Making Do with Less: An Introduction to Compressed Sensing 4
Geolocation of RF Emitters

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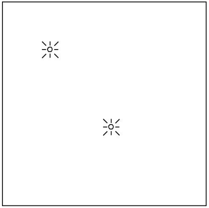
Application: Geolocation of RF Signals

A region D on the ground in which one or more (but "few") radio frequency (RF) emitters (transmitters) may be present:



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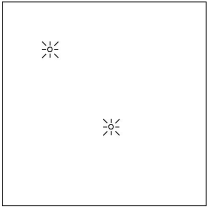

Goal: Locate the emitters and their transmission power using relatively few RF receivers.

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- Military—localizing noncooperative transmitters.
- In many cases it may be possible to use existing hardware.
- Many other "source identification" problems that don't involve RF emitters have a similar mathematical structure.

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- The rate at which the RF power decreases with distance (attenuates) is known for the environment in question.

Modeling Assumptions

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In most of what follows we'll use a rate of $1/r^k$ with $k \approx 3$ (we estimated $k = 3.5$ for the Rose football field, based on data we took). Here k is called the *pathloss exponent*.

Received Signal Strength (RSS) Model: One Emitter

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We model the received signal strength (RSS) as proportional to

$$RSS \sim \frac{p}{r^k}.$$

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for some constant C that depends on the receiver antenna/sensitivity. We can calibrate for the value of C.

Received Signal Strength (RSS) Model: Multiple Emitters

Suppose multiple emitters with powers $p_1, \ldots, p_n$ are present at coordinates $(a_1, b_1), \ldots, (a_n, b_n)$, ground level, and receiver at (x, y), altitude h.

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This is a linear expression in the powers $p_1, \ldots, p_n$.

Example

A receiver is located at position $(0.2, 0.4)$, altitude 0.2. Emitter 1 lies at position $(0.3, 0.7)$ with power p_1 and emitter 2 lies at $(0.7, 0.1)$, power p_2 (both at altitude 0). Then

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- $r_1 = \sqrt{0.1^2 + 0.3^2 + 0.2^2} \approx 0.374$, so the power received from emitter 1 is proportional to $p_1/0.374^k$. With $k = 3.1$ this is $21.06 p_1$.

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- $r_2 = \sqrt{0.5^2 + 0.3^2 + 0.2^2} \approx 0.616$, so the power received from emitter 1 is proportional to $p_2/0.616^k$. With $k = 3.1$ this is $4.48p_2$.

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- $r_2 = \sqrt{0.5^2 + 0.3^2 + 0.2^2} \approx 0.616$, so the power received from emitter 1 is proportional to $p_2/0.616^k$. With $k = 3.1$ this is $4.48p_2$.
- The total power received is

$$RSS = C\left(\frac{p_1}{0.374^k} + \frac{p_2}{0.616^k}\right) = C(21.06p_1 + 4.48p_2)$$

for some constant C.

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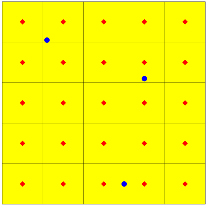
Divide the region D of interest on the ground into sub-rectangles, numbered 1 to n.



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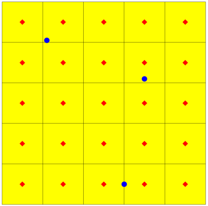

Let (a_j, b_j) be the coordinates of the center of the jth subrectangle (red diamonds). An emitter with power p_j may lie at (a_j, b_j).

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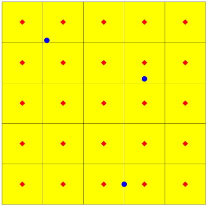
Suppose m receivers (altitude h) are at known positions (x_i, y_i), $1 \leq i \leq m$ (blue dots).



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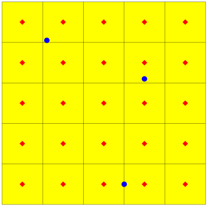

We know the coordinates (a_j, b_j) of each rectangle center, so we can compute

$$r_{ij} = \sqrt{(a_j - x_i)^2 + (b_j - y_i)^2 + h^2}.$$

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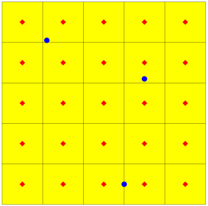

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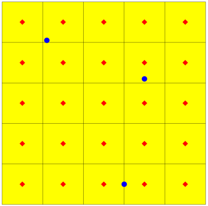

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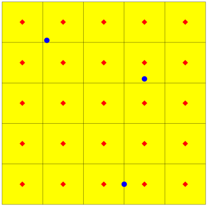

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This is a linear equation relating $p_1, \ldots, p_n$ to d_i.

Example

Consider a 2×2 grid of subrectangles on the square $D = [0, 1]^2$. Rectangles $1, 2, 3, 4$ have centers $(0.25, 0.25), (0.75, 0.25), (0.27, 0.75), (0.75, 0.75)$, respectively.

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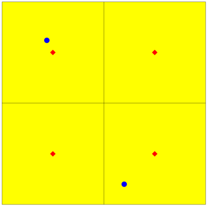
Receivers are at altitude $h = 0.1$, (x, y) locations $(0.6, 0.1)$ and $(0.22, 0.81)$.



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Example

In this case the equation

$$d_i = C \sum_{j=1}^{n} \frac{p_j}{r_{ij}^k},$$

for $1 \leq i \leq m$ (with $C = 1$) yields

$$\begin{aligned} 16.39p_1 + 77.53p_2 + 7.77p_3 + 16.39p_4 &= d_1 \\ 5.66p_1 + 2.07p_2 + 544.33p_3 + 5.66p_4 &= d_2, \end{aligned}$$

a system of $m = 2$ equations in $n = 4$ unknowns.

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More generally, from

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But n could be large ($n \approx 2500$) and m small, e.g., $m \approx 30$. The system is underdetermined.

However, if only a few emitters are present, most $p_i = 0$. It's ripe for compressed sensing!

Matrix Formulation

The resulting system of linear equations can be written as $\mathbf{Ap} = \mathbf{d}$ where $\mathbf{p} = \langle p_1, \ldots, p_n \rangle$, $\mathbf{d} = \langle d_1, \ldots, d_m \rangle$ (with $m << n$) and $\mathbf{A}$ is the $m \times n$ sensing matrix with entries (taking $C = 1$ for the moment)

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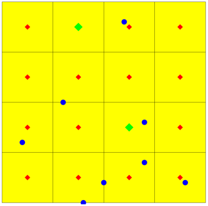
The nonzero p_j indicate the location of emitters (and their power).

A Small Example

Consider a 4×4 grid, so there are $n = 16$ subrectangles in $D = [0, 1]^2$.

Receivers are at altitude 0.1 and 8 random (x, y) coordinates.



There is an emitter at $(0.625, 0.375)$ (power 0.2) and $(0.375, 0.875)$ (power 0.17), shown in green.

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A sample of $\mathbf{A}$ and $\mathbf{d}$:

$$\mathbf{A} = \begin{bmatrix} 1.31 & 1.86 & 2.09 & 1.76 & \cdots \\ 17.0 & 235.0 & 235.0 & 17.0 & \cdots \\ 13.0 & 16.2 & 7.71 & 3.0 & \cdots \\ 31.0 & 235.0 & 47.5 & 7.94 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad \mathbf{d} = \begin{bmatrix} 12.5 \\ 6.55 \\ 6.96 \\ 2.46 \\ \vdots \end{bmatrix}$$

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Nonetheless, OMP exactly recovers the two emitters (and makes it clear there are exactly two emitters).

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This experiment is repeated 500 times.

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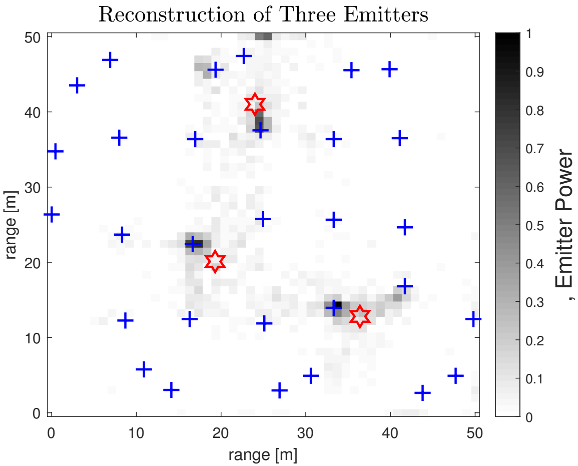

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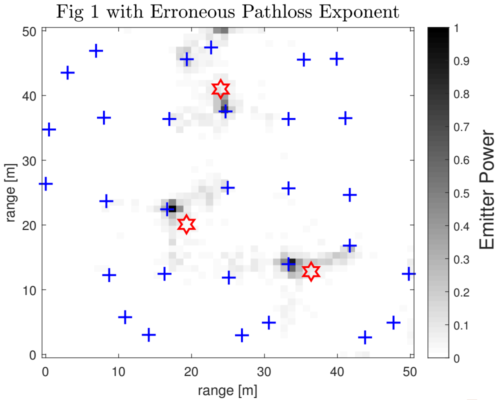
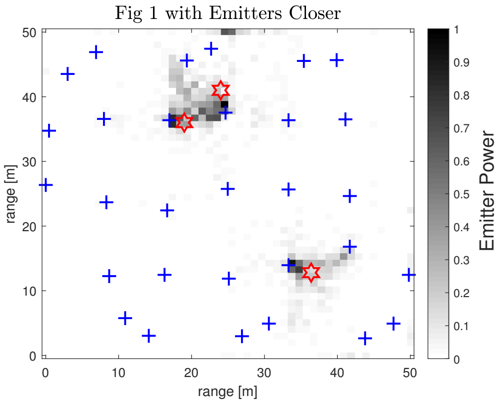
An erroneous estimate of the pathloss exponent (2.5 instead of 3.5) doesn't hurt things.



Resolution

As expected, when emitters are closer, it's more difficult to resolve them.



Resolution Analysis

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Consider a potential emitter at some location (a, b), unknown power p; the data collected would be $\mathbf{d} \in \mathbb{R}^m$.

Consider another potential nearby location for an emitter, $(\tilde{a}, \tilde{b})$, unknown power $\tilde{p}$; the data collected would be $\tilde{\mathbf{d}} \in \mathbb{R}^m$.

Resolution Analysis

With no noise present we know that $\mathbf{d} = p\mathbf{v}$ and $\tilde{\mathbf{d}} = \tilde{p}\mathbf{w}$ where

$$\mathbf{v} = \begin{bmatrix} 1/r_{11}^k \\ \vdots \\ 1/r_{m1}^k \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} 1/r_{12}^k \\ \vdots \\ 1/r_{m2}^k \end{bmatrix}$$

where r_{ij} is the distance from receiver i to emitter j and k is the pathloss exponent. We can compute $\mathbf{v}$ and $\mathbf{w}$ from the geometry of the situation.

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If $\mathbf{d}^*$ is the actual data we collect, either $\mu(\mathbf{d}^*, \mathbf{v}) = 1$ or $\mu(\mathbf{d}^*, \mathbf{w}) = 1$, so we can tell in which location the emitter lies.

Resolution Analysis

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$$\mathbf{d} = p \underbrace{\begin{bmatrix} 0.6558 \\ 0.1874 \\ 0.6143 \\ 0.5071 \end{bmatrix}}_{\mathbf{v}} \quad \tilde{\mathbf{d}} = \tilde{p} \underbrace{\begin{bmatrix} 0.6527 \\ 0.1751 \\ 0.5935 \\ 0.4859 \end{bmatrix}}_{\mathbf{w}}$$

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We" know the emitter is not at $(0.22, 0.39)$ since $\mu(\mathbf{d}^*, \mathbf{w}) = \mu(\mathbf{v}, \mathbf{w}) = 0.9998$.

Resolution Analysis

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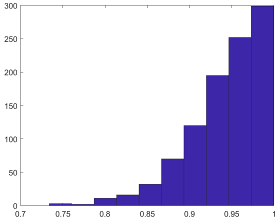
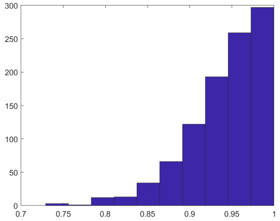
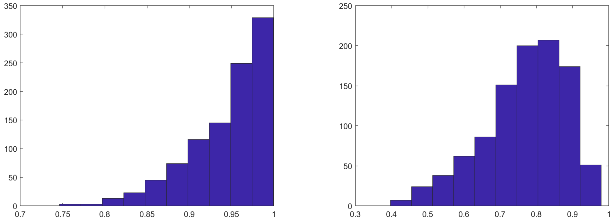
Consider an emitter a $(0.2, 0.4)$, data vector $\mathbf{d}^*$ generated with a realistic amount of noise. Histograms of the coherence of $\mathbf{d}^*$ with each of $\mathbf{v}$ and $\mathbf{w}$ (1000 trials) look like





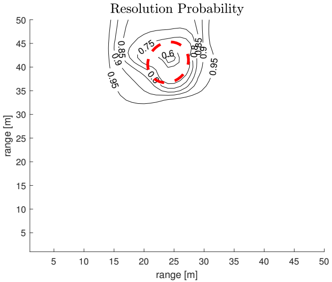
Resolution Analysis

If the potential emitter locations are more widely separated, there's hope.



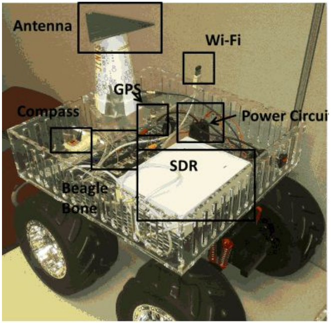
Resolution Analysis

By looking at the receiver configuration, noise level, and emitter separation one can analyze the statistical properties of the coherences, and determine when the emitters can be distinguished with high probability.



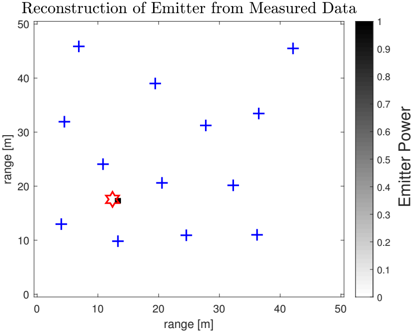
Data!

A team of Rose-Hulman seniors built apparatuses to act as emitters/receivers and collect actual data.



Data!

A single emitter reconstruction from 13 receiver data points.



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The entries $1/r_{ij}^{k}$ in the sensing matrix $\mathbf{A}$ are all positive.

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As a result, many columns of **A** are nearly parallel, and **A** has coherence close to 1.

We used a modification of OMP called "Band-Limited Locally-Optimized Orthogonal Matching Pursuit" (BLOOMP) that recognizes this and doesn't assign RF sources in nearby subrectangles.