

## The Polar Area Diagrams of Florence Nightingale

If you read the article on Florence Nightingale in “The Children’s Book of Famous Lives”<sup>1</sup> you will not learn that she had to battle with her parents to be allowed to study Mathematics. If you read the Ladybird book “Florence Nightingale”<sup>2</sup> you will not discover that she was the first woman to be elected as a Fellow of the Royal Statistical Society. In looking around for an area of research I was intrigued to discover that Florence Nightingale, who I always thought of as the “lady with the lamp”, was a competent Mathematician who created her own type of statistical diagram which she used to save thousands of soldiers from needless death.

Florence Nightingale headed a group of 38 nurses who went to clean up the hospitals for the British soldiers in the Crimea in 1854. She found that most of the deaths were due to diseases which could be prevented by basic hygiene, such as typhus and cholera. Her improvements were simple but they had an enormous effect:

“She and her nurses washed and bathed the soldiers, laundered their linens, gave them clean beds to lie in, and fed them”<sup>3</sup>.

When she returned to Britain she made a detailed report to the Government setting out what conditions were like and what needed to be done to reduce deaths in the hospitals. Nothing was done, so she tried again, making another statistical report and included in it three new statistical diagrams to make data collated by William Farr more accessible to people who could not get their minds around tables of figures. These were her polar area diagrams or rose diagrams, sometimes also known as ‘coxcombs’. The first showed how many men had died over the two years 1854-5, the second showed what proportions of men had died from wounds in battle, from disease and from other causes, the third showed how the number of deaths had decreased once “sanitary improvements”<sup>4</sup> had been introduced.

I decided I would try to recreate the second of these diagrams which is the most complicated and the most shocking. It is called “Diagram of the causes of mortality in the army in the east”. A copy of it is below:

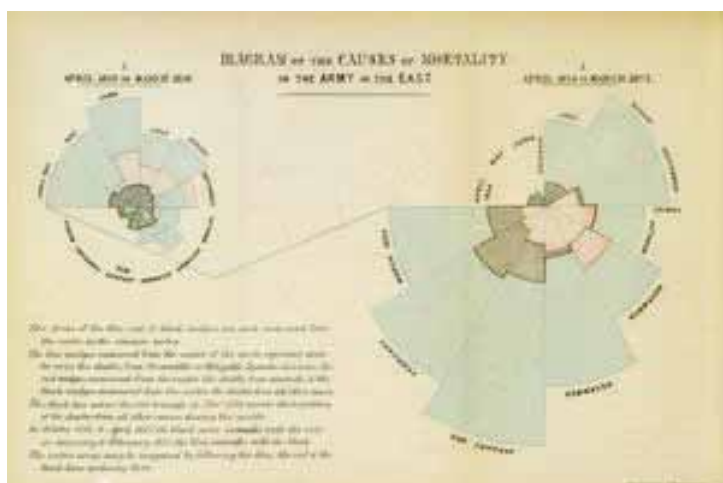


Figure 1

The basic ideas are very simple. The blue area represents deaths due to disease, the red area represents death due to wounds in battle and the black area represents death due to other causes.

I tried to find a copy of the data which this diagram represented, but I had no luck, so I decided to make sure I understood exactly how the diagram was made and to make my own version of some data which I did have to hand. Once I tried to understand the diagram in detail I found there were some problems.

### The First Problem



I wasn't sure whether the black area in a shape such as this: was supposed to be



this area



or this area

In other words, were the colours separate, or overlapping? The articles I read didn't make it clear. O'Connor says that "The area of each coloured wedge, measured from the centre as a common point, is in proportion to the statistic it represents"<sup>5</sup>, which makes it seem that all colours are wedged shaped, or sectors, so the colours overlap. However, Lienhard commented that in the November 1854 section "battle deaths take up a very small portion of each slice"<sup>6</sup>, which makes it sound as though the slice has three separate portions, and Brasseur says that "she also divided the areas within each of the wedges to show which portion of the mortality data for that month could be allotted to each cause of death"<sup>4</sup>. I decided to construct polar area diagrams for a set of data with the colours separate and with the colours overlapping to see if putting theory into practice would make it clearer to me. The data I used was taken from the IB grade distribution statistics for the past 15 years at my own institution. I used the numbers taking Higher Mathematics, Standard Mathematics and Mathematical Studies to be represented by my three colours. I took the old Mathematical Methods course to be the same as Standard Mathematics.

To fit 15 sectors into the circle I needed each arc to subtend an angle of  $\frac{2\pi}{15}$  radians at

the centre. The area of each sector would then be  $A = \frac{1}{2} \times \frac{2\pi}{15} r^2 = \frac{\pi}{15} r^2$  where  $r$  is the radius of the sector. Since the area needs to be proportional to the statistic, I needed to

find the radius, so I used  $r = \sqrt{\frac{15A}{\pi}}$  and just used a scale which would allow me to draw

a reasonable sized diagram. To create a polar area diagram with overlapping sectors I just used this formula on each of the numbers of students taking the various options.

Numbers taking Mathematics year on year

	Numbers (A)			Radius (r)		
	Higher	Studies	Standard	Higher	Studies	Standard
1995	1	24	0	2.2	10.7	0.0
1996	4	15	0	4.4	8.5	0.0
1997	8	10	0	6.2	6.9	0.0
1998	6	31	0	5.4	12.2	0.0
1999	9	17	0	6.6	9.0	0.0
2000	10	20	0	6.9	9.8	0.0
2001	4	31	1	4.4	12.2	2.2
2002	5	21	2	4.9	10.0	3.1
2003	4	15	4	4.4	8.5	4.4
2004	5	29	5	4.9	11.8	4.9
2005	1	28	0	2.2	11.6	0.0
2006	3	16	2	3.8	8.7	3.1
2007	8	13	0	6.2	7.9	0.0
2008	11	29	14	7.2	11.8	8.2
2009	10	23	15	6.9	10.5	8.5

I then used a geometric program (GeoGebra) to draw the sectors all with a common centre, each with an angle of  $\frac{2\pi}{15}$  radians, and with the radii as given in the table. I drew the Higher sectors first with the Studies on top of these, and the Standard on top of these. This was the result:

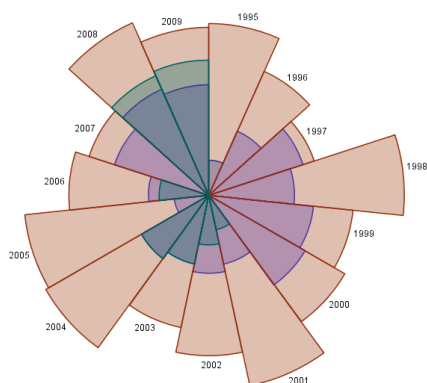


Figure 2

Polar area diagram to show students taking Mathematics at one school (colours overlapping)

Blue represents the number of students taking Higher Maths.

Brown represents the number of students taking Mathematical Studies.

Green represents the number of students taking Standard Maths.

The colours are not solid, so where colours overlap there is a different colour. The blue overlapping the brown makes a pink here, and the green overlapping the blue makes a darker green. In 2003 and in 2004 there were an equal number of students taking Higher and Standard so three separate colours cannot be seen on the diagram.

Next I worked out the radii needed if the colours were not to overlap. For this I used

cumulative areas to work out the radii.  $R1 = \sqrt{\frac{15A1}{\pi}}$ ,  $R2 = \sqrt{\frac{15(A1 + A2)}{\pi}}$  and

$$R3 = \sqrt{\frac{15(A1 + A2 + A3)}{\pi}}.$$

Numbers taking Mathematics year on year

	Numbers (A)			Radii		
	Higher (A1)	Studies (A2)	Standard (A3)	R1	R2	R3
1995	1	24	0	2.2	10.9	10.9
1996	4	15	0	4.4	9.5	9.5
1997	8	10	0	6.2	9.3	9.3
1998	6	31	0	5.4	13.3	13.3
1999	9	17	0	6.6	11.1	11.1
2000	10	20	0	6.9	12.0	12.0
2001	4	31	1	4.4	12.9	13.1
2002	5	21	2	4.9	11.1	11.6
2003	4	15	4	4.4	9.5	10.5
2004	5	29	5	4.9	12.7	13.6
2005	1	28	0	2.2	11.8	11.8
2006	3	16	2	3.8	9.5	10.0
2007	8	13	0	6.2	10.0	10.0
2008	11	29	14	7.2	13.8	16.1
2009	10	23	15	6.9	12.6	15.1

This gave a diagram with Higher numbers at the centre and Standard numbers at the edge, like this:

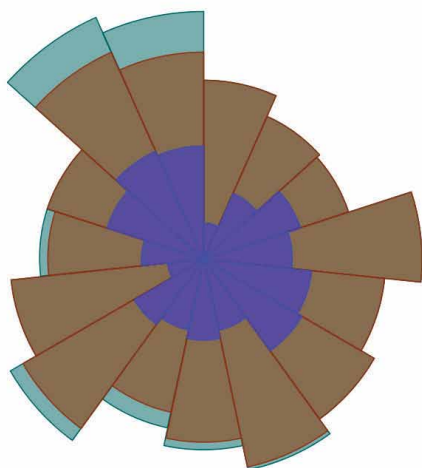


Figure 3

Polar area diagram to show students taking Mathematics at one school (colours separate)

Blue represents the number of students taking Higher Maths.

Brown represents the number of students taking Mathematical Studies.

Green represents the number of students taking Standard Maths.

This diagram is incomplete in that it has not got the dates on it, but I was interested in the basic shape it would make rather than seeing it as a finished article to represent the data. I decided to do the same thing but with Studies in the middle and Higher at the edge to see how different it would look.

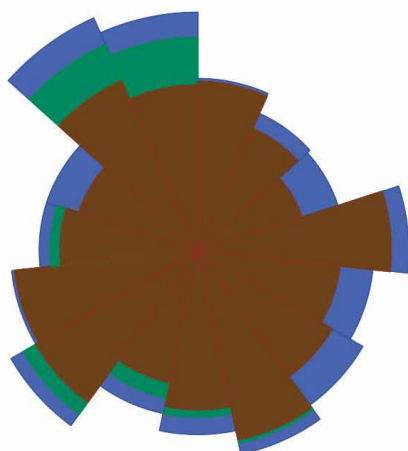


Figure 4

Polar area diagram to show students taking Mathematics at one school (colours separate)

Blue represents the number of students taking Higher Maths.

Brown represents the number of students taking Mathematical Studies.

Green represents the number of students taking Standard Maths.

This feels very different. The blue section actually looks less significant, to my eye, being put at the edges. This made me think of something else I had read in Brasseur's article, "Nightingale arranged these colored areas so that the main cause of death (and the largest sections)—deaths by disease—would be at the end of the wedges and would be more easily noticed."<sup>4</sup> I am sure that Brasseur thought that the colours were separate, and not overlapped. However, comparing my diagrams to Nightingale's original in Figure 1, I

became sure that she did mean them to be overlapped. I noticed that in the lefthand rose in figure 1 (representing the second year) there is a wedge with blue at the edge followed by a wedge with blue at the edge:



Figure 5  
A zoom in of part of figure 1

This can happen in a diagram like my figure 2 of overlapping colours, but would be impossible if the colours are separate as in figures 3 and 4. From this I deduced that the colours on the diagram must be overlapping.

### **The Second Problem**

My diagrams were unlike Nightingale's ones in that the total area of the sectors in figure 2 represented the total number of students taking the IB at this school over the 15 years. Nightingale's statistics were rates of mortality. Basically they can be thought of as percentages of soldiers who died, but, as before, when I read through the articles again, I was unsure what they were percentages of. Gill and Gill have table (Table 2) in their article with headings "No. of soldiers admitted to the hospital" and "No. (%) of soldiers who died"<sup>3</sup>. This might suggest that Nightingale was working with percentages of soldiers who were admitted into hospital. Lewi is more definite and refers to the actual statistic of one wedge of the third of Nightingale's polar area diagrams as follows: "The mortality during the first period was 192 per 1,000 hospitalized soldiers (on a yearly basis)"<sup>9</sup>. However, Brasseur refers to the statistic in a wedge of Nightingale's first diagram as being "the ratio of mortality for every 1,000 soldiers per annum in the field"<sup>4</sup>, in other words a percentage of the army actually on duty.

I decided to create a polar area diagram to act as an analogy to the possible situations as follows:

<b>Nightingale's data</b>	<b>My data</b>
Number of soldiers in the army in a month	Number of students taking the IB in a year
Number of soldiers taken to hospital	Number of students taking Maths Studies
Number of soldiers dying of wounds	Number of students gaining grade 7
Number of soldiers dying of disease	Number of students gaining grade 6
Number of soldiers dying for other reasons	Number of students gaining grade 5

My analogy of drawing a diagram showing the numbers of soldiers dying as a percentage of those admitted to hospital would then be the number of students gaining a grade above 4 as a percentage of those taking Mathematical Studies. I decided to do this one by hand, partly to prove I could, and partly to see if it would throw any extra light on the construction of the diagrams.

I gathered the data, found the percentages and used the percentages as A in the usual

formula  $r = \sqrt{\frac{15A}{\pi}}$  to find the radii needed to construct the diagram. The data is here:

Numbers gaining top three grades in Mathematical Studies

	Grade 7	Grade 6	Grade 5	Taking Studies	Total in year	As percentage of those taking Studies			Radius required for each		
						% grade 7	% grade 6	% grade 5	R7	R6	R5
1995	7	10	4	24	25	29.16667	41.66667	16.66667	11.80	14.10	8.92
1996	2	9	3	15	19	13.33333	60.00000	20.00000	7.98	16.93	9.77
1997	1	4	2	10	18	10.00000	40.00000	20.00000	6.91	13.82	9.77
1998	5	12	11	31	37	16.12903	38.70968	35.48387	8.78	13.60	13.02
1999	2	6	7	17	26	11.76471	35.29412	41.17647	7.49	12.98	14.02
2000	3	4	7	20	30	15.00000	20.00000	35.00000	8.46	9.77	12.93
2001	3	8	8	31	36	9.67742	25.80645	25.80645	6.80	11.10	11.10
2002	1	8	4	21	28	4.76190	38.09524	19.04762	4.77	13.49	9.54
2003	0	1	8	15	23	0.00000	6.66667	53.33333	0.00	5.64	15.96
2004	3	9	7	29	34	10.34483	31.03448	24.13793	7.03	12.17	10.74
2005	1	11	9	28	29	3.57143	39.28571	32.14286	4.13	13.70	12.39
2006	2	4	5	16	21	12.50000	25.00000	31.25000	7.73	10.93	12.22
2007	1	8	3	13	22	7.69231	61.53846	23.07692	6.06	17.14	10.50
2008	0	3	17	29	54	0.00000	10.34483	58.62069	0.00	7.03	16.73
2009	0	5	5	23	48	0.00000	21.73913	21.73913	0.00	10.19	10.19

And the diagram came out like this:

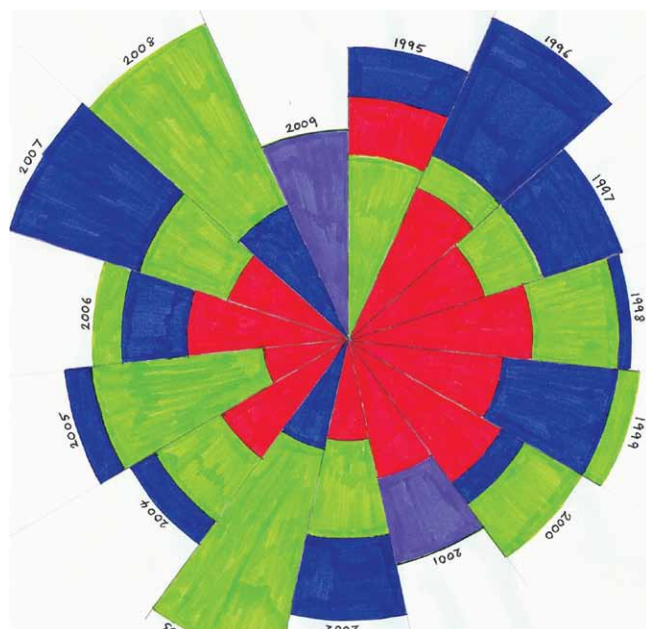


Figure 6  
Polar area diagram to show percentages of students taking Mathematical Studies who gained grades above 4  
Red represents the number of students gaining grade 7.  
Blue represents the number of students gaining grade 6.  
Green represents the number of students gaining grade 5.  
The purple areas represent coinciding numbers of students gaining grade 5 and 6.

One thing which I learnt from this exercise is that you have to be very careful about your scale and think through every move before you start if you don't want to fall off the edge of the paper! It is a far more tense experience drawing a diagram by hand because you know that one slip will make the whole diagram flawed. A computer slip can be corrected before you print out the result. My admiration for Florence Nightingale's draftsmanship was heightened by doing this.

The other thing which drawing by hand brought out was that, if you draw the arcs in in the appropriate colours, the colouring of the sectors sorts itself out. You colour from the arc inwards until you come to another arc or the centre. The only problem came when two arcs of different colours came in exactly the same place. I got around this problem by colouring these areas in a totally different colour and saying so at the side.

At this point in my research someone suggested some more possible websites to me, and following these up I found a copy of Nightingale's second diagram which was clear enough for me to read her notes, and a copy of the original data she used. The first of these was in a letter by Henry Woodbury suggesting that Nightingale got her calculations wrong and the radii represented the statistics rather than the area.<sup>7</sup> The letter had a comment posted by Ian Short which led me to an article by him<sup>8</sup> giving the data for the second diagram and explaining how it was created.

The very clear reproduction of Nightingale's second diagram in Woodbury's letter<sup>7</sup> shows that Miss Nightingale wrote beside it: "The areas of the blue, red and black wedges are each measured from the centre as the common vertex". This makes it quite clear that the colours are overlapped and so solves my first problem. She also wrote "In October 1854 & April 1855 the black area coincides with the red". She coloured the first of these in red and the second in black, but just commented on it beside the diagram to make it clear.

The article by Short<sup>8</sup> was a joy to read, although I could only work out the mathematical equations, which were written out in a way which is strange to me ( for example  $\text{\texttt{\textit{Area of sector B}}} = \frac{\pi r_B^2}{3} = 3$  ) because I already knew what they were (The example had a sector B in a diagram which I could see had

$areaB = \frac{1}{2} \frac{2\pi}{3} r_B^2 = \frac{\pi}{3} r_B^2$  ). The two things I found exciting from this article were the

table of data which Nightingale used to create the second diagram, and an explanation of what rates of mortality she used. She described these as follows; "The ratios of deaths and admissions to Force per 1000 per annum are calculated from the monthly ratios given in Dr. Smith's Table B"<sup>4</sup> and I had not been able to understand the meaning of this from the other articles. (Brasseur adds that "Dr. Smith was the late director-general of the army."<sup>4</sup>). Using Short's article I was able to work out what it meant.

I will use an example of data taken from the table in Short's article, which is in turn taken from "A contribution to the sanitary history of the British army during the late war with Russia" by Florence Nightingale of 1859<sup>8</sup>. In February 1855 the average size of the army was 30919. Of these 2120 died of 'zymotic diseases', 42 died of 'wounds & injuries' and 361 died of 'all other causes'. This gives a total of  $2120 + 42 + 361 = 2523$  deaths. 2523



out of 30919 means that  $\frac{2523}{30919} \times 1000 = 81.6003$  men died per 1000 men in the army in that month. If the size of the army had stayed at 30919, with no more men being shipped in or out, and the death rate had continued at 81.6 deaths per 1000 men per month over 12 months, the number of deaths per annum would have been  $81.6003 \times 12 = 979.2$  per 1000 men in the army. In other words 979.2 deaths per 1000 per annum.

This understanding of the units used allowed me to finally understand why O'Connor says of the death rate in January 1855, "if this rate had continued, and troops had not been replaced frequently, then disease alone would have killed the entire British army in the Crimea."<sup>5</sup> The number of deaths due to disease in January 1855 was 2761 and the

average size of the army was 32393. This gives a rate of  $\frac{2761}{32393} \times 1000 \times 12 = 1022.8$

deaths from disease per 1000 per annum. Another way of looking at it is that if 2761 had died each month from disease,  $2761 \times 12 = 33132$  would have died in 12 months, but there were only 32393 in the army!

As an aside, I noticed that O'Connor quoted the mortality rate for January 1855 as "1,023 per 10,000 being from zymotic diseases"<sup>5</sup>. Another example that we should not trust everything we see in print.

Having sorted this out I was ready to attempt my recreation of figure 1. I decided to do the right hand rose only, covering April 1854 to March 1855. The following table shows the data taken from Short's article in blue and my calculations in black:

Month	Average size of army (S)	Zymotic diseases (Z)	Wounds & injuries (W)	Other (O)	Z/S*1000*12 (Az) (1 d.p.)	Radius for Zymtotic	W/S*1000*12 (Aw) (1 d.p.)	Radius for Wounds	O/S*1000*12 (Ao) (1 d.p.)	Radius for Other
Apr-54	8571	1	0	5	1.4	2.3	0.0	0.0	7.0	5.2
May-54	23333	12	0	9	6.2	4.9	0.0	0.0	4.6	4.2
Jun-54	28333	11	0	6	4.7	4.2	0.0	0.0	2.5	3.1
Jul-54	28722	359	0	23	150.0	23.9	0.0	0.0	9.6	6.1
Aug-54	30246	828	1	30	328.5	35.4	0.4	1.2	11.9	6.7
Sep-54	30290	788	81	70	312.2	34.5	32.1	11.1	27.7	10.3
Oct-54	30643	503	132	128	197.0	27.4	51.7	14.1	50.1	13.8
Nov-54	29736	844	287	106	340.6	36.1	115.8	21.0	42.8	12.8
Dec-54	32779	1725	114	131	631.5	49.1	41.7	12.6	48.0	13.5
Jan-55	32393	2761	83	324	1022.8	62.5	30.7	10.8	120.0	21.4
Feb-55	30919	2120	42	361	822.8	56.1	16.3	7.9	140.1	23.1
Mar-55	30107	1205	32	172	480.3	42.8	12.8	7.0	68.6	16.2

Az is the death rate per 1000 per annum from disease, Aw is the death rate per 1000 per annum from wounds and Ao is the death rate per 1000 per annum from other causes. For

this diagram there are 12 divisions so each sector has an angle of  $\frac{2\pi}{12} = \frac{\pi}{6}$  and an area of

$$\frac{1}{2} \frac{\pi}{6} r^2 = \frac{\pi}{12} r^2. \text{ So for each radius } r = \sqrt{\frac{12A}{\pi}}.$$

I will show my final polar area diagram side by side with Nightingale's original version:

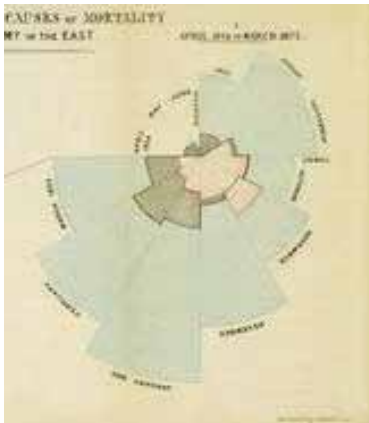
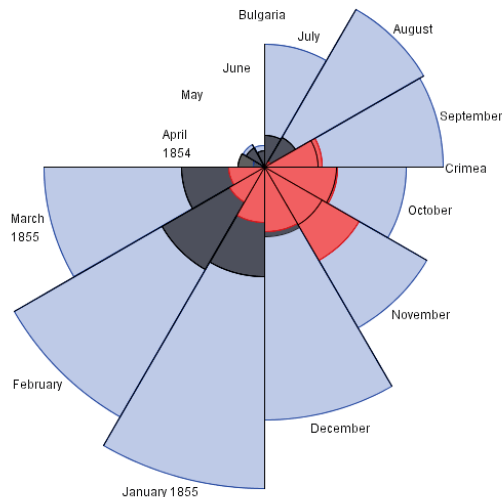


Figure 7.  
Nightingale's original  
"Diagram of the causes of  
mortality in the army in the  
east" and my recreation.



I have to admit that I felt rather proud once I had done this! However, looking at the September 1854 wedge I realised that the two diagrams didn't correspond. In Nightingale's original diagram I can see that there are more deaths from other causes than from wounds. In my version there are fewer deaths from other causes than from wounds. All other versions of the original in other articles I looked at ( Gill and Gill<sup>3</sup>, Brasseur<sup>4</sup>, O'Connor<sup>5</sup>, Woodbury<sup>7</sup>, Riddle<sup>10</sup>, Small<sup>11</sup>, Lienhard<sup>6</sup>) are as the original, but the table in Short definitely shows fewer deaths from other causes than from wounds<sup>8</sup>.

### Conclusion

I started out to try to learn how to recreate the polar area diagram which Florence Nightingale made to communicate to other people just how bad the situation was in army hospitals. This diagram shouts a need for reform. Look at it. The blue represents deaths which could be avoided with a bit of organisation and care. The red represents deaths due to the actual battles. Florence Nightingale had copies of her report containing her diagrams published at her own expense and sent them to doctors, army officers, members of parliament and the Queen. Following her persistent lobbying a commission was set up to improve military barracks and hospitals, sanitary codes were established and procedures were put in place for more organised collection of medical statistics<sup>4</sup>. It is a very shocking picture with a huge snowball of social change behind it. It has been an exciting adventure to drill down to a real understanding of its construction.

However, the biggest lesson I have learnt from this research is that you can't trust what you read. As I have argued in the main text, I am moderately sure that Brasseur thought the colours of the second diagram did not overlap<sup>4</sup>, I think O'Connor got his death rates wrong for January 1855<sup>5</sup>, and I think Short may have transcribed the data incorrectly for September 1854<sup>8</sup>. According to Brasseur, Florence Nightingale cross checked her data and was systematic about addressing objections to her analysis<sup>4</sup>. Everyone can make mistakes, and errors can propagate if we just quote what someone else says without looking for corroboration.

I have been left with a desire to find out more about this tenacious woman who wouldn't let society mould her into a genteel wife. Also, if I ever get the chance, I would like to get a look at one of the 2000 copies of "Notes on Matters Affecting the Health, Efficiency and Hospital Administration of the British Army. Founded Chiefly on the Experience of the Late War" which Florence Nightingale had published in 1858, to see the actual table of data and check the numbers for September 1854.

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