Determine Planck's Constant by Line Spectrum of Hydrogen

Suni Yao

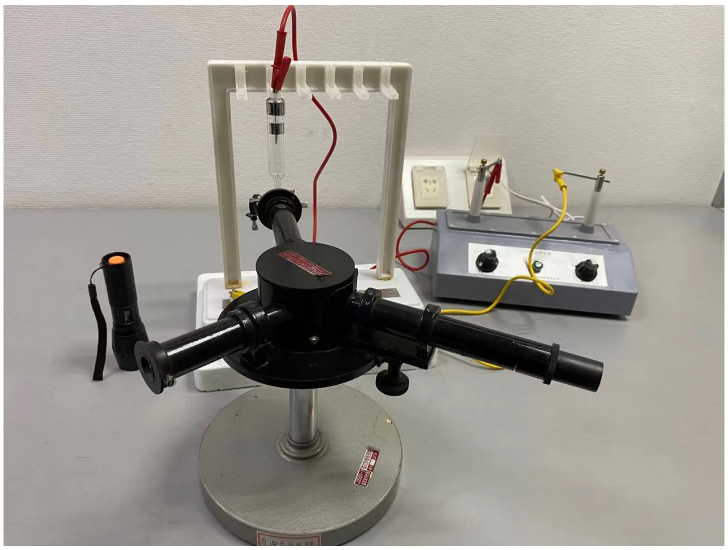
Contents

1 Aim

The aim of this experiment is to determine Planck's constant by analyzing the line spectrum of hydrogen. Using a spectroscope, we will measure the wavelengths of the lines in the visible region of the hydrogen spectrum, specifically the Balmer series. Applying quantum theory, we will then calculate Planck's constant from these measurements.

2 Equipment

- a spectroscope
- a hydrogen gas discharge tube
- an induction coil
- a torch

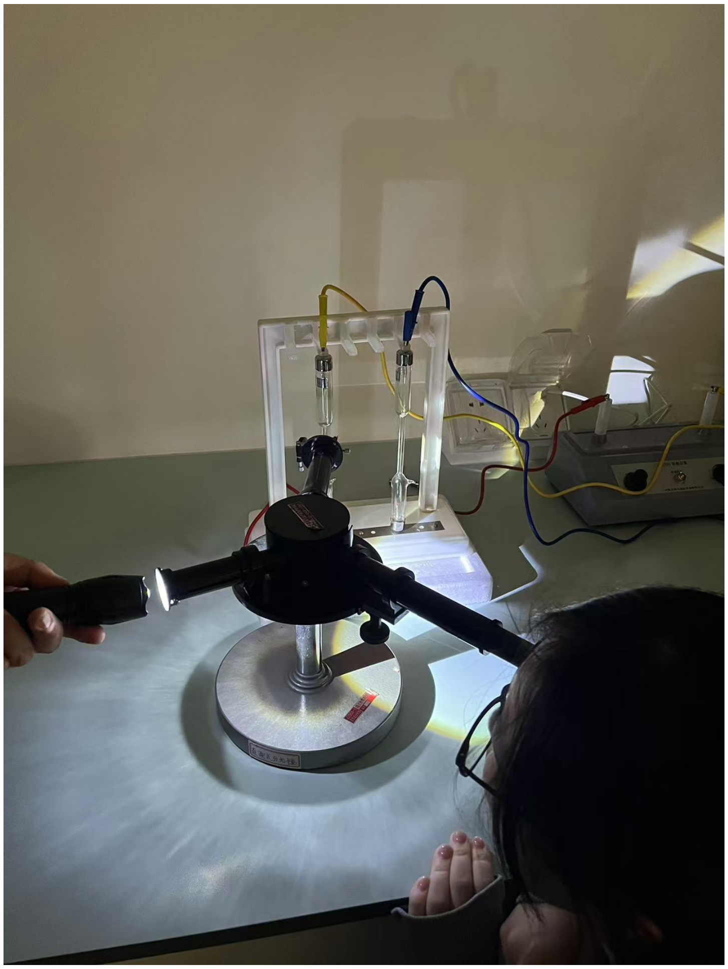


3 Data Collection

From observing from the spectroscope, we record the wavelength of the bright and distinguishable lines with uncertainties.

There are in total 2 different line spectra, and their wavelengths are listed below:

$$\lambda_1 = (520 \pm 5)\text{nm} \qquad \lambda_2 = (770 \pm 20)\text{nm}.$$



4 Data Processing

To calculate Planck's constant from the measured wavelengths in the Balmer series of the hydrogen spectrum, we will follow these steps:

The wavelengths of the spectral lines in the Balmer series of hydrogen can be described using the following equation:

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

Since we have the wavelengths known, we can calculate the energy levels n_1, n_2 according to the wavelengths by rearranging the equation to

$$n = \left(\frac{1}{4} - \frac{1}{\lambda \cdot 1.097 \cdot 10^7}\right)^{-\frac{1}{2}}$$

Therefore, plugging in the λ_1 and λ_2 values, we have

$$n_1 = 3.66 \qquad n_2 = 2.76.$$

Since we have $\Delta E = E_n - E_m = h\frac{c}{\lambda}$, $E_n = -\frac{13.6}{n^2}eV = -\frac{13.6}{n^2}\cdot 1.6\times 10^{-19}$ and E_m is unknown, we subtract the two values of ΔE and have the following equation shown

$$\begin{aligned}\Delta E_1 - \Delta E_2 &= E_{n,1} - E_{n,2}\\ &= hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)\\ &= -13.6\cdot 1.6\times 10^{-19}\cdot\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\\ \implies h &= -\frac{13.6\cdot 1.6\times 10^{-19}\cdot\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}{c\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)}\end{aligned}$$

Plugging in the values, we have

$$\begin{aligned}h &= -\frac{13.6\cdot 1.6\cdot 10^{-19}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}{3\cdot 10^8\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)}\\ &= 6.611972045\times 10^{-34}.\end{aligned}$$

We can reexpress $\frac{1}{n_1^2} - \frac{1}{n_2^2}$ as

$$\frac{1}{n_1^2} - \frac{1}{n_2^2} = \frac{1}{1.097\times 10^7}(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}) \implies \frac{\Delta(\frac{1}{n_1^2} - \frac{1}{n_2^2})}{\frac{1}{n_1^2} - \frac{1}{n_2^2}} = \frac{\Delta(\frac{1}{\lambda_1} - \frac{1}{\lambda_2})}{\frac{1}{\lambda_1} - \frac{1}{\lambda_2}}$$

Then[1] the uncertainty of Planck's constant can be calculated as

$$\begin{aligned}\frac{\Delta h}{h} &= \frac{\Delta(\frac{1}{\lambda_1} - \frac{1}{\lambda_2})}{\frac{1}{\lambda_1} - \frac{1}{\lambda_2}} + \frac{\Delta(\frac{1}{n_1^2} - \frac{1}{n_2^2})}{\frac{1}{n_1^2} - \frac{1}{n_2^2}}\\ &= 2\cdot\frac{\Delta(\frac{1}{\lambda_1} - \frac{1}{\lambda_2})}{\frac{1}{\lambda_1} - \frac{1}{\lambda_2}}\\ &= 2\cdot(\frac{\Delta\lambda_1}{\lambda_1} + \frac{\Delta\lambda_2}{\lambda_2})\\ &= 2\cdot(\frac{5}{520} + \frac{20}{770^2})\\ &= 7.11788212\%.\end{aligned}$$

So the uncertainty of the Planck constant is

$$\begin{aligned}\Delta h &= h\cdot\frac{\Delta h}{h}\\ &= 7.11788212\%\cdot 6.611972045\times 10^{-34}\\ &\approx \pm 5\times 10^{-35}\,\mathrm{J\,s}.\end{aligned}$$

[1] I realized that we don't even need the two values. *This is a bit clowning oops.*

Therefore, the Planck constant is

$$h = (6.6 \pm 0.5) \times 10^{-34}\,\mathrm{J\,s}$$

5 Conclusion

In this experiment, Planck's constant h was determined by analyzing the line spectrum of hydrogen, specifically the wavelengths corresponding to the Balmer series. Using the measured wavelengths and applying the quantum theory, we calculated the value of Planck's constant to be:

$$h = (6.6 \pm 0.5) \times 10^{-34}\,\mathrm{J\,s}$$

This result is in good agreement with the accepted value of Planck's constant, which is approximately $6.626 \times 10^{-34}\,\mathrm{J\,s}$. The slight discrepancy can be attributed to experimental uncertainties, which are inherent in any measurement process.

6 Evaluation

Uncertainty Analysis The calculated uncertainty in h was $\pm 0.5 \times 10^{-34}\,\mathrm{J\,s}$, which represents the precision of the measurements and the accuracy of the methods used in this experiment. The relatively small uncertainty reflects the precision of the spectroscopic measurements and the careful application of quantum theory.

Sources of Error The accuracy of the spectroscope used for measuring the wavelengths of the hydrogen spectral lines could have introduced slight errors. Calibration errors in the spectroscope or instrumental drift might have contributed to the uncertainty in the measurements.

The precision with which the wavelengths of the spectral lines (Balmer series) were recorded is critical in determining h. Any error in identifying the exact positions of the lines could lead to small deviations in the calculated value.

Variations in temperature, pressure, and other environmental factors might have influenced the experimental setup, potentially introducing minor inaccuracies.

Improvements for Future Experiments

- Future experiments could involve using a higher-resolution spectroscope or calibrating the instrument more precisely to reduce instrumental errors.
- Repeating the measurements for more lines in the spectrum would improve the reliability of the results and reduce the overall uncertainty.
- Careful control of environmental factors (e.g., temperature and pressure) and minimizing external light interference could further reduce errors.

Overall, while the value of Planck's constant obtained in this experiment is in close agreement with the theoretical value, improvements in experimental design and techniques could further reduce uncertainties and provide more accurate results.