The Investigation of The Relation between Sucrose Solution Concentration and Optical Rotational Angle

1. Introduction

Exploring the fascinating world of optical phenomena, one encounters the enigmatic spectacle known as the barber-pole effect, where shining polarized light into sugar water reveals diagonal stripes of color. This intriguing display arises from the unique interplay between light and the molecular structure of sucrose dissolved within the solution. While pure water does not affect the polarized light rotation inside the tube, the presence of sucrose twists and contorts the polarized light as it traverses through the medium, giving rise to the distinct diagonal stripe pattern.

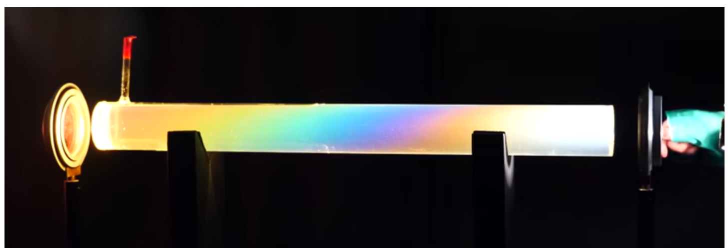


Figure 1: The light is shined from the left of the glass tube through a linear polarizing filter and gets twisted inside the sugar water, shown as diagonal stripes of colors, by 3Blue1Brown [1].

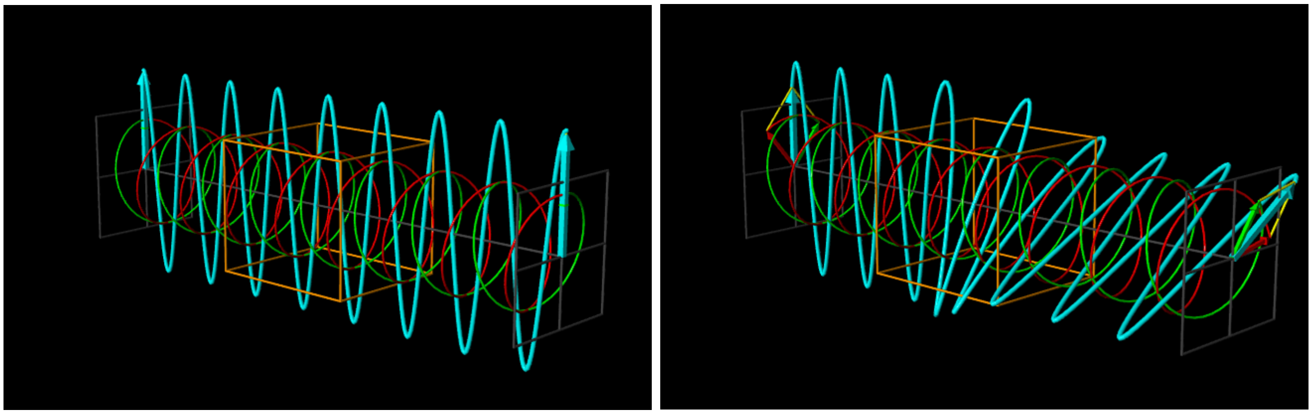
Therefore, I hypothesize that the optical rotation of the polarized light is a result of the existence of sucrose. In this captivating exploration, I am curious about how the concentration of sucrose shapes the degree of the optical rotation. Thus, the quest to understand how varying concentrations of sucrose affect the optical rotation of polarized light transcends mere scientific curiosity. Through meticulous experimentation and observation, we unravel the secrets of the barber-pole effect, illuminating the path toward a deeper understanding of the relation between sucrose solution concentration and optical rotational angle.

In this study, I investigated the effect of increasing sucrose solution concentration including 0g/L, 25g/L, 50g/L, 75g/L, 100g/L, 125g/L, 150g/L, and 175g/L on the optical rotational angle.

Physicist Augustin-Jean Fresnel explained the phenomenon of the optical rotation as a result of the different speeds of right-hand and left-hand circularly polarized light, [2].

Light can be viewed as an oscillation in the electric and magnetic fields that permeate the universe. When light is traveling in the universe, electric and magnetic fields oscillate in phase, but perpendicular to each other. A random light beam can be a superposition of electric fields (and magnetic fields) in different directions, which forms an unpolarized light. When an unpolarized light passes through a linear polarizer (polarizing filter), it blocks all directions of oscillation except one direction, restricting the polarized light to oscillating in only one specific direction.

When the linear polarizer filters the light, the light passing it is linear polarized light. The linear polarized light can be taken as a superposition of two circularly polarized states going in opposite directions (one clockwise and one anticlockwise), as shown in Figure 2(a). When the two circularly polarized components pass through the sucrose molecule, due to the asymmetry, one may hit the molecule more so it needs to pass the medium more. As we learned in class when light passes through different mediums, its travel speed changes, and the wavelength changes. Suppose the clockwise component of circular polarized light interacts more strongly with the molecule, the speed of the component is expected to slow down more than the anticlockwise component. Then, when the components pass the molecule, the anticlockwise component would have shifted up its phase relative to the clockwise component. If one circular polarized component of the polarized light is shifted forward, it causes the superposition to rotate and therefore causes the rotation in the polarized plane, [3], as shown in Figure 2(b).



(a) $n_r = n_l = 1.00$

(b) $n_r = 1.00, n_l = 1.15$

Figure 2: When two circular polarized components pass through a material that affects the left-hand circular wave (the red one) more with a larger refraction index ($n_l > n_r$). (a) shows a linear polarized wave that can be composed of two circular polarized waves in opposite rotating directions - left-hand (red) and right-hand (green); (b) Due to the refraction index difference, the left-hand circular polarized wave traveling with a lower speed, so the superposition of the two circular polarized waves rotate. The two pictures are produced by EMANIM, [4].

For optically inactive materials with molecules that hold the mirror symmetry property, the rotation will not work because this effect can be canceled by another upside-down molecule that interacts more strongly with the anticlockwise component. And after all, they will all cancel out. However, for a specific type of molecule that holds the handedness property, that is, the upside-down action will not change which circular polarized component interacts more strongly with, and they will all point to one direction so that the net rotation is formed in the polarized plane. Substances that can rotate the plane of polarized light passing through them include quartz and cinnabar as examples of optically active crystals and aqueous solutions of sugar, tartaric acid as optically active solutions, [5].

In this research, I hypothesize that there exists a proportional relationship between the angle of optical rotation and the concentration of sucrose solution.

1.1. Experimental Variables

The **independent variable** (IV) is the concentration of the sucrose/sugar solution. The solution will be made with 25g, 50g, 75g, 100g, 125g, 150g, and 175g sugar and water to where the solution is 1000ml.

The **dependent variable** (DV) will be the optical rotational angle, measured by the rotating angle of the linear polarizer at the end of the long water tank.

The **controlled variables** and the reasons are listed below:

- Initial polarizer position: the polarizer must be placed at a fixed angle θ_0. Then by measuring the other polarizer's rotation angle θ_1, we can calculate the rotation $\theta = \theta_1 - \theta_0$.

- Length of the path the polarized light has passed: the length can be controlled by fixing the length of the water tank

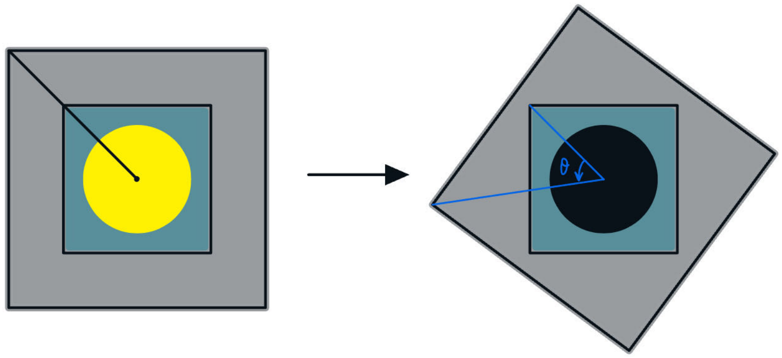


Figure 3: When $\theta_0 = 0°$, the angle rotated $\theta = \theta_1$.

used in this experiment, which is fixed to be $l = 50$cm. If the length of the path the polarized light has passed is not fixed, then the rotation of the polarized plane will be affected.

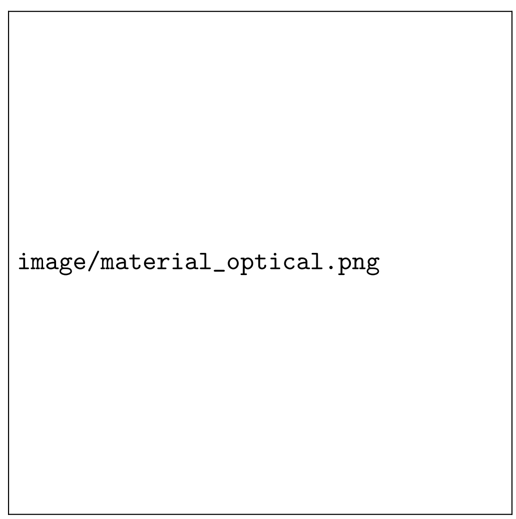
- Wavelength of light: different wavelengths of light may affect the optical rotational angle. Therefore, we fixed the same wavelength by fixing the light resource to be the same.

When a beam of light passes through one polarizer, if another polarizer is placed vertically with the first polarizer, then no light can be passed through the second polarizer and by observing from the second polarizer it will be completely dark.

2. Experiment

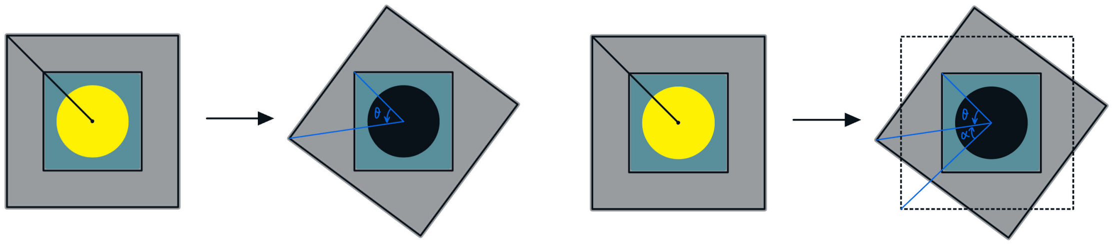
2.1. Materials for The Experiment

(a) From left to right: sucrose ($C_{12}H_{22}O_{11}$, table sugar) as the optical-active substance in the experiment, distilled water, volumetric flask (1000mL $\pm$ 0.5mL), water tank (5cm $\times$ 5cm $\times$ 50cm)



(b) From left to right: the lamp, acts as a light resource; 2 linear polarizing filters, one changes the lamp light to polarized light (polarizer), another is used to observe the intensity (analyzer); Marker, used to fix the center of both the analyzer and observer's side of the water tank to help later measurement.

We use GeoGebra, [6], to measure the optical rotational angle.



(c) (d)

Figure 4: (a) Overlap the marks on the water tank side and the analyzer. A picture is taken when the analyzer is rotated angle θ such that the light intensity is the lowest. (b) The actual optical rotational angle should be $\alpha = 90° - \theta$ because the light passing through 2 linearly polarizing filters should be the dimmest when rotated 90°. However, due to the optical rotation, it is reduced α to θ. So the optical rotational angle $\alpha = 90° - \theta$.

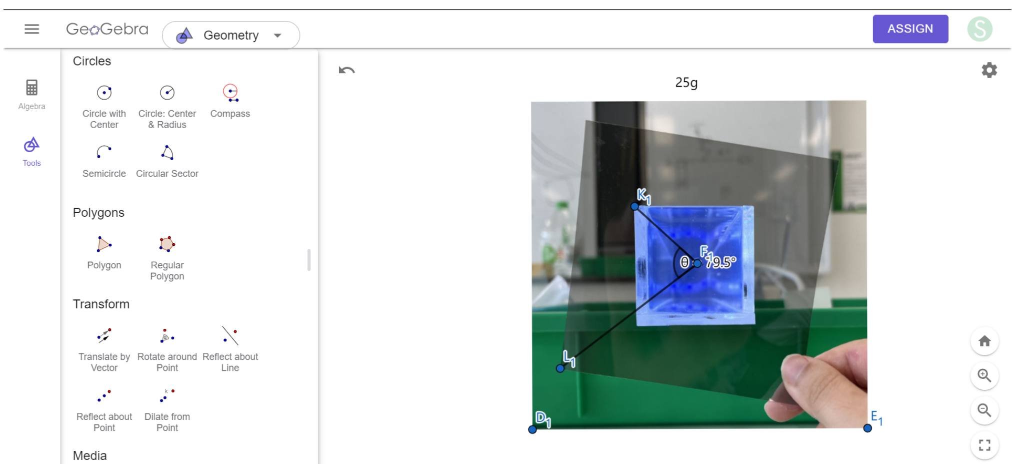


Figure 5: The picture taken is imported into GeoGebra, [6]. The two points on the photo with reference to Figure 2.1 is connected to the marked center. The optical rotational angle $\alpha = 90° - 79.5° = 10.5°$.

2.2. Experiment Setup

2.3. Procedure for The Experiment

1. Solution preparation: Weight 25g of sucrose (table sugar), and add distilled water to make the required sucrose solution. Put the solution inside the 1000ml volumetric flask, continue to add water until it reaches the 1000ml scale.
2. Apparatus setup:
   (a) Fix one polarizer to the light source by sticking it onto the light source. Keep the light source close to one side of the tank
   (b) Pour the prepared solution into the water tank until it fills the water tank
3. Optical angle measurement:
   (a) Mark both the center of the observer's side of the water tank and the center of another polarizer (the analyzer).

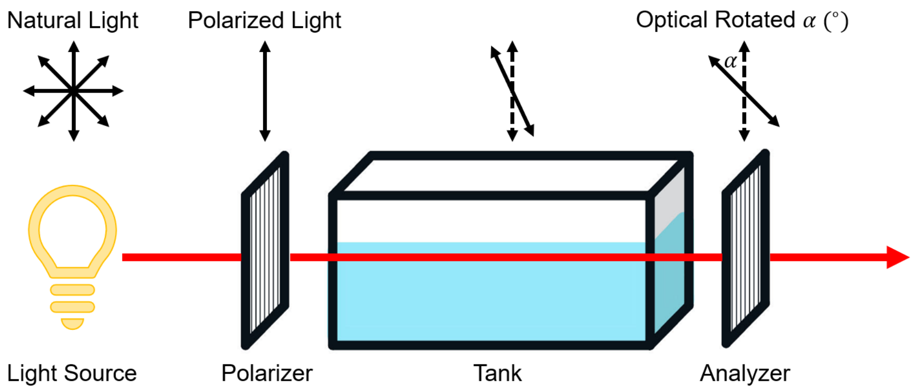


Figure 6: Schematic of the experiment setup, including the light source that creates monochromatic light with the constant wavelength, polarizer, the water tank containing sucrose solution, analyzer.

(b) Turn on the light, and overlap the analyzer and the observer's side of the water tank such that the two marked centers of the polarizer (analyzer) and the water tank side are completely overlapped.

(c) Observe the light that passed through the analyzer, rotate it until the light has the least intensity, and record the angle of rotation θ_1.

4. Repetition of the experiment for the same concentration: Change the initial position of the polarizer to 45°. Repeat Step 3c, and record the angle of rotation θ_2. Then change the initial position of the polarizer to 90°, and record the angle of rotation θ_3. Use $\theta_{1,2,3}$ to calculate $\alpha_{1,2,3}$ ($\alpha_i = 90° - \theta_i$)

5. Change the solution and repeat the above steps: Process all the following steps for different concentrations of sucrose solution by dissolving different amounts of sucrose, for mass $m =$ 0g, 50g, 75g, 100g, 125g, 150g, 175g.

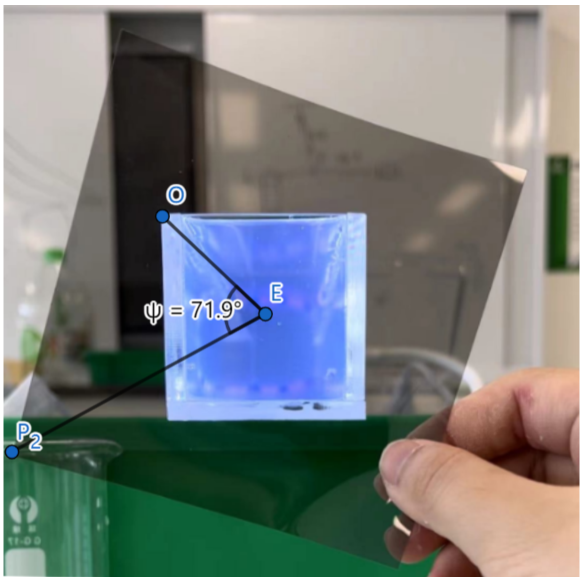


Figure 7: An example photo of measurement of one trial for $c =$ 50g/L. The angle θ_1 measured is 71.9° and $\alpha_1 = 90° - \theta_1 =$ 18.1°.

3. Data Collection and Analysis

3.1. Raw Results and Uncertainties

3.1.1. Experimental Data

c (g/mL)	α_1 (°)	α_2 (°)	α_3 (°)
0.000	0.4	0.2	-0.2
0.025	8.0	10.5	11.7
0.050	17.1	18.1	20.9
0.075	25.7	26.3	25.9
0.100	34.9	33.9	30.8
0.125	39.7	38.8	41.5
0.150	46.1	44.7	45.3
0.175	52.3	55.1	52.5

Table 1: Raw data table of the optical rotational angle and corresponding concentration of sucrose solution.

3.1.2. Uncertainties

- Concentration of sucrose solution: $\Delta c = \pm(\frac{0.1}{m_{\text{sugar}}} + \frac{0.5}{1000}) \cdot c$ – the weighing scale was precise to 1 decimal, so the absolute uncertainty of mass is ±0.1g. The error of the volumetric flask is ±0.5mL. Therefore, the percentage uncertainty of sucrose solution concentration is

$$\frac{\Delta c}{c} = \frac{\Delta m_{\text{sugar}}}{m_{\text{sugar}}} + \frac{\Delta V}{V}.$$
$$\Longrightarrow \Delta c = (\frac{\Delta m_{\text{sugar}}}{m_{\text{sugar}}} + \frac{\Delta V}{V}) \cdot c = \pm(\frac{0.1}{m_{\text{sugar}}} + \frac{0.5}{1000}) \cdot c$$

  Take $c = 0.025$g/mL as the example, the absolute error of concentration $\Delta c_{0.025}$ is

$$\Delta c_{0.025} = (\frac{0.1}{25} + \frac{0.5}{1000}) \cdot 0.025 = 0.0001125\text{g/mL} \approx 0.0001\text{g/mL}.$$

  Therefore, $c_{0.025} = (0.0250 \pm 0.0001)$g/mL.

- Angle rotated: when the picture is taken the camera may not be completely parallel to the polarizer and therefore there may exist a distortion of the picture. Therefore, we rotate the polarizer 45° and 90° and respectively record them as trail 2 and trail 3.

  We take $c = 0.025$g/mL as an example of calculating the error of the optical rotational angle $\Delta\alpha_{0.025}$ and the average optical rotational angle $\alpha_{0.025}$:

$$\Delta\alpha_{0.025} = \pm\frac{11.7 - 8.0}{2} = \pm 1.85° \approx \pm 2°$$
$$\alpha_{0.025} = \frac{8.0 + 10.5 + 11.7}{3} = 10.0667° \approx 10°$$

3.2. Processed Data

In Table 2, we list the concentration of sucrose solution and their corresponding optical rotational angle α.

concentration of sucrose solution (g/mL)	angle rotated (°)
0.0000 ± 0.0000	0.1 ± 0.3
0.0250 ± 0.0001	10 ± 2
0.0500 ± 0.0001	19 ± 2
0.0750 ± 0.0001	26.0 ± 0.3
0.1000 ± 0.0002	33 ± 2
0.1250 ± 0.0002	40 ± 1
0.1500 ± 0.0002	45.4 ± 0.7
0.1750 ± 0.0002	53 ± 1

Table 2: Processed data table, average angle rotated to the concentration of sucrose solution, with absolute uncertainties.

3.3. Graphical Analysis

3.3.1. Best-Fit Line Uncertainty Calculations

According to the processed data (Table 2), we can plot the polarized plane angle and concentration as the x and y-axes, respectively, as shown in Figure 8.

Then we calculate the gradient k and y-intercept to derive the expression of the relationship between the concentration of sucrose solution and the optical rotation of the polarized plane.

According to Figure 8(a), the equation of the best fit is $y = 295.24x + 2.52$. The uncertainty in the gradient is

$$\Delta k = \frac{\text{maximum-slope} - \text{minimum-slope}}{2} = \frac{296.21 - 294.26}{2} = \pm 0.975.$$

Therefore, the gradient and its uncertainty is

$$k \pm \Delta k = (295.24 \pm 0.975)^\circ \cdot \mathrm{g}^{-1} \cdot \mathrm{mL} \approx (295 \pm 1)^\circ \cdot \mathrm{g}^{-1} \cdot \mathrm{mL}$$

3.4. Extension

Until here, we have validated the relationship between the angle of optical rotation and concentration of the optical substance. The result is convincing, existing exploration of optical activity also stated that optical rotation is affected by the concentration of the optical substance, the path length, and the property of light and optical substance. The relationship can be expressed as

$$\alpha = [\alpha]lc,$$

where c is the concentration in g/mL, l is the path length in dm, and $[\alpha]$ is the change in optical rotational angle of monochromatic plane-polarized light, per unit distance-concentration product, as the light passes through a sample of a compound in solution, [7].

Since the gradient of the relation between concentration and optical rotational angle is derived and the length of sucrose solution the polarized light passed is 49cm = 4.9dm, the specific rotation constant $[\alpha]$ for light from the lamp in a sucrose solution can also be calculated.

We substitute the value of gradient m into the equation

$$k = \frac{\alpha}{c} = [\alpha]l \implies 295 = \alpha \cdot 4.9 \implies [\alpha] = 60.25^\circ \cdot \mathrm{cm}^2 \cdot \mathrm{g}^{-1}.$$

Since $\Delta k/k = \Delta\alpha/\alpha$, the uncertainty in α is

$$\Delta\alpha = \frac{\Delta k \cdot \alpha}{k} = \frac{0.975 \cdot 60.25}{295.24} = \pm 0.198 \approx \pm 0.2^\circ \cdot \mathrm{cm}^2 \cdot \mathrm{g}^{-1},$$

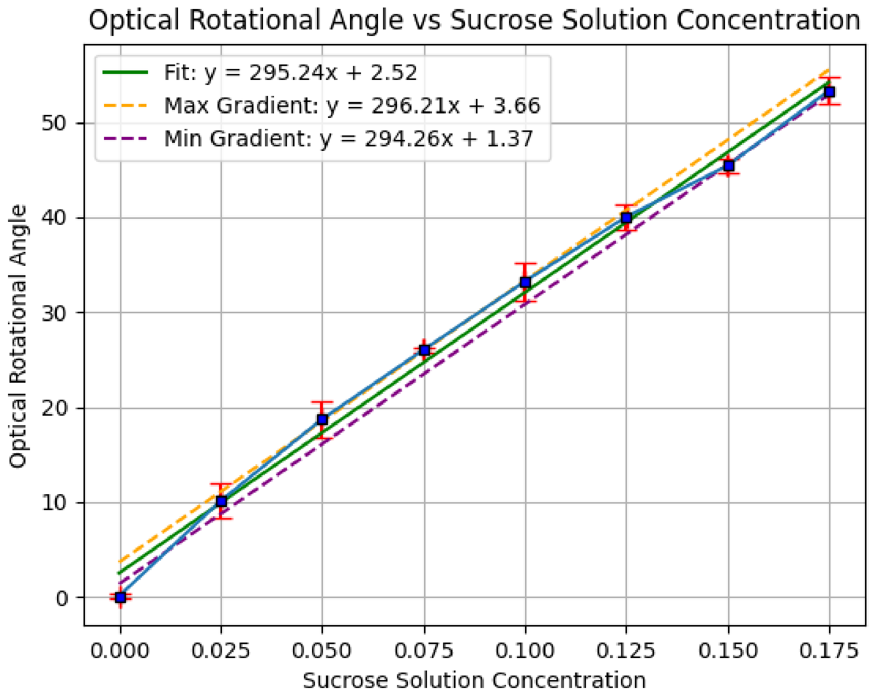


Figure 8: The relationship between sucrose solution concentration and polarized plane rotation, given best-fit line and maximum-gradient and minimum-gradient.

and with uncertainty, the specific rotation $\alpha = (60.3 \pm 0.2)^\circ \cdot \mathrm{cm}^2 \cdot \mathrm{g}^{-1}$.

By observing the range of y-intercept b, we found out that the range of y-intercept is within $1.37 \leq b \leq 3.66$. The lines that do not pass through the origin may result from a systematic error due to the acrylic material of the water tank.

4. Evaluation

The experiment has some strengths. For example, I use 2 linear polarizing filters and the water tank to validate the relation between sucrose solution and optical rotational angle instead of direct use of the polarimeter. The self-design of the experiment apparatus makes me learn more about how the polarimeter functions. Marking the center of the water tank face and polarizer eliminates the need for a protractor and streamlines the experimental process. This simplification improves convenience and efficiency, making it easier to align the polarizers accurately and conduct measurements effectively.

Despite the experiment's success, several drawbacks remain that could be addressed to reduce errors and enhance the accuracy of the experimental results.

One notable weakness is that the polarizer may not be completely parallel to the side of the water tank due to deformation. This misalignment results in varying angles (θ) when pictures are taken from different vertices of the square, leading to less accurate results. This type of error is classified as a random error. To mitigate this, θ should be measured from different vertices in a single picture, and the average should be taken. This approach can help reduce overall uncertainty.

In addition, the intensity of light is measured by the eye instead of using sensors. Determining the minimum light intensity with the naked eye is challenging, leading to potential inaccuracies and fluctuations in the θ value around its true value. This, too, is a random error. To improve accuracy, θ should be measured three times using different pictures taken in separate attempts, and the average should be calculated to minimize random error. Alternatively, using a photosensitive sensor could provide more precise measurements.

By addressing these weaknesses, the reliability and precision of the experimental outcomes can be significantly improved.

Additionally, the influence of the water tank's acrylic material has been overlooked. It is not definitively determined that the rotation in the polarized light plane is caused solely by the sucrose solution. There remains a possibility that the optical rotation is affected by both the material of the tank and the solution concentration. To validate this hypothesis, further experimentation is required. Specifically, an experiment should be conducted to measure the optical rotational angle of a fixed sucrose concentration using water tanks made of different materials. This would help isolate and identify the contribution of the tank material to the observed optical rotation.

5. Conclusion

In conclusion, our experiment successfully elucidated the relationship between sucrose concentration and the rotation of polarized light, providing valuable insights into the phenomenon of optical activity. The results demonstrated a clear and consistent proportional relationship between the concentration of sucrose solution and the rotation angle of the polarized plane. As the concentration of sucrose increased, so did the magnitude of the rotation angle, aligning with the fundamental principles of optical activity and validating the theoretical understanding of how optically active substances interact with polarized light.

By fitting our experimental data to a best-fit line, we determined the specific rotation constant $[\alpha]$ of lamp light passing through sucrose solution to be $(60.3 \pm 0.2)^{\circ}$, a value very close to the previously measured result of $[\alpha] = 66.6^{\circ}$ in [8]. This close agreement underscores the accuracy and reliability of our experimental methodology.

While our experiment yielded significant findings, it is essential to acknowledge its limitations and potential sources of error. Imperfections in measurement techniques, deformation of the polarizer, and environmental conditions may have introduced slight deviations in our results. Despite these challenges, our data remained consistent and reproducible, indicating the robustness of our experimental setup.

The property of optical activity has practical applications in various fields. For example, sucrose ($C_{12}H_{22}O_{11}$) and glucose ($C_6H_{12}O_6$) are both dextrorotary – rotating the plane of polarization to the right – while fructose ($C_6H_{12}O_6$) is levorotary, rotating the plane of polarization to the left. These differing optical properties can be utilized to measure the rate constant and the reacting speed of the reaction converting sucrose to fructose and glucose by measuring the resultant change in optical rotational angle caused by the change in concentration, [9]. This application highlights the utility of optical active property and how the concentration of optical-active substances can influence the optical rotational angle.

Through the combination of theoretical principles and empirical evidence, we have expanded our comprehension of this intricate aspect of optical science, reinforcing the significance of optical activity in both scientific research and practical applications.

References

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