# Lab Report: Simple Pendulum

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## 1 Background & Theory Explanation

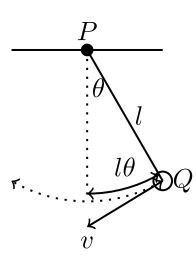
The pendulum's motion is very similar to simple harmonic motion, especially when it is moving in a small angle range.

First we prove that simple pendulum motion is simple harmonic motion by theory, and the rest of the lab report aimed on validating the relation between the period of simple pendulum motion and the pendulum length.

On the pendulum there exist two forces, gravitational force, and the tension provided by the string. The tension is perpendicular to the displacement of the pendulum, so does not affect the tangential acceleration (the tension only provided the centripetal force). We decompose the gravitational force in two direction - parallel to the displacement direction and perpendicular to the displacement direction. Only the component of gravity that is parallel to the motion contributes to the acceleration and the force $F$ satisfies the equation

$$F = mg\sin\theta,$$

where $m$ is the mass of pendulum and $\theta$ is the angle between the pendulum and the vertical line.



Here we should analyse the force in rotational motion, the force that counts is the force that contributes to the tangential motion, so

$$F = -mg\sin\theta,$$
$$a = -g\sin\theta,$$

here the minus sign addresses that the acceleration is in the opposite direction of displacement (perpendicular line is considered as 0 angular displacement).

And by applying Maclaurin expansion to $\sin\theta$, we know that

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots,$$

therefore, for small angles we have

$$\theta \approx \sin\theta.$$

As the (half) displacement $x$ is equal to $l\theta$, we can approximately have

$$x = -\frac{g}{l}a,$$
$$\omega = \sqrt{\frac{g}{l}},$$
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}.$$

So the simple pendulum is approximately SHM when $\theta$ is small. Besides, according to the theoretical analysis, hypothesis is made that the period of simple pendulum is not related to the mass of pendulum and specific angle (when the angle is small enough to nearly be SHM), and is validated by prelab.

## 2 Aim

The aim of the experiment is to validate the relation between the length of pendulum $l$ and the period $T$ when the angle $\theta$ of the pendulum is small so it is approximately a SHM.

## 3 Materials (Per Pair)

- String with steel ball at one end
- Stopwatch

## 4 Procedure

1. Mark the points on the string with the length 35cm, 30cm, 25cm, 20cm, 15cm, 10cm, and 5cm to the mass center of the steel ball
2. Tie the string such that the marks are on the other end to make sure $l = 35, 30, 25, 20, 15, 10, 5$cm.
3. Pull the rope such that there exists a small angle $\theta$ from the vertical line
4. Start the stopwatch the moment you release the ball and press the stopwatch when the ball has swung 10 times, record the time and repeat the process for 3 times.

# 5 Data Collection

| $l$ (cm) | Time 1 (s) | Time 2 (s) | Time 3 (s) |
|---|---|---|---|
| 5 | 4.56 | 4.65 | 4.72 |
| 10 | 6.44 | 6.28 | 6.28 |
| 15 | 7.71 | 7.75 | 7.66 |
| 20 | 8.78 | 8.97 | 8.88 |
| 25 | 9.84 | 9.75 | 9.90 |
| 30 | 10.91 | 10.78 | 11.06 |
| 35 | 11.25 | 11.72 | 11.87 |

Table 1: Raw data, including the length of pendulum $l$, and the 3 time recorded of the pendulum of every length swung 10 times.

This is the data table of the times measured of a 10 times swung, in next section, I will show how to process the data.

# 6 Data Processing

## 6.1 Example of Data Processing

We take the raw data for $l = 5$cm as the example of data processing. First we calculate the average time of swung 10 times,

$$\text{Average Time} = \frac{4.56 + 4.65 + 4.72}{3} = 4.64\text{s}.$$

Then, we know that human reaction time is around 250ms, so the error of average time should be $\pm 0.25\text{s} = \pm 0.3\text{s}$. So the average time of swung 10 times including the error should be $(4.6 \pm 0.3)$s.

Therefore, the period $T$, which is the time swung for 1 time, should be

$$T = \frac{4.6 \pm 0.3}{10} = (0.46 \pm 0.03)\text{s}.$$

By the same method of data processing, we can derive the processed data table showing the pendulum length $l$ and average period $T$ with error:

| $l$ (m) | Period $T$ (s) |
|---|---|
| $0.050 \pm 0.001$ | $0.46 \pm 0.03$ |
| $0.100 \pm 0.001$ | $0.63 \pm 0.03$ |
| $0.150 \pm 0.001$ | $0.77 \pm 0.03$ |
| $0.200 \pm 0.001$ | $0.89 \pm 0.03$ |
| $0.250 \pm 0.001$ | $0.98 \pm 0.03$ |
| $0.300 \pm 0.001$ | $1.09 \pm 0.03$ |
| $0.350 \pm 0.001$ | $1.16 \pm 0.03$ |

Table 2: Processed data, including the length of pendulum $l$, and the corresponding period $T$ with error.

Then I need to validate the relation between the length of pendulum $l$ and the period $T$, which should be

$$T = k \cdot \sqrt{l} \implies \ln(T) = \frac{1}{2}\ln(l) + k$$

, where $k = \ln\left(\frac{2\pi}{\sqrt{g}}\right)$. Therefore two validation can be used:

1. Plot the graph with $T$ as $y$-axis and $\sqrt{l}$ as $x$-axis, and it should turn out to be a straight line passing through origin.

2. Plot the graph with $\log(T)$ as $y$-axis and $\log(l)$ as $x$-axis, and it should turn out to be a straight line with gradient $1/2$ (and intersect $y$-axis at $(0, \frac{2\pi}{\sqrt{g}})$).

# 7 Validation 1

As we want to validate the relation through plotting $T$ and $\sqrt{l}$, we should further process $l$ to $\sqrt{l}$ and simultaneously compute the corresponding error, by applying

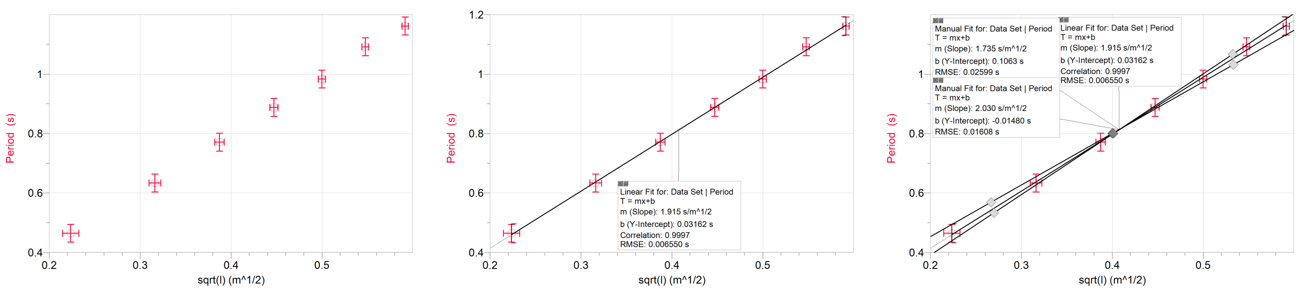
$$\frac{\Delta\sqrt{l}}{\sqrt{l}} = \frac{1}{2} \cdot \frac{\Delta l}{l},$$

we have

| $\sqrt{l}$ ($\mathrm{m}^{1/2}$) | Period $T$ (s) |
|---|---|
| $0.223 \pm 0.009$ | $0.46 \pm 0.03$ |
| $0.316 \pm 0.006$ | $0.63 \pm 0.03$ |
| $0.387 \pm 0.005$ | $0.77 \pm 0.03$ |
| $0.447 \pm 0.004$ | $0.89 \pm 0.03$ |
| $0.500 \pm 0.004$ | $0.98 \pm 0.03$ |
| $0.548 \pm 0.004$ | $1.09 \pm 0.03$ |
| $0.592 \pm 0.003$ | $1.16 \pm 0.03$ |

Table 3: Processed data, including $\sqrt{l}$ and $T$ as independent variable and dependent variable, respectively, and their errors.

By this, we plot the following diagram with independent variable $\sqrt{l}$ and dependent variable $T$:




(a) Data Points and Error Bars (b) Best Fit Line (c) Maximum- & Minimum-Gradient Lines

Therefore, we can calculate the expression of the line:

$$\begin{aligned} T &= m \cdot \sqrt{l} + b, \\ \Delta m &= \frac{2.030 - 1.735}{2} = 0.1475 \implies m \approx (1.9 \pm 0.1)\mathrm{s} \cdot \mathrm{m}^{-1/2}, \\ \Delta b &= \frac{0.1063 - (-0.01480)}{2} = 0.06055 \implies b \approx (0.03 \pm 0.06)\mathrm{s}. \end{aligned}$$

Comparing the result with the theoretical value,

$$\begin{aligned} m' &= \frac{2\pi}{\sqrt{g}} \approx 2.006, \\ b' &= 0, \end{aligned}$$

both theoretical value are within the range of the experimental value.

# 8 Validation 2

Similarly, we did a validation by taking the natural logarithm on both sides

$$T = \frac{2\pi}{\sqrt{g}}\sqrt{l},$$
$$\ln(T) = \frac{1}{2}\cdot\ln(l) + \ln(\frac{2\pi}{\sqrt{g}}).$$

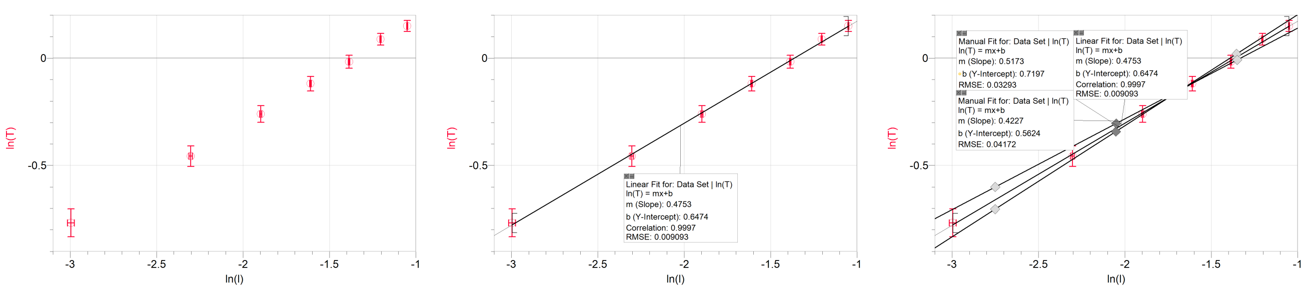
Also, when taking natural logarithm, the error should be calculated as

$$\Delta\ln(T) = \frac{\Delta T}{T},$$
$$\Delta\ln(l) = \frac{\Delta l}{l}.$$

Thus we can calculate the further processed data

| $\ln(l)$ | $\ln(T)$ |
|---|---|
| $-3.00 \pm 0.02$ | $-0.77 \pm 0.06$ |
| $-2.30 \pm 0.01$ | $-0.46 \pm 0.05$ |
| $-1.897 \pm 0.007$ | $-0.261 \pm 0.04$ |
| $-1.609 \pm 0.005$ | $-0.119 \pm 0.03$ |
| $-1.386 \pm 0.004$ | $-0.017 \pm 0.03$ |
| $-1.204 \pm 0.003$ | $0.087 \pm 0.03$ |
| $-1.050 \pm 0.003$ | $0.14 \pm 0.03$ |

Table 4: Processed data, including $\ln(l)$ and $\ln(T)$ as independent variable and dependent variable, respectively, and their errors.




(d) Data Points and Error Bars (e) Best Fit Line (f) Maximum- & Minimum-Gradient Lines

Therefore, we can calculate the expression of the line to validate the relation between $\ln(T)$ and $\ln(l)$:

$$\ln(T) = m\cdot\ln(l) + b,$$
$$\Delta m = \frac{0.5173 - 0.4227}{2} = 0.0473 \implies m \approx 0.48 \pm 0.05,$$
$$\Delta b = \frac{0.7197 - 0.4227}{2} = 0.1485 \implies b \approx 0.6 \pm 0.1.$$

We can compare this with the theoretical values which are

$$\ln(T) = \frac{1}{2} \cdot \ln(l) + \ln(\frac{2\pi}{\sqrt{g}})$$
$$m' = 0.5,$$
$$b' = \ln(\frac{2\pi}{\sqrt{g}}) \approx 0.696.$$

Both theoretical values are in the range of the experimental value.

# 9 Conclusion and Evaluation

We have done two validations and as all the theoretical values are in the range of the experimental value, our theoretical relation derived between period of a simple pendulum and the pendulum length

$$T = 2\pi \cdot \sqrt{\frac{l}{g}}$$

is acceptable.

However, as the theoretical value is only at the margin of the experimental value range, there may still exist some errors which are created when experimenting, potential reasons for the error should be

- The angle $\theta$ we lift is large and $\sin\theta \approx \theta$ is a less accurate approximation.
- The theoretical value is derived when no fiction is considered in the motion, however, there does exist fiction between air and the steel ball.
- In the hypothesis, we assume that the pendulum will move in one fixed plane, while in actual experiment, the plane the pendulum is moving is varying. In earth-fixed reference frame the pendulum plane rotates, unless the experiment is conducted on equator, as the Foucault effect stated.