{v_T}_{AU}$ /ms^{-1}	Average terminal velocity squared $(\overline{v_T})^2$ /(ms^{-1})^2	Uncertainty in average terminal velocity squared $\Delta(\overline{v_T})^2_{AU}$ /(ms^{-1})^2
0.00023	1.10	0.07	1.21	0.15
0.00046	1.50	0.06	2.25	0.19
0.00069	1.94	0.08	3.76	0.31
0.00092	2.21	0.10	4.88	0.42
0.00115	2.47	0.11	6.10	0.52

The notation below is: *AU* = absolute uncertainty and *RU* = relative uncertainty.

Sample calculations for the mass of 0.00023 kg are shown here:

Commented [A26]: Data analysis: The candidate correctly propagated percentage by doubling for the square. Good detail.

average terminal velocity

$$\overline{v}_T = \frac{\sum v_T}{N} = \frac{1.11+1.07+1.11+1.10+1.11}{5} = 1.10\text{ms}^{-1}$$

absolute uncertainty in average terminal velocity

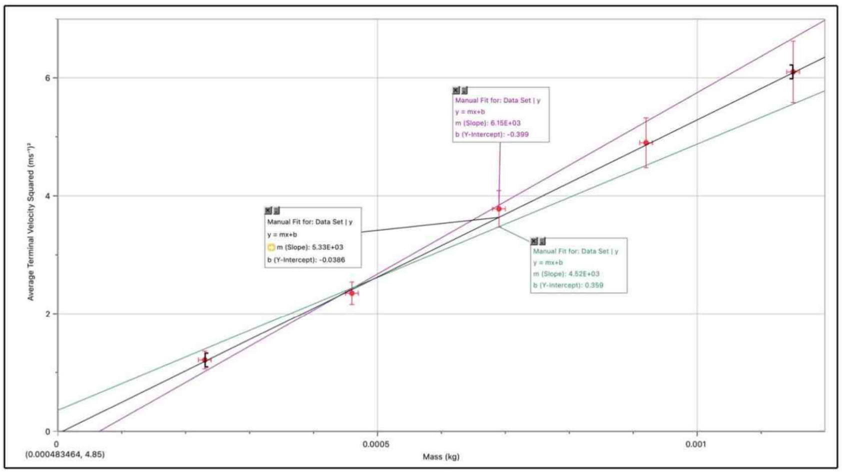
$$\Delta\overline{v}_{T_{AU}} = \frac{\sum \Delta v_T}{N} = \frac{0.08+0.06+0.08+0.05+0.06}{5} = \pm 0.066\text{ms}^{-1}$$

absolute uncertainty in average terminal velocity squared

$$\Delta\left(\overline{v}_T\right)^2_{AU} = \Delta\overline{v}_{T_{RU}} \times 2 \times \left(\overline{v}_T\right)^2 = \left(\frac{0.066}{1.10}\right) \times 2 \times 1.21 = \pm 0.15\left(\text{ms}^{-1}\right)^2$$

Next, I graph the average terminal velocities against mass and determine a linear line best fit. Below, the black line represents the line of best fit, the purple line represent the maximum slope and the green line represent the minimum slope.

<u>Graph of Average Terminal Velocity Squared vs Mass</u>



Commented [A27]: Data analysis/conclusion: This graph and the preceding analysis nicely address the data analysis requirements. The quality data, properly presented, answers the research question.

The equation of best fit is $y = 5330\,x - 0.0386$. The uncertainty in the gradient of the best fit line is:

Commented [A28]: Data analysis: The "x" in the best-fit linear equation is the gradient, but here (in the following calculation) the symbol "m" represents the gradient. On page 2 and in the earlier equations, "m" represented mass. The "m" in "v^2/m" is not the gradient. With this in mind we can still follow the text.

$$\Delta m_{AU} = \frac{\text{maximum slope} - \text{minimum slope}}{2} = \frac{6150 - 4520}{2} = \pm 815$$

Therefore the gradient and it uncertainty and then solve for the constant b.

Commented [A29]: Data analysis: The gradient uncertainty range is excessive but workable. The candidate could have followed the physics skill statement about determining the maximum and minimum gradients by eye taking into account all the uncertainty bars (and not used just the first and last error bars, extreme points, which yield too wide a range).

$$m \pm \Delta m = \left(5330 \pm 815\right) \mathrm{m^2\,s^{-2}\,kg^{-1}}$$

Now we substitute the value of the slope into the equation:

$$slope = \frac{v^2}{m} = \frac{g}{b} \quad \rightarrow \quad 5330 = \frac{9.81}{b} \quad \rightarrow \quad b = 0.00184 \pm 0.00028 \mathrm{kgm^{-1}}$$

The uncertainty in b is:

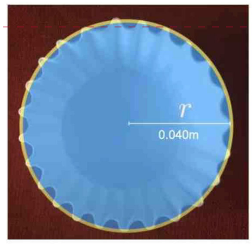
$$\Delta b_{AU} = \Delta m_{RU} \times b = \frac{815}{5330} \times 0.00184 = 0.000281\ \mathrm{kg\ m^{-1}}$$

From the drag constant I can calculate the drag coefficient, C_d:

Commented [A30]: Conclusion: The details and process here justify the conclusion.

$$C_d = \frac{2b}{\rho A} = \frac{0.00368}{1.28A}$$

Next I calculate the cross-sectional area of my liner, as shown. The measurement of diameter was done using a 15 cm ruler, and it was 0.08 m ± 0.0005 m. The uncertainty here is insignificant and will be ignored. The area A is:



Commented [A31]: Data analysis: The calculation of area is nicely presented.

$$A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \pi\left(0.040\right)^2 = 0.0050\,\mathrm{m}^2$$

Substituting this into the drag coefficient equation, we find:

$$C_d = \frac{0.00368\,\mathrm{kg\,m}^{-1}}{\left(1.28\,\mathrm{kg\,m}^{-3}\right)\left(0.0050\,\mathrm{m}^2\right)} = 0.57 \pm 0.09$$

The uncertainty is

$$\Delta C_{d_{AU}} = \Delta b_{RU} \times C_d = \frac{0.00028}{0.00184} \times 0.57 = \pm 0.09$$

Again, **the drag coefficient is 0.57 ± 0.09.** This value has a range from 0.48 to 0.66. The units cancel out, and the coefficient is a dimensionless quantity. My results are well within the range of those value listed in Wikipedia for various shapes of object. Unfortunately, cupcake liners are not listed but a half sphere is 0.42, a cone 0.50.

Commented [A32]: Data analysis/conclusion: The candidate has a 16% uncertainty here. However, the textbook value for cupcake liners is not listed (and is unique as the candidate tells us). The 0.57 value is realistic and acceptable. The candidate has justified their result in comparison to accepted scientific context.

Conclusion

Commented [A33]: Conclusion: A most thorough appreciation of the validity of the conclusion.

The processed data suggests a strong linear relationship between average terminal velocity squared and mass, thus supporting the scientific theory behind it. Unfortunately, a literature value for the drag coefficient of a cupcake liner cannot be found hence the accuracy of the derived drag coefficient in relation to a known value cannot be evaluated. The uncertainty on the derived drag coefficient is 16% and this is quite high, suggesting that the result is reliable only to some extent. However, intuitively, the true value of the drag coefficient should be slightly higher than what I obtained, because drag coefficients for non-streamlined objects are around 1, for example, the drag coefficient of a flat plate is 1.28, but for a streamlined object like a bullet, it's only 0.295 (Nancy Hall). A cupcake liner falls more towards the non-streamlined category and hence will have a fairly large drag coefficient, but slightly lower than 1.

Commented [A34]: Evaluation: The candidate shares their insight about the quality of their conclusion.

Commented [A35]: Conclusion: An interesting and insightful argument.

Evaluation Table

Weakness	How it affects my results	Type of error	Improvement
The uncertainty in time was quite large since the drop was recorded at a relatively low FPS.	This reduces the reliability of my results as it creates a large uncertainty on the average terminal velocity squared, which is one of the sources of uncertainty in the derived drag coefficient.	Random error	Record at a higher frame per second e.g. 120fps, which makes the uncertainty in time to also be insignificant hence significantly improving the reliability of the derived drag coefficient due to the exceptionally low random and systematic errors.
Unable to click the exact same spot on the cupcake liner at each frame.	In Figure 2, the frame by frame analysis required me to click the same spot on the cupcake liner at each frame, but this was extremely challenging as the cursor was quite large and the object was exceptionally small in the field of view. This creates some inaccuracy in the position vs time graph, which affects the derived terminal velocity and hence the drag coefficient.	Random error	Much larger cupcake liners can be used, which would not only eliminate this issue, but also allow for the utilisation of a motion sensor. The larger cupcake liners would have a much higher chance of landing directly on top of it as they have more inertia and momentum hence small disturbances in the air flow wouldn't affect their path.

Commented [A36]: Evaluation: The first issue does not seem too important. The best-fit linear line is already well known. The second issue is indeed important. Procedures more than the methodology are addressed here—these are specific, however.

Bibliography

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